Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

4MA1/1H November 2021 Worked Solutions

November 2021 paper rebuilt with the worked solution after each question.

20
paper questions

2020–2026
52 past papers

Single-paper solutions
checked by Dr Eslam



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PAST PAPER

4MA1/1H November 2021

November 2021

20	100	2 h
questions	marks	exam time

Each question is followed by its worked solution and final answer.

4MA1/1H NOVEMBER 2021

Question 1

3 marks | Expanding Brackets

1 (a) Simplify $e^8 \div e^2$

.....

(1)

(b) Expand and simplify $(x - 3)(x + 1)$

.....

(2)

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H November 2021

1 **(a)**

(a) $e^8 \div e^2 = e^6$.

2 **(b)**

(b) $(x-3)(x+1) = x^2 - 2x - 3$.

✓ **FINAL ANSWER**

(a) e^6, (b) $x^2 - 2x - 3$

4MA1/1H NOVEMBER 2021

Question 2

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

2 Here is a right-angled triangle.

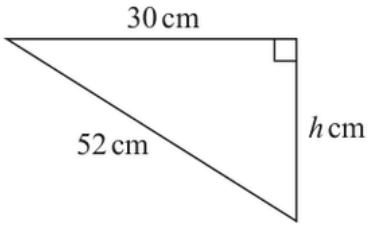


Diagram **NOT** accurately drawn

Calculate the value of h.
Give your answer correct to 3 significant figures.

h = ...

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H November 2021

1 Use trigonometry

$$h^2 + 30^2 = 52^2$$

$$h^2 = 52^2 - 30^2 = 1804$$

$$h = \sqrt{1804} = 42.473\ldots$$

FINAL ANSWER

42.5 cm to 3 significant figures.

4MA1/1H NOVEMBER 2021

Question 3

4 marks | Probability Toolkit

3 There are 54 fish in a tank.
Some of the fish are white and the rest of the fish are red.

Jeevan takes at random a fish from the tank.
The probability that he takes a white fish is $\frac{4}{9}$

(a) Work out the number of white fish originally in the tank.

.....

(2)

Jeevan puts the fish he took out, back into the tank.
He puts some more white fish into the tank.

Jeevan takes at random a fish from the tank.
The probability that he takes a white fish is now $\frac{1}{2}$

(b) Work out the number of white fish Jeevan put into the tank.

.....

(2)

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H November 2021

1 Calculate probability

(a) Let the number of white fish be w.

$$\frac{w}{54} = \frac{4}{9}$$

$$w = 54 \times \frac{4}{9} = 24$$

2 (b) There are still 54

(b) There are still 54 fish, and now

$$\frac{\text{white fish}}{54} = \frac{1}{2}$$

$$\text{white fish} = 27$$

3 **Jeevan added**

Jeevan added

$$27 - 24 = 3$$

✓ **FINAL ANSWER**

(a) 24 white fish, (b) 3 white fish.

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Question 4

5 marks | Perimeter & Area

4 The diagram shows the front of a wooden door with a semicircular glass window.

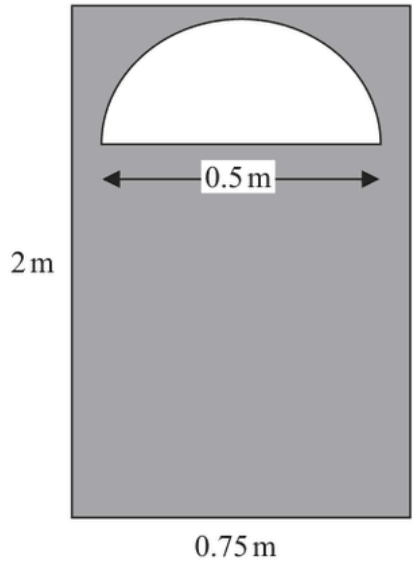


Diagram **NOT** accurately drawn

Julie wants to apply 2 coats of wood varnish to the front of the door, shown shaded in the diagram.

250 millilitres of wood varnish covers $4\,m^2$ of the wood.

Work out how many millilitres of wood varnish Julie will need.
Give your answer correct to the nearest millilitre.

..... millilitres

(Total for Question 4 is 5 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H November 2021

1 **Area of the rectangular door**

Area of the rectangular door:

$$0.75 \times 2 = 1.5 \text{ m}^2$$

2 **The semicircular window has diameter**

The semicircular window has diameter 0.5 m, so

$$r = 0.25 \text{ m}$$

3 Area of the semicircular window

Area of the semicircular window:

$$\frac{1}{2}\pi(0.25)^2 = 0.098174\ldots \text{ m}^2$$

4 Shaded area for one coat

Shaded area for one coat:

$$1.5 - 0.098174\ldots = 1.401825\ldots \text{ m}^2$$

5 Calculate area

For 2 coats:

$$2 \times 1.401825\ldots = 2.80365\ldots \text{ m}^2$$

6 Calculate area

Since 250 ml covers 4 m^2, varnish needed is

$$\frac{2.80365\ldots}{4} \times 250 = 175.22\ldots$$

FINAL ANSWER

175 millilitres.

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Question 5

5 marks | Forming & Solving Equations

5 Yasmin has some identical rectangular tiles.
Each tile is L cm by W cm.

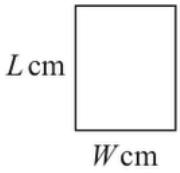


Diagram **NOT** accurately drawn

Using 9 of her tiles, Yasmin makes rectangle $ABCD$, shown in the diagram below.

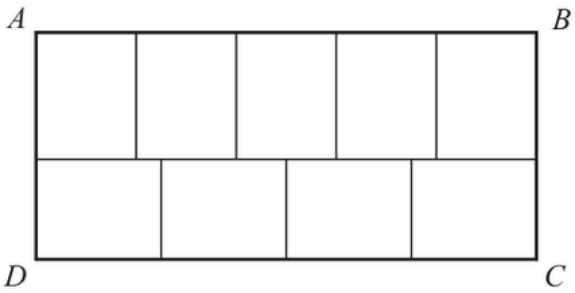


Diagram **NOT** accurately drawn

The area of $ABCD$ is $1620\,\text{cm}^2$

Work out the value of L and the value of W.

$L =$ ..... $W =$ .....

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H November 2021

1 The top row has 5

The top row has 5 tile widths and the bottom row has 4 tile lengths, so

$$5W = 4L$$

$$L = \frac{5}{4}W$$

2 Use trigonometry

The rectangle has width $5W$ and height $L + W$:

$$5W(L + W) = 1620$$

3 **Substitute**

Substitute $L = \frac{5}{4}W$:

$$5W\left(\frac{5}{4}W + W\right) = 1620$$

$$5W\left(\frac{9}{4}W\right) = 1620$$

$$\frac{45}{4}W^2 = 1620$$

$$W^2 = 144$$

$$W = 12, \qquad L = \frac{5}{4}(12) = 15$$

✓ **FINAL ANSWER**

$L = 15\ \text{cm}$, $W = 12\ \text{cm}$.

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Question 6

5 marks | Simultaneous Equations

6 Alison buys 5 apples and 3 pears for a total cost of $1.96
Greg buys 3 apples and 2 pears for a total cost of $1.22

Michael buys 10 apples and 10 pears.

Work out how much Michael pays for his 10 apples and 10 pears.
Show your working clearly.

$.....

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H November 2021

1 Solve simultaneous equations

Let a be the cost of one apple and p be the cost of one pear.

$$5a + 3p = 1.96$$

$$3a + 2p = 1.22$$

2 Multiply the first equation by

Multiply the first equation by 2:

$$10a + 6p = 3.92$$

3 Multiply the second equation by

Multiply the second equation by 3:

$$9a + 6p = 3.66$$

4 Subtract

Subtract:

$$a = 0.26$$

5 **Solve simultaneous equations**

Then

$$3(0.26) + 2p = 1.22$$

$$2p = 0.44$$

$$p = 0.22$$

---

6 **Solve simultaneous equations**

Michael buys 10 apples and 10 pears:

$$10(0.26) + 10(0.22) = 4.80$$

✓ **FINAL ANSWER**

$\$4.80$.

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Question 7

3 marks | Prime Factors, HCF & LCM

7 Write 3.6×10^3 as a product of powers of its prime factors.
Show your working clearly.

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H November 2021

1 Find the LCM

$$3.6 \times 10^3 = 3600$$

$$3600 = 36 \times 100$$

$$36 = 2^2 \times 3^2, \qquad 100 = 2^2 \times 5^2$$

2 Find the LCM

So

$$3600 = 2^4 \times 3^2 \times 5^2$$

FINAL ANSWER

$2^4 \times 3^2 \times 5^2$

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Question 8

3 marks | Percentages

8 In 2018, the population of Sydney was 5.48 million.
This was 22% of the total population of Australia.

Work out the total population of Australia in 2018
Give your answer correct to 3 significant figures.

..... million

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H November 2021

1 Calculate percentage

Sydney's population was 22% of Australia's total population.

$$0.22 \times \text{total} = 5.48$$

$$\text{total} = \frac{5.48}{0.22} = 24.909\ldots$$

2 Calculate percentage

Correct to 3 significant figures,

$$24.909\ldots \approx 24.9$$

FINAL ANSWER

24.9 million

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Question 9

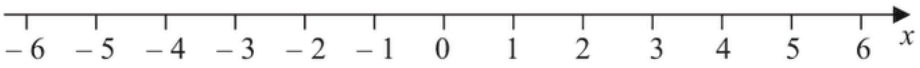
5 marks | Inequalities (Solving & Graphing)

9 (i) Solve the inequalities $-7 \leqslant 2x - 3 < 5$

.....

(3)

(ii) On the number line, represent the solution set to part (i)



(2)

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H November 2021

1 Solve inequality

$$-7 \leq 2x - 3 < 5$$

2 Add 3 to each part

Add 3 to each part:

$$-4 \leq 2x < 8$$

3 Divide by 2

Divide by 2:

$$-2 \leq x < 4$$

4 Use a closed circle

On the number line, use a closed circle at -2, an open circle at 4, and shade between them.

✓ FINAL ANSWER

$-2 \leq x < 4$.

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Question 10

5 marks | Standard & Compound Units

10 A solid aluminium cylinder has radius 10 cm and height h cm.

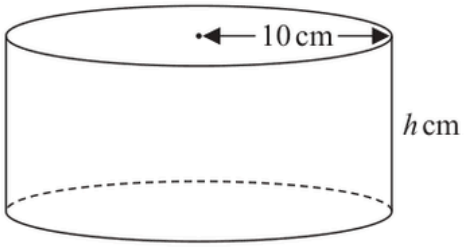


Diagram **NOT** accurately drawn

The mass of the cylinder is 5.4 kg.
The density of aluminium is 0.0027 kg/cm^3

Calculate the value of h.
Give your answer correct to one decimal place.

$h =$ .....

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H November 2021

1 Calculate volume

The mass is $5.4\ \text{kg}$, and the density is $0.0027\ \text{kg/cm}^3$.

$$\text{volume} = \frac{\text{mass}}{\text{density}} = \frac{5.4}{0.0027} = 2000\ \text{cm}^3$$

2 Evaluate fraction

For the cylinder,

$$V = \pi r^2 h$$

$$2000 = \pi(10)^2 h$$

$$h = \frac{2000}{100\pi} = 6.366\ldots$$

FINAL ANSWER

$6.4\ \text{cm}$.

Question 11

5 marks | Cumulative Frequency Diagrams

11 The table gives information about the times taken by 90 runners to complete a 10 km race.

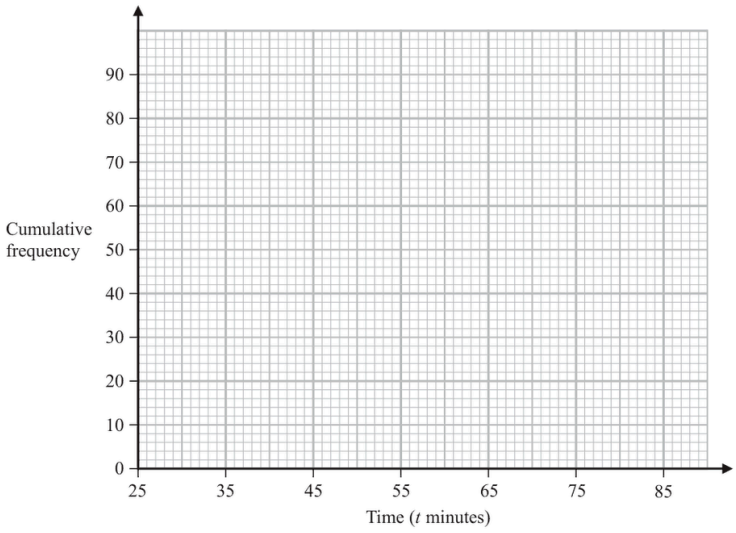
Time (t minutes)	Frequency
$25 < t \leqslant 35$	12
$35 < t \leqslant 45$	24
$45 < t \leqslant 55$	28
$55 < t \leqslant 65$	12
$65 < t \leqslant 75$	10
$75 < t \leqslant 85$	4

(a) Complete the cumulative frequency table.

Time (t minutes)	Cumulative frequency
$25 < t \leqslant 35$	12
$25 < t \leqslant 45$	
$25 < t \leqslant 55$	
$25 < t \leqslant 65$	
$25 < t \leqslant 75$	
$25 < t \leqslant 85$	

(1)

(b) On the grid below, draw a cumulative frequency graph for your table.



(2)

Any runner who completed the race in a time T minutes such that $42 < T \leqslant 52$ minutes was awarded a silver medal.

(c) Use your graph to find an estimate for the number of runners who were awarded a silver medal.

..... runners

(2)

(Total for Question 11 is 5 marks)

1 **(a) Running totals**

$$12,\ 36,\ 64,\ 76,\ 86,\ 90$$

2 **(b) Plot at the upper class boundaries**

Plot $(35, 12), (45, 36), (55, 64), (65, 76), (75, 86), (85, 90)$, start at $(25, 0)$, and join with a smooth curve.

3 **(c) Read the curve at 52 and 42 minutes**

At 52 minutes: about 56. At 42 minutes: about 29.

4 **(c) Silver medals**

$$56 - 29 \approx 27 \text{ runners}$$

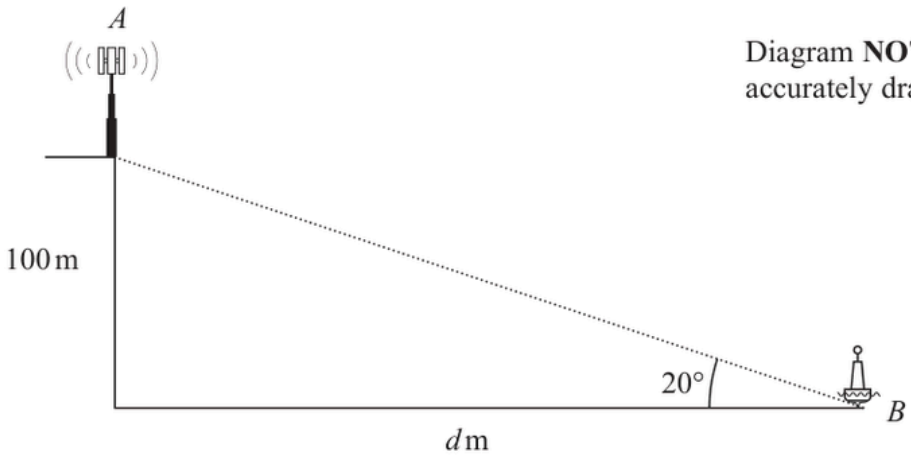
✓ **FINAL ANSWER**

(a) $12, 36, 64, 76, 86, 90$ (c) about 27

Question 12

6 marks | Right-Angled Triangles - Pythagoras & Trigonometry

12 The diagram shows a vertical cliff with a vertical radio mast on top of the cliff and a buoy in the sea.



The height of the cliff is 100 metres.
The buoy is at the point B that is d metres from the base of the cliff.
The angle of elevation from B to the top of the cliff is 20°

(a) Calculate the value of d.
Give your answer correct to 3 significant figures.

$d =$ .....

(3)

The point A at the top of the radio mast is vertically above the top of the cliff.
The angle of elevation from B to A is 25°

(b) Calculate the height of the radio mast.
Give your answer correct to 3 significant figures.

..... m

(3)

(Total for Question 12 is 6 marks)

1 (a)

(a)

$$\tan 20° = \frac{100}{d}$$

$$d = \frac{100}{\tan 20°} = 274.747\ldots$$

2 Use trigonometry

So

$$d = 275 \text{ m}$$

3 (b) The angle of elevation

(b) The angle of elevation to the top of the radio mast is $25°$.

$$\tan 25° = \frac{100 + h}{274.747\ldots}$$

$$100 + h = 274.747\ldots \tan 25° = 128.116\ldots$$

$$h = 28.116\ldots$$

FINAL ANSWER

(a) 275 m, (b) 28.1 m.

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Question 13

3 marks | Rounding, Estimation & Bounds

13 Here is a triangle XYZ.

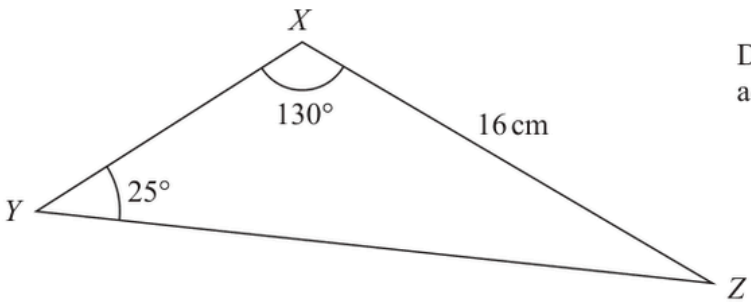


Diagram **NOT** accurately drawn

The length XZ and the angles YXZ and XYZ are each given correct to 2 significant figures.

Calculate the upper bound for the length YZ.
Give your answer correct to one decimal place.
Show your working clearly.

..... cm

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H November 2021

1 Use trigonometry

Using the sine rule,

$$\frac{YZ}{\sin X} = \frac{XZ}{\sin Y}$$

2 Find upper bound

For the upper bound of YZ:

$$XZ < 16.5, \quad X < 25.5°, \quad Y < 135°$$

$$YZ_{\text{upper}} = \frac{16.5 \sin 25.5°}{\sin 135°} = 10.0457\ldots$$

3 Use trigonometry

Correct to one decimal place,

$$10.0457\ldots \approx 10.0$$

FINAL ANSWER

10.0 cm

Question 14

8 marks | Vectors

14 *ABCDEF* and *GHIJKL* are regular hexagons each with centre *O*.

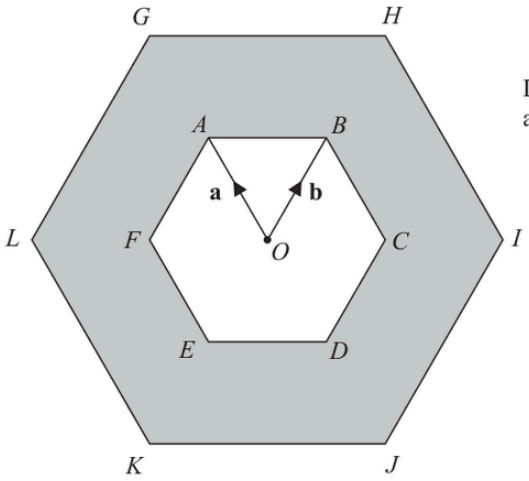


Diagram **NOT** accurately drawn

GHIJKL is an enlargement of *ABCDEF*, with centre *O* and scale factor 2

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

(a) Write the following vectors, in terms of **a** and **b**.
Simplify your answers.

(i) $\overrightarrow{AB}$

.....

(1)

(ii) $\overrightarrow{KI}$

.....

(2)

(iii) $\overrightarrow{LD}$

.....

(2)

The triangle *OAB* has an area of 5 cm^2

(b) Calculate the area of the shaded region.

..... cm^2

(3)

(Total for Question 14 is 8 marks)

1 **Use vectors**

$$\overrightarrow{OA} = \mathbf{a}, \qquad \overrightarrow{OB} = \mathbf{b}$$

2 **(a)(i)**

(a)(i)

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \mathbf{b} - \mathbf{a}$$

3 **Use vectors**

For a regular hexagon,

$$\overrightarrow{OC} = \mathbf{b} - \mathbf{a}, \quad \overrightarrow{OD} = -\mathbf{a}, \quad \overrightarrow{OE} = -\mathbf{b}, \quad \overrightarrow{OF} = \mathbf{a} - \mathbf{b}$$

4 **The outer hexagon has scale**

The outer hexagon has scale factor $\mathbf{2}$, so

$$\overrightarrow{OI} = 2(\mathbf{b} - \mathbf{a}), \quad \overrightarrow{OK} = -2\mathbf{b}, \quad \overrightarrow{OL} = 2(\mathbf{a} - \mathbf{b})$$

5 **(ii)**

(ii)

$$\overrightarrow{KI} = \overrightarrow{OI} - \overrightarrow{OK} = 2(\mathbf{b} - \mathbf{a}) - (-2\mathbf{b}) = 4\mathbf{b} - 2\mathbf{a}$$

6 **(iii)**

(iii)

$$\overrightarrow{LD} = \overrightarrow{OD} - \overrightarrow{OL} = -\mathbf{a} - 2(\mathbf{a} - \mathbf{b}) = 2\mathbf{b} - 3\mathbf{a}$$

7 **(b) The inner hexagon is**

(b) The inner hexagon is made from 6 equal triangles like OAB:

$$\text{inner area} = 6 \times 5 = 30$$

8 **The outer hexagon is an**

The outer hexagon is an enlargement with scale factor $\mathbf{2}$, so area scale factor is $\mathbf{2^2 = 4}$:

$$\text{outer area} = 4 \times 30 = 120$$

$$\text{shaded area} = 120 - 30 = 90$$

FINAL ANSWER

(i) $\mathbf{b} - \mathbf{a}$, (ii) $4\mathbf{b} - 2\mathbf{a}$, (iii) $2\mathbf{b} - 3\mathbf{a}$, (b) $90\ \text{cm}^2$.

Question 15

8 marks | Tree Diagrams & Conditional Probability

15 Magnus and Garry play 2 games of chess against each other.

The probability that Magnus beats Garry in any game is $\frac{2}{9}$

The probability that any game between Magnus and Garry is drawn is $\frac{4}{9}$

The result of any game is independent of the result of any other game.

(a) Complete the probability tree diagram.



(2)

For each game of chess,

the winner gets 2 points and the loser gets 0 points,
when the game is drawn, each player gets 1 point.

(b) Work out the probability that, after 2 games, Magnus and Garry have the same number of points.

(3)

Magnus and Garry now play a third game of chess.

(c) Work out the probability that, after 3 games, Magnus and Garry have the same number of points.

(3)

(Total for Question 15 is 8 marks)

1 **(a) Complete the tree**

Magnus loses: $1 - \frac{2}{9} - \frac{4}{9} = \frac{3}{9}$.

Every second-game branch repeats $\frac{2}{9}$, $\frac{4}{9}$, $\frac{3}{9}$ (the games are independent).

2 **(b) Equal points after 2 games**

Two draws, or one win and one loss (either order):

$$\left(\frac{4}{9}\right)^2 + 2 \times \frac{2}{9} \times \frac{3}{9} = \frac{16}{81} + \frac{12}{81} = \frac{28}{81}$$

3 **(c) Equal points after 3 games**

Each player needs 3 of the 6 points: three draws, or one win, one draw and one loss ($3! = 6$ orders).

4 **(c) Add the cases**

$$\left(\frac{4}{9}\right)^3 + 6 \times \frac{2}{9} \times \frac{4}{9} \times \frac{3}{9} = \frac{64}{729} + \frac{144}{729} = \frac{208}{729}$$

✓ **FINAL ANSWER**

(a) $\frac{3}{9}$; $\frac{2}{9}$, $\frac{4}{9}$, $\frac{3}{9}$ on every branch (b) $\frac{28}{81}$ (c) $\frac{208}{729}$

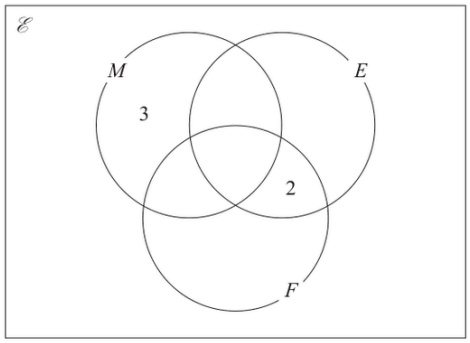
Question 16

5 marks | Set Notation & Venn Diagrams

16 There are 32 students in a class.
In one term these 32 students each took a test in Maths (M), in English (E) and in French (F).

25 students passed the test in Maths.
20 students passed the test in English.
14 students passed the test in French.
18 students passed the tests in Maths and English.
11 students passed the tests in Maths and French.
4 students failed all three tests.
x students passed all three tests.

The incomplete Venn diagram gives some more information about the results of the 32 students.

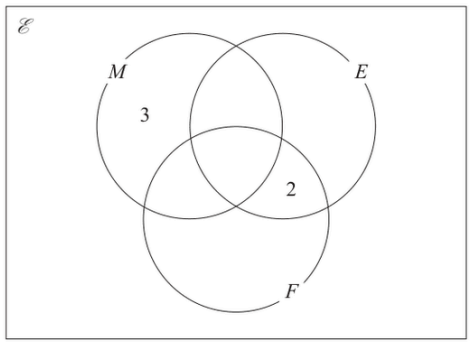


(a) Use all the given information about the results of students who passed the test in Maths to find the value of x.

$x =$ .....

(2)

(b) Use your value of x to complete the Venn diagram to show the number of students in each subset.



(2)

A student who passed the test in Maths is chosen at random.

(c) Find the probability that this student failed the test in French.

.....

(1)

(Total for Question 16 is 5 marks)

1 **(a) Use the Maths circle**

Maths only $= 3$, Maths and English only $= 18 - x$, Maths and French only $= 11 - x$, all three $= x$.

$$3 + (18 - x) + (11 - x) + x = 25 \;\Rightarrow\; 32 - x = 25 \;\Rightarrow\; x = 7$$

2 **(b) Fill every region**

$M \cap E$ only $= 11$, $M \cap F$ only $= 4$, centre $= 7$, $E \cap F$ only $= 2$.

$$E \text{ only} = 20 - 11 - 7 - 2 = 0, \qquad F \text{ only} = 14 - 4 - 7 - 2 = 1$$

Outside: 4. (Check: $3 + 11 + 4 + 7 + 0 + 2 + 1 + 4 = 32$.)

3 **(c) Passed Maths, failed French**

Maths only and Maths-and-English only: $3 + 11 = 14$ of the 25.

$$\frac{14}{25}$$

✓ **FINAL ANSWER**

(a) $x = 7$ (b) M only 3, $M \cap E$ only 11, $M \cap F$ only 4, centre 7, $E \cap F$ only 2, E only 0, F only 1, outside 4 (c) $\frac{14}{25}$

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Question 17

8 marks | Rearranging Formulae

17 (a) Factorise $6y^2 - y - 5$

.....

(2)

(b) Make f the subject of $w = \dfrac{2f + 3}{8 - f}$

.....

(3)

(c) Express $4x^2 - 8x + 7$ in the form $a(x + b)^2 + c$ where a, b and c are integers.

.....

(3)

(Total for Question 17 is 8 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H November 2021

1 **(a)**

(a)

$$6y^2 - y - 5 = (6y + 5)(y - 1)$$

2 **(b)**

(b)

$$w = \frac{2f + 3}{8 - f}$$

$$w(8 - f) = 2f + 3$$

$$8w - 3 = f(w + 2)$$

$$f = \frac{8w - 3}{w + 2}$$

3 **(c)**

(c)

$$4x^2 - 8x + 7 = 4(x^2 - 2x) + 7$$

$$= 4\left((x-1)^2 - 1\right) + 7$$

$$= 4(x-1)^2 + 3$$

FINAL ANSWER

$(6y+5)(y-1)$, $f = \frac{8w-3}{w+2}$, $4(x-1)^2 + 3$

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Question 18

3 marks | Recurring Decimals

18 $0.4\dot{x}$ is a recurring decimal.
x is a whole number such that $1 \leqslant x \leqslant 9$

Find, in terms of x, the recurring decimal $0.4\dot{x}$ as a fraction.
Give your fraction in its simplest form.
Show clear algebraic working.

(Total for Question 18 is 3 marks)

WORKED SOLUTION

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1 Let

Let

$$y = 0.4\dot{x}$$

2 Simplify fraction

So

$$y = 0.4xxx\ldots$$

3 Simplify fraction

Multiply by 10 and 100:

$$10y = 4.xxx\ldots$$

$$100y = 4x.xxx\ldots$$

4 Subtract

Subtract:

$$100y - 10y = (4x.xxx\ldots) - (4.xxx\ldots)$$

$$90y = x + 36$$

5 Simplify fraction

Therefore

$$y = \frac{x + 36}{90}$$

FINAL ANSWER

$$\frac{x+36}{90}$$

Question 19

6 marks | Algebraic Proof

19 *ABCED* is a five-sided shape.

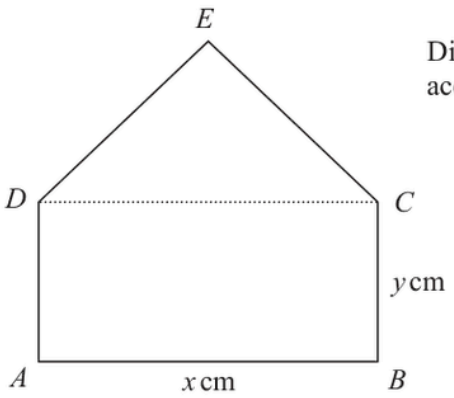


Diagram **NOT** accurately drawn

ABCD is a rectangle.
CED is an equilateral triangle.

$AB = x$ cm $BC = y$ cm

The perimeter of *ABCED* is 100 cm.
The area of *ABCED* is R cm^2

(a) Show that $R = \frac{x}{4}\left(200 - \left[6 - \sqrt{3}\right]x\right)$

(3)

(b) (i) Find the value of x for which R has its maximum value.

Give your answer in the form $\frac{p}{q - \sqrt{3}}$ where p and q are integers.

$x =$ .....

(2)

(ii) Explain why the maximum value of R is given by this value of x.

.....

(1)

(Total for Question 19 is 6 marks)

1 **Rearrange formula**

The perimeter is made from three sides of length x and two sides of length y:

$$3x + 2y = 100$$

2 **Rearrange formula**

So

$$y = \frac{100 - 3x}{2}$$

3 **The area is the area**

The area is the area of the rectangle plus the area of the equilateral triangle:

$$R = xy + \frac{\sqrt{3}}{4}x^2$$

4 **Substitute**

Substitute $y = \frac{100 - 3x}{2}$:

$$R = x\left(\frac{100 - 3x}{2}\right) + \frac{\sqrt{3}}{4}x^2$$

$$R = 50x - \frac{3}{2}x^2 + \frac{\sqrt{3}}{4}x^2$$

$$R = \frac{x}{4}\left(200 - (6 - \sqrt{3})x\right)$$

5 **Rearrange formula**

For the maximum, write

$$R = -\frac{6 - \sqrt{3}}{4}x^2 + 50x$$

6 **The maximum occurs at**

The maximum occurs at

$$x = \frac{-b}{2a} = \frac{-50}{2\left(-\frac{6-\sqrt{3}}{4}\right)}$$

$$x = \frac{100}{6 - \sqrt{3}}$$

7 **Solve quadratic equation**

The coefficient of x^2 is negative, so the quadratic graph opens downwards and this vertex gives a maximum.

✓ **FINAL ANSWER**

$R = \frac{x}{4}\left(200 - (6 - \sqrt{3})x\right)$, and $x = \frac{100}{6 - \sqrt{3}}$.

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Question 20

7 marks | Linear Graphs

20 The straight line **L** passes through point A $(-6, 2)$ and point B $(5, 3)$
The straight line **M** is perpendicular to **L** and passes through the midpoint of A and B.
The line **M** intersects the line $x = -1$ at point C.

Calculate the area of triangle ABC.

(Total for Question 20 is 7 marks)

WORKED SOLUTION

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1 **Calculate value**

$$A = (-6, 2), \qquad B = (5, 3)$$

2 **The midpoint of is**

The midpoint of AB is

$$\left(\frac{-6+5}{2}, \frac{2+3}{2}\right) = \left(-\frac{1}{2}, \frac{5}{2}\right)$$

3 **The gradient of is**

The gradient of AB is

$$\frac{3-2}{5-(-6)} = \frac{1}{11}$$

4 **Find the gradient**

So the perpendicular gradient is

$$-11$$

5 **Evaluate fraction**

Line M passes through $\left(-\frac{1}{2}, \frac{5}{2}\right)$, so

$$y - \frac{5}{2} = -11\left(x + \frac{1}{2}\right)$$

6 **Evaluate fraction**

At point C, $x = -1$:

$$y - \frac{5}{2} = -11\left(-\frac{1}{2}\right)$$

$$y - \frac{5}{2} = \frac{11}{2}$$

$$y = 8$$

---

7 **Calculate value**

So

$$C = (-1, 8)$$

---

8 **Use vectors**

Now use vectors from A:

$$\vec{AB} = (11, 1), \qquad \vec{AC} = (5, 6)$$

---

9 **Area of triangle is**

Area of triangle ABC is

$$\frac{1}{2}|11(6) - 1(5)|$$

$$= \frac{1}{2}(61)$$

$$= 30.5$$

✓ **FINAL ANSWER**

30.5