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Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

4MA1/2H November 2021 Worked Solutions

November 2021 paper rebuilt with the worked solution after each question.

22	2020–2026	Single-paper solutions
paper questions	52 past papers	checked by Dr Eslam



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PAST PAPER

4MA1/2H November 2021

November 2021

22	100	2 h
questions	marks	exam time

Each question is followed by its worked solution and final answer.

4MA1/2H NOVEMBER 2021

Question 1

4 marks | Algebraic Roots & Indices

1 (a) Write down the value of m, given that $3^4 \times 3^5 = 3^m$

$m =$ .....

(1)

(b) Write down the value of n, given that $(5^3)^7 = 5^n$

$n =$ .....

(1)

(c) Find the value of p, given that $\dfrac{7^8 \times 7^2}{7^p} = 7^6$

$p =$ .....

(2)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H November 2021

1 **Use index laws**

(a) $3^4 \times 3^5 = 3^9$, so $m = 9$.

2 **Use index laws**

(b) $(5^3)^7 = 5^{21}$, so $n = 21$.

3 **(c)**

(c)

$$\frac{7^8 \times 7^2}{7^p} = 7^{10-p} = 7^6$$

4 **Use index laws**

So $10 - p = 6$, hence $p = 4$.

✓ **FINAL ANSWER**

$m = 9$, $n = 21$, $p = 4$

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Question 2

4 marks | Forming & Solving Equations

2 Here are two rectangles, rectangle A and rectangle B.

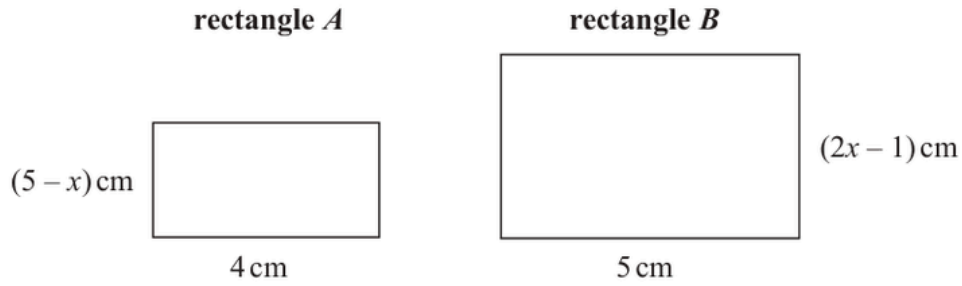


Diagram **NOT** accurately drawn

The area of rectangle B is twice the area of rectangle A.

Work out the value of x.
Show your working clearly.

$x =$ .....

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H November 2021

1 Area of rectangle A

Area of rectangle A:

$$4(5 - x)$$

2 Area of rectangle B

Area of rectangle B:

$$5(2x - 1)$$

3 **Rectangle B has twice the**

Rectangle B has twice the area of rectangle A:

$$5(2x - 1) = 2 \cdot 4(5 - x)$$

$$10x - 5 = 40 - 8x$$

$$18x = 45$$

$$x = 2.5$$

FINAL ANSWER

$x = 2.5$.

4MA1/2H NOVEMBER 2021

Question 3

5 marks | Averages from Frequency Tables

3 The table gives information about the amounts of money, in euros, that 70 of Anjali's friends spent last Saturday.

Money spent (S euros)	Frequency
$0 < S \leqslant 8$	6
$8 < S \leqslant 16$	14
$16 < S \leqslant 24$	19
$24 < S \leqslant 32$	25
$32 < S \leqslant 40$	6

One of Anjali's 70 friends is going to be chosen at random.

(a) Find the probability that this friend spent more than 24 euros last Saturday.

.....

(1)

(b) Work out an estimate for the mean amount of money spent by Anjali's friends last Saturday. Give your answer correct to 2 decimal places.

..... euros

(4)

(Total for Question 3 is 5 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H November 2021

1 **More than euros**

More than 24 euros:

$$25 + 6 = 31$$

2 **Total frequency**

Total frequency:

$$6 + 14 + 19 + 25 + 6 = 70$$

$$P(S > 24) = \frac{31}{70}$$

3 **Use midpoints**

Use midpoints $4, 12, 20, 28, 36$.

$$4(6) + 12(14) + 20(19) + 28(25) + 36(6) = 1488$$

$$\text{estimated mean} = \frac{1488}{70} = 21.257\ldots$$

✓ **FINAL ANSWER**

$\frac{31}{70}$, estimated mean 21.3 euros.

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Question 4

4 marks | Similar Shapes

4 *ABC* and *DEF* are similar triangles.

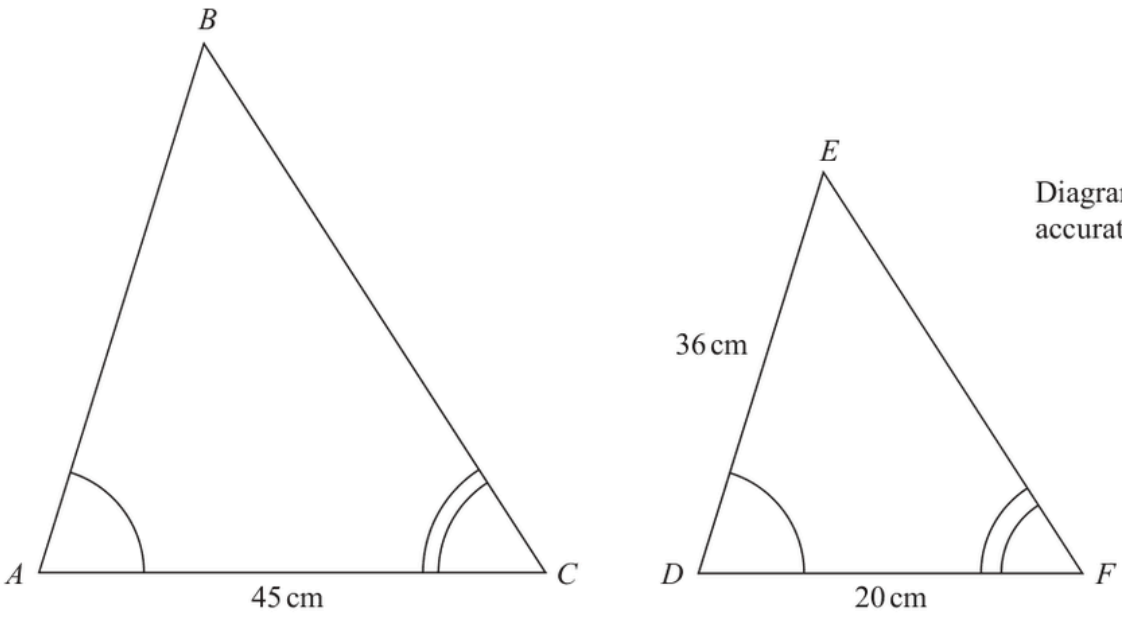


Diagram **NOT** accurately drawn

(a) Work out the length of *AB*.

..... cm

(2)

Given that *BC* = 54 cm,

(b) work out the length of *EF*.

..... cm

(2)

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H November 2021

1 The scale factor from triangle

The scale factor from triangle DEF to triangle ABC is

$$\frac{AC}{DF} = \frac{45}{20} = 2.25$$

$$AB = DE \times 2.25 = 36 \times 2.25 = 81$$

2 **Evaluate fraction**

Given $BC = 54$,

$$EF = \frac{54}{2.25} = 24$$

FINAL ANSWER

$AB = 81\ \text{cm}$, and $EF = 24\ \text{cm}$.

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Question 5

5 marks | Angles in Polygons & Parallel Lines

5 The diagram shows a regular octagon $ABCDHIJK$ and a pentagon $DEFGH$.

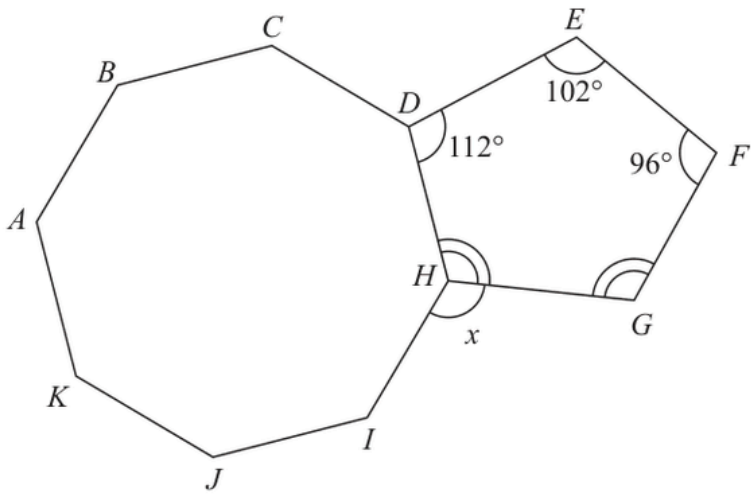


Diagram **NOT** accurately drawn

Angle GHD = angle FGH.

Work out the size of the angle marked x.
Show your working clearly.

.....°

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H November 2021

1 Find the gradient

For pentagon $DEFGH$, the interior angle sum is

$$(5-2)\times 180 = 540°$$

2 Let

Let

$$\angle GHD = \angle FGH = a$$

3 **Find the gradient**

Then

$$112 + 102 + 96 + a + a = 540$$

$$2a = 230$$

$$a = 115°$$

---

4 **A regular octagon has interior**

A regular octagon has interior angle

$$\frac{(8-2) \times 180}{8} = 135°$$

---

5 **Find the gradient**

At H, angles around a point add to $360°$:

$$x = 360° - 135° - 115° = 110°$$

✓ **FINAL ANSWER**

$110°$.

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Question 6

3 marks | Reverse Percentages

6 Victor buys 12 bottles of apple juice for a total cost of $21
Victor sells all 12 bottles at $2.45 each bottle.

Work out Victor's percentage profit.

.....%

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H November 2021

1 Total selling price

Total selling price:

$$12 \times 2.45 = 29.40$$

2 Profit

Profit:

$$29.40 - 21 = 8.40$$

3 Percentage profit

Percentage profit:

$$\frac{8.40}{21} \times 100 = 40$$

FINAL ANSWER

40%

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Question 7

4 marks | Compound Interest & Depreciation

7 Ali and Badia each have 25 000 dollars to invest.

Cyclone Bank	Tornado Bank
Invest 25 000 dollars 4.5% compound interest per year for 3 years	Invest 25 000 dollars Receive 1150 dollars interest each year for 3 years

Ali invests in the Cyclone Bank for 3 years.
Badia invests in the Tornado Bank for 3 years.

By the end of the 3 years, Ali will have received more interest than Badia.

How much more?
Show your working clearly.
Give your answer correct to the nearest dollar.

..... dollars

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H November 2021

1 Ali s interest from Cyclone

Ali's interest from Cyclone Bank is

$$25000(1.045)^3 - 25000 = 3529.153\ldots$$

2 Badia s interest from Tornado

Badia's interest from Tornado Bank is

$$1150 \times 3 = 3450$$

3 Difference

Difference:

$$3529.153\ldots - 3450 = 79.153\ldots$$

4 Use compound interest

Correct to the nearest dollar,

$$79.153\ldots \approx 79$$

FINAL ANSWER

$79

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Question 8

4 marks | Factorising

8 (a) Simplify $(3x^2y)^0$

.....

(1)

(b) (i) Factorise $x^2 - 5x - 36$

.....

(2)

(ii) Hence solve $x^2 - 5x - 36 = 0$

.....

(1)

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H November 2021

1 **(a)**

(a) $(3x^2y)^0 = 1$.

2 **(b)(i)**

(b)(i)

$$x^2 - 5x - 36 = (x - 9)(x + 4)$$

3 **Factorise expression**

(b)(ii) $x = 9$ or $x = -4$.

✓ **FINAL ANSWER**

1, $(x - 9)(x + 4)$, $x = 9$ or $x = -4$

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Question 9

5 marks | Standard Form

9 A rainwater tank contains 2.4×10^7 raindrops.
The rainwater tank also contains 1.75×10^6 bacteria.

(a) Work out the number of bacteria per raindrop in the tank.
Give your answer in standard form correct to 2 significant figures.

.....

(3)

A drop of rainwater contains 5.01×10^{21} atoms.

In a drop of rainwater the number of atoms is 3 times the number of molecules.

(b) Work out the number of molecules in the rainwater tank.
Give your answer in standard form correct to one significant figure.

..... molecules

(2)

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H November 2021

1 Convert standard form

For part (a), divide the number of bacteria by the number of raindrops:

$$\frac{1.75 \times 10^6}{2.4 \times 10^7} = \frac{1.75}{2.4} \times 10^{-1}$$

$$= 0.072916\ldots$$

2 Correct to 2 significant figures

Correct to 2 significant figures:

$$7.3 \times 10^{-2}$$

3 Convert standard form

For part (b), one drop contains 5.01×10^{21} atoms.

4 **Convert standard form**

The number of molecules in one drop is

$$\frac{5.01 \times 10^{21}}{3} = 1.67 \times 10^{21}$$

5 **Convert standard form**

There are 2.4×10^{7} drops, so the number of molecules in the tank is

$$(1.67 \times 10^{21})(2.4 \times 10^{7}) = 4.008 \times 10^{28}$$

6 **Correct to one significant figure**

Correct to one significant figure:

$$4 \times 10^{28}$$

✓ **FINAL ANSWER**

(a) 7.3×10^{-2}, (b) 4×10^{28} molecules

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Question 10

4 marks | Perimeter & Area

10 ABC is an isosceles triangle with $BA = BC$.

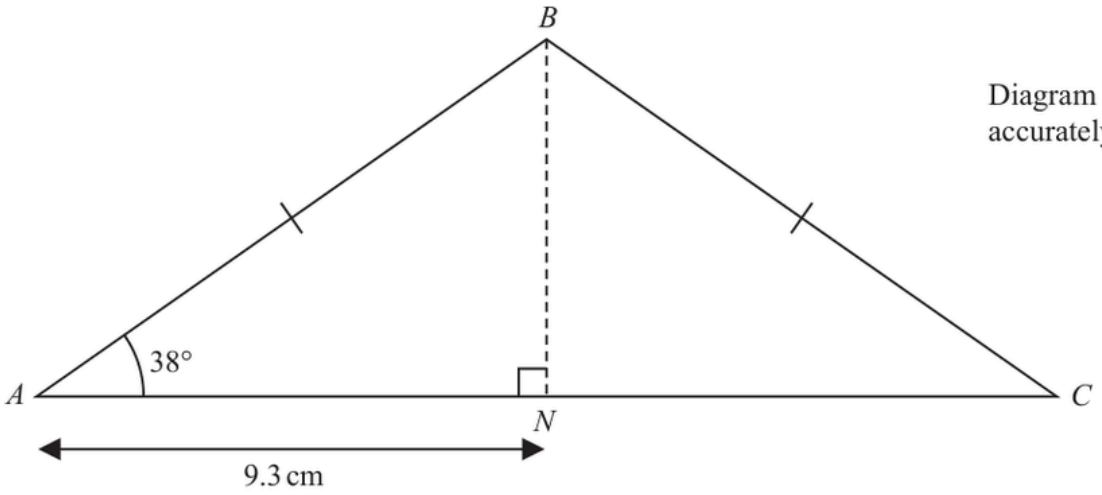


Diagram **NOT** accurately drawn

N is the point on AC such that $AN = 9.3$ cm and BN is perpendicular to AC.

Work out the perimeter of triangle ABC.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H November 2021

1 Use trigonometry

In right-angled triangle ABN,

$$\cos 38° = \frac{9.3}{AB}$$

$$AB = \frac{9.3}{\cos 38°} = 11.802\ldots$$

2 Calculate area

Since ABC is isosceles,

$$BC = AB$$

3 Calculate area

Also,

$$AC = 2 \times 9.3 = 18.6$$

4 Perimeter

Perimeter:

$$11.802\ldots + 11.802\ldots + 18.6 = 42.204\ldots$$

FINAL ANSWER

$42.2\ \text{cm}$.

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Question 11

3 marks | Circle Theorems

11

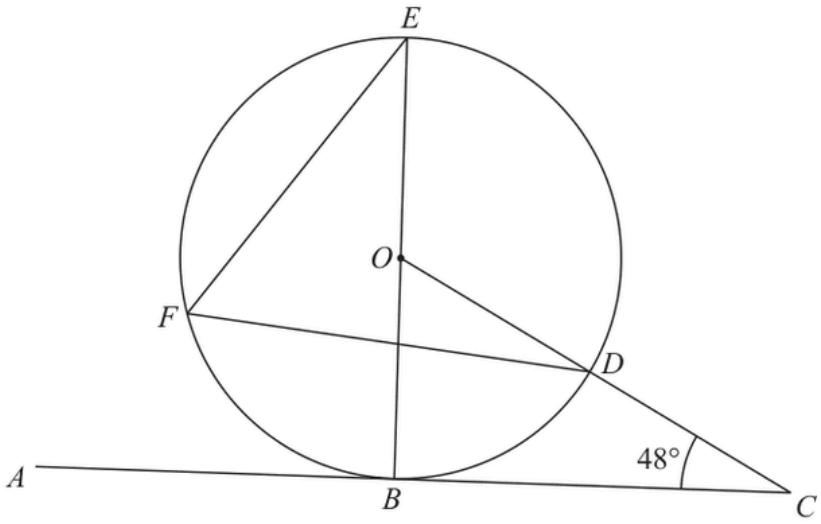


Diagram **NOT** accurately drawn

B, D, E and F are points on a circle, centre O.
ABC is a tangent to the circle.
ODC is a straight line.

BOE is a diameter of the circle.

Angle $BCD = 48°$

Find the size of angle DFE.

.....°

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H November 2021

1 The tangent at is perpendicular

The tangent at B is perpendicular to the radius OB, so

$$\angle OBC = 90°$$

2 Use trigonometry

In triangle OBC,

$$\angle BOC = 180° - 90° - 48° = 42°$$

3 **Use trigonometry**

Since O, D, C are on a straight line,

$$\angle BOD = 42°$$

---

4 **Use trigonometry**

BOK is a diameter, so BOE is a straight line and

$$\angle DOE = 180° - 42° = 138°$$

---

5 **The angle at the circumference**

The angle at the circumference is half the angle at the centre:

$$\angle DFE = \frac{138°}{2} = 69°$$

✓ **FINAL ANSWER**

$69°$.

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Question 12

7 marks | Expanding Brackets

12 (a) Simplify $(64p^9q^{12})^{\frac{2}{3}}$

.....

(2)

(b) Write as a single fraction $\frac{2}{3x} + \frac{4}{5x} - \frac{9}{10x}$

Give your answer in its simplest form.

.....

(2)

(c) Expand and simplify $4x(x - 5)(2x + 3)$

Show your working clearly.

.....

(3)

(Total for Question 12 is 7 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H November 2021

1 **(a)**

(a)

$$(64p^9q^{12})^{2/3} = 16p^6q^8$$

2 **(b)**

(b)

$$\frac{2}{3x} + \frac{4}{5x} - \frac{9}{10x} = \frac{20 + 24 - 27}{30x} = \frac{17}{30x}$$

3 **(c)**

(c)

$$4x(x-5)(2x+3) = 4x(2x^2 - 7x - 15)$$

$$= 8x^3 - 28x^2 - 60x$$

✓ **FINAL ANSWER**

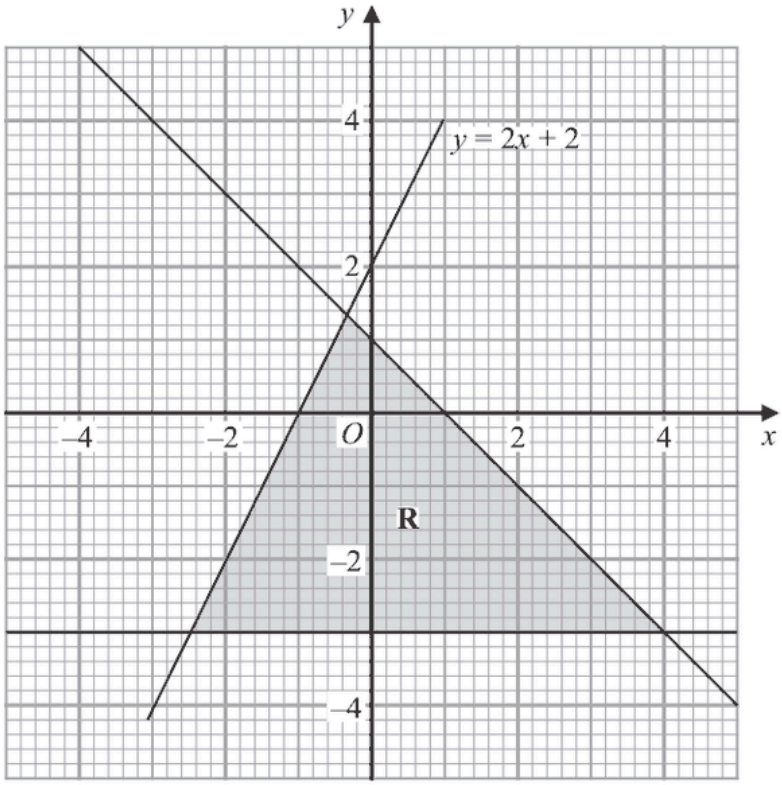
(a) $16p^6q^8$, (b) $\frac{17}{30x}$, (c) $8x^3 - 28x^2 - 60x$

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Question 13

3 marks | Inequalities (Solving & Graphing)

13



The region **R**, shown shaded in the diagram, is bounded by three straight lines.

Write down the three inequalities that define **R**.

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2H November 2021

1 **The bottom boundary is so**

The bottom boundary is

$$y = -3$$

so

$$y \geq -3$$

2 **The rising boundary is so**

The rising boundary is

$$y = 2x + 2$$

so the region has

$$y \leq 2x + 2$$

---

3 **Calculate value**

The falling boundary goes through $(0, 1)$ and $(1, 0)$, so

$$x + y = 1$$

---

4 **The region is below this**

The region is below this line:

$$x + y \leq 1$$

✓ **FINAL ANSWER**

$y \geq -3$, $y \leq 2x + 2$, $x + y \leq 1$.

Question 14

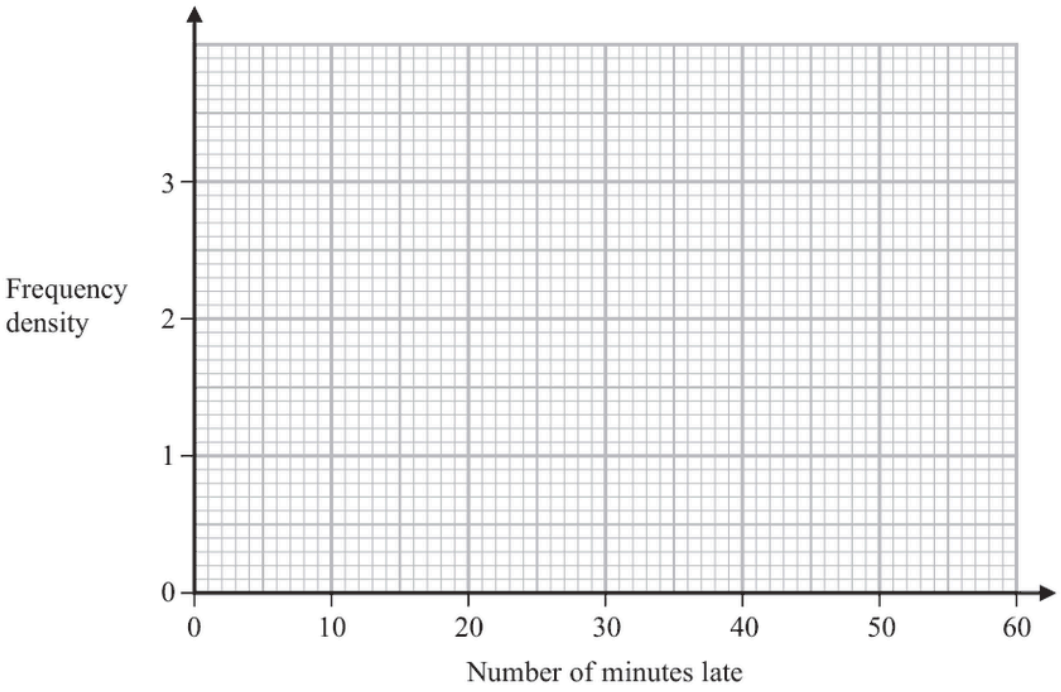
5 marks | Histograms

14 Manuel collected information about the flights that arrived late at an airport last month.

The table gives information about the number of minutes that these flights were late.

Minutes late (L minutes)	Frequency
$0 < L \leqslant 10$	8
$10 < L \leqslant 15$	13
$15 < L \leqslant 25$	19
$25 < L \leqslant 40$	24
$40 < L \leqslant 60$	6

(a) On the grid, draw a histogram for this information.



(3)

Manuel selected at random a flight that was late by 25 minutes or less from his results.

(b) Work out an estimate for the probability that this flight was late by 5 minutes or less.

.....

(2)

(Total for Question 14 is 5 marks)

1 **The histogram densities are**

The histogram densities are

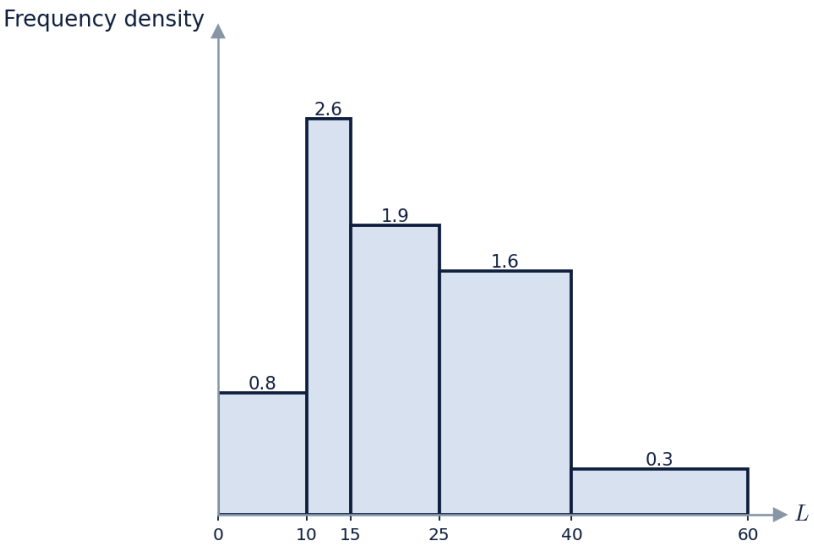
$$0 < L \leq 10 : \frac{8}{10} = 0.8$$

$$10 < L \leq 15 : \frac{13}{5} = 2.6$$

$$15 < L \leq 25 : \frac{19}{10} = 1.9$$

$$25 < L \leq 40 : \frac{24}{15} = 1.6$$

$$40 < L \leq 60 : \frac{6}{20} = 0.3$$



2 **Use histogram**

Given $L \leq 25$, the total frequency is

$$8 + 13 + 19 = 40$$

3 **Use histogram**

For $L \leq 5$, estimate half of the first class:

$$5(0.8) = 4$$

$$P(L \leq 5 \mid L \leq 25) = \frac{4}{40} = \frac{1}{10}$$

FINAL ANSWER

histogram densities $0.8, 2.6, 1.9, 1.6, 0.3$, and probability $\frac{1}{10}$.

4MA1/2H NOVEMBER 2021

Question 15

6 marks | Functions

15 The functions f and g are such that

$$f(x) = 2x - 3$$

$$g(x) = \frac{x}{3x + 1}$$

(a) State the value of x that cannot be included in any domain of g

.....

(1)

(b) Find gf(x)
Simplify your answer.

gf(x) = ...

(2)

(c) Express the inverse function g^{-1} in the form $g^{-1}(x) = ...$

$g^{-1}(x)$ = ...

(3)

(Total for Question 15 is 6 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H November 2021

1 **Evaluate fraction**

$$f(x) = 2x - 3, \qquad g(x) = \frac{x}{3x + 1}$$

2 **(a) The denominator of cannot**

(a) The denominator of g cannot be zero:

$$3x + 1 = 0$$

$$x = -\frac{1}{3}$$

3 **(b)**

(b)

$$gf(x) = g(f(x))$$

$$gf(x) = \frac{2x-3}{3(2x-3)+1}$$

$$gf(x) = \frac{2x-3}{6x-8}$$

---

4 **(c) Let**

(c) Let

$$y = \frac{x}{3x+1}$$

$$y(3x+1) = x$$

$$3xy + y = x$$

$$x(3y-1) = -y$$

$$x = \frac{y}{1-3y}$$

---

5 **Evaluate fraction**

So

$$g^{-1}(x) = \frac{x}{1-3x}$$

✓ **FINAL ANSWER**

Excluded value $-\frac{1}{3}$, $gf(x) = \frac{2x-3}{6x-8}$, and $g^{-1}(x) = \frac{x}{1-3x}$.

4MA1/2H NOVEMBER 2021

Question 16

3 marks | Tree Diagrams & Conditional Probability

16 A box contains 15 counters.

There are 4 red counters, 5 green counters and the rest are yellow counters.

Niklas takes at random a counter from the box and writes down the colour of his counter.
He then puts the counter back into the box.

Sasha then takes at random a counter from the box and writes down the colour of her counter.

Work out the probability that the counters taken by Niklas and Sasha both have the same colour.

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/2H November 2021

1 Count the colours

4 red, 5 green and $15 - 9 = 6$ yellow. The counter is replaced, so the second pick has the same probabilities.

2 Both the same colour

$$\left(\frac{4}{15}\right)^2 + \left(\frac{5}{15}\right)^2 + \left(\frac{6}{15}\right)^2 = \frac{16 + 25 + 36}{225} = \frac{77}{225}$$

FINAL ANSWER

$$\frac{77}{225}$$

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Question 17

3 marks | Surds

17 Express $\frac{8}{\sqrt{5}-1}$ in the form $\sqrt{a}+b$ where a and b are integers.

Show each stage of your working clearly.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H November 2021

1 Simplify surd

$$\frac{8}{\sqrt{5}-1}$$

2 Rationalise the denominator

Rationalise the denominator:

$$\frac{8}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{8(\sqrt{5}+1)}{5-1}$$

$$= 2(\sqrt{5}+1) = 2\sqrt{5}+2$$

3 Simplify surd

In the form $\sqrt{a}+b$,

$$2\sqrt{5}+2 = \sqrt{20}+2$$

FINAL ANSWER

$\sqrt{20}+2$

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Question 18

5 marks | Sine & Cosine Rules

18 Here is a quadrilateral $ABCD$.

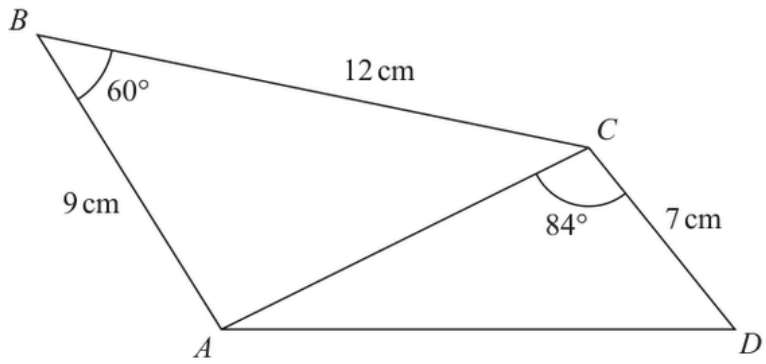


Diagram **NOT** accurately drawn

Calculate the area of quadrilateral $ABCD$.
Give your answer correct to 3 significant figures.
Show your working clearly.

..... cm^2

(Total for Question 18 is 5 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H November 2021

1 Area of triangle

Area of triangle ABC:

$$\frac{1}{2}(9)(12)\sin 60° = 46.765\ldots$$

2 Find using the cosine rule

Find AC using the cosine rule:

$$AC^2 = 9^2 + 12^2 - 2(9)(12)\cos 60°$$

$$AC = 10.816\ldots$$

3 Area of triangle

Area of triangle ACD:

$$\frac{1}{2}(10.816\ldots)(7)\sin 84° = 37.642\ldots$$

4 **Total area**

Total area:

$$46.765\ldots + 37.642\ldots = 84.407\ldots$$

✓ **FINAL ANSWER**

$84.4\ \text{cm}^2$.

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Question 19

6 marks | Linear Graphs

19 The straight line **L** has equation $x - y = 3$
The curve **C** has equation $3x^2 - y^2 + xy = 9$

L and **C** intersect at the points P and Q.

Find the coordinates of the midpoint of PQ.
Show clear algebraic working.

(..... , ...)

(Total for Question 19 is 6 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H November 2021

1 **Line is**

Line L is

$$x - y = 3$$

2 **Calculate value**

So

$$y = x - 3$$

3 **The curve is**

The curve is

$$3x - y + y^2 = 9$$

4 **Substitute**

Substitute $y = x - 3$:

$$3x - (x - 3) + (x - 3)^2 = 9$$

$$2x + 3 + x^2 - 6x + 9 = 9$$

$$x^2 - 4x + 3 = 0$$

$$(x - 1)(x - 3) = 0$$

5 **Calculate value**

So $x = 1$ or $x = 3$.

6 **Calculate value**

If $x = 1$, then $y = -2$. If $x = 3$, then $y = 0$.

7 **The midpoint of and is**

The midpoint of $(1, -2)$ and $(3, 0)$ is

$$\left(\frac{1+3}{2}, \frac{-2+0}{2}\right) = (2, -1)$$

FINAL ANSWER

$(2, -1)$.

4MA1/2H NOVEMBER 2021

Question 20

5 marks | Sequences

20 Here are the first four terms of an arithmetic series.

$$k \quad \frac{3k}{4} \quad \frac{k}{2} \quad \frac{k}{4}$$

Given that the 15th term of the series is $(90 + 2k)$,

calculate the sum of the first 30 terms of the series.

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H November 2021

1 The first four terms are

The first four terms are

$$k, \quad \frac{3k}{4}, \quad \frac{k}{2}, \quad \frac{k}{4}$$

2 The common difference is

The common difference is

$$d = -\frac{k}{4}$$

3 The 15th term is

The 15th term is

$$u_{15} = k + 14\left(-\frac{k}{4}\right)$$

$$u_{15} = k - \frac{7k}{2} = -\frac{5k}{2}$$

4 **Given**

Given

$$-\frac{5k}{2} = 90 + 2k$$

$$-5k = 180 + 4k$$

$$k = -20$$

---

5 **Evaluate fraction**

So

$$a = -20, \quad d = 5$$

$$S_{30} = \frac{30}{2}(2(-20) + 29(5))$$

$$S_{30} = 15(105) = 1575$$

✓ **FINAL ANSWER**

1575

Question 21

6 marks | Transformations of Graphs

21 The curve **C** has equation $y = f(x)$ where $f(x) = 9 - 3(x + 2)^2$
The point A is the maximum point on **C**.

(a) Write down the coordinates of A.

(..... , ...)

(1)

The curve **C** is transformed to the curve **S** by a translation of $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

(b) Find an equation for the curve **S**.

.....

(1)

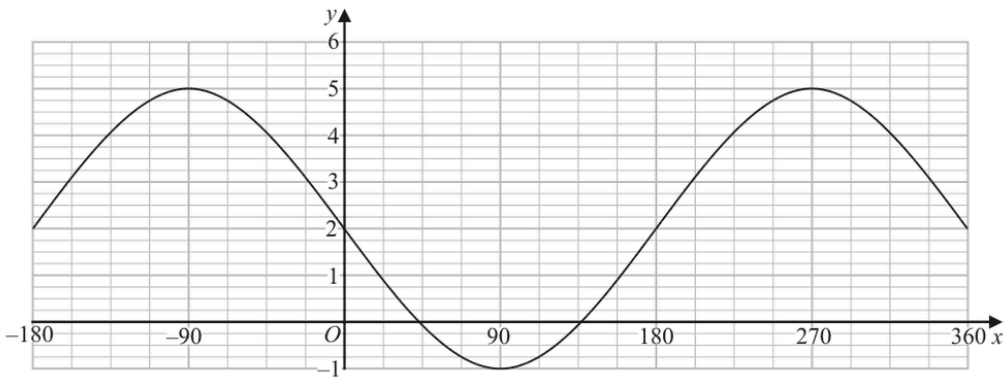
The curve **C** is transformed to the curve **T**.
The curve **T** has equation $y = 3(x + 2)^2 - 9$

(c) Describe fully the transformation that maps curve **C** onto curve **T**.

.....

(1)

The graph of $y = a\cos(x - b)° + c$ for $-180 \leqslant x \leqslant 360$ is drawn on the grid below.



(d) Find the value of a, the value of b and the value of c.

$a =$ .....

$b =$ .....

$c =$ .....

(3)

(Total for Question 21 is 6 marks)

1 **Calculate value**

$$f(x) = 9 - 3(x+2)^2$$

2 **This is in completed square**

This is in completed square form, so the maximum point is

$$(-2, 9)$$

3 **Use matrices**

For the translation

$$\begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

4 **the graph moves 4 units**

the graph moves 4 units right. Replace x by $x - 4$:

$$y = 9 - 3((x-4)+2)^2$$

$$y = 9 - 3(x-2)^2$$

5 **The curve has equation**

The curve T has equation

$$y = 3(x+2)^2 - 9$$

6 **This is**

This is

$$y = -\left(9 - 3(x+2)^2\right)$$

7 **Apply transformation**

So C is reflected in the x-axis.

8 **Use trigonometry**

For the cosine graph, the maximum is 5 and the minimum is -1.

$$a = \frac{5 - (-1)}{2} = 3$$

$$c = \frac{5 + (-1)}{2} = 2$$

9 **Use trigonometry**

For

$$y = a\cos(x - b)^\circ + c$$

10 **Read the graph**

a maximum occurs when $x - b = 0$. The graph has a maximum at $x = 270°$, so

$$b = 270$$

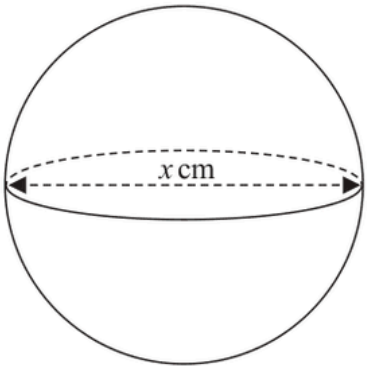
✓ **FINAL ANSWER**

$A = (-2, 9)$, $y = 9 - 3(x - 2)^2$, reflection in the x-axis, $a = 3,\ b = 270,\ c = 2$

Question 22

6 marks | Volume & Surface Area

22 The diagram shows a sphere of diameter x cm and a pyramid $ABCDE$ with a horizontal rectangular base $BCDE$.



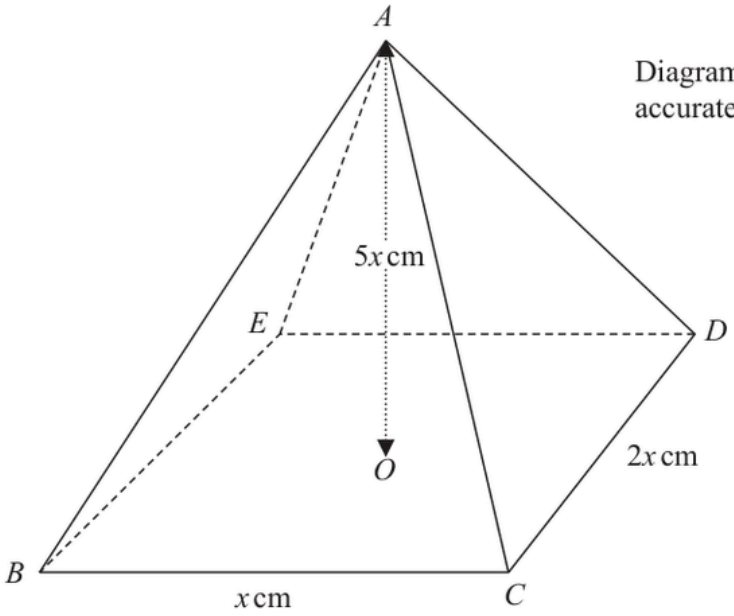


Diagram **NOT** accurately drawn

The vertex A of the pyramid is vertically above the centre O of the base so that $AB = AC = AD = AE$.

$BC = x$ cm, $CD = 2x$ cm and $AO = 5x$ cm.

The volume of the sphere is 288π cm^3

Calculate the total surface area of the pyramid.
Give your answer correct to the nearest cm^2

..... cm^2

(Total for Question 22 is 6 marks)

1 **Use trigonometry**

The sphere has diameter x, so its radius is

$$\frac{x}{2}$$

2 **The volume of the sphere**

The volume of the sphere is $288\pi\ \text{cm}^3$:

$$\frac{4}{3}\pi\left(\frac{x}{2}\right)^3 = 288\pi$$

$$\frac{\pi x^3}{6} = 288\pi$$

$$x^3 = 1728$$

$$x = 12$$

3 **The rectangular base of the**

The rectangular base of the pyramid has dimensions x and $2x$, so its area is

$$x(2x) = 2x^2$$

4 **Use trigonometry**

For the two faces with base x, the slant height is

$$\sqrt{(5x)^2 + x^2} = x\sqrt{26}$$

5 **Their total area is**

Their total area is

$$2\left(\frac{1}{2}x \cdot x\sqrt{26}\right) = x^2\sqrt{26}$$

6 **Use trigonometry**

For the two faces with base $2x$, the slant height is

$$\sqrt{(5x)^2 + \left(\frac{x}{2}\right)^2} = \frac{x\sqrt{101}}{2}$$

7 **Their total area is**

Their total area is

$$2\left(\frac{1}{2}(2x)\cdot\frac{x\sqrt{101}}{2}\right)=x^2\sqrt{101}$$

8 **Total surface area**

Total surface area:

$$2x^2+x^2\sqrt{26}+x^2\sqrt{101}$$

9 **Substitute**

Substitute $x=12$:

$$144(2+\sqrt{26}+\sqrt{101})=2469.440\ldots$$

10 **Correct to the nearest**

Correct to the nearest cm^2:

$$2469$$

✓ **FINAL ANSWER**

$$\boxed{2469\ \text{cm}^2}$$