Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

4MA1/1H May 2021 Worked Solutions

May 2021 paper rebuilt with the worked solution after each question.

26	2020–2026	Single-paper solutions
paper questions	52 past papers	checked by Dr Eslam



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PAST PAPER

4MA1/1H May 2021

May 2021

26	100	2 h
questions	marks	exam time

Each question is followed by its worked solution and final answer.

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Question 1

3 marks | Standard & Compound Units

1 A plane flew from Madrid to Dubai.

The distance the plane flew was 5658 km.
The flight time was 8 hours 12 minutes.

Work out the average speed of the plane.

..... km/h

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H May 2021

1 Evaluate fraction

$$8\text{ h }12\text{ min} = 8 + \frac{12}{60} = 8.2\text{ hours}$$

$$\text{average speed} = \frac{5658}{8.2} = 690$$

FINAL ANSWER

690 km/h.

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Question 2

2 marks | Sequences

2 Here are the first 4 terms of an arithmetic sequence.

85 79 73 67

Find an expression, in terms of n, for the nth term of the sequence.

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H May 2021

1 The sequence is

The sequence is

$$85,\ 79,\ 73,\ 67$$

2 The common difference is

The common difference is -6.

$$\text{nth term} = 85 + (n-1)(-6)$$

$$= 85 - 6n + 6 = 91 - 6n$$

FINAL ANSWER

$91 - 6n$.

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Question 3

4 marks | Perimeter & Area

3

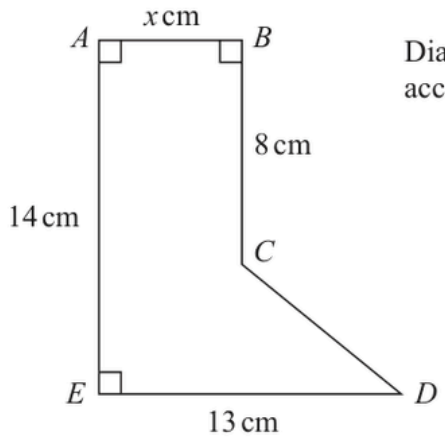


Diagram **NOT** accurately drawn

The diagram shows the shape $ABCDE$.

The area of the shape is $91.8\,\text{cm}^2$

Work out the value of x.

$x =$ .....

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H May 2021

1 Split the shape into a

Split the shape into a rectangle and a triangle.

2 Rectangle area

Rectangle area:

$$14x$$

3 Use trigonometry

The triangle has base $13 - x$ and height 6, so its area is

$$\frac{1}{2}(13 - x)(6) = 39 - 3x$$

4 **Total area**

Total area:

$$14x + 39 - 3x = 91.5$$

$$11x = 52.5$$

$$x = 4.772\ldots$$

✓ **FINAL ANSWER**

$4.77\ \text{cm}$.

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Question 4

3 marks | Ratio Toolkit

4 On a farm there are chickens, ducks and pigs.

The ratio of the number of chickens to the number of ducks is $7:2$
The ratio of the number of ducks to the number of pigs is $5:9$
There are 36 pigs on the farm.

Work out the number of chickens on the farm.

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H May 2021

1 Ducks pigs

Ducks : pigs $= \mathbf{5 : 9}$.

2 Split the ratio

There are $\mathbf{36}$ pigs, so

$$9 \text{ parts} = 36$$

$$1 \text{ part} = 4$$

$$\text{ducks} = 5 \times 4 = 20$$

3 Chickens ducks

Chickens : ducks $= \mathbf{7 : 2}$.

$$2 \text{ parts} = 20$$

$$1 \text{ part} = 10$$

$$\text{chickens} = 7 \times 10 = 70$$

FINAL ANSWER

70

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Question 5

6 marks | Rearranging Formulae

5 (a) Expand and simplify $3x(2x + 3) - x(3x + 5)$

.....

(2)

(b) Make t the subject of the formula $p = at - d$

.....

(2)

Given that $\frac{w^5 \times w^n}{w^3} = w^{10}$

(c) work out the value of n.

$n =$ .....

(2)

(Total for Question 5 is 6 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H May 2021

1 **(a)**

(a)

$$3x(2x+3) - x(3x+5) = 6x^2 + 9x - 3x^2 - 5x$$

$$= 3x^2 + 4x$$

2 **(b)**

(b)

$$p = at - d$$

$$p + d = at$$

$$t = \frac{p+d}{a}$$

3 **(c)**

(c)

$$\frac{w^5 \times w^n}{w^3} = w^{n+2} = w^{10}$$

4 **Rearrange formula**

So $n + 2 = 10$, hence $n = 8$.

FINAL ANSWER

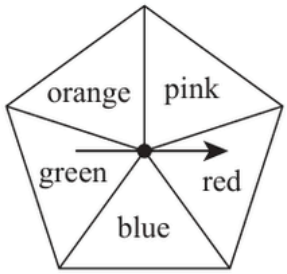
(a) $3x^2 + 4x$, (b) $t = \frac{p+d}{a}$, (c) $n = 8$

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Question 6

5 marks | Probability Toolkit

6 Grace has a biased 5-sided spinner.



Grace is going to spin the arrow on the spinner once.

The table below gives the probabilities that the spinner will land on red or on blue or on green.

Colour	Red	Blue	Green	Orange	Pink
Probability	0.20	0.12	0.08		

The probability that the spinner will land on orange is 3 times the probability that the spinner will land on pink.

(a) Work out the probability that the spinner will land on orange.

.....

(3)

Grace spins the arrow on the spinner 150 times.

(b) Work out an estimate for the number of times the spinner lands on blue.

.....

(2)

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H May 2021

1 **(a) Set up with the missing probabilities**

Let $P(\text{pink}) = x$, so $P(\text{orange}) = 3x$.

$$0.20 + 0.12 + 0.08 + 3x + x = 1 \Rightarrow 4x = 0.60 \Rightarrow x = 0.15$$

2 **(a) Probability of orange**

$$3x = 0.45$$

3 (b) Estimate for blue

$$150 \times 0.12 = 18$$

FINAL ANSWER

(a) 0.45 (b) 18

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Question 7

4 marks | Inequalities (Solving & Graphing)

7 $-4 \leqslant 2y < 6$

y is an integer.

(a) Write down all the possible values of y.

.....

(2)

(b) Solve the inequality $7t - 3 \leqslant 2t + 31$

Show your working clearly.

.....

(2)

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H May 2021

1 **(a)**

(a)

$$-4 \leq 2y < 6$$

2 **Divide by 2**

Divide by 2:

$$-2 \leq y < 3$$

3 **Calculate value**

Since y is an integer,

$$y = -2, -1, 0, 1, 2$$

4 **(b)**

(b)

$$7t - 3 \leq 2t + 31$$

$$5t \leq 34$$

$$t \leq \frac{34}{5}$$

FINAL ANSWER

$y = -2, -1, 0, 1, 2$, and $t \leq \frac{34}{5}$.

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Question 8

4 marks | Standard Form

8 The table shows the populations of five countries.

Country	Population
China	1.4×10^9
Germany	8.2×10^7
Sweden	9.9×10^6
Fiji	9.1×10^5
Malta	4.3×10^5

(a) Work out the difference between the population of China and the population of Germany.
Give your answer in standard form.

.....

(2)

Given that

$$\text{population of Fiji} = \frac{1}{k} \times \text{population of Sweden}$$

(b) work out the value of k.
Give your answer correct to the nearest whole number.

$k =$ .....

(2)

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H May 2021

1 **Convert standard form**

For part (a),

$$1.4 \times 10^9 = 1400000000$$

$$8.2 \times 10^7 = 82000000$$

2 **Difference**

Difference:

$$1400000000 - 82000000 = 1318000000$$

$$1318000000 = 1.318 \times 10^{9}$$

---

3 **Convert standard form**

For part (b),

$$\text{population of Fiji} = \frac{1}{k} \times \text{population of Sweden}$$

---

4 **Convert standard form**

So

$$k = \frac{\text{population of Sweden}}{\text{population of Fiji}}$$

$$k = \frac{9.9 \times 10^{6}}{9.1 \times 10^{5}} = 10.879\ldots$$

---

5 **Convert standard form**

To the nearest whole number,

$$k = 11$$

✓ **FINAL ANSWER**

(a) 1.318×10^{9}, (b) 11

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Question 9

5 marks | Factorising

9 (a) Factorise fully $25a^4c^7d + 45a^9c^3h$

.....

(2)

(b) Solve $(2x + 5)^2 = (2x + 3)(2x - 1)$

$x =$ .....

(3)

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H May 2021

1 **(a)**

(a)

$$25a^4c^7d + 45a^9c^3h = 5a^4c^3(5c^4d + 9a^5h)$$

2 **(b)**

(b)

$$(2x + 5)^2 = (2x + 3)(2x - 1)$$

$$4x^2 + 20x + 25 = 4x^2 + 4x - 3$$

$$16x = -28$$

$$x = -\frac{7}{4}$$

✓ **FINAL ANSWER**

(a) $5a^4c^3(5c^4d + 9a^5h)$, (b) $x = -\frac{7}{4}$

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Question 10

3 marks | Statistics Toolkit

10 Jethro has sat 5 tests.
Each test was marked out of 100 and Jethro's mean mark for the 5 tests is 74

Jethro has to sit one more test that is also to be marked out of 100

Jethro wants his mean mark for all 6 tests to be at least 77

Work out the least mark that Jethro needs to get for the last test.

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H May 2021

1 The total for the first

The total for the first 5 tests is

$$5 \times 74 = 370$$

2 Calculate value

For an average of at least 77 after 6 tests, the total must be at least

$$6 \times 77 = 462$$

3 Calculate value

So the last test score must be at least

$$462 - 370 = 92$$

FINAL ANSWER

at least 92.

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Question 11

4 marks | Algebraic Roots & Indices

11 $\sqrt{2} \times 16 = 2^x$

(a) Find the value of x.
Show your working clearly.

$x =$ .....

(2)

$$\frac{(11^{-6})^5}{11^4} = 11^n$$

(b) Find the value of n.
Show your working clearly.

$n =$ .....

(2)

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H May 2021

1 **Simplify surd**

$$\sqrt{2} \times 16 = 2^x$$

2 **Write both factors as powers**

Write both factors as powers of 2:

$$\sqrt{2} = 2^{1/2}, \quad 16 = 2^4$$

3 **Use index laws**

So

$$2^{1/2} \times 2^4 = 2^{9/2}$$

4 **Use index laws**

Therefore

$$x = \frac{9}{2}$$

5 **Use index laws**

For part (b),

$$\frac{(11^{-6})^5}{11^4} = \frac{11^{-30}}{11^4} = 11^{-30-4} = 11^{-34}$$

6 **Use index laws**

So

$$n = -34$$

FINAL ANSWER

(a) $x = \frac{9}{2}$, (b) $n = -34$.

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Question 12

2 marks | Circles, Arcs & Sectors

12 The diagram shows a sector of a circle with radius 7 cm.

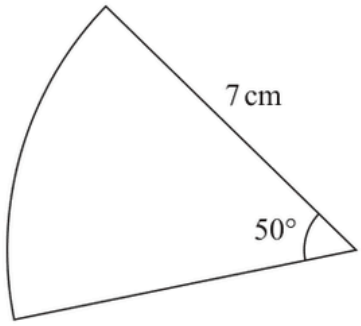


Diagram **NOT** accurately drawn

Work out the length of the arc of the sector.
Give your answer correct to one decimal place.

..... cm

(Total for Question 12 is 2 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H May 2021

1 **Evaluate fraction**

$$\text{arc length} = \frac{50}{360} \times 2\pi(7)$$

$$= 6.108\ldots$$

FINAL ANSWER

6.1 cm.

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Question 13

3 marks | Expanding Brackets

13 Expand and simplify $4x(3x + 1)(2x - 3)$
Show your working clearly.

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H May 2021

1 Expand brackets

$$4x(3x+1)(2x-3) = 4x(6x^2 - 7x - 3)$$

$$= 24x^3 - 28x^2 - 12x$$

FINAL ANSWER

$24x^3 - 28x^2 - 12x$

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Question 14

2 marks | Statistics Toolkit

14 Here is the number of goals that Henri's team scored one summer in each water polo match.

5 8 9 11 13 13 14 15 16 17 20

Find the interquartile range of the numbers of goals.
Show your working clearly.

(Total for Question 14 is 2 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H May 2021

1 **The goals in order are**

The goals in order are

$$5,\ 8,\ 9,\ 11,\ 13,\ 13,\ 14,\ 15,\ 16,\ 17,\ 20$$

$$Q_1 = 9, \qquad Q_3 = 16$$

$$\text{IQR} = 16 - 9 = 7$$

FINAL ANSWER

7.

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Question 15

4 marks | Circle Theorems

15 P, Q and R are points on a circle, centre O.
TRV is the tangent to the circle at R.

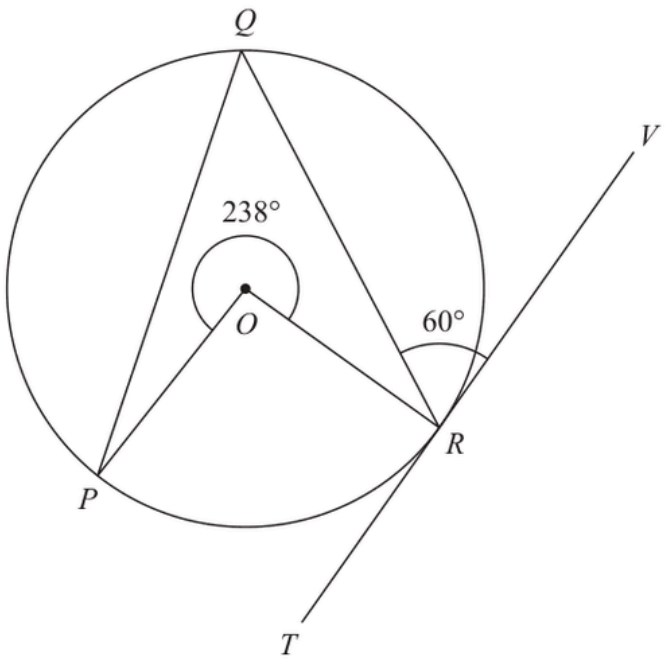


Diagram **NOT** accurately drawn

Reflex angle $POR = 238°$
Angle $QRV = 60°$

Calculate the size of angle OPQ.
Give a reason for each stage of your working.

.....°

(Total for Question 15 is 4 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H May 2021

1 **Use trigonometry**

The reflex angle POR is $238°$, so the smaller angle is

$$360° - 238° = 122°$$

2 **Use trigonometry**

Since $OP = OR$, triangle OPR is isosceles:

$$\angle OPR = \frac{180° - 122°}{2} = 29°$$

3 **Use trigonometry**

By the alternate segment theorem,

$$\angle QPR = 60°$$

---

4 **Use trigonometry**

Therefore

$$\angle QPO = 60° - 29° = 31°$$

✓ **FINAL ANSWER**

$31°$.

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Question 16

2 marks | Recurring Decimals

16 Use algebra to show that the recurring decimal $0.28\dot{1}\dot{3} = \frac{557}{1980}$

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H May 2021

1 Let

Let

$$x = 0.28131313\ldots$$

2 Calculate value

Then

$$100x = 28.131313\ldots$$

3 Calculate value

and

$$10000x = 2813.131313\ldots$$

4 Subtract

Subtract:

$$10000x - 100x = 2813.131313\ldots - 28.131313\ldots$$

$$9900x = 2785$$

$$x = \frac{2785}{9900} = \frac{557}{1980}$$

FINAL ANSWER

$0.28131313\ldots = \frac{557}{1980}$.

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Question 17

3 marks | Algebraic Proof

17 Using algebra, prove that, given any 3 consecutive even numbers, the difference between the square of the largest number and the square of the smallest number is always 8 times the middle number.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H May 2021

1 Let the three consecutive even

Let the three consecutive even numbers be

$$2n-2, \quad 2n, \quad 2n+2$$

2 The middle number is

The middle number is $2n$.

3 The difference between the square

The difference between the square of the largest and the square of the smallest is

$$(2n+2)^2-(2n-2)^2$$

$$=(4n^2+8n+4)-(4n^2-8n+4)$$

$$=16n$$

4 Calculate value

Since $8 \times \text{middle number} = 8(2n) = 16n$, the result is 8 times the middle number.

FINAL ANSWER

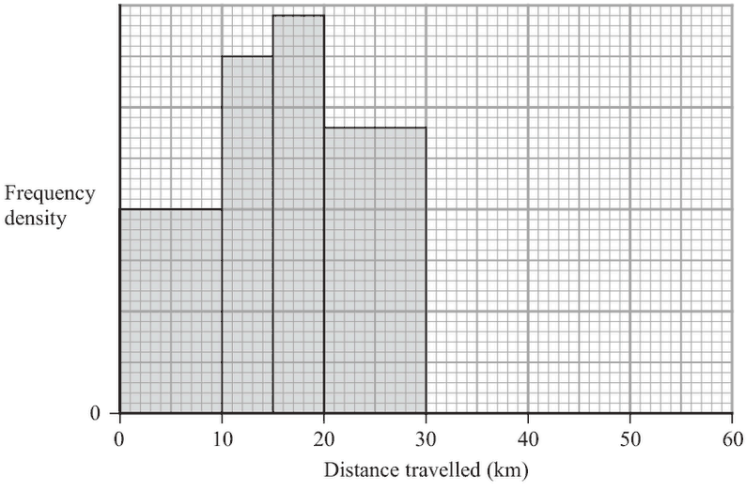
The difference is always 8 times the middle number.

Question 18

5 marks | Histograms

18 The table and histogram give information about the distance travelled, in order to get to work, by each person working in a large store.

Distance (d km)	Frequency
$0 \leqslant d < 10$	40
$10 \leqslant d < 15$	
$15 \leqslant d < 20$	
$20 \leqslant d < 30$	
$30 \leqslant d < 60$	30



Using the information in the table and in the histogram,

(a) complete the table,

(2)

(b) complete the histogram.

(1)

One of the people working in the store is chosen at random.

(c) Work out an estimate for the probability that the distance travelled by this person, in order to get to work, was greater than 25 km.

.....

(2)

(Total for Question 18 is 5 marks)

1 **Use histogram**

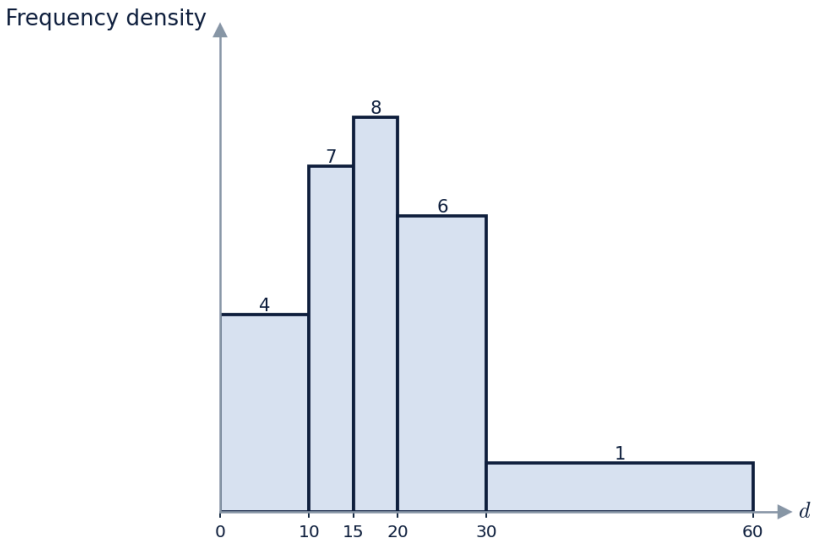
The $0 \leq d < 10$ class has frequency 40, so its density is

$$\frac{40}{10} = 4$$

2 **Use histogram**

From the histogram, the missing frequencies are

$$10 \leq d < 15 : \ 35, \qquad 15 \leq d < 20 : \ 40, \qquad 20 \leq d < 30 : \ 60$$



3 **Together with the table**

Together with the table:

$$40 + 35 + 40 + 60 + 30 = 205$$

4 **Use histogram**

For $d > 25$:

$$5(6) + 30 = 60$$

$$P(d > 25) = \frac{60}{205} = \frac{12}{41}$$

FINAL ANSWER

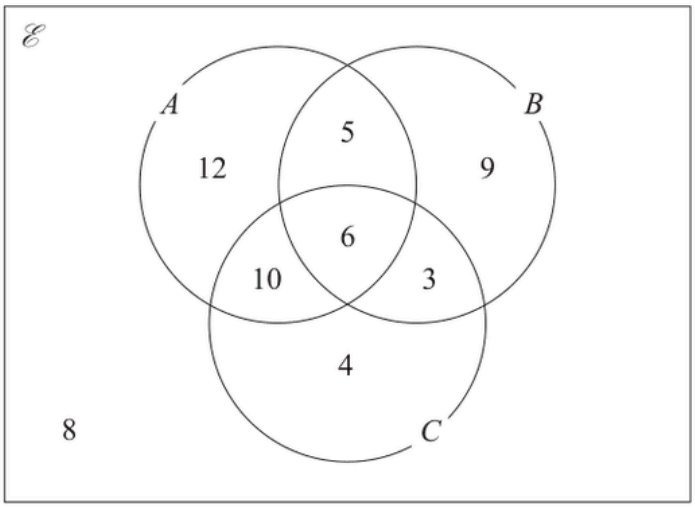
completed frequencies $35, 40, 60$, missing histogram density 1, and probability $\frac{12}{41}$.

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Question 19

3 marks | Set Notation & Venn Diagrams

19 The Venn diagram shows a universal set, $\mathscr{E}$ and sets A, B and C.



12, 5, 9, 10, 6, 3, 4 and 8 represent the **numbers** of elements.

Find

(i) $n(A \cup B)$

.....

(1)

(ii) $n(A' \cap B')$

.....

(1)

(iii) $n([A \cap B] \cup C)$

.....

(1)

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H May 2021

1 **From the Venn diagram**

From the Venn diagram:

2 **only is only is only**

- A only is 12
- $A \cap B$ only is 5
- B only is 9
- $A \cap C$ only is 10
- all three is 6
- $B \cap C$ only is 3
- C only is 4
- outside is 8

---

3 **Add every region inside**

For $A \cup B$, add every region inside A or B:

$$12 + 5 + 9 + 10 + 6 + 3 = 45$$

---

4 **Add the regions outside both**

For $A' \cap B'$, add the regions outside both A and B:

$$4 + 8 = 12$$

---

5 **Add and all**

For $(A \cap B) \cup C$, add $A \cap B$ and all of C:

$$5 + 6 + 10 + 3 + 4 = 28$$

✓ **FINAL ANSWER**

(i) 45, (ii) 12, (iii) 28

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Question 20

3 marks | Rounding, Estimation & Bounds

20 $P = \dfrac{t - w}{y}$

t = 9.7 correct to 1 decimal place

w = 5.9 correct to 1 decimal place

y = 3 correct to 1 significant figure

Calculate the upper bound for the value of P.
Show your working clearly.

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H May 2021

1 Estimate the value

$$P = \frac{t - w}{y}$$

2 Find upper bound

For the upper bound, make the numerator large and the denominator small.

$$t < 9.75, \quad w \geq 5.85, \quad y \geq 2.5$$

$$P_{\text{upper}} = \frac{9.75 - 5.85}{2.5} = 1.56$$

FINAL ANSWER

1.56

4MA1/1H MAY 2021

Question 21

4 marks | Algebraic Fractions

21 Given that $x = \dfrac{5}{9y + 5}$ and that $y = \dfrac{5}{5a - 2}$

find an expression for x in terms of a.
Give your expression as a single fraction in its simplest form.

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H May 2021

1 Given

Given

$$x = \frac{5}{9y + 5}$$

2 Use trigonometry

and

$$y = \frac{5}{5a - 2}$$

3 Use cosine rule

Substitute $y = \frac{5}{5a-2}$ into the expression for x:

$$x = \frac{5}{9\left(\frac{5}{5a-2}\right) + 5}$$

$$x = \frac{5}{\frac{45}{5a-2} + 5}$$

4 **Write the denominator as a**

Write the denominator as a single fraction:

$$\frac{45}{5a-2}+5=\frac{45+5(5a-2)}{5a-2}$$

$$=\frac{45+25a-10}{5a-2}$$

$$=\frac{25a+35}{5a-2}$$

---

5 **Use trigonometry**

So

$$x=5\div\frac{25a+35}{5a-2}$$

$$x=5\times\frac{5a-2}{25a+35}$$

$$x=\frac{5(5a-2)}{5(5a+7)}$$

$$x=\frac{5a-2}{5a+7}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{5a-2}{5a+7}}$$

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Question 22

4 marks | 3D Pythagoras & Trigonometry

22 The diagram shows a triangular prism $ABCDEF$ with a horizontal base $ABEF$.

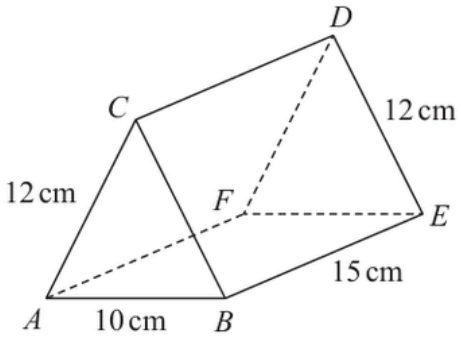


Diagram **NOT** accurately drawn

$AC = BC = FD = ED = 12\,\text{cm}$ $AB = 10\,\text{cm}$ $BE = 15\,\text{cm}$

Calculate the size of the angle between AD and the base $ABEF$.
Give your answer correct to 3 significant figures.

.....°

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H May 2021

1 Calculate value

The triangular end is isosceles, with

$$AC = BC = 12, \qquad AB = 10$$

2 The perpendicular height of the

The perpendicular height of the triangle is found by halving AB:

$$AM = 5$$

$$CM^2 = 12^2 - 5^2 = 144 - 25 = 119$$

$$CM = \sqrt{119}$$

3 The projection of on the

The projection of AD on the horizontal base goes from A to the midpoint of FE. Its length is

$$\sqrt{15^2 + 5^2} = \sqrt{250}$$

4 Use trigonometry

So, for the angle θ between AD and the base,

$$\tan\theta = \frac{\sqrt{119}}{\sqrt{250}}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{119}}{\sqrt{250}}\right) = 34.602\ldots$$

FINAL ANSWER

$34.6°$

4MA1/1H MAY 2021

Question 23

5 marks | Sequences

23 The sum of the first N terms of an arithmetic series, S, is 292
The 2nd term of S is 8.5
The 5th term of S is 13

Find the value of N.
Show clear algebraic working.

$N =$ .....

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H May 2021

1 **The 2nd term is**

The 2nd term is

$$a + d = 8.5$$

2 **The 5th term is**

The 5th term is

$$a + 4d = 13$$

3 **Subtract**

Subtract:

$$3d = 4.5$$

$$d = 1.5$$

$$a = 8.5 - 1.5 = 7$$

4 **Use**

Use

$$S_N = \frac{N}{2}(2a + (N-1)d)$$

$$292 = \frac{N}{2}(14 + 1.5(N-1))$$

$$292 = \frac{N}{2}(1.5N + 12.5)$$

$$3N^2 + 25N - 1168 = 0$$

$$N = \frac{-25 + 121}{6} = 16$$

✓ **FINAL ANSWER**

$N = 16$

4MA1/1H MAY 2021

Question 24

6 marks | Functions

24 The functions f and g are defined as

$$f(x) = 5x^2 - 10x + 7 \qquad \text{where } x \geqslant 1$$

$$g(x) = 7x - 6$$

(a) Find fg(2)

.....

(2)

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) = ...$

$f^{-1}(x) =$ .....

(4)

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H May 2021

1 **(a)**

(a)

$$g(2) = 7(2) - 6 = 8$$

$$fg(2) = f(8)$$

$$f(8) = 5(8)^2 - 10(8) + 7$$

$$f(8) = 247$$

2 **(b)**

(b)

$$f(x) = 5x^2 - 10x + 7$$

$$f(x) = 5(x - 1)^2 + 2$$

3 **Let**

Let

$$y = 5(x-1)^2 + 2$$

$$(x-1)^2 = \frac{y-2}{5}$$

---

4 **Find inverse function**

Since the domain is $x \geq 1$, take the positive square root:

$$x = 1 + \sqrt{\frac{y-2}{5}}$$

✓ **FINAL ANSWER**

(a) 247, (b) $f^{-1}(x) = 1 + \sqrt{\dfrac{x-2}{5}}$

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Question 25

6 marks | Circles, Arcs & Sectors

25 The diagram shows two circles such that the region **R**, shown shaded in the diagram, is the region common to both circles.

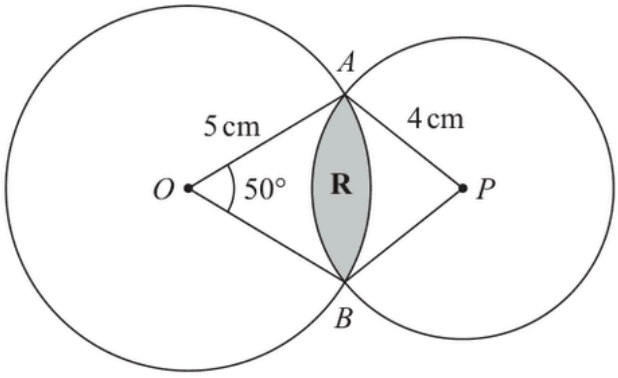


Diagram **NOT** accurately drawn

One of the circles has centre O and radius 5 cm.
The other circle has centre P and radius 4 cm.
Angle $AOB = 50°$

Calculate the area of region **R**.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 25 is 6 marks)

WORKED SOLUTION

Question 25 · 4MA1/1H May 2021

1 **Use trigonometry**

For the circle with centre O:

$$\angle AOB = 50°, \qquad OA = OB = 5$$

2 **The half chord length is**

The half chord length is

$$\frac{AB}{2} = 5\sin 25°$$

3 **Use trigonometry**

For the circle with centre P, let

$$\angle APB = \theta$$

---

4 **Use trigonometry**

The radius is 4, so

$$4\sin\left(\frac{\theta}{2}\right) = 5\sin 25°$$

$$\sin\left(\frac{\theta}{2}\right) = \frac{5\sin 25°}{4}$$

$$\theta = 63.7777\ldots°$$

---

5 **The common region is made**

The common region is made from two minor segments.

---

6 **Use trigonometry**

For the circle with centre O:

$$\text{segment area} = \frac{50}{360}\pi(5)^2 - \frac{1}{2}(5)^2\sin 50° = 1.3328\ldots$$

---

7 **Use trigonometry**

For the circle with centre P:

$$\text{segment area} = \frac{\theta}{360}\pi(4)^2 - \frac{1}{2}(4)^2\sin\theta$$

$$= 1.7284\ldots$$

---

8 **Total area of region**

Total area of region R:

$$1.3328\ldots + 1.7284\ldots = 3.0611\ldots$$

✓ **FINAL ANSWER**

$\boxed{3.06\ \text{cm}^2}$

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Question 26

5 marks | Vectors

26 $OACB$ is a trapezium.

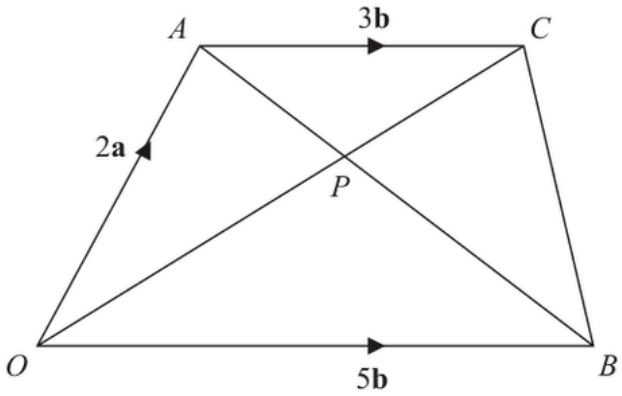


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = \mathbf{2a}$ $\overrightarrow{OB} = \mathbf{5b}$ $\overrightarrow{AC} = \mathbf{3b}$

The diagonals, OC and AB, of the trapezium intersect at the point P.

Find and simplify an expression, in terms of **a** and **b**, for $\overrightarrow{OP}$
Show your working clearly.

$\overrightarrow{OP} =$ .....

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4MA1/1H May 2021

1 **Use vectors**

$$\overrightarrow{OA} = \mathbf{2a}, \quad \overrightarrow{OB} = \mathbf{5b}, \quad \overrightarrow{AC} = \mathbf{3b}$$

2 **Use vectors**

So

$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = \mathbf{2a} + \mathbf{3b}$$

3 **Use vectors**

Since P lies on OC, let

$$\overrightarrow{OP} = t(2\mathbf{a} + 3\mathbf{b})$$

4 **Use vectors**

Since P also lies on AB, let

$$\overrightarrow{OP} = \overrightarrow{OA} + s\overrightarrow{AB}$$

5 **Use vectors**

Now

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 5\mathbf{b} - 2\mathbf{a}$$

6 **Use vectors**

So

$$\overrightarrow{OP} = 2\mathbf{a} + s(5\mathbf{b} - 2\mathbf{a})$$

$$\overrightarrow{OP} = (2 - 2s)\mathbf{a} + 5s\mathbf{b}$$

7 **Use vectors**

Compare this with

$$t(2\mathbf{a} + 3\mathbf{b}) = 2t\mathbf{a} + 3t\mathbf{b}$$

8 **Equating coefficients**

Equating coefficients:

$$2t = 2 - 2s$$

$$3t = 5s$$

9 **Use vectors**

From $t = 1 - s$,

$$3(1 - s) = 5s$$

$$3 = 8s$$

$$s = \frac{3}{8}$$

---

10 **Use vectors**

So

$$t = 1 - \frac{3}{8} = \frac{5}{8}$$

---

11 **Use vectors**

Therefore

$$\overrightarrow{OP} = \frac{5}{8}(2\mathbf{a} + 3\mathbf{b})$$

$$\overrightarrow{OP} = \frac{5}{4}\mathbf{a} + \frac{15}{8}\mathbf{b}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{5}{4}\mathbf{a} + \frac{15}{8}\mathbf{b}}$$