E

Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

4MA1/2HR June 2023 Worked Solutions

June 2023 paper rebuilt with the worked solution after each question.

26	2020–2026	Single-paper solutions
paper questions	52 past papers	checked by Dr Eslam



Dr Eslam Ahmed
Assistant Lecturer, Faculty of Engineering, Cairo University
eliteigcse.com · free for every student

PAST PAPER

4MA1/2HR June 2023

June 2023

26	100	2 h
questions	marks	exam time

Each question is followed by its worked solution and final answer.

4MA1/2HR JUNE 2023

Question 1

4 marks | Expanding Brackets

1 $P = m^2 - 4c$

(a) Work out the value of P when $m = -5$ and $c = 3$

$P =$ .....

(2)

(b) Expand and simplify $(x + 5)(x - 7)$

.....

(2)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR June 2023

1 **Expand brackets**

$$P = m^2 - 4c = (-5)^2 - 4(3) = 25 - 12 = 13$$

$$(x + 5)(x - 7) = x^2 - 7x + 5x - 35 = x^2 - 2x - 35$$

FINAL ANSWER

$P = 13$, and $x^2 - 2x - 35$

4MA1/2HR JUNE 2023

Question 2

4 marks | Prime Factors, HCF & LCM

2 Sandeep wants to buy some packets of pens and some boxes of pencils for his stationery shop.

Each packet of pens contains 9 pens.
Each box of pencils contains 12 pencils.

Each packet of pens costs £7.60
Each box of pencils costs £4.80

Sandeep can only buy full packets of pens and full boxes of pencils.
He wants to buy exactly the same number of pens as pencils.

Work out the minimum amount Sandeep needs to pay.

£.....

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/2HR June 2023

1 Find the LCM

Pens come in packs of 9 and pencils come in boxes of 12.

2 Find the LCM

The smallest equal number of pens and pencils is

$$\text{LCM}(9, 12) = 36$$

3 Find the LCM

So Sandeep buys

$$\text{4 packs of pens}$$

4 Find the LCM

and

$$\text{3 boxes of pencils}$$

5 Cost

Cost:

$$4(7.60) + 3(4.80) = 30.40 + 14.40 = 44.80$$

FINAL ANSWER

$\pounds 44.80$.

4MA1/2HR JUNE 2023

Question 3

3 marks | Standard & Compound Units

3 Anjali travels on the Eurostar train from Paris to Amsterdam.

The distance the train travels between Paris and Amsterdam is 515 km.
The time taken by the train to travel between Paris and Amsterdam is 3 hours 18 minutes.

Work out the average speed of the train.
Give your answer in km/h correct to the nearest whole number.

..... km/h

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2HR June 2023

1 Time hours minutes

Time $=$ 3 hours 18 minutes:

$$3 + \frac{18}{60} = 3.3 \text{ hours}$$

$$\text{average speed} = \frac{515}{3.3} = 156.06\ldots$$

FINAL ANSWER

156 km/h.

4MA1/2HR JUNE 2023

Question 4

2 marks | Sequences

4 Here are the first four terms of an arithmetic sequence.

38 31 24 17

Find an expression, in terms of n, for the nth term of the sequence.

(Total for Question 4 is 2 marks)

WORKED SOLUTION

Question 4 · 4MA1/2HR June 2023

1 The sequence is

The sequence is

$$38,\ 31,\ 24,\ 17$$

2 The common difference is

The common difference is -7.

$$\text{nth term} = 38 + (n-1)(-7)$$

$$= 38 - 7n + 7 = 45 - 7n$$

FINAL ANSWER

$45 - 7n$.

4MA1/2HR JUNE 2023

Question 5

4 marks | Perimeter & Area

5 A field is in the shape of a trapezium.

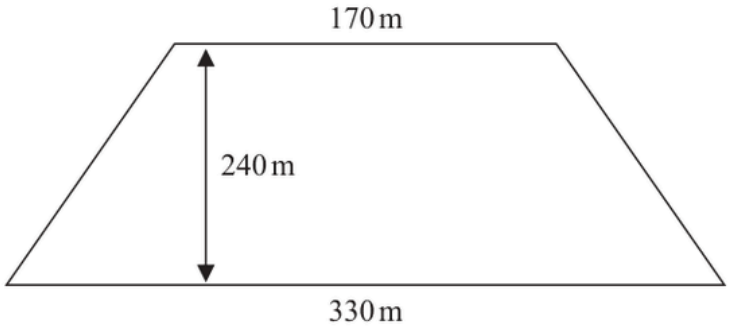


Diagram **NOT** accurately drawn

The field is sold for a price of \$49 650

Given that 1 hectare = 10 000 m²

work out the average price of the field per hectare.

\$.....

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2HR June 2023

1 Calculate area

$$\text{area} = \frac{1}{2}(170 + 330) \times 240 = 60000 \text{ m}^2$$

$$60000 \text{ m}^2 = 6 \text{ hectares}$$

$$\text{price per hectare} = 49650 \div 6 = 8275$$

FINAL ANSWER

\$8275.

4MA1/2HR JUNE 2023

Question 6

7 marks | Reverse Percentages

6 In his previous job, Pierre was paid 400 euros in total for working a 5-day week.

In his new job, Pierre is paid 14 euros per hour.
In his new job, Pierre works for 7 hours each day for a 5-day week.

(a) Work out the percentage increase in the amount that Pierre is paid for a 5-day week.

..... %
(4)

Marie changes her job.

Her salary decreases by 6%
Her new salary is 23 030 euros.

(b) Work out Marie's salary before she changes her job.

..... euros
(3)

(Total for Question 6 is 7 marks)

WORKED SOLUTION

Question 6 · 4MA1/2HR June 2023

1 **Pierre s old weekly pay**

Pierre's old weekly pay is 400 euros.

2 **His new weekly pay is**

His new weekly pay is

$$14 \times 7 \times 5 = 490$$

3 **The increase is**

The increase is

$$490 - 400 = 90$$

4 **Percentage increase**

Percentage increase:

$$\frac{90}{400} \times 100 = 22.5\%$$

5 **Marie s new salary is**

Marie's new salary is after a 6% decrease, so it is 94% of her old salary.

$$0.94 \times \text{old salary} = 23030$$

$$\text{old salary} = 23030 \div 0.94 = 24500$$

✓ **FINAL ANSWER**

Pierre's pay increased by 22.5%. Marie's old salary was 24500 euros.

4MA1/2HR JUNE 2023

Question 7

2 marks | Algebraic Roots & Indices

7 (a) Simplify $(4^{-2})^0$

.....

(1)

$3^{-14} \times 3^8 = 3^m$

(b) Find the value of m

$m =$ .....

(1)

(Total for Question 7 is 2 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR June 2023

1 Use index laws

Any non-zero expression to the power 0 is 1:

$$(4^{-2})^0 = 1$$

2 Use index laws

For part (b),

$$3^{-14} \times 3^8 = 3^{-14+8} = 3^{-6}$$

3 Use index laws

So

$$m = -6$$

FINAL ANSWER

(a) 1, (b) $m = -6$.

4MA1/2HR JUNE 2023

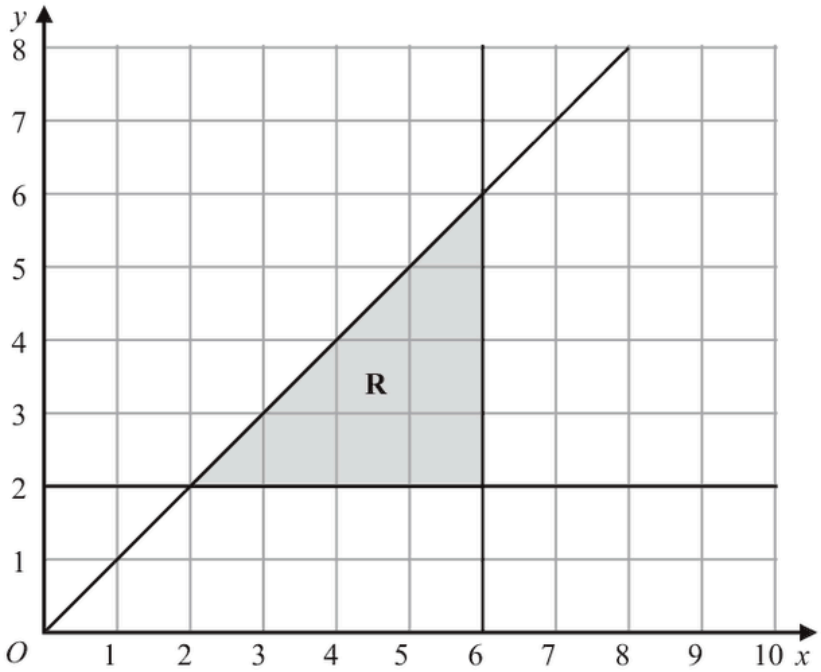
Question 8

5 marks | Inequalities (Solving & Graphing)

8 (a) Solve $9 - 4x > 17$

.....

(2)



(b) Write down the three inequalities that represent the shaded region **R**

.....

(3)

(Total for Question 8 is 5 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR June 2023

1 **Read the graph**

For part (a),

$$9 - 4x > 17$$

$$-4x > 8$$

$$x < -2$$

2 **Read the graph**

For part (b), the region is above $y = 2$, left of $x = 6$, and below $y = x$:

$$y \geq 2$$

$$x \leq 6$$

$$y \leq x$$

✓ **FINAL ANSWER**

(a) $x < -2$. (b) $y \geq 2$, $x \leq 6$, $y \leq x$.

4MA1/2HR JUNE 2023

Question 9

6 marks | Volume & Surface Area

9 The diagram shows a rectangular sheet of metal $ABCD$

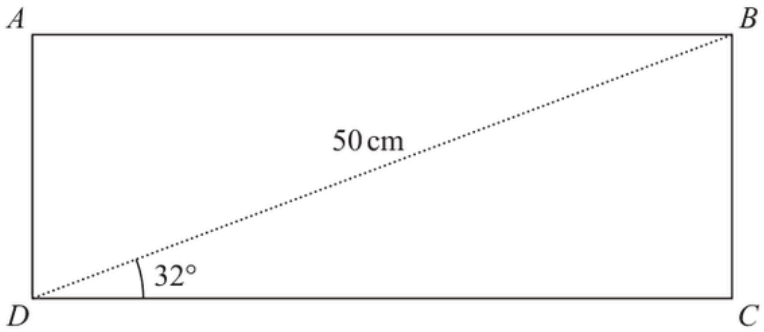


Diagram **NOT** accurately drawn

$BD = 50\,\text{cm}$ and angle $BDC = 32°$

Nasser joins side AD to side BC to form a cylinder.

BC is the height of the cylinder.
DC is the circumference of the cross section of the cylinder.

Work out the volume, in cm^3, of the cylinder.
Give your answer correct to 3 significant figures.

..... cm^3

(Total for Question 9 is 6 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR June 2023

1 **Use trigonometry**

In triangle DBC,

$$DC = 50\cos 32° = 42.402\ldots, \qquad BC = 50\sin 32° = 26.496\ldots$$

2 The circumference of the cylinder

The circumference of the cylinder is DC, so

$$2\pi r = DC$$

$$r = \frac{42.402\ldots}{2\pi}$$

3 Volume

Volume:

$$V = \pi r^2 h = \pi\left(\frac{42.402\ldots}{2\pi}\right)^2 (26.496\ldots) = 3790.974\ldots$$

FINAL ANSWER

$3.79 \times 10^3\ \mathrm{cm}^3$ to 3 significant figures.

4MA1/2HR JUNE 2023

Question 10

3 marks | Statistics Toolkit

10 Gemara works as a taxi driver.
Last week, he recorded the following information about the distances he drove.

For the 5 days from Monday to Friday, the mean number of kilometres he drove was 104
For the 7 days from Monday to Sunday, the mean number of kilometres he drove was 127

On Saturday, Gemara drove 132 kilometres.

Work out the number of kilometres he drove on Sunday.

..... kilometres

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2HR June 2023

1 Total for Monday to Friday

Total for Monday to Friday:

$$5 \times 104 = 520$$

2 Total for Monday to Sunday

Total for Monday to Sunday:

$$7 \times 127 = 889$$

3 Calculate value

So the total for Saturday and Sunday is

$$889 - 520 = 369$$

4 Calculate value

Saturday was $132\ \text{km}$, so

$$369 - 132 = 237$$

FINAL ANSWER

$237\ \text{km}$.

4MA1/2HR JUNE 2023

Question 11

2 marks | Algebraic Roots & Indices

11 Express $\left(\frac{m^6k^{10}}{25}\right)^{\frac{3}{2}}$ in the form $\frac{m^ak^b}{c}$ where a, b and c are integers to be found.

(Total for Question 11 is 2 marks)

WORKED SOLUTION

Question 11 · 4MA1/2HR June 2023

1 Convert standard form

$$\left(\frac{m^6k^{10}}{25}\right)^{\frac{3}{2}} = \frac{(m^6)^{\frac{3}{2}}(k^{10})^{\frac{3}{2}}}{25^{\frac{3}{2}}}$$

$$(m^6)^{\frac{3}{2}} = m^9$$

$$(k^{10})^{\frac{3}{2}} = k^{15}$$

2 Convert standard form

Also,

$$25^{\frac{3}{2}} = (\sqrt{25})^3 = 5^3 = 125$$

3 Convert standard form

So

$$\left(\frac{m^6k^{10}}{25}\right)^{\frac{3}{2}} = \frac{m^9k^{15}}{125}$$

✓ **FINAL ANSWER**

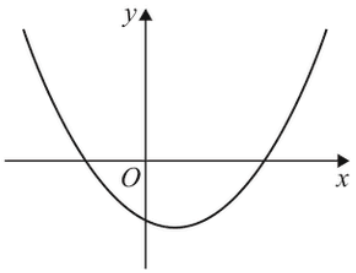
$$\frac{m^9k^{15}}{125}$$

Question 12

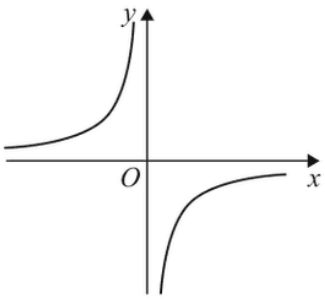
3 marks | Graphs of Functions

12 Here are six graphs.

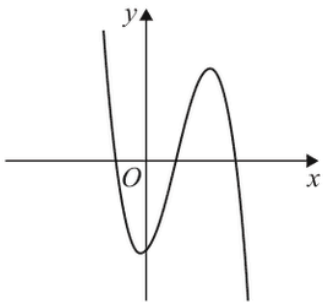
Graph **A**



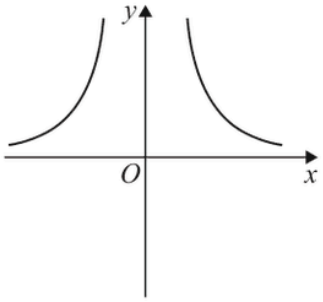
Graph **B**



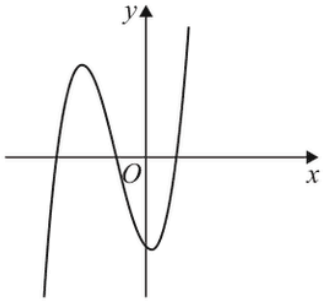
Graph **C**



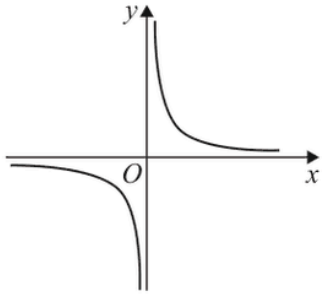
Graph **D**



Graph **E**



Graph **F**



Write down the letter of the graph of

(a) $y = \frac{10}{x^2}$

.....

(1)

(b) $y = x - 3 + 3x^2 - x^3$

.....

(1)

(c) $y = -\frac{3}{x}$

.....

(1)

(Total for Question 12 is 3 marks)

1 **Solve equation**

For

$$y = \frac{10}{x^2}$$

the graph is positive on both sides and symmetrical in the y-axis.

2 **This is Graph D**

This is Graph D.

3 **Solve equation**

For

$$y = x - 3 + 3x^2 - x^3$$

the leading term is $-x^3$, so it is a cubic falling to the right with two turning points.

4 **This is Graph C**

This is Graph C.

5 **Solve equation**

For

$$y = -\frac{3}{x}$$

the graph is a negative reciprocal graph in quadrants 2 and 4.

6 **This is Graph B**

This is Graph B.

✓ **FINAL ANSWER**

D, C, B.

4MA1/2HR JUNE 2023

Question 13

3 marks | Compound Interest & Depreciation

13 Feruzi invests 80 000 Kenyan shillings (KES)
He invests the money for 3 years at $x\%$ compound interest each year.

At the end of 3 years, the total interest he receives is 6151.25 KES

Work out the value of x

$x =$ .....

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2HR June 2023

1 The final value is

The final value is

$$80000 + 6151.25 = 86151.25$$

2 Let the annual compound interest

Let the annual compound interest rate be $x\%$.

$$80000\left(1 + \frac{x}{100}\right)^3 = 86151.25$$

$$1 + \frac{x}{100} = \sqrt[3]{\frac{86151.25}{80000}} = 1.025$$

$$x = 2.5$$

FINAL ANSWER

$x = 2.5$

4MA1/2HR JUNE 2023

Question 14

4 marks | Statistics Toolkit

14 Akari played a computer game eleven times.
Here are her scores.

25 20 28 27 26 22 23 29 20 29 26

(a) Find the interquartile range of her scores.

.....

(3)

Machi played the same computer game eleven times.

The interquartile range for Machi's scores was 9

(b) Who had the more consistent scores, Akari or Machi?
Give a reason for your answer.

.....

.....

(1)

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/2HR June 2023

1 **Akari s scores in order**

Akari's scores in order are

$$20,\ 20,\ 22,\ 23,\ 25,\ 26,\ 26,\ 27,\ 28,\ 29,\ 29$$

$$Q_1 = 22, \qquad Q_3 = 28$$

$$\text{IQR} = 28 - 22 = 6$$

2 **Machi s interquartile range is**

Machi's interquartile range is $\mathbf{9}$.

3 **Akari is more consistent because**

Akari is more consistent because her interquartile range is smaller.

FINAL ANSWER

Akari's IQR is $\mathbf{6}$, and Akari is more consistent.

4MA1/2HR JUNE 2023

Question 15

3 marks | Probability Toolkit

15 Osvaldo has a biased coin.
He spins the coin three times.

The probability that the coin lands on a head three times is $\frac{27}{64}$

Work out the probability that the coin will land on a tail three times.

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/2HR June 2023

1 Find the probability of a head

$$P(H)^3 = \frac{27}{64} \Rightarrow P(H) = \sqrt[3]{\frac{27}{64}} = \frac{3}{4}$$

2 Three tails

$P(T) = 1 - \frac{3}{4} = \frac{1}{4}$, so

$$P(TTT) = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$$

FINAL ANSWER

$$\frac{1}{64}$$

4MA1/2HR JUNE 2023

Question 16

3 marks | Algebraic Proof

16 Show that $\dfrac{2\sqrt{3}}{\sqrt{3}-1}$ can be written in the form $a+\sqrt{a}$ where a is an integer.

Show your working clearly.

(Total for Question 16 is 3 marks)

WORKED SOLUTION

Question 16 · 4MA1/2HR June 2023

1 Simplify surd

$$\frac{2\sqrt{3}}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{2\sqrt{3}(\sqrt{3}+1)}{3-1}$$

$$=\frac{6+2\sqrt{3}}{2}=3+\sqrt{3}$$

2 This is in the form

This is in the form $a+\sqrt{a}$, where $a=3$.

FINAL ANSWER

$3+\sqrt{3}$, so $a=3$.

4MA1/2HR JUNE 2023

Question 17

4 marks | Rearranging Formulae

17 Make x the subject of $y = \sqrt[3]{\dfrac{6+5x}{x+4}}$

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/2HR June 2023

1 Rearrange formula

$$y = \sqrt[3]{\frac{6+5x}{x+4}}$$

$$y^3 = \frac{6+5x}{x+4}$$

$$y^3(x+4) = 6+5x$$

$$x(y^3-5) = 6-4y^3$$

FINAL ANSWER

$$x = \frac{6-4y^3}{y^3-5}$$

4MA1/2HR JUNE 2023

Question 18

3 marks | Circle Theorems

18

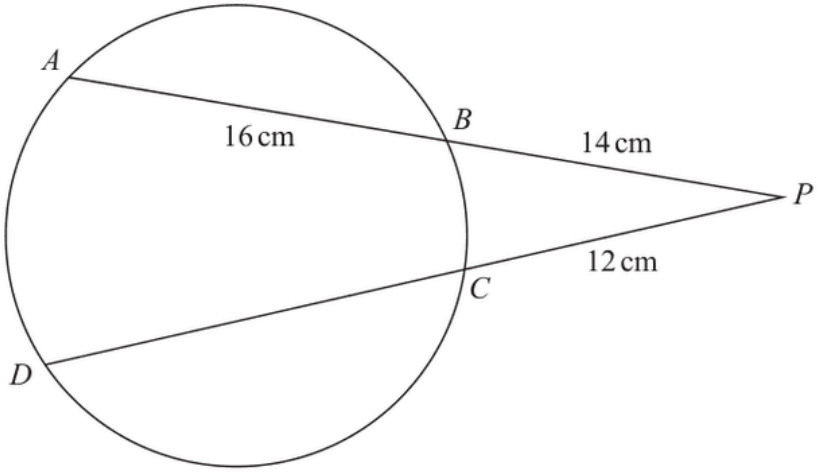


Diagram **NOT** accurately drawn

A, B, C and D are points on a circle.

ABP and DCP are straight lines.

$AB = 16\,\text{cm}$ $BP = 14\,\text{cm}$ $CP = 12\,\text{cm}$

Work out the length of DC

..... cm

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR June 2023

1 Use circle theorem

From the external point P, two secants meet the circle.

2 Use circle theorem

For the upper secant,

$$PB = 14, \qquad PA = 14 + 16 = 30$$

3 Use circle theorem

For the lower secant,

$$PC = 12, \qquad PD = 12 + DC$$

4 **Use circle theorem**

Using the secant theorem:

$$PB \times PA = PC \times PD$$

$$14 \times 30 = 12(12 + DC)$$

$$420 = 144 + 12DC$$

$$276 = 12DC$$

$$DC = 23$$

✓ **FINAL ANSWER**

$DC = 23$ cm.

4MA1/2HR JUNE 2023

Question 19

4 marks | Probability Toolkit

19 30 adults booked to stay in a hotel.

19 adults booked breakfast
15 adults booked dinner
4 adults did not book breakfast or dinner

Some adults booked breakfast **and** dinner.

Meihui chooses at random two of the 30 adults.

Work out the probability that these two adults each booked breakfast **and** dinner.

(Total for Question 19 is 4 marks)

WORKED SOLUTION

Question 19 · 4MA1/2HR June 2023

1 **Adults who booked breakfast and dinner**

$30 - 4 = 26$ adults booked at least one meal.

$$n(B \cap D) = 19 + 15 - 26 = 8$$

2 **Choose two of them (without replacement)**

$$\frac{8}{30} \times \frac{7}{29} = \frac{56}{870} = \frac{28}{435}$$

✓ **FINAL ANSWER**

$$\frac{28}{435}$$

4MA1/2HR JUNE 2023

Question 20

2 marks | Circle Theorems

20 A, B and C are points on a circle.

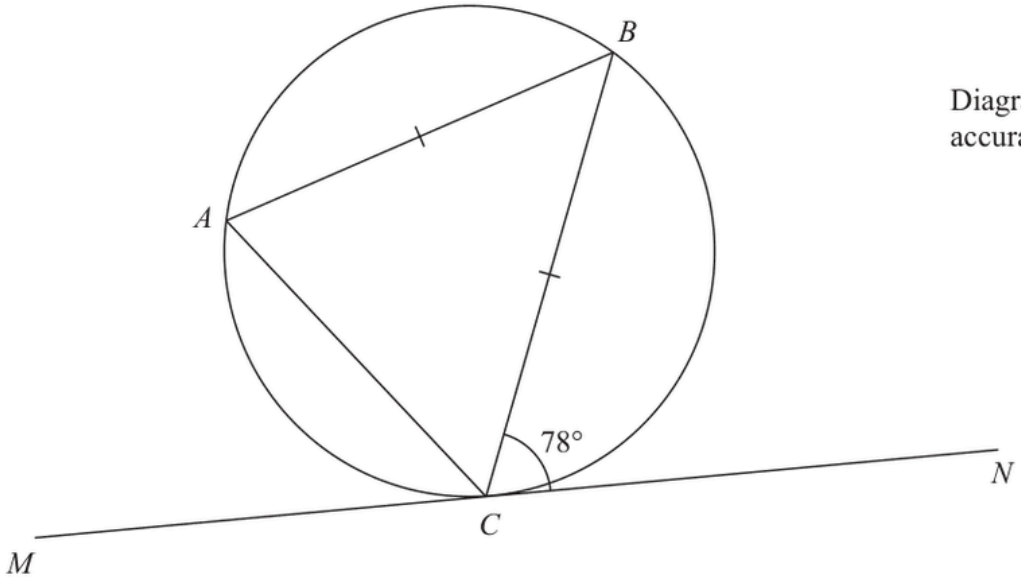


Diagram **NOT** accurately drawn

MN is the tangent to the circle at C

$AB = CB$

Angle $BCN = 78°$

Find the size of angle ABC

.....°

(Total for Question 20 is 2 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR June 2023

1 **The angle between the tangent**

The angle between the tangent CN and the chord CB is $78°$.

2 **Use trigonometry**

By the alternate segment theorem,

$$\angle BAC = 78°$$

3 **Calculate value**

Also,

$$AB = CB$$

---

4 **Use trigonometry**

so triangle ABC is isosceles and the base angles are equal:

$$\angle BAC = \angle ACB = 78^\circ$$

---

5 **Use trigonometry**

Using the angle sum of a triangle:

$$\angle ABC = 180^\circ - 78^\circ - 78^\circ$$

$$\angle ABC = 24^\circ$$

✓ **FINAL ANSWER**

$\boxed{24^\circ}$

4MA1/2HR JUNE 2023

Question 21

5 marks | Linear Graphs

21 Work out the coordinates of the points of intersection of

$$y - 2x = 1 \quad \text{and} \quad y^2 + xy = 7$$

Show clear algebraic working.

(..... , ...)

(..... , ...)

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR June 2023

1 **The line is**

The line is

$$y - 2x = 1$$

2 **Calculate value**

So

$$y = 2x + 1$$

3 **Calculate value**

Substitute into

$$y^2 + xy = 7$$

$$(2x + 1)^2 + x(2x + 1) = 7$$

$$4x^2 + 4x + 1 + 2x^2 + x = 7$$

$$6x^2 + 5x - 6 = 0$$

4 **Factorise**

Factorise:

$$(3x - 2)(2x + 3) = 0$$

5 **Evaluate fraction**

So

$$x = \frac{2}{3} \quad \text{or} \quad x = -\frac{3}{2}$$

---

6 **Evaluate fraction**

Using $y = 2x + 1$:

$$x = \frac{2}{3} \Rightarrow y = \frac{7}{3}$$

$$x = -\frac{3}{2} \Rightarrow y = -2$$

✓ **FINAL ANSWER**

$\left(\frac{2}{3}, \frac{7}{3}\right)$ and $\left(-\frac{3}{2}, -2\right)$

Question 22

5 marks | 3D Pythagoras & Trigonometry

22 Here is a cuboid $ABCDEFGH$

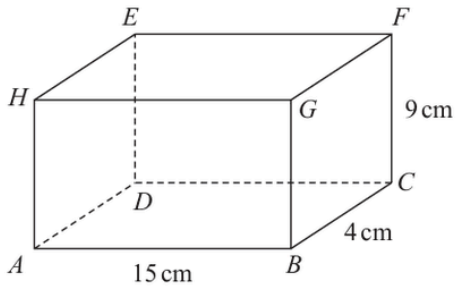


Diagram **NOT** accurately drawn

$AB = 15\,\text{cm}$ $BC = 4\,\text{cm}$ $CF = 9\,\text{cm}$

(a) Work out the length of BE
Give your answer correct to 3 significant figures.

..... cm

(2)

Here is a cuboid $PQRSTUVW$

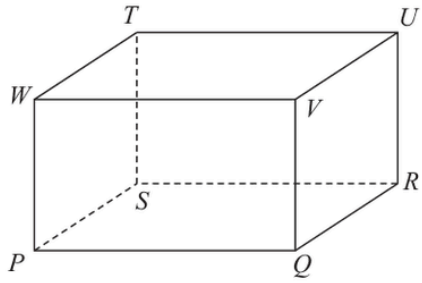


Diagram **NOT** accurately drawn

$PR = 42\,\text{cm}$

The size of the angle between PU and the plane $PQRS$ is 30°

M is the midpoint of PR

(b) Work out the size of angle UMR
Give your answer correct to 3 significant figures.

.....°

(3)

(Total for Question 22 is 5 marks)

1 **Find the gradient**

For part (a), BE is the space diagonal using the three perpendicular lengths:

$$AB = 15, \qquad BC = 4, \qquad CF = 9$$

$$BE^2 = 15^2 + 4^2 + 9^2$$

$$BE^2 = 225 + 16 + 81 = 322$$

$$BE = \sqrt{322} = 17.9 \text{ cm}$$

2 **Use trigonometry**

For part (b), the projection of PU onto the plane $PQRS$ is PR.

$$PR = 42$$

3 **The angle between and the**

The angle between PU and the plane is $30°$, so

$$\tan 30° = \frac{RU}{PR}$$

$$RU = 42 \tan 30°$$

4 **Find the midpoint**

Since M is the midpoint of PR,

$$MR = 21$$

5 **Use trigonometry**

In right triangle MRU,

$$\tan \angle UMR = \frac{RU}{MR}$$

$$\tan \angle UMR = \frac{42 \tan 30°}{21}$$

$$\tan \angle UMR = 2 \tan 30°$$

$$\angle UMR = 49.1°$$

FINAL ANSWER

$BE = 17.9\ \text{cm}$, $\angle UMR = 49.1°$

4MA1/2HR JUNE 2023

Question 23

5 marks | Sequences

23 Here are the first three terms of an arithmetic sequence.

$$8p \qquad 7p - 3 \qquad 4p + 2$$

The sum of the first n terms of the sequence is -1914

Work out the value of n
Show your working clearly.

$n =$ .....

(Total for Question 23 is 5 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR June 2023

1 The first three terms are

The first three terms are

$$8p, \quad 7p - 3, \quad 4p + 2$$

2 Find the nth term

For an arithmetic sequence, consecutive differences are equal:

$$(7p - 3) - 8p = (4p + 2) - (7p - 3)$$

$$-p - 3 = -3p + 5$$

$$2p = 8$$

$$p = 4$$

3 Find the nth term

So the first term is

$$8p = 32$$

4 **Find common difference**

and the common difference is

$$(7p - 3) - 8p = 25 - 32 = -7$$

---

5 **The sum of the first**

The sum of the first n terms is

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$-1914 = \frac{n}{2}(64 - 7(n-1))$$

$$-1914 = \frac{n}{2}(71 - 7n)$$

$$-3828 = n(71 - 7n)$$

$$7n^2 - 71n - 3828 = 0$$

$$n = \frac{71 \pm 335}{14}$$

---

6 **Only the positive value is**

Only the positive value is possible:

$$n = 29$$

✓ **FINAL ANSWER**

$n = 29$

4MA1/2HR JUNE 2023

Question 24

5 marks | Volume & Surface Area

24 The surface area of sphere **A** is nine times the surface area of sphere **B**
The difference between the volume of sphere **A** and the volume of sphere **B** is 117π cm^3

Find the radius of the smaller sphere.
Show your working clearly.

..... cm

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2HR June 2023

1 **Let the radius of the**

Let the radius of the smaller sphere be r.

2 **Let the radius of the**

Let the radius of the larger sphere be R.

3 **Surface area of a sphere**

Surface area of a sphere is

$$4\pi r^2$$

4 **The surface area of sphere**

The surface area of sphere A is nine times the surface area of sphere B, so

$$4\pi R^2 = 9(4\pi r^2)$$

$$R^2 = 9r^2$$

$$R = 3r$$

5 **The difference between the volumes**

The difference between the volumes is 117π:

$$\frac{4}{3}\pi(3r)^3 - \frac{4}{3}\pi r^3 = 117\pi$$

$$\frac{4}{3}\pi(27r^3 - r^3) = 117\pi$$

$$\frac{104}{3}\pi r^3 = 117\pi$$

---

6 **Cancel**

Cancel π:

$$\frac{104}{3}r^3 = 117$$

$$r^3 = \frac{117 \times 3}{104}$$

$$r^3 = \frac{27}{8}$$

$$r = \frac{3}{2}$$

✓ **FINAL ANSWER**

$\boxed{1.5\text{ cm}}$

4MA1/2HR JUNE 2023

Question 25

6 marks | Linear Graphs

25 The straight line with equation $y - 2x = 7$ is the perpendicular bisector of the line AB where A is the point with coordinates $(j, 7)$ and B is the point with coordinates $(6, k)$

Find the coordinates of the midpoint of the line AB
Show clear algebraic working.

(..... , ...)

(Total for Question 25 is 6 marks)

WORKED SOLUTION

Question 25 · 4MA1/2HR June 2023

1 The perpendicular bisector is

The perpendicular bisector is

$$y - 2x = 7$$

2 Calculate value

so

$$y = 2x + 7$$

3 The gradient of the perpendicular

The gradient of the perpendicular bisector is 2, so the gradient of AB is

$$-\frac{1}{2}$$

4 **Evaluate fraction**

Since $A = (j, 7)$ and $B = (6, k)$,

$$\frac{k-7}{6-j} = -\frac{1}{2}$$

$$2(k-7) = -(6-j)$$

$$j = 2k - 8$$

---

5 **The midpoint of is**

The midpoint of AB is

$$\left(\frac{j+6}{2}, \frac{7+k}{2}\right)$$

---

6 **Substitute**

Substitute $j = 2k - 8$:

$$\left(k-1, \frac{k+7}{2}\right)$$

---

7 **Find the midpoint**

The midpoint lies on $y = 2x + 7$, so

$$\frac{k+7}{2} = 2(k-1) + 7$$

$$k + 7 = 4k + 10$$

$$k = -1$$

---

8 **Calculate value**

Then

$$j = 2(-1) - 8 = -10$$

---

9 **Find the midpoint**

Therefore the midpoint is

$$\left(\frac{-10+6}{2}, \frac{7+(-1)}{2}\right) = (-2, 3)$$

FINAL ANSWER

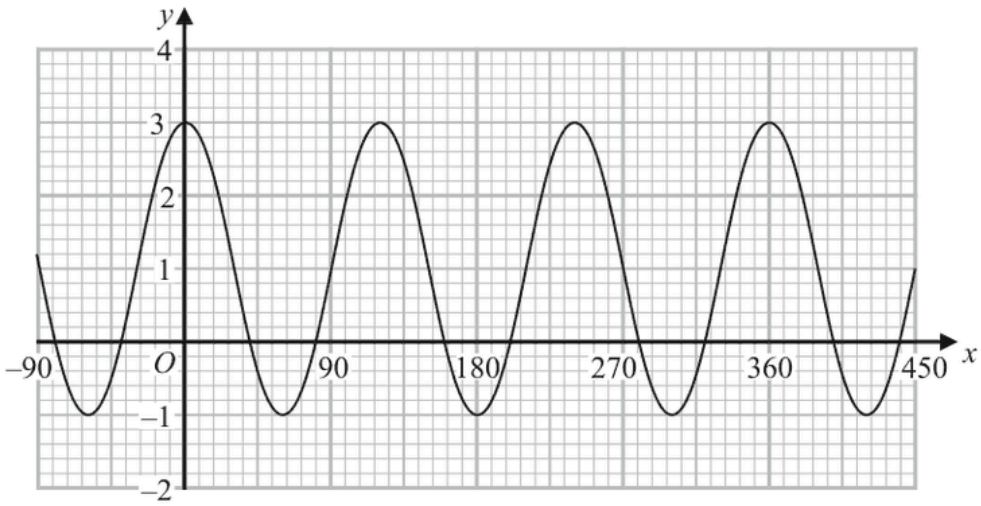
$$\boxed{(-2, 3)}$$

4MA1/2HR JUNE 2023

Question 26

3 marks | Graphs of Functions

26 Here is a sketch of the curve with equation $y = a\cos bx° + c$ where $-90 \leqslant x \leqslant 450$



Find the value of a, the value of b and the value of c

$a =$ .....

$b =$ .....

$c =$ .....

(Total for Question 26 is 3 marks)

WORKED SOLUTION

Question 26 · 4MA1/2HR June 2023

1 The curve is

The curve is

$$y = a\cos bx° + c$$

2 Use trigonometry

From the graph, the maximum value is

$$3$$

3 Use trigonometry

and the minimum value is

$$-1$$

4 **The amplitude is half the**

The amplitude is half the difference:

$$a = \frac{3 - (-1)}{2} = 2$$

5 **The midline is the average**

The midline is the average:

$$c = \frac{3 + (-1)}{2} = 1$$

6 **The distance between consecutive maxima**

The distance between consecutive maxima is $120°$, so the period is

$$120°$$

7 **Use trigonometry**

For $y = a\cos bx° + c$, the period is

$$\frac{360°}{b}$$

8 **Use trigonometry**

So

$$\frac{360}{b} = 120$$

$$b = 3$$

✓ **FINAL ANSWER**

$$\boxed{a = 2,\ b = 3,\ c = 1}$$