Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

4MA1/1H May 2023 Worked Solutions

May 2023 paper rebuilt with the worked solution after each question.

23	2020–2026	Single-paper solutions
paper questions	52 past papers	checked by Dr Eslam



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PAST PAPER

4MA1/1H May 2023

May 2023

23	100	2 h
questions	marks	exam time

Each question is followed by its worked solution and final answer.

4MA1/1H MAY 2023

Question 1

3 marks | Ratio Toolkit

1 Last season, the number of goals scored by Arjun, by Simon and by Kath for their football team were in the ratios $2:5:8$

Simon scored 12 more goals than Arjun.

Work out the number of goals scored by Kath.

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H May 2023

1 The ratio is

The ratio is

$$\text{Arjun} : \text{Simon} : \text{Kath} = 2 : 5 : 8$$

2 Simon scored more than Arjun

Simon scored 12 more than Arjun.

$$5 - 2 = 3 \text{ parts}$$

$$3 \text{ parts} = 12$$

$$1 \text{ part} = 4$$

3 Kath scored

Kath scored

$$8 \times 4 = 32$$

FINAL ANSWER

32

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Question 2

3 marks | Statistics Toolkit

2 The table gives information about the number of minutes that Abby spent walking each day in September.

Number of minutes (M)	Frequency
$0 < M \leqslant 30$	5
$30 < M \leqslant 60$	6
$60 < M \leqslant 90$	8
$90 < M \leqslant 120$	9
$120 < M \leqslant 150$	2

Work out an estimate for the total number of minutes that Abby spent walking in September.

..... minutes

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H May 2023

1 **Use midpoints**

Use midpoints $15, 45, 75, 105, 135$.

$$15(5) + 45(6) + 75(8) + 105(9) + 135(2) = 2160$$

✓ **FINAL ANSWER**

2160 minutes.

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Question 3

4 marks | Reverse Percentages

3 Nanette buys 60 notebooks for a total cost of 780 dirhams.

Nanette sells 70% of the notebooks for 22 dirhams each.
She sells the remaining notebooks for 19 dirhams each.

Work out Nanette's percentage profit.
Give your answer correct to 3 significant figures.

.....%

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H May 2023

1 Number sold for 22 dirhams

Number sold for 22 dirhams:

$$70\% \times 60 = 42$$

2 Number remaining

Number remaining:

$$60 - 42 = 18$$

3 Total selling price

Total selling price:

$$42 \times 22 + 18 \times 19 = 924 + 342 = 1266$$

4 Profit

Profit:

$$1266 - 780 = 486$$

5 Percentage profit

Percentage profit:

$$\frac{486}{780} \times 100 = 62.307\ldots\%$$

FINAL ANSWER

62.3%

Question 4

8 marks | Transformations

4 The diagram shows a sketch of triangle **A** and triangle **B** on a coordinate grid.

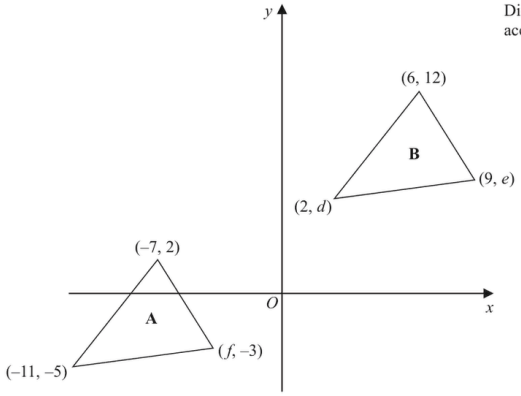


Diagram **NOT** accurately drawn

(a) Given that triangle **A** has been translated to give triangle **B**, work out the value of d, the value of e and the value of f

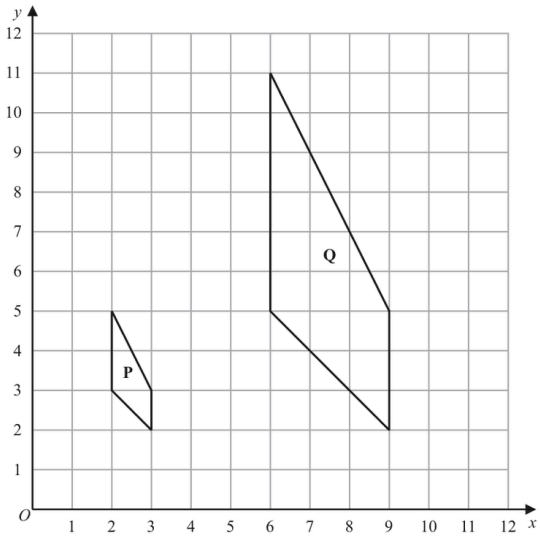
$d =$ .....

$e =$ .....

$f =$ .....

(3)

The diagram shows shape **P** and shape **Q** drawn on a grid.



(b) Describe fully the single transformation that maps shape **P** onto shape **Q**

.....

(3)

(c) On the grid above, rotate shape **P** 90° clockwise about (3, 5)
Label your shape **R**

(2)

(Total for Question 4 is 8 marks)

1 **Use matrices**

(a) The translation from $(-7, 2)$ to $(6, 12)$ is

$$\begin{pmatrix} 13 \\ 10 \end{pmatrix}$$

2 **Use Transformations tag correct**

So

$$(-11, -5) \mapsto (2, 5)$$

3 **Use Transformations tag correct**

and

$$(f, -3) \mapsto (9, 7)$$

4 **Apply transformation**

Therefore

$$d = 5, \qquad e = 7, \qquad f = -4$$

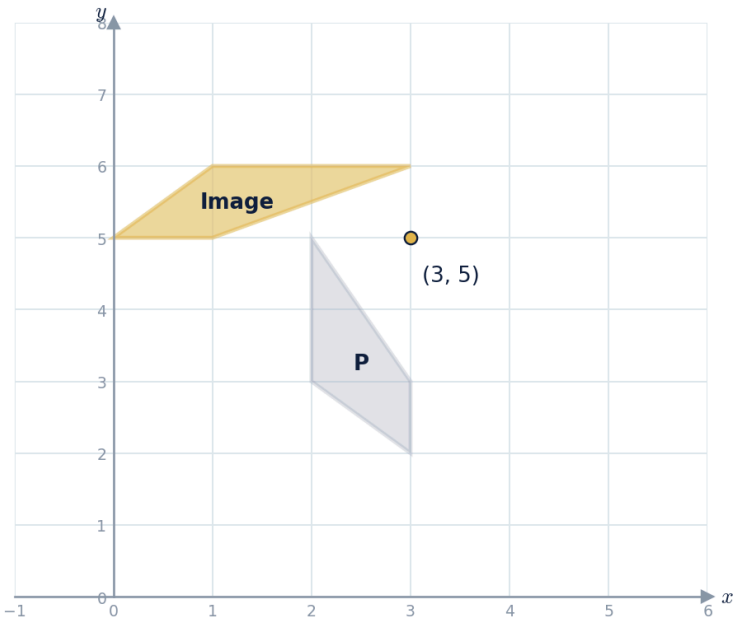
5 **(b) Shape maps to by**

(b) Shape P maps to Q by an enlargement with scale factor 3 and centre $(0, 2)$.

6 **(c) Rotate clockwise about**

(c) Rotate P $90°$ clockwise about $(3, 5)$. The image has vertices

$$(3, 6),\ (1, 6),\ (0, 5),\ (1, 5)$$



FINAL ANSWER

(a) $d = 5,\ e = 7,\ f = -4$; (b) enlargement, scale factor 3, centre $(0, 2)$; (c) vertices $(3, 6), (1, 6), (0, 5), (1, 5)$.

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Question 5

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

5 The diagram shows a shaded shape $AEBCD$ made by removing triangle AEB from rectangle $ABCD$

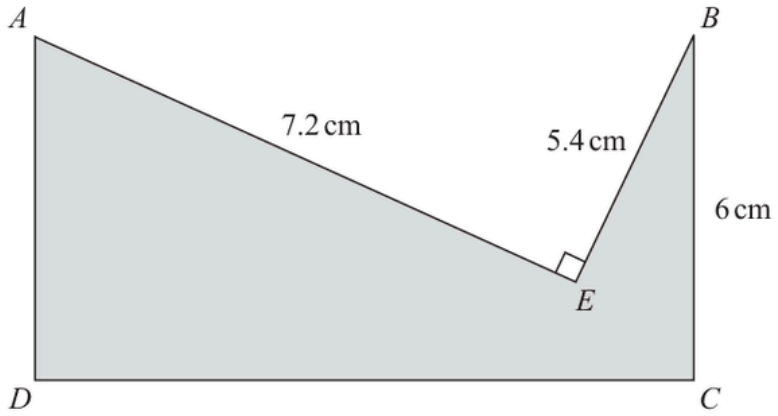


Diagram **NOT** accurately drawn

$AE = 7.2\,\text{cm}$ $BE = 5.4\,\text{cm}$ $BC = 6\,\text{cm}$ angle $AEB = 90°$

Work out the perimeter of the shaded shape.

..... cm

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H May 2023

1 **Triangle is right angled**

Triangle AEB is right-angled:

$$AB^2 = 7.2^2 + 5.4^2 = 81$$

$$AB = 9$$

2 **Use trigonometry**

For rectangle $ABCD$,

$$DC = 9, \qquad AD = BC = 6$$

3 **Perimeter of the shaded shape**

Perimeter of the shaded shape:

$$7.2 + 5.4 + 6 + 9 + 6 = 33.6$$

FINAL ANSWER

$33.6\ \text{cm}$.

4MA1/1H MAY 2023

Question 6

8 marks | Factorising

6 (a) Simplify $(2c^4d^7)^3$

.....

(2)

(b) Find the value of $5y^0$ where $y > 0$

.....

(1)

(c) Factorise fully $16a^2b^3 + 20a^3b$

.....

(2)

(d) (i) Factorise $x^2 + 9x - 22$

.....

(2)

(ii) Hence solve $x^2 + 9x - 22 = 0$

.....

(1)

(Total for Question 6 is 8 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H May 2023

1 **(a)**

(a)

$$(2c^4d^7)^3 = 8c^{12}d^{21}$$

2 **Factorise expression**

(b) $5y^0 = 5$, since $y^0 = 1$.

3 **(c)**

(c)

$$16a^2b^3 + 20a^3b = 4a^2b(4b^2 + 5a)$$

4 **(d)(i)**

(d)(i)

$$x^2 + 9x - 22 = (x + 11)(x - 2)$$

5 **Factorise expression**

(d)(ii) $x = -11$ or $x = 2$.

✓ **FINAL ANSWER**

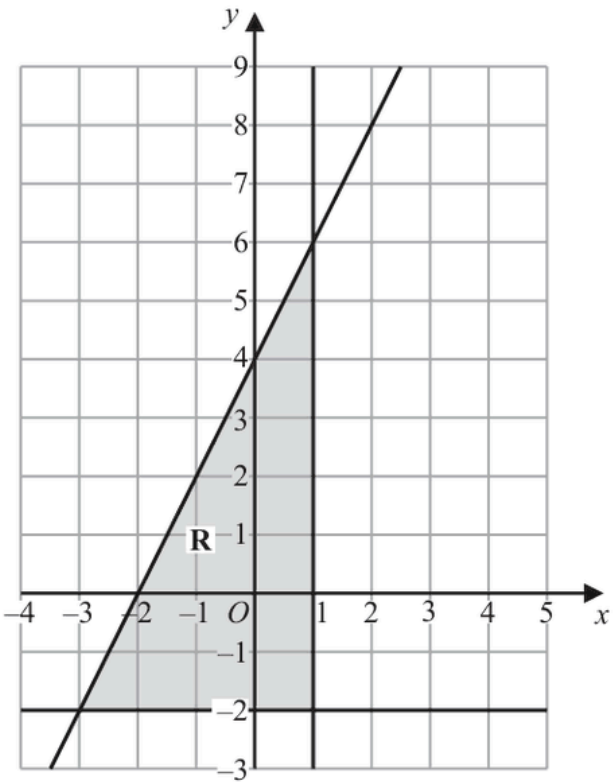
$8c^{12}d^{21}$, 5, $4a^2b(4b^2 + 5a)$, $(x + 11)(x - 2)$, $x = -11$ or $x = 2$

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Question 7

4 marks | Inequalities (Solving & Graphing)

7



The region **R**, shown shaded in the diagram, is bounded by three straight lines.

Find the inequalities that define **R**

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H May 2023

1 **The vertical boundary is so**

The vertical boundary is

$$x = 1$$

so the region has

$$x \leq 1$$

2 **The horizontal boundary is so**

The horizontal boundary is

$$y = -2$$

so

$$y \geq -2$$

---

3 **Find the gradient**

The sloping line passes through $(0, 4)$ and has gradient 2, so

$$y = 2x + 4$$

---

4 **The shaded region is below**

The shaded region is below this line:

$$y \leq 2x + 4$$

✓ **FINAL ANSWER**

$x \leq 1$, $y \geq -2$, $y \leq 2x + 4$.

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Question 8

4 marks | Prime Factors, HCF & LCM

8 (a) Write 300 as a product of its prime factors.
Show your working clearly.

.....

(2)

$A = 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$B = 2 \times 2 \times 3 \times 3 \times 3 \times 5$

(b) Find the lowest common multiple (LCM) of $5A$ and $7B$
Show your working clearly.

.....

(2)

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H May 2023

1 **Find the LCM**

First write 300 as a product of prime factors:

$$300 = 3 \times 100 = 3 \times 2^2 \times 5^2$$

$$300 = 2^2 \times 3 \times 5^2$$

2 **Find the LCM**

Now

$$A = 2^3 \times 3^2 \times 5$$

3 **Find the LCM**

and

$$B = 2^2 \times 3^3 \times 5$$

4 **Find the LCM**

So

$$5A = 2^3 \times 3^2 \times 5^2$$

5 **Find the LCM**

and

$$7B = 2^2 \times 3^3 \times 5 \times 7$$

---

6 **Find the LCM**

For the LCM of $5A$ and $7B$, take the highest power of every prime:

$$\text{LCM} = 2^3 \times 3^3 \times 5^2 \times 7$$

✓ **FINAL ANSWER**

(a) $2^2 \times 3 \times 5^2$, (b) $2^3 \times 3^3 \times 5^2 \times 7$

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Question 9

3 marks | Simultaneous Equations

9 Solve the simultaneous equations

$$2x + 9y = 14.5$$
$$7x + 3y = 8$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H May 2023

1 Solve simultaneous equations

$$2x + 9y = 14.5$$

$$7x + 3y = 8$$

2 Multiply the second equation by

Multiply the second equation by 3:

$$21x + 9y = 24$$

3 Subtract the first equation

Subtract the first equation:

$$19x = 9.5$$

$$x = 0.5$$

4 **Solve simultaneous equations**

Substitute into $7x + 3y = 8$:

$$7(0.5) + 3y = 8$$

$$3.5 + 3y = 8$$

$$y = 1.5$$

✓ **FINAL ANSWER**

$x = 0.5$, $y = 1.5$.

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Question 10

2 marks | Statistics Toolkit

10 Here are the test marks of 15 students.

7 10 14 15 16 17 18 19 20 20 23 25 30 39 40

Find the interquartile range of these marks.

(Total for Question 10 is 2 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H May 2023

1 The marks in order are

The marks in order are

$$7,\ 10,\ 14,\ 15,\ 16,\ 17,\ 18,\ 19,\ 20,\ 20,\ 23,\ 25,\ 30,\ 39,\ 40$$

$$Q_1 = 15, \qquad Q_3 = 25$$

$$\text{IQR} = 25 - 15 = 10$$

FINAL ANSWER

10.

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Question 11

6 marks | Differentiation & Turning Points

11 The curve **C** has equation $y = 4x^3 + x^2 - 20x$

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} =$ .....

(2)

(b) Find the x coordinates of the points on **C** where the gradient is 4
Show clear algebraic working.

.....

(4)

(Total for Question 11 is 6 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H May 2023

1 **Find the gradient**

$$y = 4x^3 + x^2 - 20x$$

2 **(a)**

(a)

$$\frac{dy}{dx} = 12x^2 + 2x - 20$$

3 **Find the gradient**

(b) The gradient is 4, so

$$12x^2 + 2x - 20 = 4$$

$$12x^2 + 2x - 24 = 0$$

$$6x^2 + x - 12 = 0$$

4 **Factorise**

Factorise:

$$(3x - 4)(2x + 3) = 0$$

5 **Find the gradient**

So

$$x = \frac{4}{3} \quad \text{or} \quad x = -\frac{3}{2}$$

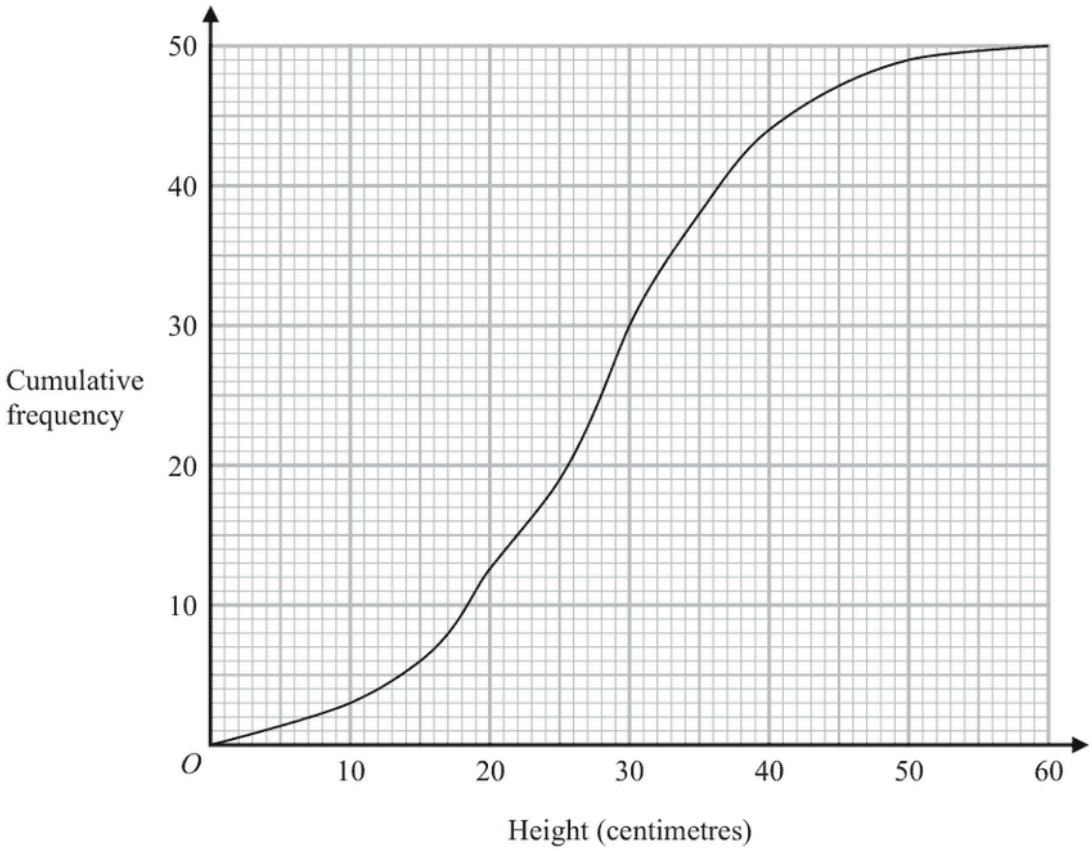
FINAL ANSWER

$\frac{dy}{dx} = 12x^2 + 2x - 20$, and $x = \frac{4}{3}$ or $x = -\frac{3}{2}$.

Question 12

4 marks | Cumulative Frequency Diagrams

12 The cumulative frequency graph shows information about the heights, in centimetres, of 50 plants in a flowerbed.



(a) Use the graph to find an estimate for the median height of these plants.

..... centimetres

(1)

(b) Use the graph to find the frequency for the class interval $30 < \text{Height} \leqslant 40$

.....

(1)

(c) Use the graph to find an estimate for the number of plants with a height greater than 35 centimetres.

.....

(2)

(Total for Question 12 is 4 marks)

1

(a) Median

There are 50 plants, so read across from 25: the curve reaches 25 at about 28.5 cm.

2

(b) Frequency for 30 < height ≤ 40

Read the curve at both ends of the class: about 44 at 40 cm and 30 at 30 cm.

$$44 - 30 = 14$$

3

(c) Taller than 35 cm

At 35 cm the curve reads about 37, so about $50 - 37 = 13$ plants.

✓ **FINAL ANSWER**

(a) about 28.5 cm (b) 14 (c) about 13

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Question 13

3 marks | Linear Graphs

13 A is the point with coordinates $(-5, 12)$

B is the point with coordinates $(19, -48)$

Find an equation of the straight line that passes through the points A and B

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H May 2023

1 Calculate value

$$A = (-5, 12), \qquad B = (19, -48)$$

2 The gradient is

The gradient is

$$\frac{-48 - 12}{19 - (-5)} = \frac{-60}{24} = -\frac{5}{2}$$

3 Evaluate fraction

Using point $(-5, 12)$:

$$y - 12 = -\frac{5}{2}(x + 5)$$

4 Multiply by 2

Multiply by 2:

$$2y - 24 = -5x - 25$$

$$5x + 2y + 1 = 0$$

FINAL ANSWER

$5x + 2y + 1 = 0$.

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Question 14

3 marks | Factorising

14 Factorise fully $50g^2 - 18$

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H May 2023

1 Factorise expression

$$50g^2 - 18 = 2(25g^2 - 9)$$

$$= 2(5g - 3)(5g + 3)$$

FINAL ANSWER

$2(5g - 3)(5g + 3)$

4MA1/1H MAY 2023

Question 15

6 marks | Standard Form

15 (a) $\sqrt{2} \div \dfrac{8^3}{16^{\frac{3}{2}}} = 2^n$

Work out the value of n
Show your working clearly.

$n =$ .....

(3)

(b) Find 4% of 4.5×10^{157}
Give your answer in standard form.

.....

(3)

(Total for Question 15 is 6 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H May 2023

1 Convert standard form

$$\sqrt{2} = 2^{\frac{1}{2}}$$

$$8^3 = (2^3)^3 = 2^9$$

$$16^{\frac{3}{2}} = (2^4)^{\frac{3}{2}} = 2^6$$

2 Convert standard form

So

$$\frac{8^3}{16^{\frac{3}{2}}} = \frac{2^9}{2^6} = 2^3$$

3 Convert standard form

Therefore

$$\sqrt{2} \div \frac{8^3}{16^{\frac{3}{2}}} = 2^{\frac{1}{2}} \div 2^3$$

$$= 2^{\frac{1}{2}-3} = 2^{-\frac{5}{2}}$$

4 **Convert standard form**

So

$$n = -\frac{5}{2}$$

---

5 **Convert standard form**

For part (b),

$$4\% = 0.04 = 4 \times 10^{-2}$$

$$0.04(4.5 \times 10^{157}) = 18 \times 10^{155}$$

$$= 1.8 \times 10^{156}$$

✓ **FINAL ANSWER**

(a) $n = -\frac{5}{2}$, (b) 1.8×10^{156}

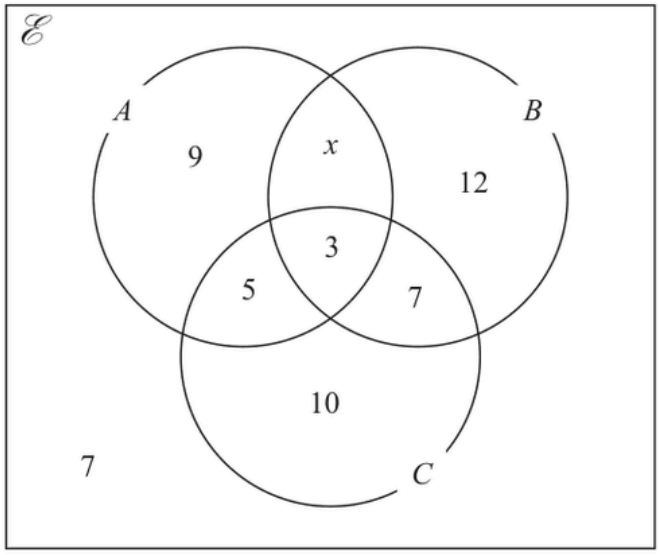
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Question 16

3 marks | Set Notation & Venn Diagrams

16 The Venn diagram shows a universal set $\mathscr{E}$ and sets A, B and C

The numbers and the letter x represent **numbers** of elements.



Given that $n(A \cup B) = 42$

(a) find the value of x

$x =$ .....

(1)

(b) Find $n(A')$

.....

(1)

(c) Find $n(B' \cap C)$

.....

(1)

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H May 2023

1 **The regions inside are**

The regions inside $A \cup B$ are

$$9 + x + 12 + 5 + 3 + 7$$

2 **We are told that**

We are told that

$$n(A \cup B) = 42$$

3 **Calculate value**

So

$$36 + x = 42$$

$$x = 6$$

4 **Add every region not inside**

For A', add every region not inside A:

$$12 + 7 + 10 + 7 = 36$$

5 **Add the regions**

For $B' \cap C$, add the regions that are in C but not in B:

$$5 + 10 = 15$$

✓ **FINAL ANSWER**

(a) $x = 6$, (b) 36, (c) 15

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Question 17

4 marks | Functions

17 The functions g and h are such that

$$g(x) = \frac{11}{2x - 5}$$

$$h(x) = x^2 + 4 \qquad x \geqslant 0$$

(a) What value of x must be excluded from any domain of g?

.....

(1)

(b) Solve $gh(x) = 1$

.....

(3)

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H May 2023

1 **Find inverse function**

$$g(x) = \frac{11}{2x - 5}, \qquad h(x) = x^2 + 4, \quad x \geq 0$$

2 **(a) The denominator of cannot**

(a) The denominator of $g(x)$ cannot be zero.

$$2x - 5 = 0$$

$$x = \frac{5}{2}$$

3 **Find inverse function**

So $x = \frac{5}{2}$ must be excluded.

4 **(b)**

(b)

$$gh(x) = g(h(x)) = \frac{11}{2(x^2+4)-5}$$

$$= \frac{11}{2x^2+3}$$

---

5 **Solve**

Solve

$$\frac{11}{2x^2+3} = 1$$

$$2x^2 + 3 = 11$$

$$2x^2 = 8$$

$$x^2 = 4$$

---

6 **Solve inequality**

Since $x \geq 0$,

$$x = 2$$

✓ **FINAL ANSWER**

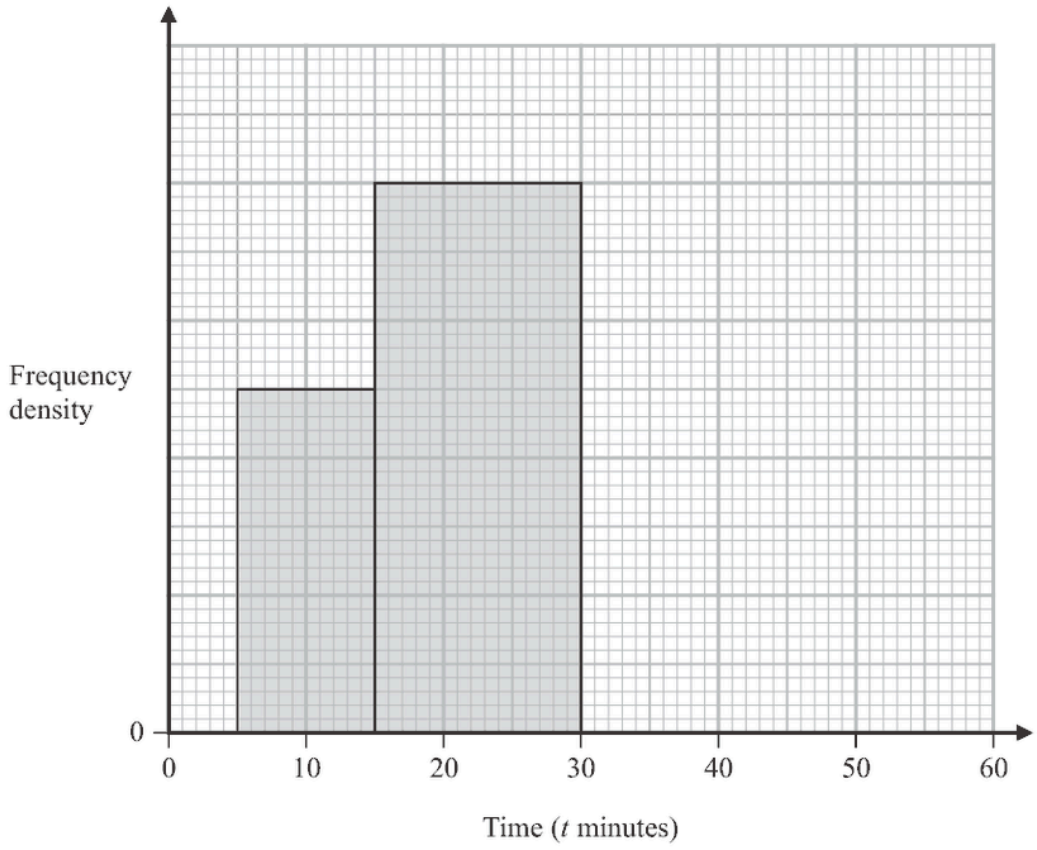
Exclude $x = \frac{5}{2}$, and $x = 2$.

Question 18

4 marks | Histograms

18 The incomplete table and incomplete histogram give information about the times, in minutes, that 140 people waited at a station for a train.

Time (t minutes)	Frequency
$0 < t \leqslant 5$	23
$5 < t \leqslant 15$	
$15 < t \leqslant 30$	
$30 < t \leqslant 40$	18
$40 < t \leqslant 60$	14



Complete the table and the histogram.

(Total for Question 18 is 4 marks)

1 **The total frequency is**

The total frequency is 140.

2 **The known frequencies are**

The known frequencies are

$$23 + 18 + 14 = 55$$

3 **Use histogram**

So the two missing frequencies have total

$$140 - 55 = 85$$

4 **Use histogram**

From the histogram, the missing frequencies are

$$5 < t \leq 15 : \ 25, \qquad 15 < t \leq 30 : \ 60$$

5 **The missing histogram densities are**

The missing histogram densities are

$$0 < t \leq 5 : \frac{23}{5} = 4.6$$

$$30 < t \leq 40 : \frac{18}{10} = 1.8$$

$$40 < t \leq 60 : \frac{14}{20} = 0.7$$

FINAL ANSWER

missing frequencies $25, 60$, and missing densities $4.6, 1.8, 0.7$.

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Question 19

5 marks | Vectors

19

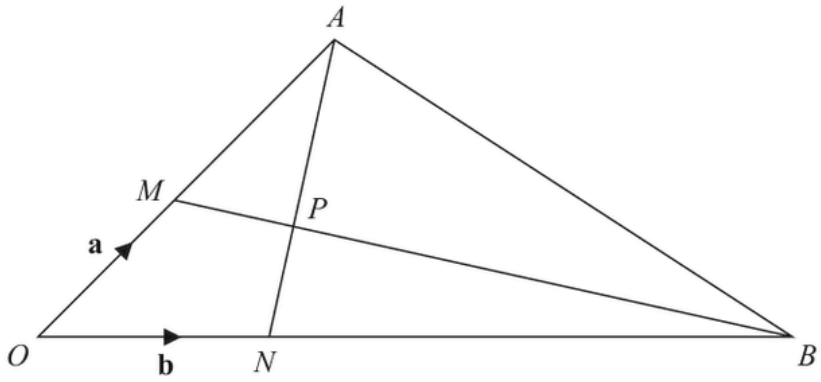


Diagram **NOT** accurately drawn

OMA, *ONB*, *MPB* and *NPA* are straight lines.

M is the midpoint of *OA*

$ON : NB = 1 : 5$

$\overrightarrow{OM} = \mathbf{a}$ $\overrightarrow{ON} = \mathbf{b}$

(a) Find in terms of **a** and **b** the vector $\overrightarrow{AN}$

.....

(1)

(b) Use a vector method to find the ratio $AP : PN$

$AP : PN =$ .....

(4)

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H May 2023

1 **Split the ratio**

Since M is the midpoint of OA,

$$\overrightarrow{OA} = 2a$$

2 **Split the ratio**

Also,

$$\overrightarrow{ON} = b$$

3 **Split the ratio**

So

$$\overrightarrow{AN} = \overrightarrow{ON} - \overrightarrow{OA} = b - 2a$$

4 **Let**

Let

$$\overrightarrow{AP} = \lambda \overrightarrow{AN}$$

5 **Split the ratio**

Then

$$\overrightarrow{OP} = 2a + \lambda(b - 2a) = (2 - 2\lambda)a + \lambda b$$

6 **Split the ratio**

Since $ON : NB = 1 : 5$,

$$\overrightarrow{OB} = 6b$$

7 **Split the ratio**

Point P is also on MB, so

$$\overrightarrow{OP} = a + \mu(6b - a) = (1 - \mu)a + 6\mu b$$

8 **Compare coefficients**

Compare coefficients:

$$2 - 2\lambda = 1 - \mu$$

$$\lambda = 6\mu$$

9 **Split the ratio**

From $\mu = 2\lambda - 1$,

$$\lambda = 6(2\lambda - 1)$$

$$11\lambda = 6$$

$$\lambda = \frac{6}{11}$$

---

10 **Split the ratio**

So

$$AP : PN = \frac{6}{11} : \frac{5}{11} = 6 : 5$$

✓ **FINAL ANSWER**

(a) $b - 2a$, (b) $6 : 5$

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Question 20

6 marks | Sequences

20 The sum of the first 80 terms of an arithmetic series, S, is 470

The 75th term of S is 14.5

The sum of the first X terms of S is 171

Work out the value of X
Show your working clearly.

$X =$ .....

(Total for Question 20 is 6 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H May 2023

1 Evaluate fraction

$$S_{80} = 470$$

$$40(2a + 79d) = 470$$

$$2a + 79d = \frac{47}{4}$$

2 The 75th term is

The 75th term is

$$a + 74d = 14.5 = \frac{29}{2}$$

3 Double it

Double it:

$$2a + 148d = 29$$

4 **Subtract**

Subtract:

$$69d = 29 - \frac{47}{4} = \frac{69}{4}$$

$$d = \frac{1}{4}$$

$$a + 74\left(\frac{1}{4}\right) = \frac{29}{2}$$

$$a = -4$$

---

5 **Evaluate fraction**

Now

$$S_X = 171$$

$$171 = \frac{X}{2}\left(2(-4) + (X-1)\frac{1}{4}\right)$$

$$171 = \frac{X(X-33)}{8}$$

$$X^2 - 33X - 1368 = 0$$

$$X = \frac{33 + 81}{2} = 57$$

✓ **FINAL ANSWER**

$X = 57$

4MA1/1H MAY 2023

Question 21

2 marks | Transformations of Graphs

21 A curve has equation $y = f(x)$

There is only one turning point on the curve.
The coordinates of this turning point are $(6, 5)$

Write down the coordinates of the turning point on the curve with equation

(a) $y = f(x - 4)$

(..... , ...)

(1)

(b) $y = f(3x)$

(..... , ...)

(1)

(Total for Question 21 is 2 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H May 2023

1 The turning point of is

The turning point of $y = f(x)$ is $(6, 5)$.

2 Calculate value

For

$$y = f(x - 4)$$

3 the graph moves 4 units

the graph moves 4 units right:

$$(6, 5) \to (10, 5)$$

4 Calculate value

For

$$y = f(3x)$$

5 **the coordinate is divided by**

the x-coordinate is divided by 3:

$$(6, 5) \rightarrow (2, 5)$$

✓ **FINAL ANSWER**

$(10, 5)$ and $(2, 5)$

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Question 22

6 marks | Circles, Arcs & Sectors

22 The diagram shows two circles with centre O and a regular pentagon $ABCDE$

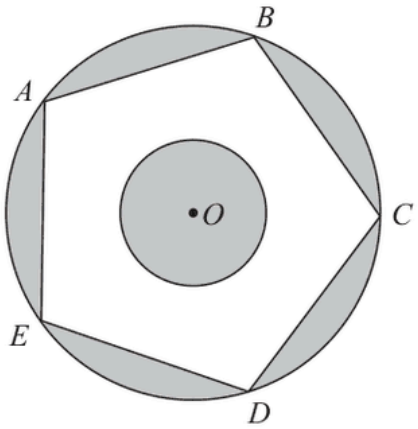


Diagram **NOT** accurately drawn

A, B, C, D and E are points on the larger circle.
The pentagon has sides of length 8 cm.

The diagram is shaded such that

shaded area = unshaded area

Work out the radius of the smaller circle.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 22 is 6 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H May 2023

1 **The regular pentagon has side**

The regular pentagon has side length 8 cm.

2 **Use trigonometry**

For a regular pentagon,

$$R = \frac{s}{2\sin 36°}$$

3 **Use trigonometry**

where R is the radius of the larger circle.

$$R = \frac{8}{2\sin 36^\circ} = 6.8052\ldots$$

4 **Area of the larger circle**

Area of the larger circle:

$$\pi R^2 = 145.4898\ldots$$

5 **Area of the regular pentagon**

Area of the regular pentagon:

$$\frac{5s^2}{4\tan 36^\circ} = \frac{5(8)^2}{4\tan 36^\circ} = 110.1106\ldots$$

6 **Let the radius of the**

Let the radius of the smaller circle be r.

7 **The shaded area is**

The shaded area is:

$$\text{larger circle area} - \text{pentagon area} + \pi r^2$$

8 **The unshaded area is**

The unshaded area is:

$$\text{pentagon area} - \pi r^2$$

9 **Simplify surd**

These are equal, so

$$145.4898 - 110.1106 + \pi r^2 = 110.1106 - \pi r^2$$

$$2\pi r^2 = 2(110.1106) - 145.4898$$

$$\pi r^2 = 37.3657\ldots$$

$$r = \sqrt{\frac{37.3657\ldots}{\pi}} = 3.4487\ldots$$

✓ **FINAL ANSWER**

$$\boxed{3.45\text{ cm}}$$

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Question 23

5 marks | Area & Volume of Similar Shapes

23 A frustum is made by removing a small square-based pyramid from a similar large squared-based pyramid as shown in the diagram.

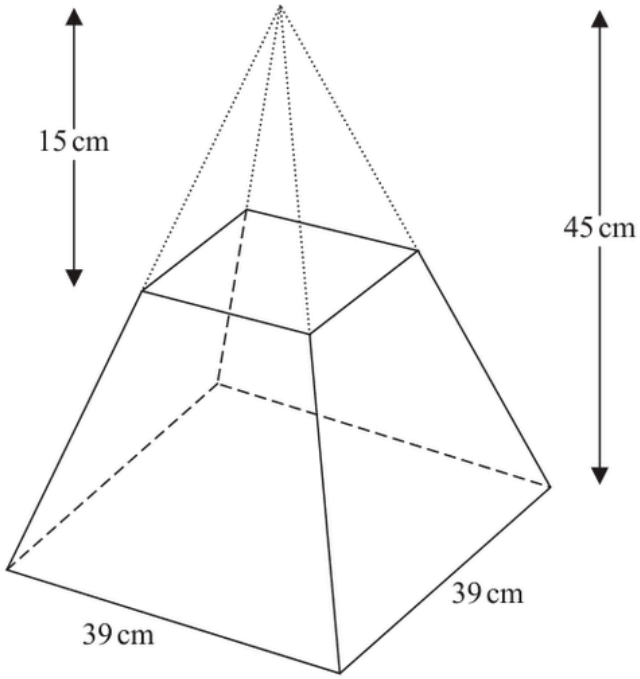


Diagram **NOT** accurately drawn

The height of the small pyramid is 15 cm.
The height of the large pyramid is 45 cm.
The square base of the large pyramid has side length 39 cm.

Work out the **total** surface area of the frustum.
Give your answer correct to the nearest whole number.

..... cm^2

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H May 2023

1 The small and large square

The small and large square-based pyramids are similar.

2 **Their heights are**

Their heights are

$$15 : 45 = 1 : 3$$

---

3 **Square of the frustum**

So the side length of the top square of the frustum is

$$\frac{39}{3} = 13 \text{ cm}$$

---

4 **The vertical height of the**

The vertical height of the frustum is

$$45 - 15 = 30 \text{ cm}$$

---

5 **Evaluate fraction**

For one trapezium face, the difference between the half-side lengths is

$$\frac{39 - 13}{2} = 13 \text{ cm}$$

---

6 **Simplify surd**

So the slant height of the trapezium face is

$$\sqrt{30^2 + 13^2} = \sqrt{1069}$$

---

7 **Area of one trapezium face**

Area of one trapezium face:

$$\frac{1}{2}(39 + 13)\sqrt{1069} = 26\sqrt{1069}$$

---

8 **Area of four trapezium faces**

Area of four trapezium faces:

$$4(26\sqrt{1069}) = 104\sqrt{1069}$$

---

9 **Add the top and bottom**

Add the top and bottom square faces:

$$13^2 + 39^2 = 169 + 1521 = 1690$$

---

10 **Total surface area**

Total surface area:

$$1690 + 104\sqrt{1069} = 5090.338\ldots$$

---

11 Correct to the nearest whole

Correct to the nearest whole number:

$$5090$$

FINAL ANSWER

$$\boxed{5090\ \text{cm}^2}$$