Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

4WM2/01 Specimen Worked Solutions

Specimen 0 paper rebuilt with the worked solution after each question.

24	2020–2026	Single-paper solutions
paper questions	52 past papers	checked by Dr Eslam



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PAST PAPER

4WM2/01 Specimen

Specimen 0

24	100	2 h
questions	marks	exam time

Each question is followed by its worked solution and final answer.

4WM2/01 SPECIMEN

Question 1

3 marks | Statistics Toolkit

1 Here are five cards, where x represents a number.

15	7	−2	23	x

The mean of the five numbers is 12

Work out the value of x

$x =$ .....

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4WM2/01 Specimen

1 **Calculate statistic**

The mean is 12, so the total of the five numbers is

$$5 \times 12 = 60$$

2 **Calculate value**

The known numbers add to

$$15 + 7 - 2 + 23 = 43$$

---

3 **Calculate value**

Therefore

$$43 + x = 60$$

$$x = 17$$

✓ **FINAL ANSWER**

$x = 17$.

Question 2

4 marks | Prime Factors, HCF & LCM

2 (a) Find the highest common factor (HCF) of 56 and 84
Show your working clearly.

.....

(2)

(b) Find the lowest common multiple (LCM) of 60 and 72
Show your working clearly.

.....

(2)

(Total for Question 2 is 4 marks)

1 **Find the LCM**

$$56 = 2^3 \times 7, \qquad 84 = 2^2 \times 3 \times 7$$

2 **The common prime factors are**

The common prime factors are 2^2 and 7, so

$$\text{HCF} = 2^2 \times 7 = 28$$

3 **Find the LCM**

Also

$$60 = 2^2 \times 3 \times 5, \qquad 72 = 2^3 \times 3^2$$

4 **Find the LCM**

Use the highest powers of each prime:

$$\text{LCM} = 2^3 \times 3^2 \times 5 = 360$$

✓ **FINAL ANSWER**

HCF $= 28$, LCM $= 360$.

4WM2/01 SPECIMEN

Question 3

2 marks | Algebraic Roots & Indices

3 $\frac{2^k}{4^n} = 2^x$

Find an expression for x in terms of k and n

$x =$ .....

(Total for Question 3 is 2 marks)

WORKED SOLUTION

Question 3 · 4WM2/01 Specimen

1 **Use index laws**

$$4^n = (2^2)^n = 2^{2n}$$

2 **Use index laws**

So

$$\frac{2^k}{4^n} = \frac{2^k}{2^{2n}} = 2^{k-2n}$$

3 **Use index laws**

Since this equals 2^x,

$$x = k - 2n$$

✓ **FINAL ANSWER**

$x = k - 2n$.

Question 4

4 marks | Angles in Polygons & Parallel Lines

4 The diagram shows parts of three regular polygons, **A**, **B** and **C**, meeting at a point.

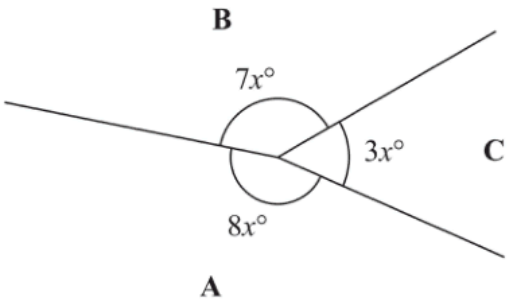


Diagram **NOT** accurately drawn

Polygon **B** has n sides.
Work out the value of n

$n =$ .....

(Total for Question 4 is 4 marks)

1 **Angles around a point add**

Angles around a point add to $360°$:

$$8x + 7x + 3x = 360$$

$$18x = 360$$

$$x = 20$$

2 **The interior angle of polygon**

The interior angle of polygon B is

$$7x = 7(20) = 140°$$

3 **Find the gradient**

For a regular n-sided polygon,

$$\frac{180(n-2)}{n} = 140$$

$$180n - 360 = 140n$$

$$40n = 360$$

$$n = 9$$

FINAL ANSWER

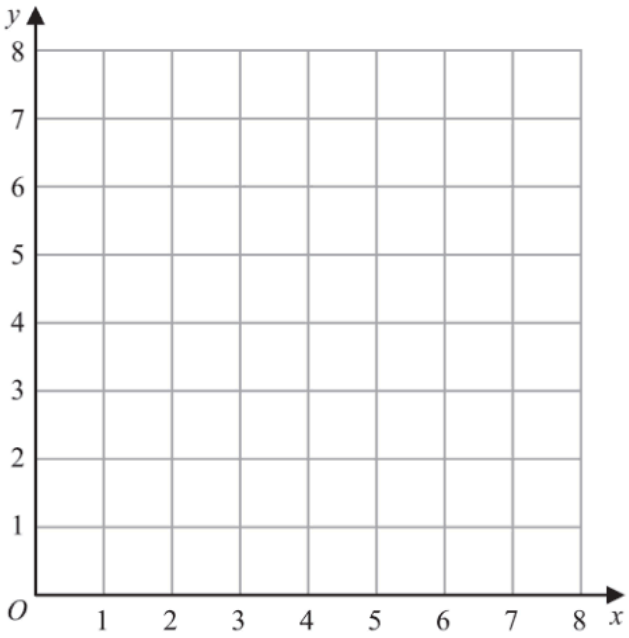
$n = 9$.

Question 5

4 marks | Inequalities (Solving & Graphing)

5 (a) On the grid, draw and label with its equation the straight line with equation

(i) $y = 1$ (ii) $x = 2$ (iii) $x + y = 7$



(3)

(b) Show, by shading on the grid, the region that satisfies **all three** of the inequalities

$y \geqslant 1$ $x \geqslant 2$ $x + y \leqslant 7$

Label the region **R**

(1)

(Total for Question 5 is 4 marks)

1 Draw the three boundary lines

Draw the three boundary lines:

$$y = 1, \qquad x = 2, \qquad x + y = 7$$

2 Read the graph

For $x + y = 7$, use points such as $(0, 7)$ and $(7, 0)$.

3 The required region is

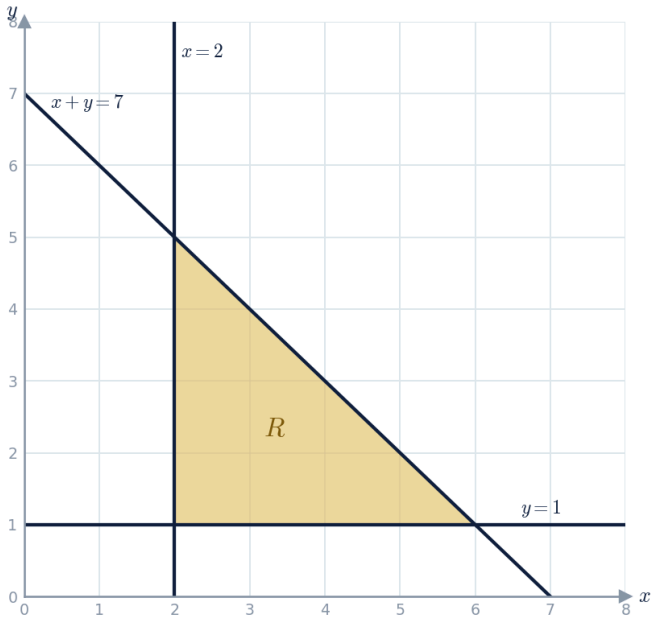
The required region is:

$$y \geq 1, \qquad x \geq 2, \qquad x + y \leq 7$$

4 Read the graph

So shade the triangular region above $y = 1$, to the right of $x = 2$, and below the line $x + y = 7$. Its vertices are

$$(2, 1), \quad (2, 5), \quad (6, 1)$$



FINAL ANSWER

the shaded region R is the triangle with vertices $(2, 1)$, $(2, 5)$, and $(6, 1)$.

4WM2/01 SPECIMEN

Question 6

3 marks | Statistics Toolkit

6 Here are some integers where $a < b < c < d$

$a \quad b \quad c \quad d \quad d \quad d$

The mode of the integers is 9
The range of the integers is 4
The median of the integers is 8

Work out the value of a, the value of b, the value of c and the value of d

$a =$ .....

$b =$ .....

$c =$ .....

$d =$ .....

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4WM2/01 Specimen

1 Calculate statistic

The mode is 9, and d appears three times, so

$$d = 9$$

2 Calculate statistic

The range is 4, so

$$d - a = 4$$

$$9 - a = 4$$

$$a = 5$$

3 Calculate statistic

There are six values, so the median is the mean of the 3rd and 4th values:

$$\frac{c+d}{2} = 8$$

$$\frac{c+9}{2} = 8$$

$$c = 7$$

4 Solve inequality

Since $a < b < c < d$, the remaining integer is $b = 6$.

FINAL ANSWER

$a = 5,\ b = 6,\ c = 7,\ d = 9$.

Question 7

4 marks | Standard & Compound Units

7 A cylinder is placed on the ground.

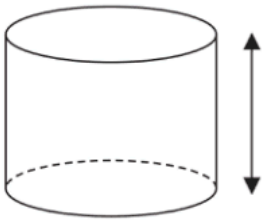


Diagram **NOT** accurately drawn

The height of the cylinder is 18 cm

The force exerted by the cylinder on the ground is 72 newtons.
The pressure on the ground due to the cylinder is 1.4 newtons/cm^2

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the volume of the cylinder.
Give your answer correct to 3 significant figures.

..... cm^3

(Total for Question 7 is 4 marks)

1 **Calculate area**

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

2 **Calculate area**

So the area of the base is

$$\text{area} = \frac{72}{1.4} = 51.428571\ldots$$

3 **The volume is**

The volume is

$$51.428571\ldots \times 18 = 925.714285\ldots$$

4 **Calculate volume**

Correct to 3 significant figures,

$$925.714\ldots \approx 926$$

✓ **FINAL ANSWER**

$926\ \text{cm}^3$.

4WM2/01 SPECIMEN

Question 8

6 marks | Compound Interest & Depreciation

8 In 2021, the value of Asha's apartment was 634 400 euros.
The value of Asha's apartment had increased by 4% from its value in 2020

(a) Work out the value of Asha's apartment in 2020

..... euros
(3)

Pam bought a boat.

In each year after Pam bought the boat, the value of the boat depreciated by 15%

(b) Work out the total percentage by which the value of the boat had depreciated by the end of the second year after Pam bought the boat.

..... %
(3)

(Total for Question 8 is 6 marks)

WORKED SOLUTION

Question 8 · 4WM2/01 Specimen

1 **Use compound interest**

For part (a), the 2021 value is 104% of the 2020 value.

$$\text{2020 value} = \frac{634400}{1.04} = 610000$$

2 **Use compound interest**

For part (b), after one year the multiplier is 0.85. After two years:

$$0.85^2 = 0.7225$$

3 **Use compound interest**

So 72.25% of the value remains, and the total depreciation is

$$100\% - 72.25\% = 27.75\%$$

✓ **FINAL ANSWER**

610000 euros; 27.75%.

4WM2/01 SPECIMEN

Question 9

2 marks | Standard Form

9 (a) Write 0.000 089 in standard form.

.....

(1)

(b) Write 8.34×10^4 as an ordinary number.

.....

(1)

(Total for Question 9 is 2 marks)

WORKED SOLUTION

Question 9 · 4WM2/01 Specimen

1 Convert standard form

$$0.000089 = 8.9 \times 10^{-5}$$

2 Convert standard form

Also

$$8.34 \times 10^4 = 83400$$

FINAL ANSWER

8.9×10^{-5}, 83400.

Question 10

5 marks | Ratio Toolkit

10 Payel makes 300 celebration cards so that

$$\text{number of birthday cards} : \text{number of anniversary cards} : \text{number of congratulations cards} = 7 : 5 : 3$$

$\frac{2}{5}$ of the birthday cards have numbers on them.

36% of the anniversary cards have numbers on them.
None of the congratulations cards have numbers on them.

Work out what fraction of the 300 cards have numbers on them.
Give your answer in its simplest form.

.....

(Total for Question 10 is 5 marks)

1 **The total number of parts**

The total number of parts is

$$7 + 5 + 3 = 15$$

2 **Each part is**

Each part is

$$300 \div 15 = 20$$

3 **Split the ratio**

So the numbers of cards are:

$$140, \quad 100, \quad 60$$

4 **The birthday cards with numbers**

The birthday cards with numbers are

$$\frac{2}{5} \times 140 = 56$$

5 **The anniversary cards with numbers**

The anniversary cards with numbers are

$$36\% \times 100 = 36$$

6 **Split the ratio**

So the total with numbers is

$$56 + 36 = 92$$

7 **The fraction of all cards**

The fraction of all cards is

$$\frac{92}{300} = \frac{23}{75}$$

✓ **FINAL ANSWER**

$\frac{23}{75}$.

4WM2/01 SPECIMEN

Question 11

3 marks | Simultaneous Equations

11 Solve the simultaneous equations

$$7x + 3y = 3$$
$$3x - y = 7$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4WM2/01 Specimen

1 **Solve simultaneous equations**

From

$$3x - y = 7$$

$$y = 3x - 7$$

2 **Solve simultaneous equations**

Substitute into $7x + 3y = 3$:

$$7x + 3(3x - 7) = 3$$

$$16x - 21 = 3$$

$$16x = 24$$

$$x = 1.5$$

---

3 **Solve simultaneous equations**

Then

$$y = 3(1.5) - 7 = -2.5$$

✓ **FINAL ANSWER**

$x = 1.5,\ y = -2.5$.

Question 12

5 marks | Percentages

12 Zimo is going on holiday.
He makes 3 separate payments to cover the total cost of his holiday.

The following table shows how much money Zimo pays to the holiday company.

Payment	Amount paid
Payment 1	$\frac{2}{5}$ of the total cost
Payment 2	45% of the total cost
Payment 3	£405

Work out how much Zimo has to pay for Payment 2

£.....

(Total for Question 12 is 5 marks)

1 **Calculate percentage**

$$\frac{2}{5} = 40\%$$

2 **Calculate percentage**

So Payments 1 and 2 together are

$$40\% + 45\% = 85\%$$

3 **This means Payment 3 is**

This means Payment 3 is

$$100\% - 85\% = 15\%$$

4 **Calculate percentage**

So

$$15\% = 405$$

$$1\% = 27$$

5 **Payment 2 is**

Payment 2 is 45%:

$$45 \times 27 = 1215$$

FINAL ANSWER

$\$1215$.

4WM2/01 SPECIMEN

Question 13

4 marks | Functions

13 The function f is defined as

$$f : x \mapsto \frac{2x}{x-6}$$

(a) Find f(10)

.....

(1)

(b) Express the inverse function f^{-1} in the form $f^{-1} : x \mapsto \ldots$

$f^{-1} : x \mapsto$ .....

(3)

WORKED SOLUTION

Question 13 · 4WM2/01 Specimen

1 **Evaluate fraction**

$$f(10) = \frac{2(10)}{10-6} = \frac{20}{4} = 5$$

2 **Let**

For the inverse, let

$$y = \frac{2x}{x-6}$$

$$y(x-6) = 2x$$

$$xy - 6y = 2x$$

$$x(y-2) = 6y$$

$$x = \frac{6y}{y-2}$$

---

3 **Evaluate fraction**

So

$$f^{-1} : x \mapsto \frac{6x}{x-2}$$

✓ **FINAL ANSWER**

5, and $f^{-1} : x \mapsto \frac{6x}{x-2}$.

Question 14

5 marks | Circle Theorems

14 A, B, C and D are points on a circle, centre O

EBF is the tangent to the circle at B

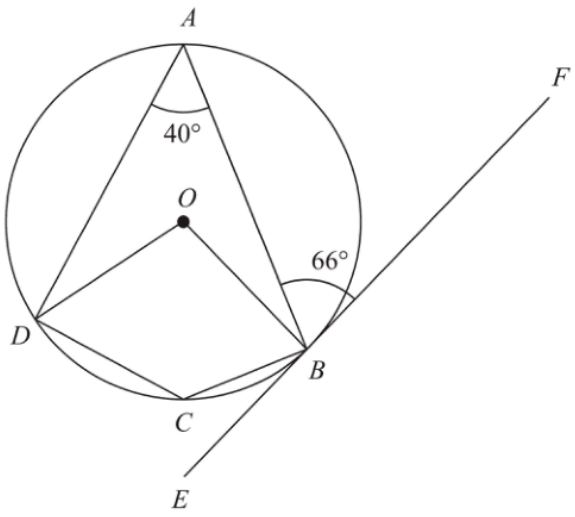


Diagram **NOT** accurately drawn

(a) (i) Work out the size of angle DCB

.....

(1)

(ii) Give a reason for your answer to (a)(i)

.....

.....

(1)

(b) Work out the size of angle ADO

..... °

(3)

(Total for Question 14 is 5 marks)

1 **Use trigonometry**

In cyclic quadrilateral $ADCB$, opposite angles add to $180°$.

$$\angle DCB = 180° - 40° = 140°$$

2 **Reason opposite angles in a**

Reason: opposite angles in a cyclic quadrilateral add to $180°$.

3 **Use trigonometry**

By the alternate segment theorem,

$$\angle ADB = 66°$$

4 **Use trigonometry**

In triangle ADB,

$$\angle ABD = 180° - 40° - 66° = 74°$$

5 **The angle at the centre**

The angle at the centre is twice the angle at the circumference standing on the same chord AD:

$$\angle AOD = 2(74°) = 148°$$

6 **Use trigonometry**

Since $OA = OD$, triangle AOD is isosceles:

$$\angle ADO = \frac{180° - 148°}{2} = 16°$$

FINAL ANSWER

$\angle DCB = 140°$; reason: opposite angles in a cyclic quadrilateral; $\angle ADO = 16°$.

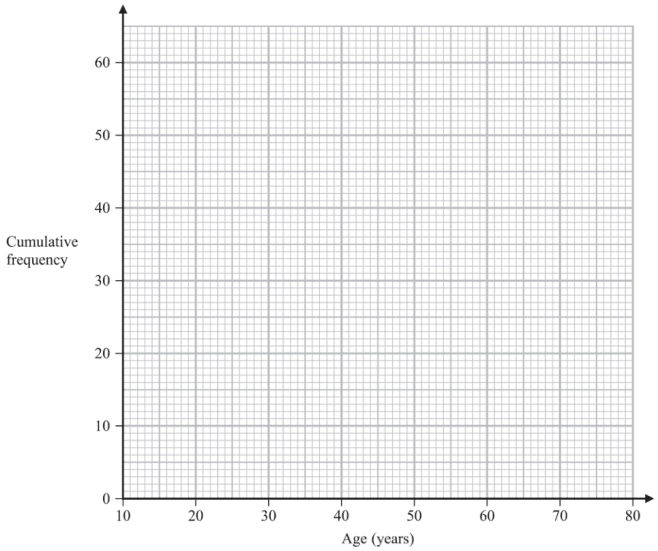
Question 15

7 marks | Cumulative Frequency Diagrams

15 The cumulative frequency table shows information about the ages of 60 people who went to a gym on Saturday.

Age (a years)	Cumulative frequency
$10 < a \leqslant 20$	13
$10 < a \leqslant 30$	36
$10 < a \leqslant 40$	42
$10 < a \leqslant 50$	47
$10 < a \leqslant 60$	52
$10 < a \leqslant 70$	56
$10 < a \leqslant 80$	60

(a) On the grid, draw a cumulative frequency graph for the information in the table.



(2)

(b) Use your graph to find an estimate for the median of the ages of these people.

..... years

(1)

(c) Use your graph to find an estimate for the interquartile range of the ages of these people.

..... years

(2)

(d) Use your graph to find an estimate for the number of these people who are older than 55 years.

.....

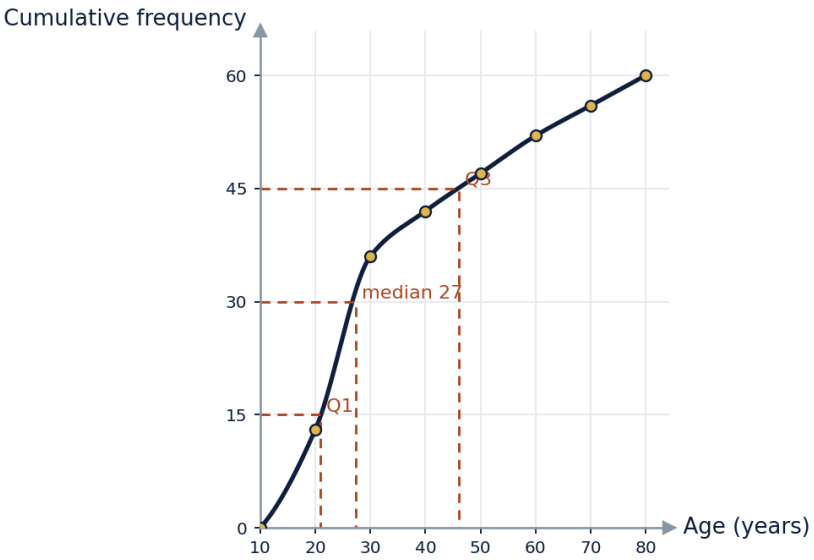
(2)

(Total for Question 15 is 7 marks)

1 **Plot the cumulative frequency points**

Plot the cumulative frequency points and join with a smooth increasing curve:

$$(10, 0),\ (20, 13),\ (30, 36),\ (40, 42),\ (50, 47),\ (60, 52),\ (70, 56),\ (80, 60)$$



2 **Use cumulative frequency**

There are 60 people, so the median is read at cumulative frequency 30.

3 **Convert standard form**

Using linear interpolation between $(20, 13)$ and $(30, 36)$:

$$20 + \frac{30 - 13}{36 - 13} \times 10 \approx 27.4$$

4 **Use cumulative frequency**

So the median is about 27 years.

5 **Use cumulative frequency**

For the interquartile range, read Q_1 at cumulative frequency 15 and Q_3 at cumulative frequency 45:

$$Q_1 \approx 20.9, \qquad Q_3 \approx 46$$

$$\text{IQR} \approx 46 - 20.9 = 25.1$$

6 **Use cumulative frequency**

So the IQR is about 25 years.

7 **Use cumulative frequency**

At age 55, the cumulative frequency is about 49.5, so the number older than 55 is

$$60 - 49.5 \approx 10.5$$

8 **This gives about people**

This gives about 11 people.

✓ **FINAL ANSWER**

median about 27 years, IQR about 25 years, about 11 people older than 55.

4WM2/01 SPECIMEN

Question 16

4 marks | Direct & Inverse Proportion

16 M is directly proportional to h^3
$M = 4$ when $h = 0.5$

Find the value of h when $M = 500$

$h =$ .....

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4WM2/01 Specimen

1 **Find inverse function**

Since M is directly proportional to h^3,

$$M = kh^3$$

2 **Use when**

Use $M = 4$ when $h = 0.5$:

$$4 = k(0.5)^3$$

$$4 = 0.125k$$

$$k = 32$$

---

3 **Use**

Now use $M = 500$:

$$500 = 32h^3$$

$$h^3 = 15.625$$

$$h = 2.5$$

✓ **FINAL ANSWER**

$h = 2.5$.

Question 17

5 marks | Differentiation & Turning Points

17 A particle P moves along a straight line.
The fixed point O lies on this line.

The displacement of P from O at time t seconds, $t \geqslant 1$, is s metres where

$$s = 4t^2 + \frac{125}{t}$$

The velocity of P at time t seconds, $t \geqslant 1$, is v m/s

Work out the distance of P from O at the instant when $v = 0$

..... m

(Total for Question 17 is 5 marks)

1 **Differentiate**

$$s = 4t^2 + \frac{125}{t}$$

2 **Differentiate to find velocity**

Differentiate to find velocity:

$$v = \frac{ds}{dt} = 8t - \frac{125}{t^2}$$

3 **Set**

Set $v = 0$:

$$8t - \frac{125}{t^2} = 0$$

$$8t^3 = 125$$

$$t^3 = \frac{125}{8}$$

$$t = 2.5$$

4 **Differentiate**

Now find s:

$$s = 4(2.5)^2 + \frac{125}{2.5}$$

$$s = 25 + 50 = 75$$

✓ **FINAL ANSWER**

75 m.

4WM2/01 SPECIMEN

Question 18

3 marks | Inequalities (Solving & Graphing)

18 Solve the inequality $2y^2 - 7y - 30 \leqslant 0$
Show your working clearly.

.....

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4WM2/01 Specimen

1 **Factorise**

Factorise:

$$2y^2 - 7y - 30 = (2y + 5)(y - 6)$$

2 **Solve inequality**

So

$$(2y + 5)(y - 6) \leq 0$$

3 **The roots are**

The roots are

$$y = -\frac{5}{2}, \qquad y = 6$$

---

4 **Solve quadratic equation**

The quadratic opens upwards, so it is less than or equal to zero between the roots.

✓ **FINAL ANSWER**

$-\frac{5}{2} \leq y \leq 6$.

Question 19

4 marks | Area & Volume of Similar Shapes

19 The diagram shows two similar metal statues.

Diagram **NOT** accurately drawn



A **B**

The volume of statue **B** is 20% less than the volume of statue **A**
The surface area of statue **B** is k% less than the surface area of statue **A**

Work out the value of k
Give your answer correct to 3 significant figures.

$k =$ .....

(Total for Question 19 is 4 marks)

1 **The volume of statue is**

The volume of statue B is 80% of the volume of statue A, so the volume scale factor is

$$0.8$$

2 **Calculate area**

For similar solids:

$$\text{linear scale factor} = \sqrt[3]{0.8}$$

3 **The surface area scale factor**

The surface area scale factor is therefore

$$\left(\sqrt[3]{0.8}\right)^2 = 0.8^{2/3}$$

$$0.8^{2/3} = 0.861773\ldots$$

4 **Convert standard form**

So the percentage decrease in surface area is

$$(1 - 0.861773\ldots) \times 100 = 13.8226\ldots$$

5 **Calculate area**

Correct to 3 significant figures,

$$k = 13.8$$

✓ **FINAL ANSWER**

$k = 13.8$.

Question 20

5 marks | Simultaneous Equations

20 Solve the simultaneous equations

$$x - 2y = 3$$
$$x^2 - y^2 + 2x = 10$$

Show clear algebraic working.

(Total for Question 20 is 5 marks)

1 **Solve simultaneous equations**

From

$$x - 2y = 3$$

$$x = 2y + 3$$

2 **Solve simultaneous equations**

Substitute into $x^2 - y^2 + 2x = 10$:

$$(2y + 3)^2 - y^2 + 2(2y + 3) = 10$$

$$4y^2 + 12y + 9 - y^2 + 4y + 6 = 10$$

$$3y^2 + 16y + 5 = 0$$

$$(3y + 1)(y + 5) = 0$$

3 **Solve simultaneous equations**

So

$$y = -\frac{1}{3} \quad \text{or} \quad y = -5$$

4 **Solve simultaneous equations**

If $y = -\frac{1}{3}$,

$$x = 2\left(-\frac{1}{3}\right) + 3 = \frac{7}{3}$$

5 **Solve simultaneous equations**

If $y = -5$,

$$x = 2(-5) + 3 = -7$$

✓ **FINAL ANSWER**

$\left(\frac{7}{3}, -\frac{1}{3}\right)$ and $(-7, -5)$.

Question 21

3 marks | Algebraic Fractions

21 $a = \dfrac{14}{3x - 7}$ $\quad x = \dfrac{7}{4y - 3}$

Express a in the form $\dfrac{py + q}{ry + s}$ where p, q, r and s are integers.

Give your answer in its simplest form.

$a =$ .....

(Total for Question 21 is 3 marks)

1 **Simplify fraction**

$$a = \frac{14}{3x - 7}, \qquad x = \frac{7}{4y - 3}$$

2 **Simplify fraction**

Substitute for x:

$$a = \frac{14}{3\left(\frac{7}{4y-3}\right) - 7}$$

3 **Simplify the denominator**

Simplify the denominator:

$$3\left(\frac{7}{4y - 3}\right) - 7 = \frac{21}{4y - 3} - 7$$

$$= \frac{21 - 7(4y - 3)}{4y - 3} = \frac{42 - 28y}{4y - 3}$$

4 **Simplify fraction**

So

$$a = \frac{14}{\frac{42-28y}{4y-3}} = \frac{14(4y - 3)}{42 - 28y}$$

$$a = \frac{4y - 3}{3 - 2y}$$

✓ **FINAL ANSWER**

$a = \dfrac{4y - 3}{3 - 2y}$.

4WM2/01 SPECIMEN

Question 22

5 marks | Volume & Surface Area

22 A solid is made from a cone and a hemisphere.

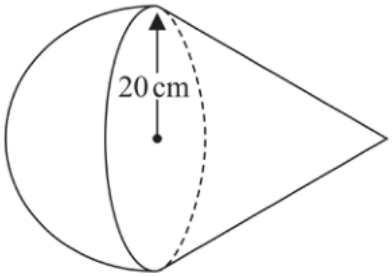


Diagram **NOT** accurately drawn

The circular plane face of the hemisphere coincides with the circular base of the cone.
The radius of the hemisphere and the radius of the circular base of the cone are both 20 cm

The curved surface area of the cone is $580\pi\,\text{cm}^2$

The volume of the solid is $k\pi\,\text{cm}^3$

Work out the exact value of k

$k =$ .....

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4WM2/01 Specimen

1 **Calculate surface area**

The curved surface area of a cone is

$$\pi rl$$

2 **Calculate area**

Here $r = 20$ and $\pi rl = 580\pi$, so

$$20l = 580$$

$$l = 29$$

3 **Use Pythagoras to find the**

Use Pythagoras to find the height of the cone:

$$h^2 = 29^2 - 20^2$$

$$h^2 = 841 - 400 = 441$$

$$h = 21$$

---

4 **Volume of the cone**

Volume of the cone:

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (20)^2 (21) = 2800\pi$$

---

5 **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi r^3 = \frac{2}{3}\pi (20)^3 = \frac{16000}{3}\pi$$

---

6 **Total volume**

Total volume:

$$2800\pi + \frac{16000}{3}\pi = \frac{24400}{3}\pi$$

---

7 **Calculate area**

Since the volume is $k\pi$,

$$k = \frac{24400}{3}$$

✓ **FINAL ANSWER**

$k = \frac{24400}{3}$.

Question 23

5 marks | Vectors

23

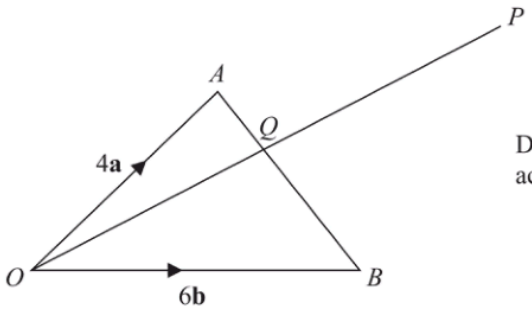


Diagram **NOT** accurately drawn

OAB is a triangle.
Q is the point on AB such that OQP is a straight line.

$\overrightarrow{OA} = \mathbf{4a}$ $\overrightarrow{OB} = \mathbf{6b}$ $\overrightarrow{AP} = \mathbf{2a} + \mathbf{8b}$

Using a vector method, find the ratio $AQ : QB$

$AQ : QB =$ .....

(Total for Question 23 is 5 marks)

1 **Use vectors**

$$\overrightarrow{OA} = 4a, \qquad \overrightarrow{OB} = 6b, \qquad \overrightarrow{AP} = 2a + 8b$$

2 **Use vectors**

So

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$

$$\overrightarrow{OP} = 4a + (2a + 8b) = 6a + 8b$$

3 **Use vectors**

Since O, Q, P are collinear, let

$$\overrightarrow{OQ} = \lambda(6a + 8b)$$

4 **Use vectors**

Also Q lies on AB. Let

$$\overrightarrow{AQ} = \mu\overrightarrow{AB}$$

5 **Use vectors**

Then

$$\overrightarrow{AB} = 6b - 4a$$

$$\overrightarrow{OQ} = 4a + \mu(6b - 4a)$$

$$\overrightarrow{OQ} = (4 - 4\mu)a + 6\mu b$$

6 **Use vectors**

Compare coefficients with

$$\lambda(6a + 8b) = 6\lambda a + 8\lambda b$$

7 **Use vectors**

So

$$4 - 4\mu = 6\lambda, \qquad 6\mu = 8\lambda$$

8 **Use vectors**

From $6\mu = 8\lambda$,

$$\lambda = \frac{3}{4}\mu$$

---

9 **Substitute**

Substitute:

$$4 - 4\mu = 6\left(\frac{3}{4}\mu\right)$$

$$4 - 4\mu = \frac{9}{2}\mu$$

$$8 = 17\mu$$

$$\mu = \frac{8}{17}$$

---

10 **Use vectors**

Therefore

$$AQ : QB = \frac{8}{17} : \frac{9}{17} = 8 : 9$$

✓ **FINAL ANSWER**

$AQ : QB = 8 : 9$.

Question 24

5 marks | Sequences

24 The sum of the first 10 terms of an arithmetic series is 4 times the sum of the first 5 terms of the same series.
The 8th term of this series is 45

Find the first term of this series.
Show clear algebraic working.

.....

(Total for Question 24 is 5 marks)

1 Let the first term be

Let the first term be a and the common difference be d.

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

2 Find the nth term

So

$$S_{10} = 5(2a + 9d)$$

3 Find the nth term

and

$$S_5 = \frac{5}{2}(2a + 4d)$$

4 Given

Given $S_{10} = 4S_5$:

$$5(2a + 9d) = 4\left(\frac{5}{2}(2a + 4d)\right)$$

$$10a + 45d = 20a + 40d$$

$$5d = 10a$$

$$d = 2a$$

5 The 8th term is

The 8th term is 45:

$$a + 7d = 45$$

6 Substitute

Substitute $d = 2a$:

$$a + 14a = 45$$

$$15a = 45$$

$$a = 3$$

FINAL ANSWER

the first term is $\mathbf{3}$.