Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

4MA1/2H November 2023 Worked Solutions

November 2023 paper rebuilt with the worked solution after each question.

24	2020–2026	Single-paper solutions
paper questions	52 past papers	checked by Dr Eslam



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PAST PAPER

4MA1/2H November 2023

November 2023

24	100	2 h
questions	marks	exam time

Each question is followed by its worked solution and final answer.

4MA1/2H NOVEMBER 2023

Question 1

4 marks | Averages from Frequency Tables

1 The table shows information about the lengths, in minutes, of 50 telephone calls.

Length of telephone call (m minutes)	Frequency
$0 < m \leqslant 5$	8
$5 < m \leqslant 10$	2
$10 < m \leqslant 15$	6
$15 < m \leqslant 20$	4
$20 < m \leqslant 25$	12
$25 < m \leqslant 30$	18

(a) Write down the modal class.

.....

(1)

(b) Work out an estimate for the total length, in minutes, of these telephone calls.

..... minutes

(3)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H November 2023

1 The modal class is the

The modal class is the class with the greatest frequency:

$$25 < m \leq 30$$

2 Use midpoints

Use midpoints $2.5, 7.5, 12.5, 17.5, 22.5, 27.5$.

$$2.5(8) + 7.5(2) + 12.5(6) + 17.5(4) + 22.5(12) + 27.5(18) = 945$$

FINAL ANSWER

modal class $25 < m \leq 30$, estimated total 945 minutes.

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Question 2

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

2 The diagram shows triangle ABC and triangle ECD

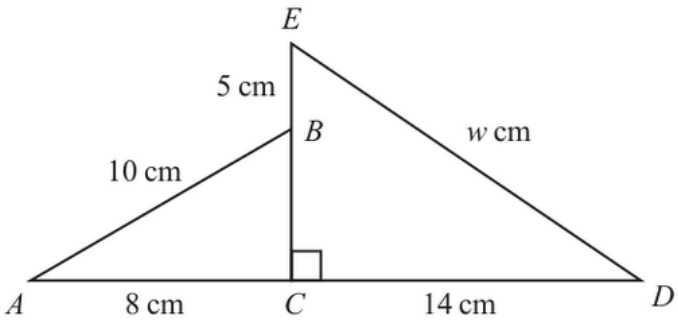


Diagram **NOT** accurately drawn

ACD and EBC are straight lines.

$AB = 10$ cm $\quad AC = 8$ cm $\quad EB = 5$ cm $\quad CD = 14$ cm $\quad ED = w$ cm

Work out the value of w
Give your answer correct to one decimal place.

$w =$ .....

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H November 2023

1 **Use trigonometry**

In triangle ABC,

$$BC^2 = 10^2 - 8^2$$

$$BC^2 = 36$$

$$BC = 6$$

2 **Use trigonometry**

So

$$EC = EB + BC = 5 + 6 = 11$$

3 **Use trigonometry**

In triangle ECD,

$$w^2 = 11^2 + 14^2$$

$$w^2 = 317$$

$$w = 17.804\ldots$$

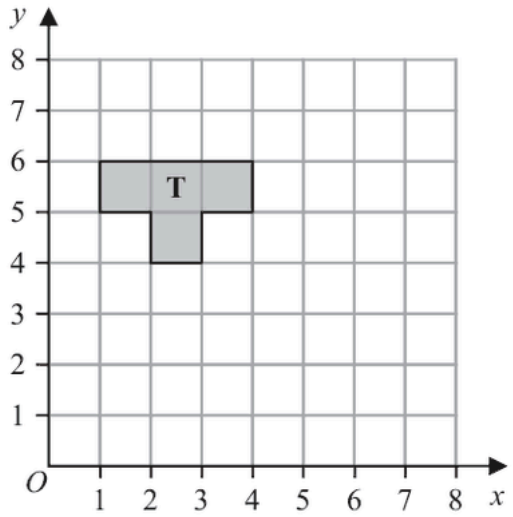
✓ **FINAL ANSWER**

$w = 17.8\ \text{cm}$ to 1 decimal place.

Question 3

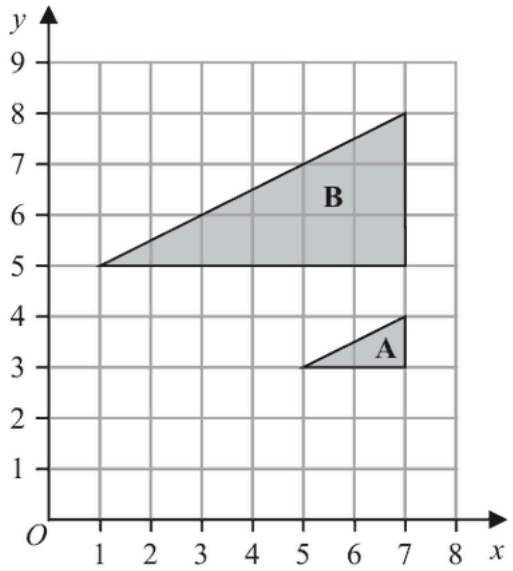
5 marks | Transformations

3



(a) Reflect shape **T** in the line $y = x$

(2)



(b) Describe fully the single transformation that maps triangle **A** onto triangle **B**

.....

.....

(3)

(Total for Question 3 is 5 marks)

1 (a) Reflection in swaps the

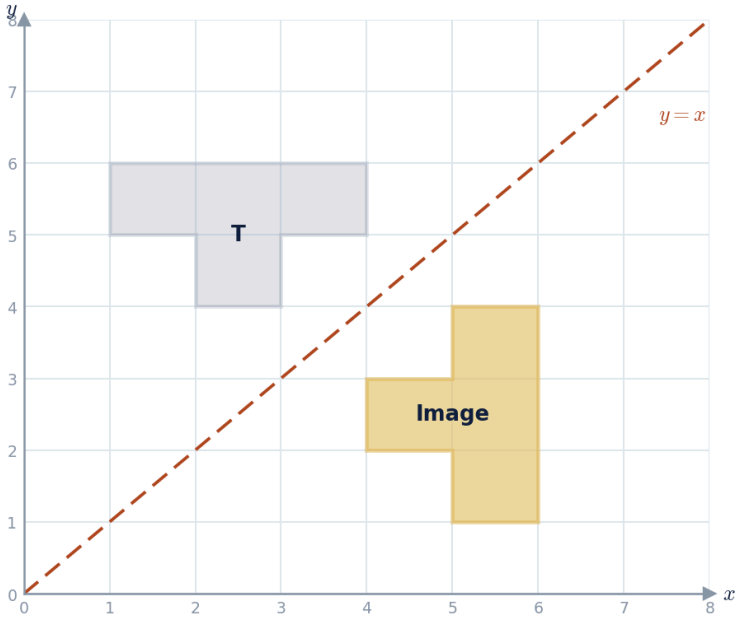
(a) Reflection in $y = x$ swaps the coordinates:

$$(x, y) \mapsto (y, x)$$

2 The reflected T shape has

The reflected T-shape has vertices

$$(6,1), (6,4), (5,4), (5,3), (4,3), (4,2), (5,2), (5,1)$$



3 Use trigonometry

(b) Triangle A is mapped to B by an enlargement with scale factor 3 and centre $(7, 2)$.

FINAL ANSWER

(a) reflected vertices $(6,1), (6,4), (5,4), (5,3), (4,3), (4,2), (5,2), (5,1)$; (b) enlargement, scale factor 3, centre $(7, 2)$.

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Question 4

8 marks | Algebraic Roots & Indices

4 (a) Solve $\frac{2x+5}{6} = 2x - 5$

Show clear algebraic working.

$x =$ .....

(3)

(b) Simplify $h^{15} \div h^3$

.....

(1)

(c) Simplify fully $\left(2g^3k^5\right)^4$

.....

(2)

(d) Given that $\frac{y^5 \times y^n}{y^7} = y^{12}$

work out the value of n

$n =$ .....

(2)

(Total for Question 4 is 8 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H November 2023

1 **(a)**

(a)

$$\frac{2x+5}{6} = 2x - 5$$

$$2x + 5 = 12x - 30$$

$$35 = 10x$$

$$x = \frac{7}{2}$$

2 **(b)**

(b)

$$h^{15} \div h^3 = h^{15-3} = h^{12}$$

---

3 **(c)**

(c)

$$(2g^3k^5)^4 = 2^4g^{12}k^{20} = 16g^{12}k^{20}$$

---

4 **(d)**

(d)

$$\frac{y^5 \times y^n}{y^7} = y^{5+n-7} = y^{n-2}$$

---

5 **Use index laws**

Given that this equals y^{12},

$$n - 2 = 12$$

$$n = 14$$

FINAL ANSWER

$x = \frac{7}{2}$, h^{12}, $16g^{12}k^{20}$, and $n = 14$.

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Question 5

3 marks | Ratio Toolkit

5 Avril bakes a cake.

She uses flour, butter and sugar such that

weight of flour : weight of butter = 6 : 5
weight of butter : weight of sugar = 3 : 2

Avril uses 120 grams of sugar.

Work out the weight of flour Avril uses.

..... grams

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H November 2023

1 Butter sugar

Butter : sugar $= \mathbf{3 : 2}$.

2 Split the ratio

Sugar is 120 g, so

$$2 \text{ parts} = 120$$

$$1 \text{ part} = 60$$

$$\text{butter} = 3 \times 60 = 180$$

3 Flour butter

Flour : butter $= \mathbf{6 : 5}$.

$$5 \text{ parts} = 180$$

$$1 \text{ part} = 36$$

$$\text{flour} = 6 \times 36 = 216$$

FINAL ANSWER

216 grams

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Question 6

3 marks | Algebraic Proof

6 Show that $3\frac{3}{7} \div 2\frac{2}{3} = 1\frac{2}{7}$

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H November 2023

1 Evaluate fraction

$$3\frac{3}{7} = \frac{24}{7}, \qquad 2\frac{2}{3} = \frac{8}{3}$$

$$3\frac{3}{7} \div 2\frac{2}{3} = \frac{24}{7} \div \frac{8}{3}$$

$$= \frac{24}{7} \times \frac{3}{8} = \frac{9}{7} = 1\frac{2}{7}$$

FINAL ANSWER

$1\frac{2}{7}$.

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Question 7

3 marks | Compound Interest & Depreciation

7 Hermione buys a boat for $26 800
The value of the boat depreciates by 8% each year.

Work out the value of the boat at the end of 3 years.
Give your answer correct to the nearest dollar.

$.....

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H November 2023

1 A depreciation of gives multiplier

A depreciation of 8% gives multiplier

$$0.92$$

2 Use compound interest

After 3 years,

$$26800(0.92)^3 = 20868.8384$$

3 Use compound interest

Correct to the nearest dollar,

$$20868.8384 \approx 20869$$

✓ **FINAL ANSWER**

$20869

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Question 8

3 marks | Statistics Toolkit

8 The mean number of goals scored by a hockey team in 8 matches is 6
The team plays 2 more matches and scores k goals in each match.
The mean number of goals scored by the hockey team in the 10 matches is 7

Work out the value of k

$k =$ .....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H November 2023

1 The total for the first

The total for the first 8 matches is

$$8 \times 6 = 48$$

2 The total for all 10

The total for all 10 matches is

$$10 \times 7 = 70$$

3 Compare density

So the last two matches have total

$$70 - 48 = 22$$

4 The last two scores are

The last two scores are k and $k + 10$:

$$k + (k + 10) = 22$$

$$2k + 10 = 22$$

$$k = 6$$

FINAL ANSWER

$k = 6$.

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Question 9

2 marks | Linear Graphs

9 A straight line passes through the points with coordinates (0, –3) and (2, 0)

Find an equation of the line.

(Total for Question 9 is 2 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H November 2023

1 Solve equation

The line passes through $(0, -3)$ and $(2, 0)$.

2 The gradient is

The gradient is

$$\frac{0-(-3)}{2-0} = \frac{3}{2}$$

3 Solve equation

The y-intercept is -3, so

$$y = \frac{3}{2}x - 3$$

FINAL ANSWER

$y = \frac{3}{2}x - 3$.

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Question 10

5 marks | Perimeter & Area

10 The diagram shows a hexagon $ABCDEF$

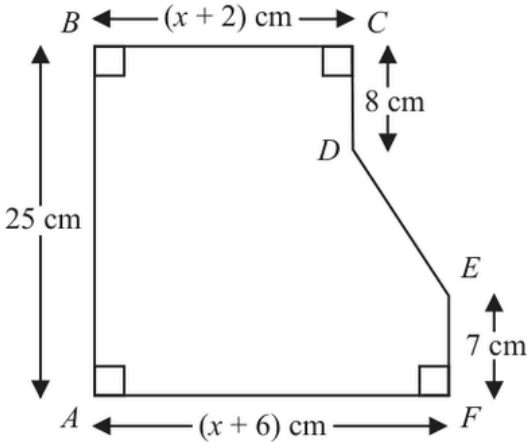


Diagram **NOT** accurately drawn

$AB = 25$ cm $\quad BC = (x + 2)$ cm $\quad CD = 8$ cm $\quad EF = 7$ cm $\quad AF = (x + 6)$ cm

The area of hexagon $ABCDEF$ is 258 cm²

Work out the value of x

$x =$ .....

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H November 2023

1 The area of the hexagon

The area of the hexagon can be found from coordinates or by splitting the shape.

2 Calculate area

It simplifies to

$$\text{area} = 25x + 98$$

3 **Calculate area**

Given the area is $258\ \text{cm}^2$,

$$25x + 98 = 258$$

$$25x = 160$$

$$x = 6.4$$

FINAL ANSWER

$x = 6.4$.

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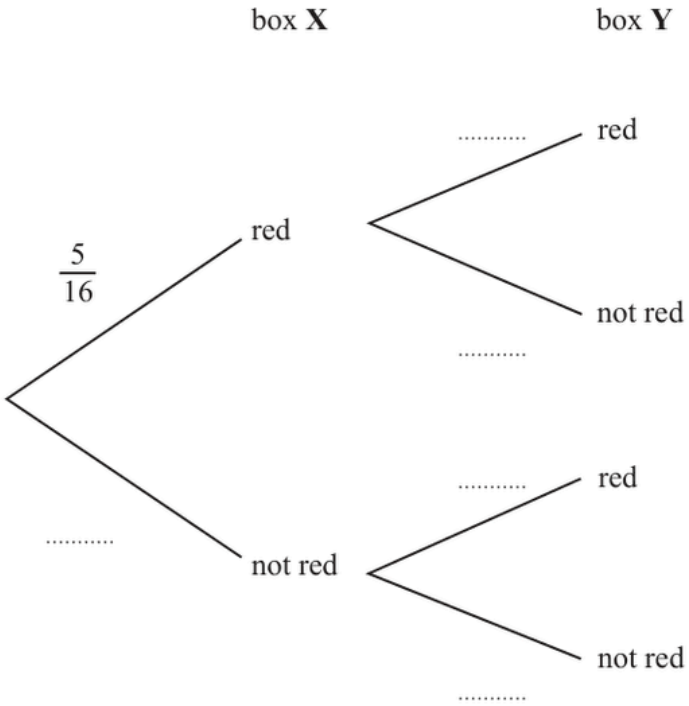
Question 11

5 marks | Tree Diagrams & Conditional Probability

11 Sid has 2 boxes of crayons, box **X** and box **Y**

5 of the 16 crayons in box **X** are red.
7 of the 20 crayons in box **Y** are red.
Sid takes at random one crayon from box **X** and one crayon from box **Y**

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that Sid takes two crayons that are red or two crayons that are not red.

.....

(3)

(Total for Question 11 is 5 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H November 2023

1 **(a) Complete the tree**

Box X: not red $\frac{11}{16}$.

Box Y (on both branches): red $\frac{7}{20}$, not red $\frac{13}{20}$.

2 **(b) Both red or both not red**

$$\frac{5}{16} \times \frac{7}{20} + \frac{11}{16} \times \frac{13}{20} = \frac{35}{320} + \frac{143}{320} = \frac{178}{320} = \frac{89}{160}$$

✓ **FINAL ANSWER**

(a) $\frac{11}{16}$; $\frac{7}{20}$, $\frac{13}{20}$ on both branches (b) $\frac{89}{160}$

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Question 12

4 marks | Algebraic Roots & Indices

12 $2^7 \times 4^5 = 4^x$

(a) Calculate the value of x

$x =$ .....

(2)

(b) Simplify fully $\left(125p^6y^{24}\right)^{\frac{2}{3}}$

.....

(2)

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H November 2023

1 **(a)**

(a)

$$2^7 \times 4^5 = 2^7 \times (2^2)^5 = 2^{17}$$

2 **Use index laws**

Also

$$4^x = (2^2)^x = 2^{2x}$$

3 **Use index laws**

So

$$2x = 17$$

$$x = \frac{17}{2}$$

4 **(b)**

(b)

$$(125p^6y^{24})^{2/3} = 125^{2/3}p^4y^{16}$$

$$= 25p^4y^{16}$$

FINAL ANSWER

(a) $x = \frac{17}{2}$, (b) $25p^4y^{16}$

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Question 13

2 marks | Statistics Toolkit

13 Robert asked 11 people how many meetings they attended last week.

Here are the results in numerical order.

1 2 4 6 6 8 11 12 13 14 17

Find the interquartile range of the number of meetings.

(Total for Question 13 is 2 marks)

WORKED SOLUTION

Question 13 · 4MA1/2H November 2023

1 The results in order are

The results in order are

$$1,\ 2,\ 4,\ 6,\ 6,\ 8,\ 11,\ 12,\ 13,\ 14,\ 17$$

2 There are 11 values

There are 11 values.

$$Q_1 = 4, \qquad Q_3 = 13$$

$$\text{IQR} = 13 - 4 = 9$$

FINAL ANSWER

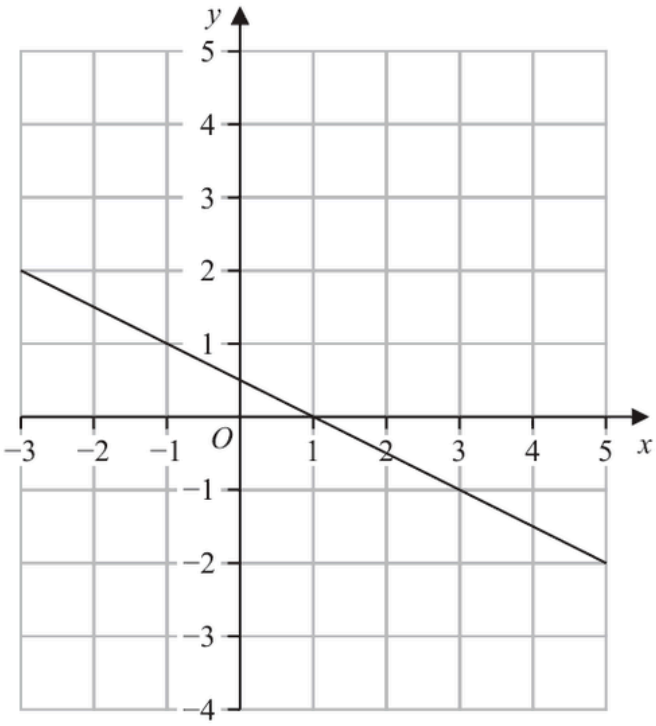
9.

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Question 14

3 marks | Simultaneous Equations

14 Here is the graph of the equation $2y + x = 1$ drawn on a grid.



By drawing another straight line on the grid, solve the simultaneous equations

$$y - x - 2 = 0$$
$$2y + x = 1$$

$x =$ .....

$y =$ .....

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H November 2023

1 **The equations are**

The equations are

$$y - x - 2 = 0$$

$$2y + x = 1$$

2 **Solve simultaneous equations**

From the first equation,

$$y = x + 2$$

3 **Substitute into the second equation**

Substitute into the second equation:

$$2(x + 2) + x = 1$$

$$3x + 4 = 1$$

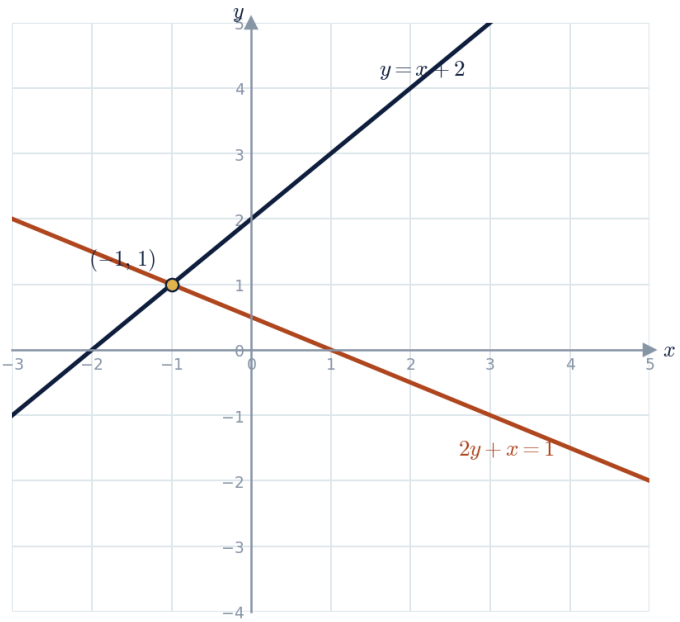
$$3x = -3$$

$$x = -1$$

4 **Solve simultaneous equations**

Then

$$y = -1 + 2 = 1$$



FINAL ANSWER

$x = -1, y = 1.$

4MA1/2H NOVEMBER 2023

Question 15

5 marks | Algebraic Proof

15 (a) Use algebra to show that $0.3\dot{7}\dot{2} = \frac{41}{110}$

(2)

(b) Express $\frac{\sqrt{125} + \sqrt{80}}{\sqrt{3}}$ in the form $\sqrt{n}$ where n is an integer.

Show your working clearly.

.....

(3)

(Total for Question 15 is 5 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H November 2023

1 **(a) Let**

(a) Let

$$x = 0.3727272\ldots$$

2 **Calculate value**

Then

$$10x = 3.727272\ldots$$

3 **Calculate value**

and

$$1000x = 372.727272\ldots$$

4 **Subtract**

Subtract:

$$990x = 369$$

$$x = \frac{369}{990} = \frac{41}{110}$$

5 (b)

(b)

$$\frac{\sqrt{125}+\sqrt{80}}{\sqrt{3}} = \frac{5\sqrt{5}+4\sqrt{5}}{\sqrt{3}}$$

$$= \frac{9\sqrt{5}}{\sqrt{3}}$$

---

6 Rationalise

Rationalise:

$$\frac{9\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{15}}{3} = 3\sqrt{15}$$

$$3\sqrt{15} = \sqrt{135}$$

FINAL ANSWER

$0.3727272\ldots = \frac{41}{110}$, and $n = 135$.

4MA1/2H NOVEMBER 2023

Question 16

3 marks | Expanding Brackets

16 Expand and simplify $(2x + 3)(x - 5)(x + 4)$

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/2H November 2023

1 Expand brackets

$$(x - 5)(x + 4) = x^2 - x - 20$$

$$(2x + 3)(x^2 - x - 20) = 2x^3 + x^2 - 43x - 60$$

FINAL ANSWER

$2x^3 + x^2 - 43x - 60$

4MA1/2H NOVEMBER 2023

Question 17

3 marks | Rounding, Estimation & Bounds

17 $P = a(c + y)$

$a =$ 8.3 correct to 2 significant figures
$c =$ 2 correct to 1 significant figure
$y =$ 15 correct to the nearest 5

Work out the upper bound for the value of P
Show your working clearly.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H November 2023

1 Estimate the value

$$P = a(c + y)$$

2 Find upper bound

For the upper bound, use the upper bounds of all three values.

$$a < 8.35, \quad c < 2.5, \quad y < 17.5$$

$$P_{\text{upper}} = 8.35(2.5 + 17.5) = 167$$

FINAL ANSWER

167

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Question 18

4 marks | Differentiation & Turning Points

18 A particle is moving along a straight line that passes through the fixed point O
The displacement, s metres, of the particle from O at time t seconds is given by

$$s = 2t^3 - 5t^2 + 6t - 5$$

Find the value of t when the acceleration of the particle is 5 m/s²

$t =$ .....

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H November 2023

1 Split the ratio

$$s = 2t^3 - 5t^2 + 6t - 5$$

2 Velocity is

Velocity is

$$\frac{ds}{dt} = 6t^2 - 10t + 6$$

3 Acceleration is

Acceleration is

$$\frac{d^2s}{dt^2} = 12t - 10$$

4 Split the ratio

The acceleration is 5, so

$$12t - 10 = 5$$

$$12t = 15$$

$$t = \frac{15}{12} = \frac{5}{4}$$

FINAL ANSWER

$t = \frac{5}{4}$ seconds

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Question 19

7 marks | Functions

19 The functions f and g are such that

$$f: x \mapsto 5x + 7$$

$$g: x \mapsto \frac{5}{2x - 9}$$

(a) State which value of x cannot be included in any domain of g

.....

(1)

(b) Find fg(4)

.....

(2)

The function h is such that

$$h: x \mapsto 3x^2 - 12x + 8 \quad \text{where } x > 2$$

(c) Express the inverse function h^{-1} in the form $h^{-1}: x \mapsto ...$

$h^{-1}: x \mapsto$ .....

(4)

(Total for Question 19 is 7 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H November 2023

1 **Evaluate fraction**

$$f(x) = 5x + 7, \qquad g(x) = \frac{5}{2x - 9}$$

2 **(a) The denominator of cannot**

(a) The denominator of g cannot be zero:

$$2x - 9 = 0$$

$$x = \frac{9}{2}$$

3 **(b)**

(b)

$$g(4) = \frac{5}{8-9} = -5$$

$$fg(4) = f(-5) = 5(-5) + 7 = -18$$

---

4 **(c)**

(c)

$$h(x) = 3x^2 - 12x + 8, \qquad x > 2$$

---

5 **Complete the square**

Complete the square:

$$h(x) = 3(x-2)^2 - 4$$

---

6 **Let**

Let

$$y = 3(x-2)^2 - 4$$

$$y + 4 = 3(x-2)^2$$

$$x - 2 = \sqrt{\frac{y+4}{3}}$$

---

7 **Simplify surd**

Since $x > 2$, take the positive root:

$$x = 2 + \sqrt{\frac{y+4}{3}}$$

---

8 **Simplify surd**

So

$$h^{-1}(x) = 2 + \sqrt{\frac{x+4}{3}}$$

✓ **FINAL ANSWER**

Excluded value $\frac{9}{2}$, $fg(4) = -18$, and $h^{-1}(x) = 2 + \sqrt{\frac{x+4}{3}}$.

4MA1/2H NOVEMBER 2023

Question 20

4 marks | Circles, Arcs & Sectors

20 The diagram shows equilateral triangle ABC with sides of length 10 cm.
A circle is drawn inside the triangle.

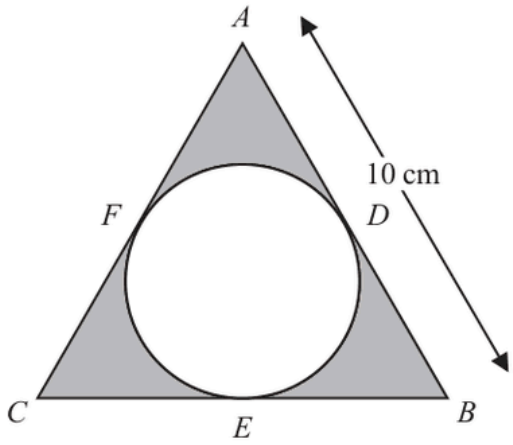


Diagram **NOT** accurately drawn

D, E and F are points on the circle.

ADB, BEC and CFA are tangents to the circle.

Calculate the total area of the regions shown shaded in the diagram.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H November 2023

1 Use trigonometry

The triangle is equilateral with side length 10 cm.

2 Area of the triangle

Area of the triangle:

$$\frac{\sqrt{3}}{4}(10)^2 = 25\sqrt{3}$$

3 Use trigonometry

For an equilateral triangle, the inradius is

$$r = \frac{s\sqrt{3}}{6}$$

4 **Simplify surd**

So

$$r = \frac{10\sqrt{3}}{6} = \frac{5\sqrt{3}}{3}$$

5 **Area of the circle**

Area of the circle:

$$\pi r^2 = \pi\left(\frac{5\sqrt{3}}{3}\right)^2 = \frac{25\pi}{3}$$

6 **The shaded area is**

The shaded area is

$$25\sqrt{3} - \frac{25\pi}{3} = 17.1213\ldots$$

✓ **FINAL ANSWER**

$\boxed{17.1\ \text{cm}^2}$

4MA1/2H NOVEMBER 2023

Question 21

5 marks | Linear Graphs

21 The line with equation $x + 2y = 5$ intersects the curve with equation $x^2 + 3y^2 = 13$ at the points A and B

Find the coordinates of A and the coordinates of B
Show clear algebraic working.

(..... , ...)

(..... , ...)

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H November 2023

1 The line is

The line is

$$x + 2y = 5$$

2 Calculate value

So

$$x = 5 - 2y$$

3 Calculate value

Substitute into

$$x^2 + 3y^2 = 13$$

$$(5 - 2y)^2 + 3y^2 = 13$$

$$25 - 20y + 4y^2 + 3y^2 = 13$$

$$7y^2 - 20y + 12 = 0$$

4 **Factorise**

Factorise:

$$(7y - 6)(y - 2) = 0$$

5 **Evaluate fraction**

So

$$y = \frac{6}{7} \quad \text{or} \quad y = 2$$

6 **Evaluate fraction**

Using $x = 5 - 2y$:

$$y = \frac{6}{7} \Rightarrow x = \frac{23}{7}$$

$$y = 2 \Rightarrow x = 1$$

✓ **FINAL ANSWER**

$\left(\frac{23}{7}, \frac{6}{7}\right)$ and $(1, 2)$

Question 22

5 marks | Vectors

22

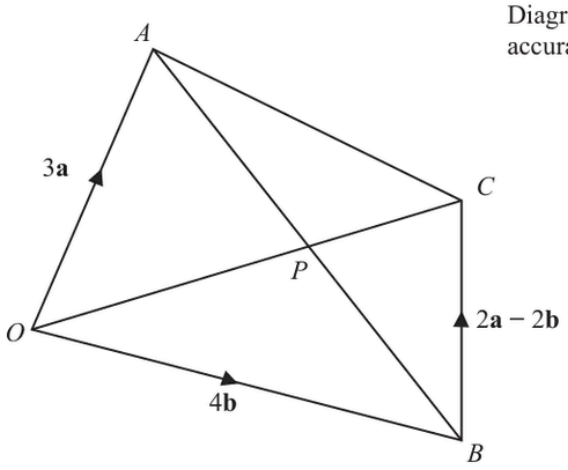


Diagram **NOT** accurately drawn

$OACB$ is a quadrilateral.

$\overrightarrow{OA} = \mathbf{3a}$ $\overrightarrow{OB} = \mathbf{4b}$ $\overrightarrow{BC} = \mathbf{2a} - \mathbf{2b}$

(a) (i) Find the vector $\overrightarrow{OC}$ in terms of **a** and **b**
Simplify your answer.

$\overrightarrow{OC} =$ .....

(1)

(ii) Find the vector $\overrightarrow{AB}$ in terms of **a** and **b**

$\overrightarrow{AB} =$ .....

(1)

The point P lies on AB and on OC

(b) Using a vector method, find the ratio $AP : PB$
Show your working clearly.

.....

(3)

(Total for Question 22 is 5 marks)

1 **Split the ratio**

$$\overrightarrow{OA} = 3a, \quad \overrightarrow{OB} = 4b, \quad \overrightarrow{BC} = 2a - 2b$$

2 **Part (a)**

Part (a):

$$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = 4b + (2a - 2b) = 2a + 2b$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 4b - 3a$$

3 **Split the ratio**

For part (b), let P divide AB so that

$$\overrightarrow{AP} = \mu\overrightarrow{AB}$$

4 **Split the ratio**

Then

$$\overrightarrow{OP} = 3a + \mu(4b - 3a) = (3 - 3\mu)a + 4\mu b$$

5 **Split the ratio**

Since P lies on OC, also let

$$\overrightarrow{OP} = \lambda(2a + 2b)$$

6 **Equating coefficients**

Equating coefficients:

$$3 - 3\mu = 2\lambda, \quad 4\mu = 2\lambda$$

7 **Split the ratio**

From $4\mu = 2\lambda$, $\lambda = 2\mu$.

$$3 - 3\mu = 4\mu$$

$$\mu = \frac{3}{7}$$

8 **Split the ratio**

So $AP : PB = \frac{3}{7} : \frac{4}{7}$.

FINAL ANSWER

$\overrightarrow{OC} = 2\boldsymbol{a} + 2\boldsymbol{b}$, $\overrightarrow{AB} = 4\boldsymbol{b} - 3\boldsymbol{a}$, and $AP : PB = 3 : 4$

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Question 23

5 marks | Area & Volume of Similar Shapes

23 Here is a frustum of a cone.
The frustum is made by removing a small cone from a similar large cone.

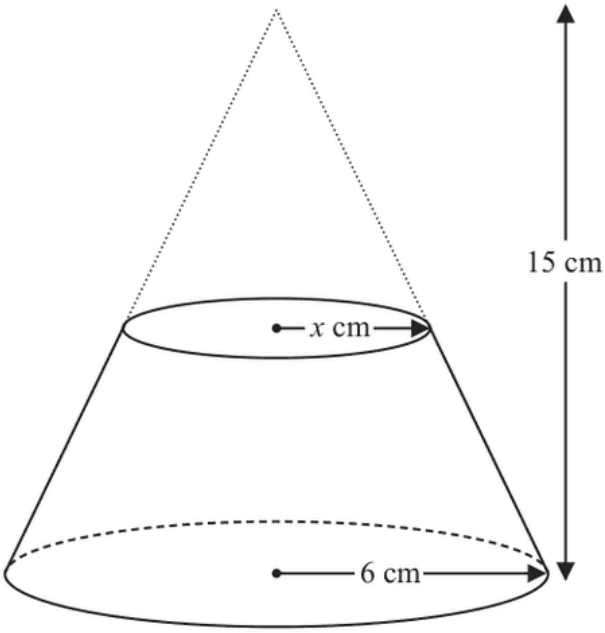


Diagram **NOT** accurately drawn

The height of the large cone is 15 cm.

The radius of the base of the large cone is 6 cm.
The radius of the base of the small cone is x cm.

Given that the volume of the frustum is $\frac{4212}{25}\pi$ cm^3

work out the value of x
Show clear algebraic working.

$x =$ .....

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H November 2023

1 **The large cone has height**

The large cone has height $\mathbf{15}$ cm and radius $\mathbf{6}$ cm.

2 **The small cone has radius**

The small cone has radius x. Since the cones are similar,

$$\frac{\text{radius}}{\text{height}} = \frac{6}{15} = \frac{2}{5}$$

3 **Evaluate fraction**

So for the small cone,

$$\frac{x}{h} = \frac{2}{5}$$

$$h = \frac{5}{2}x$$

4 **Volume of the large cone**

Volume of the large cone:

$$\frac{1}{3}\pi(6)^2(15) = 180\pi$$

5 **Volume of the small cone**

Volume of the small cone:

$$\frac{1}{3}\pi x^2\left(\frac{5}{2}x\right) = \frac{5}{6}\pi x^3$$

6 **The frustum volume is**

The frustum volume is

$$\frac{4212}{25}\pi$$

7 **Evaluate fraction**

So

$$180\pi - \frac{5}{6}\pi x^3 = \frac{4212}{25}\pi$$

8 **Cancel**

Cancel π:

$$180 - \frac{5}{6}x^3 = \frac{4212}{25}$$

$$\frac{5}{6}x^3 = 180 - \frac{4212}{25}$$

$$\frac{5}{6}x^3 = \frac{288}{25}$$

$$x^3 = \frac{1728}{125}$$

$$x = \frac{12}{5} = 2.4$$

✓ **FINAL ANSWER**

$\boxed{2.4 \text{ cm}}$

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Question 24

5 marks | Algebraic Fractions

24 Solve $\frac{45x^3-80x}{3x^2+x-4}\times\left(\frac{1}{3x-4}+\frac{1}{x}\right)=\frac{4(x+2)}{5x-8}$

Show clear algebraic working.

$x =$ .....

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H November 2023

1 Solve quadratic equation

$$\frac{45x^3-80x}{3x^2+x-4}\left(\frac{1}{3x-4}+\frac{1}{x}\right)=\frac{4(x+2)}{5x-8}$$

2 Factorise

Factorise:

$$45x^3-80x=5x(3x-4)(3x+4)$$

$$3x^2+x-4=(3x+4)(x-1)$$

3 Solve quadratic equation

Also,

$$\frac{1}{3x-4}+\frac{1}{x}=\frac{x+3x-4}{x(3x-4)}=\frac{4(x-1)}{x(3x-4)}$$

4 Solve quadratic equation

So the left side becomes

$$\frac{5x(3x-4)(3x+4)}{(3x+4)(x-1)}\times\frac{4(x-1)}{x(3x-4)}=20$$

5 **Solve quadratic equation**

Therefore,

$$20 = \frac{4(x+2)}{5x-8}$$

$$20(5x-8) = 4(x+2)$$

$$100x - 160 = 4x + 8$$

$$96x = 168$$

$$x = \frac{7}{4}$$

✓ **FINAL ANSWER**

$x = \frac{7}{4}$