Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

4MA1/2HR June 2024 Worked Solutions

June 2024 paper rebuilt with the worked solution after each question.

26	2020–2026	Single-paper solutions
paper questions	52 past papers	checked by Dr Eslam



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PAST PAPER

4MA1/2HR June 2024

June 2024

26	100	2 h
questions	marks	exam time

Each question is followed by its worked solution and final answer.

4MA1/2HR JUNE 2024

Question 1

3 marks | Prime Factors, HCF & LCM

1 Write 1400 as a product of powers of its prime factors.
Show your working clearly.

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR June 2024

1 Find the LCM

$$1400 = 14 \times 100$$

$$1400 = (2 \times 7)(2^2 \times 5^2)$$

$$1400 = 2^3 \times 5^2 \times 7$$

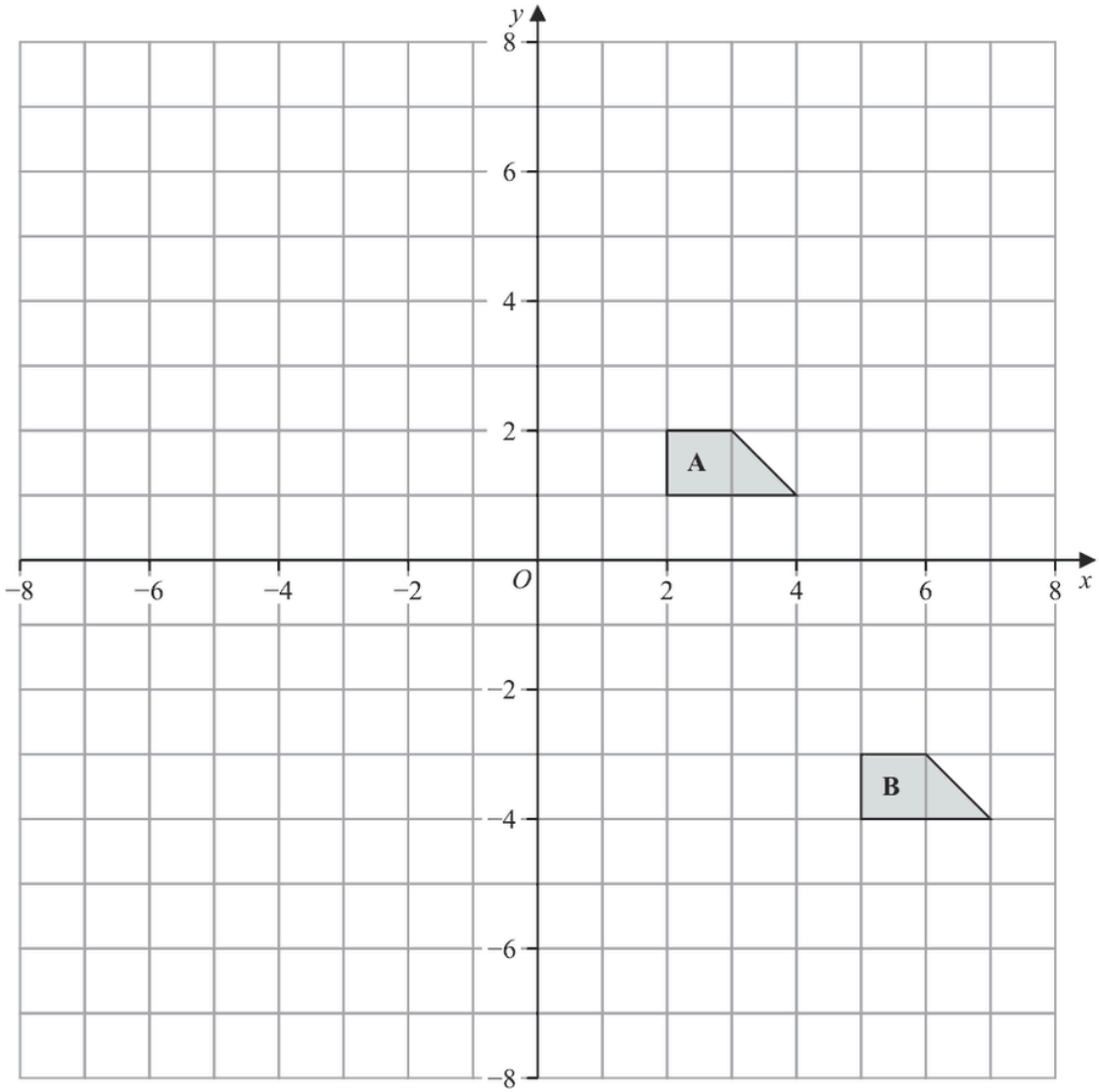
FINAL ANSWER

$2^3 \times 5^2 \times 7$

Question 2

4 marks | Transformations

2



(a) Describe fully the single transformation that maps shape **A** onto shape **B**

.....

.....

(2)

(b) On the grid above, rotate shape **A** 180° about (−1, 0)
Label your shape **C**

(2)

(Total for Question 2 is 4 marks)

1 **(a) Shape moves to shape**

(a) Shape A moves to shape B by the vector

$$\begin{pmatrix} 3 \\ -5 \end{pmatrix}$$

2 **Use matrices**

So the transformation is a translation by $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$.

3 **(b) For a rotation about**

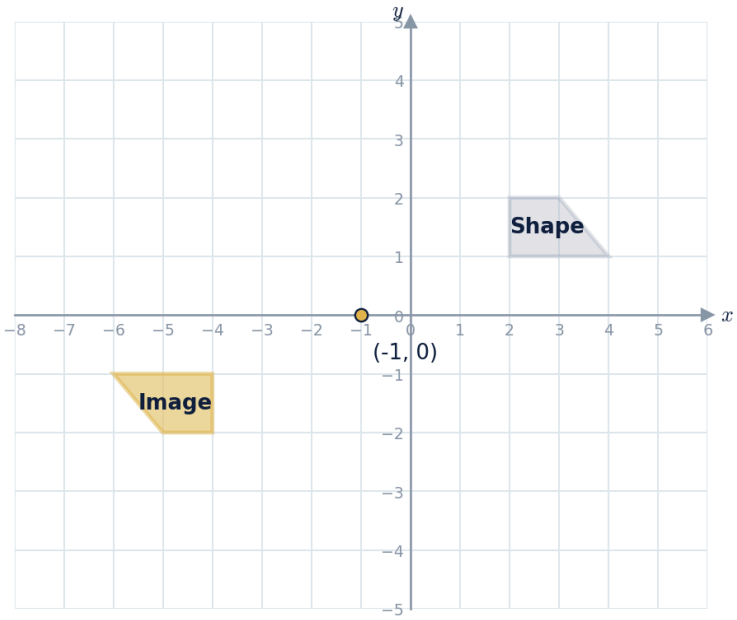
(b) For a $180°$ rotation about $(-1, 0)$,

$$(x, y) \mapsto (-2 - x, \ -y)$$

4 **The image has vertices**

The image has vertices

$$(-4, -1),\ (-6, -1),\ (-5, -2),\ (-4, -2)$$



FINAL ANSWER

(a) translation by $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$; (b) vertices $(-4, -1), (-6, -1), (-5, -2), (-4, -2)$.

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Question 3

2 marks | Statistics Toolkit

3 Here is a list of four numbers written in ascending order of size

$$x \quad x \quad y \quad 15$$

where x and y are integers.

The numbers have

a median of 12.5
a range of 4

Find the value of x and the value of y

$x =$ .....

$y =$ .....

(Total for Question 3 is 2 marks)

WORKED SOLUTION

Question 3 · 4MA1/2HR June 2024

1 The range is

The range is 4:

$$15 - x = 4$$

$$x = 11$$

2 The median is

The median is 12.5:

$$\frac{x + y}{2} = 12.5$$

$$x + y = 25$$

$$y = 25 - 11 = 14$$

FINAL ANSWER

$x = 11,\ y = 14.$

Question 4

5 marks | Set Notation & Venn Diagrams

4 $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{\text{factors of } 6\}$
$B = \{\text{prime numbers}\}$

(a) List the members of the set

(i) $A \cup B$

.....

(1)

(ii) A'

.....

(1)

Harpreet states that $A \cap B = \varnothing$

Harpreet is incorrect.

(b) Explain why.

.....

(1)

C is a set with 4 members such that

the set $A \cap C$ has 2 members
the set $B \cap C$ has 2 members

Set $A \cap C$ and set $B \cap C$ have no members in common.

(c) List the 4 members of set C

.....

(2)

(Total for Question 4 is 5 marks)

1 **Calculate value**

$$\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{1, 2, 3, 6\}$$

$$B = \{2, 3, 5, 7\}$$

2 **Calculate value**

So

$$A \cup B = \{1, 2, 3, 5, 6, 7\}$$

3 **Calculate value**

and

$$A' = \{4, 5, 7, 8, 9, 10\}$$

4 **Harpreet is not correct because**

Harpreet is not correct because 2 and 3 are in both A and B.

$$A \cap B = \{2, 3\}$$

5 **Calculate value**

For C, it has 4 members, $n(A \cap C) = 2$, $n(B \cap C) = 2$, and no member is in both of those overlaps. So choose the two elements of A only and the two elements of B only:

$$C = \{1, 5, 6, 7\}$$

✓ **FINAL ANSWER**

(a)(i) $\{1, 2, 3, 5, 6, 7\}$, (a)(ii) $\{4, 5, 7, 8, 9, 10\}$, (b) Harpreet is incorrect, (c) $C = \{1, 5, 6, 7\}$

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Question 5

4 marks | Circles, Arcs & Sectors

5 The diagram shows the design for a badge, which will be made using wire.
The design is a circle inside a square $ABCD$

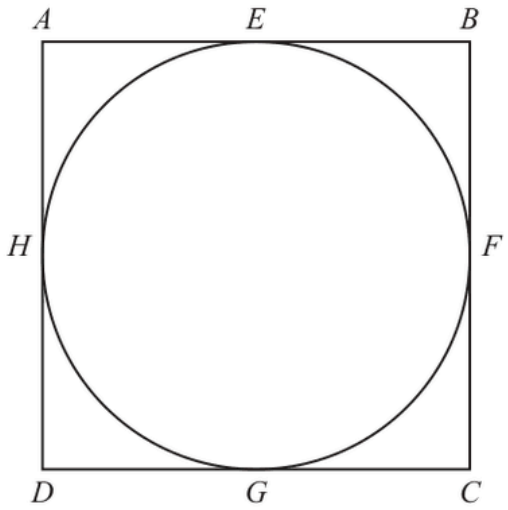


Diagram **NOT** accurately drawn

The circle touches the square at the points E, F, G and H

The area of the square is $81\ \text{cm}^2$

Calculate the total length of wire that will be needed to make the square and the circle.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2HR June 2024

1 **Calculate area**

The square has area $81\ \text{cm}^2$, so its side length is

$$\sqrt{81} = 9\ \text{cm}$$

2 **Perimeter of the square**

Perimeter of the square:

$$4 \times 9 = 36$$

3 **Calculate value**

The circle has diameter $9\ \text{cm}$, so its circumference is

$$\pi d = 9\pi$$

4 **Total wire length**

Total wire length:

$$36 + 9\pi = 64.274\ldots$$

✓ **FINAL ANSWER**

$64.3\ \text{cm}$ to 3 significant figures.

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Question 6

8 marks | Linear Equations

6 (a) Solve $\frac{2f}{3} = 4f - 17$

Show clear algebraic working.

$f =$ .....

(3)

(b) Simplify $(e + 12)^0$ where $e > 0$

.....

(1)

(c) Simplify fully $\frac{12a^4h^6}{4ah^2}$

.....

(2)

(d) Factorise fully $20x^5y + 12x^3y^4$

.....

(2)

(Total for Question 6 is 8 marks)

WORKED SOLUTION

Question 6 · 4MA1/2HR June 2024

1 (a)

(a)

$$\frac{2f}{3} = 4f - 17$$

$$2f = 12f - 51$$

$$10f = 51$$

$$f = \frac{51}{10}$$

2 **(b)**

(b) $(e+12)^0 = 1$.

---

3 **(c)**

(c)

$$\frac{12a^4h^6}{4ah^2} = 3a^3h^4$$

---

4 **(d)**

(d)

$$20x^5y + 12x^3y^4 = 4x^3y(5x^2 + 3y^3)$$

✓ **FINAL ANSWER**

$f = \frac{51}{10}$, 1, $3a^3h^4$, $4x^3y(5x^2 + 3y^3)$

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Question 7

2 marks | Algebraic Roots & Indices

7 $\dfrac{3^{-2}\times 3^{5}}{3^{10}} = 3^{n}$

Find the value of n

$n =$ .....

(Total for Question 7 is 2 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR June 2024

1 Use index laws

$$\frac{3^{-2} \times 3^{5}}{3^{10}} = 3^{n}$$

2 Use index laws

Use index laws:

$$\frac{3^{-2+5}}{3^{10}} = 3^{3-10} = 3^{-7}$$

3 Use index laws

So

$$n = -7$$

FINAL ANSWER

$n = -7$.

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Question 8

3 marks | Reverse Percentages

8 In a sale, all normal prices are reduced by 17%

The sale price of a fridge is 6225 rupees.

Work out the normal price of the fridge.

..... rupees

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR June 2024

1 **A reduction of means the**

A reduction of 17% means the sale price is 83% of the normal price.

$$0.83 \times \text{normal price} = 6225$$

$$\text{normal price} = \frac{6225}{0.83} = 7500$$

FINAL ANSWER

7500 rupees

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Question 9

4 marks | Standard Form

9 (a) Write 6.04×10^5 as an ordinary number.

.....

(1)

(b) Write 0.000 07 in standard form.

.....

(1)

(c) Work out $\dfrac{7.6 \times 10^{10}}{4 \times 10^5 - 2 \times 10^4}$

Give your answer in standard form.

.....

(2)

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR June 2024

1 **Convert standard form**

$$6.04 \times 10^5 = 604000$$

$$0.00007 = 7 \times 10^{-5}$$

2 **Convert standard form**

For part (c), first simplify the denominator:

$$4 \times 10^5 - 2 \times 10^4 = 400000 - 20000 = 380000$$

$$380000 = 3.8 \times 10^5$$

3 **Convert standard form**

Then

$$\frac{7.6 \times 10^{10}}{3.8 \times 10^5} = 2 \times 10^5$$

FINAL ANSWER

(a) 604000, (b) 7×10^{-5}, (c) 2×10^{5}

Question 10

5 marks | Perimeter & Area

10 The diagram shows a hexagon $ABCDEF$

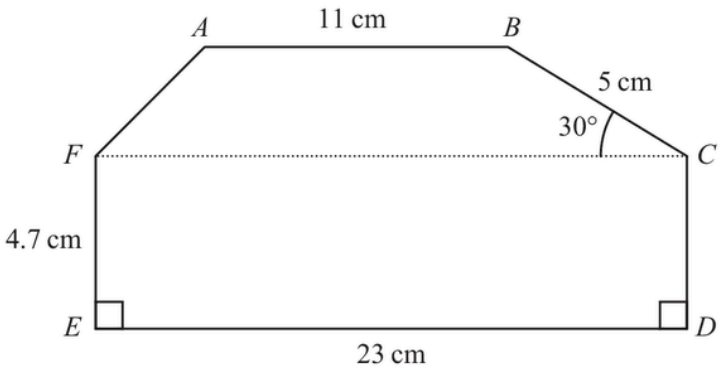


Diagram **NOT** accurately drawn

Angle $BCF = 30°$
AB, FC and ED are parallel.

Calculate the area of $ABCDEF$
Show your working clearly.

..... cm^2

(Total for Question 10 is 5 marks)

1 **The rectangle has area**

The rectangle $EDCF$ has area

$$23 \times 4.7 = 108.1$$

2 **The height of the top**

The height of the top trapezium is

$$5 \sin 30° = 2.5$$

3 **Area of the top trapezium**

Area of the top trapezium:

$$\frac{1}{2}(11 + 23) \times 2.5 = 42.5$$

4 **Total area**

Total area:

$$108.1 + 42.5 = 150.6$$

FINAL ANSWER

$150.6\ \text{cm}^2$.

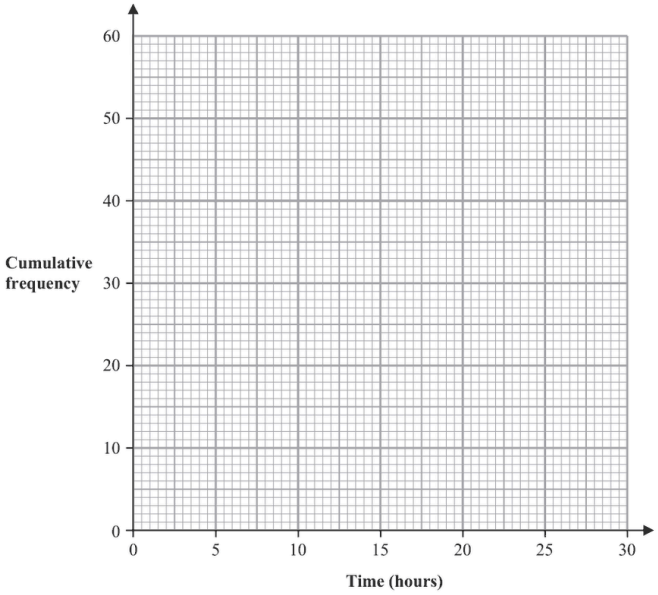
Question 11

7 marks | Cumulative Frequency Diagrams

11 The cumulative frequency table gives information about the time, in hours, that each of 60 workers spent working from home in one week.

Time (t hours)	Cumulative frequency
$0 < t \leqslant 5$	6
$0 < t \leqslant 10$	17
$0 < t \leqslant 15$	27
$0 < t \leqslant 20$	42
$0 < t \leqslant 25$	53
$0 < t \leqslant 30$	60

(a) On the grid below, draw a cumulative frequency graph for the information in the table.



(2)

(b) Use your graph to find an estimate for the interquartile range of the times.

..... hours

(2)

25 workers spent more than W hours working from home.

(c) Use your graph to find an estimate for the value of W

$W =$ .....

(2)

One of the 60 workers is chosen at random.
This worker spent H hours working from home.

(d) Find the probability that $5 < H \leqslant 10$

.....

(1)

(Total for Question 11 is 7 marks)

1

(a) Plot the table

Plot $(5, 6), (10, 17), (15, 27), (20, 42), (25, 53), (30, 60)$, start at $(0, 0)$, and join with a smooth curve.

2

(b) Read the quartiles

Lower quartile at $\frac{60}{4} = 15$: about 9 hours. Upper quartile at 45: about 21.4 hours.

$$\text{IQR} \approx 21.4 - 9.1 \approx 12.3 \text{ hours}$$

(Answers from about 11 to 14 are accepted, depending on the curve.)

3

(c) 25 workers above W

So $60 - 25 = 35$ workers spent W hours or less. Reading across from 35: $W \approx 17.7$.

4

(d) Between 5 and 10 hours

$17 - 6 = 11$ workers, so

$$P(5 < H \leq 10) = \frac{11}{60}$$

✓ **FINAL ANSWER**

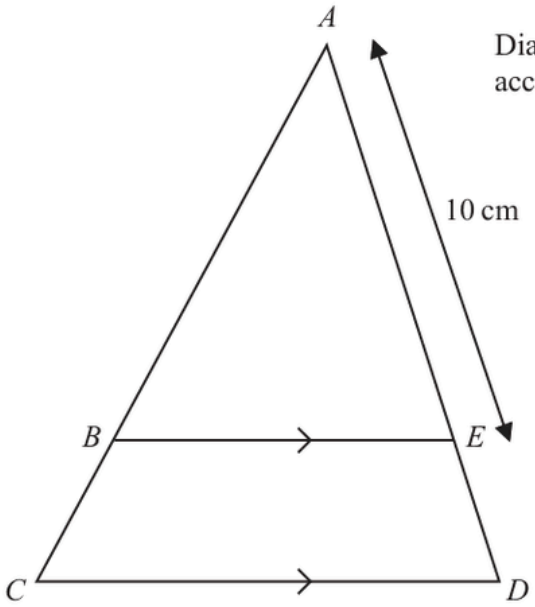
(b) about 12.3 hours (c) $W \approx 17.7$ (d) $\frac{11}{60}$

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Question 12

4 marks | Similar Shapes

12



In the diagram, ABC and AED are straight lines.
BE is parallel to CD

$$AE = 10\,\text{cm} \qquad \text{and} \qquad CD = 1.5 \times BE$$

(a) Work out the length of ED

..... cm

(2)

$AB = (2x + 5)\,\text{cm}$ and $BC = (3x - 5)\,\text{cm}$

(b) Work out the value of x

$x =$ .....

(2)

(Total for Question 12 is 4 marks)

WORKED SOLUTION

Question 12 · 4MA1/2HR June 2024

1 **Find the gradient**

Since $BE \parallel CD$, triangles ABE and ACD are similar.

2 **Find the gradient**

Also

$$CD = 1.5BE$$

so

$$\frac{BE}{CD} = \frac{1}{1.5} = \frac{2}{3}$$

---

3 **Find the gradient**

Therefore

$$\frac{AE}{AD} = \frac{2}{3}$$

---

4 **Find the gradient**

Given $AE = 10$,

$$\frac{10}{AD} = \frac{2}{3}$$

$$AD = 15$$

---

5 **Find the gradient**

So

$$ED = AD - AE = 15 - 10 = 5$$

---

6 **Find the gradient**

For part (b),

$$AB = 2x + 5, \qquad BC = 3x - 5$$

$$AC = AB + BC = 5x$$

---

7 **Find the gradient**

Using similarity,

$$\frac{AB}{AC} = \frac{2}{3}$$

$$\frac{2x+5}{5x} = \frac{2}{3}$$

$$3(2x+5) = 10x$$

$$6x + 15 = 10x$$

$$x = 3.75$$

✓ **FINAL ANSWER**

(a) $ED = 5\ \text{cm}$, (b) $x = 3.75$.

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Question 13

3 marks | Circles, Arcs & Sectors

13 OAB is a sector of a circle with centre O and radius r cm.

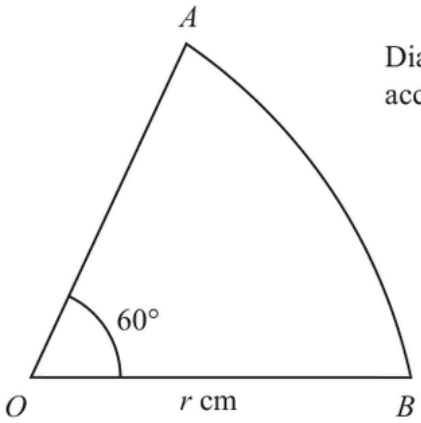


Diagram **NOT** accurately drawn

Angle $AOB = 60°$
The perimeter of the sector is P cm.

Find a formula for P in terms of r
Give your answer in the form $P = r(c\pi + k)$ where c and k are values to be found.

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2HR June 2024

1 The arc length is

The arc length is

$$\frac{60}{360} \times 2\pi r = \frac{\pi r}{3}$$

2 The perimeter is two radii

The perimeter is two radii plus the arc:

$$P = 2r + \frac{\pi r}{3}$$

$$P = r\left(\frac{\pi}{3} + 2\right)$$

FINAL ANSWER

$P = r\left(\frac{1}{3}\pi + 2\right)$.

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Question 14

3 marks | Probability Toolkit

14 Adriana is going to roll a biased dice and spin a biased coin.

The probability that the coin will land on Heads is 0.8
The probability that the dice will land on 6 and the coin will land on Heads is 0.24

Work out the probability that the dice will land on 6 and the coin will land on Tails.

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/2HR June 2024

1 Find the probability of a 6

The dice and the coin are independent:

$$P(6) \times 0.8 = 0.24 \Rightarrow P(6) = 0.3$$

2 6 and Tails

$P(\text{Tails}) = 1 - 0.8 = 0.2$, so

$$0.3 \times 0.2 = 0.06$$

FINAL ANSWER

0.06

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Question 15

3 marks | Sine & Cosine Rules

15

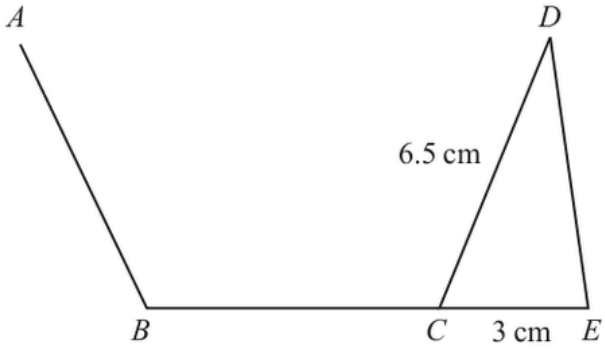


Diagram **NOT** accurately drawn

AB, *BC* and *CD* are three sides of a regular pentagon and *CDE* is a triangle.
BCE is a straight line.

$CD = 6.5\,\text{cm}$ $CE = 3\,\text{cm}$

Work out the area of triangle *CDE*
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/2HR June 2024

1 **A regular pentagon has interior**

A regular pentagon has interior angle

$$\frac{(5-2)\times 180}{5} = 108^\circ$$

2 **Use trigonometry**

Since BCE is a straight line,

$$\angle DCE = 180^\circ - 108^\circ = 72^\circ$$

3 **Area of triangle is**

Area of triangle CDE is

$$\frac{1}{2}ab\sin C$$

$$\text{Area} = \frac{1}{2}(6.5)(3)\sin 72°$$

$$\text{Area} = 9.27\ldots$$

✓ **FINAL ANSWER**

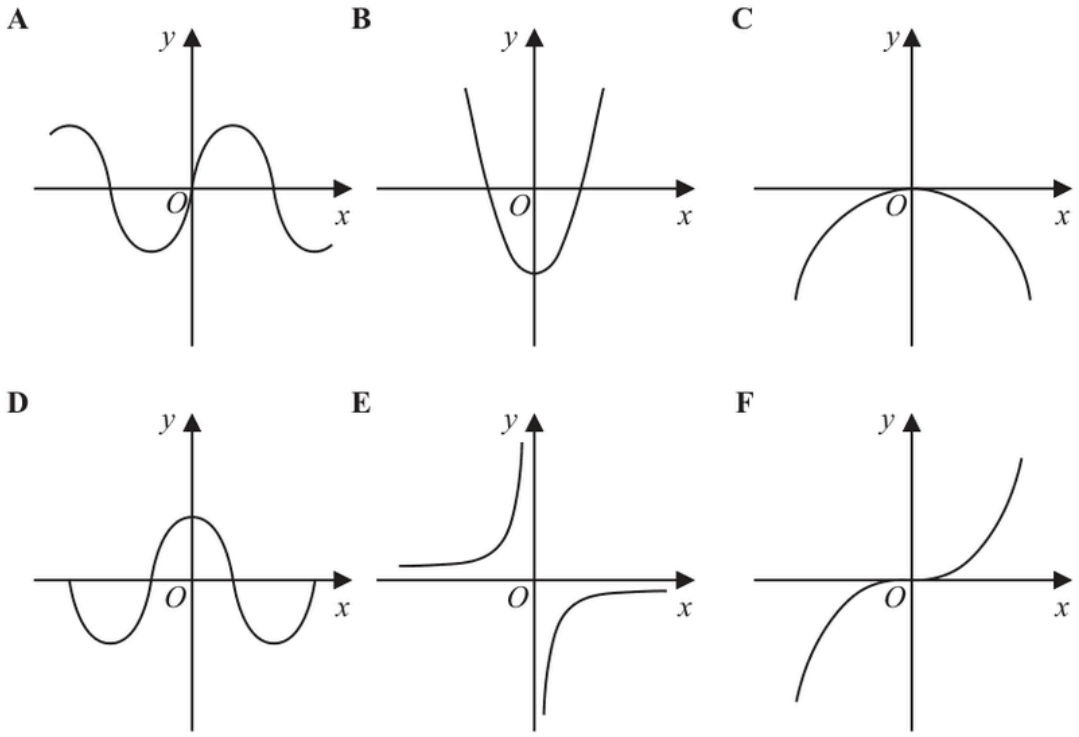
$9.27\ \text{cm}^2$ to 3 significant figures.

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Question 16

2 marks | Graphs of Functions

16 Here are six graphs.



Write down the letter of the graph that could have the equation

(i) $y = -\frac{1}{x}$

.....

(1)

(ii) $y = \sin x°$

.....

(1)

(Total for Question 16 is 2 marks)

WORKED SOLUTION

Question 16 · 4MA1/2HR June 2024

1 **(i) has branches in quadrants**

(i)

$$y = -\frac{1}{x}$$

has branches in quadrants II and IV, so it is **Graph E**.

2 **(ii) is the sine shaped**

(ii)

$$y = \sin x^\circ$$

is the sine-shaped graph through the origin, so it is **Graph A**.

✓ **FINAL ANSWER**

(i) E, (ii) A.

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Question 17

3 marks | Functions

17 $f(x) = \dfrac{x}{2x-4}$ $\quad g(x) = 3x + 1$

Given that $fg(k) = 2$

work out the value of k

$k =$ .....

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2HR June 2024

1 Evaluate fraction

$$f(x) = \frac{k}{2x-4}, \qquad g(x) = 3x + 1$$

2 Given

Given

$$fg(k) = 2$$

3 This means

This means

$$f(g(k)) = 2$$

4 Calculate value

Now

$$g(k) = 3k + 1$$

5 **Evaluate fraction**

So

$$f(g(k)) = \frac{k}{2(3k+1)-4} = \frac{k}{6k-2}$$

$$\frac{k}{6k-2} = 2$$

$$k = 12k - 4$$

$$11k = 4$$

$$k = \frac{4}{11}$$

FINAL ANSWER

$k = \frac{4}{11}$.

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Question 18

2 marks | Algebraic Proof

18 Use algebra to show that $0.\dot{3}0\dot{6} = \frac{34}{111}$

(Total for Question 18 is 2 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR June 2024

1 Let

Let

$$x = 0.\dot{3}0\dot{6}$$

2 Calculate value

Then

$$1000x = 306.\dot{3}0\dot{6}$$

3 Subtract

Subtract:

$$1000x - x = 306$$

$$999x = 306$$

$$x = \frac{306}{999} = \frac{34}{111}$$

FINAL ANSWER

$0.\dot{3}0\dot{6} = \frac{34}{111}$.

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Question 19

3 marks | Rounding, Estimation & Bounds

19 Aviv goes on a cycle journey.

For the cycle journey

average speed = 19 km/h correct to the nearest whole number

time = 1.5 hours correct to one decimal place

Work out the upper bound for the distance Aviv travels.
Give your answer correct to 3 significant figures.

..... km

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2HR June 2024

1 Distance is found using

Distance is found using

$$\text{distance} = \text{speed} \times \text{time}$$

2 Find upper bound

For the upper bound,

$$\text{speed} < 19.5, \quad \text{time} < 1.55$$

$$\text{distance}_{\text{upper}} = 19.5 \times 1.55 = 30.225$$

3 Estimate the value

Correct to 3 significant figures,

$$30.225 \approx 30.2$$

FINAL ANSWER

30.2 km

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Question 20

4 marks | Inequalities (Solving & Graphing)

20 Solve $6x^2 - 7x - 20 > 0$
Show clear algebraic working.

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR June 2024

1 Solve quadratic equation

$$6x^2 - 7x - 20 > 0$$

2 Factorise

Factorise:

$$6x^2 - 7x - 20 = (3x + 4)(2x - 5)$$

3 The critical values are

The critical values are

$$x = -\frac{4}{3} \quad \text{and} \quad x = \frac{5}{2}$$

4 Solve quadratic equation

Since the coefficient of x^2 is positive, the quadratic is greater than 0 outside the roots.

✓ **FINAL ANSWER**

$$x < -\frac{4}{3} \text{ or } x > \frac{5}{2}$$

4MA1/2HR JUNE 2024

Question 21

4 marks | Linear Graphs

21 $ABCD$ is a square.

The point A has coordinates $(-5, 2)$
The point B has coordinates $(3, 5)$

Find an equation of the line that passes through B and C
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR June 2024

1 Calculate value

$$A = (-5, 2), \qquad B = (3, 5)$$

2 The gradient of is

The gradient of AB is

$$\frac{5-2}{3-(-5)} = \frac{3}{8}$$

3 Find the gradient

Since $ABCD$ is a square, BC is perpendicular to AB.

4 Find the gradient

So the gradient of BC is

$$-\frac{8}{3}$$

5 The line through is

The line through $B = (3, 5)$ is

$$y - 5 = -\frac{8}{3}(x - 3)$$

6 **Multiply by 3**

Multiply by 3:

$$3y - 15 = -8x + 24$$

$$8x + 3y - 39 = 0$$

✓ **FINAL ANSWER**

$8x + 3y - 39 = 0$

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Question 22

5 marks | Simultaneous Equations

22 Solve the simultaneous equations

$$x^2 + y^2 = y + 11$$
$$y = 3x - 1$$

Show clear algebraic working.

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2HR June 2024

1 Calculate value

$$x^2 + y^2 = y + 11$$

$$y = 3x - 1$$

2 Substitute

Substitute:

$$x^2 + (3x - 1)^2 = 3x - 1 + 11$$

$$x^2 + 9x^2 - 6x + 1 = 3x + 10$$

$$10x^2 - 9x - 9 = 0$$

$$(5x + 3)(2x - 3) = 0$$

3 Evaluate fraction

So

$$x = -\frac{3}{5} \quad \text{or} \quad x = \frac{3}{2}$$

4 **Find**

Find y:

$$x = -\frac{3}{5} \Rightarrow y = -\frac{14}{5}$$

$$x = \frac{3}{2} \Rightarrow y = \frac{7}{2}$$

✓ **FINAL ANSWER**

$(x, y) = \left(-\frac{3}{5}, -\frac{14}{5}\right)$ or $\left(\frac{3}{2}, \frac{7}{2}\right)$

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Question 23

2 marks | Transformations of Graphs

23 A curve has equation $y = f(x)$
The coordinates of the minimum point on this curve are $(6, -3)$

Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x) + 10$

(.....,.....)
(1)

(ii) $y = f(3x)$

(.....,.....)
(1)

(Total for Question 23 is 2 marks)

WORKED SOLUTION

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1 The minimum point of is

The minimum point of $y = f(x)$ is $(6, -3)$.

2 Calculate value

For

$$y = f(x) + 10$$

3 the graph moves 10 units

the graph moves 10 units up:

$$(6, -3) \to (6, 7)$$

4 Calculate value

For

$$y = f(3x)$$

5 the coordinate is divided by

the x-coordinate is divided by 3:

$$(6, -3) \to (2, -3)$$

FINAL ANSWER

$(6, 7)$ and $(2, -3)$

Question 24

5 marks | Area & Volume of Similar Shapes

24 The diagram shows a solid, **S**, made from a cone and a hemisphere.

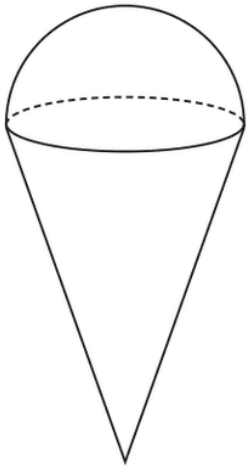


Diagram **NOT** accurately drawn

The centre of the circular face of the cone coincides with the centre of the flat surface of the hemisphere.

The radius of the circular face of the cone, x cm, is equal to the radius of the hemisphere.

The total height of **S** is 4 × the radius of the hemisphere.

A separate sphere has radius kx cm.
The volume of this sphere is 12.5 × the volume of **S**

(a) Work out the value of k

$k =$ .....

(4)

A solid, **T**, is similar to solid **S**

The volume of **T** is 512 × the volume of **S**

The total surface area of **T** is d × the total surface area of **S**

(b) Find the value of d

$d =$ .....

(1)

(Total for Question 24 is 5 marks)

1 **The hemisphere has radius**

The hemisphere has radius x, so its height is x.

2 **The total height of solid**

The total height of solid S is $4x$, so the cone height is

$$4x - x = 3x$$

3 **Volume of the cone**

Volume of the cone:

$$\frac{1}{3}\pi x^2(3x) = \pi x^3$$

4 **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi x^3$$

5 **Calculate volume**

So the volume of solid S is

$$\pi x^3 + \frac{2}{3}\pi x^3 = \frac{5}{3}\pi x^3$$

6 **Calculate volume**

The sphere has radius kx, so its volume is

$$\frac{4}{3}\pi(kx)^3 = \frac{4}{3}\pi k^3 x^3$$

7 **This is times the volume**

This is 12.5 times the volume of S:

$$\frac{4}{3}\pi k^3 x^3 = 12.5\left(\frac{5}{3}\pi x^3\right)$$

8 **Cancel**

Cancel $\frac{1}{3}\pi x^3$:

$$4k^3 = 62.5$$

$$k^3 = 15.625 = \frac{125}{8}$$

$$k = 2.5$$

---

9 **Calculate volume**

For solid T, the volume scale factor is 512.

$$512 = 8^3$$

---

10 **Calculate area**

So the linear scale factor is 8, and the surface area scale factor is

$$8^2 = 64$$

✓ **FINAL ANSWER**

$$\boxed{k = 2.5,\ d = 64}$$

Question 25

6 marks | Vectors

25 $OPQR$ is a parallelogram.

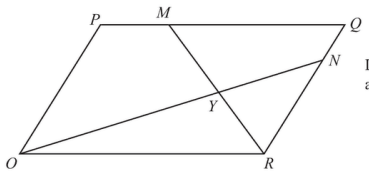


Diagram **NOT** accurately drawn

$\overrightarrow{OP} = 2\mathbf{a}$ and $\overrightarrow{OR} = 3\mathbf{b}$

The point M lies on PQ such that $PM = \frac{1}{4}PQ$

The point N lies on RQ such that $RN = \frac{4}{5}RQ$

(a) Find, in terms of **a** and **b**, giving your answers in simplest form

(i) $\overrightarrow{ON}$

.....

(1)

(ii) $\overrightarrow{MR}$

.....

(1)

MR and ON intersect at the point Y

Given that

$$OY = k \times ON$$

(b) use a vector method to find the value of k

$k =$ .....

(4)

(Total for Question 25 is 6 marks)

1 **Calculate value**

$$\overrightarrow{OP} = 2a, \qquad \overrightarrow{OR} = 3b$$

2 **Calculate value**

So

$$\overrightarrow{OQ} = 2a + 3b$$

3 **Evaluate fraction**

Since $PM : PQ = 1 : 4$,

$$\overrightarrow{PM} = \frac{1}{4}\overrightarrow{PQ}$$

$$\overrightarrow{PM} = \frac{1}{4}(3b) = \frac{3}{4}b$$

$$\overrightarrow{OM} = 2a + \frac{3}{4}b$$

4 **Evaluate fraction**

Since $RN : RQ = 4 : 5$,

$$\overrightarrow{RN} = \frac{4}{5}\overrightarrow{RQ}$$

$$\overrightarrow{RN} = \frac{4}{5}(2a) = \frac{8}{5}a$$

$$\overrightarrow{ON} = 3b + \frac{8}{5}a$$

5 **Evaluate fraction**

So

$$\overrightarrow{MR} = \overrightarrow{OR} - \overrightarrow{OM}$$

$$\overrightarrow{MR} = 3b - \left(2a + \frac{3}{4}b\right)$$

$$\overrightarrow{MR} = -2a + \frac{9}{4}b$$

---

6 **Let be on**

Let Y be on ON, so

$$\overrightarrow{OY} = k\overrightarrow{ON} = k\left(\frac{8}{5}a + 3b\right)$$

---

7 **Evaluate fraction**

Also Y is on MR, so

$$\overrightarrow{OY} = \overrightarrow{OM} + t\overrightarrow{MR}$$

$$\overrightarrow{OY} = 2a + \frac{3}{4}b + t\left(-2a + \frac{9}{4}b\right)$$

---

8 **Evaluate fraction**

Compare coefficients of a and b:

$$\frac{8}{5}k = 2 - 2t$$

$$3k = \frac{3}{4} + \frac{9}{4}t$$

---

9 **Solve equation**

From the first equation,

$$t = 1 - \frac{4}{5}k$$

---

10 **Substitute into the second equation**

Substitute into the second equation:

$$3k = \frac{3}{4} + \frac{9}{4}\left(1 - \frac{4}{5}k\right)$$

$$3k = 3 - \frac{9}{5}k$$

$$\frac{24}{5}k = 3$$

$$k = \frac{5}{8}$$

✓ **FINAL ANSWER**

$\overrightarrow{ON} = \frac{8}{5}a + 3b$, $\overrightarrow{MR} = -2a + \frac{9}{4}b$, and $k = \frac{5}{8}$

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Question 26

4 marks | Algebraic Fractions

26 Write $4 - \left[(3x - 5) \div \dfrac{3x^2 + x - 10}{4x - 1}\right]$ as a single fraction in its simplest form.

(Total for Question 26 is 4 marks)

WORKED SOLUTION

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1 Use trigonometry

$$4 - \left[(3x - 5) \div \frac{3x^2 + x - 10}{4x - 1}\right]$$

2 Factor the quadratic

Factor the quadratic:

$$3x^2 + x - 10 = (3x - 5)(x + 2)$$

3 Use trigonometry

So the expression inside the square brackets is

$$(3x - 5) \div \frac{(3x - 5)(x + 2)}{4x - 1}$$

4 Dividing by a fraction means

Dividing by a fraction means multiplying by its reciprocal:

$$(3x - 5) \times \frac{4x - 1}{(3x - 5)(x + 2)}$$

5 Cancel

Cancel $3x - 5$:

$$\frac{4x - 1}{x + 2}$$

6 Use trigonometry

Therefore the full expression is

$$4 - \frac{4x - 1}{x + 2}$$

7 Use the common denominator

Use the common denominator $x+2$:

$$\frac{4(x+2)-(4x-1)}{x+2}$$

$$=\frac{4x+8-4x+1}{x+2}$$

$$=\frac{9}{x+2}$$

FINAL ANSWER

$$\boxed{\frac{9}{x+2}}$$