Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

4MA1/2H June 2022 Worked Solutions

June 2022 paper rebuilt with the worked solution after each question.

24	2020–2026	Single-paper solutions
paper questions	52 past papers	checked by Dr Eslam



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PAST PAPER

4MA1/2H June 2022

June 2022

24	100	2 h
questions	marks	exam time

Each question is followed by its worked solution and final answer.

4MA1/2H JUNE 2022

Question 1

3 marks | Statistics Toolkit

1 Here are some integers where $a < b < c < d$

$$a \quad b \quad c \quad d \quad d \quad d$$

The mode of the integers is 9
The median of the integers is 8
The range of the integers is 4

Work out the value of a, the value of b, the value of c and the value of d

$a =$ .....

$b =$ .....

$c =$ .....

$d =$ .....

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H June 2022

1 **Calculate statistic**

The mode is 9, so

$$d = 9$$

2 **The range is**

The range is 4:

$$9 - a = 4$$

$$a = 5$$

3 **Calculate statistic**

The median is 8, so

$$\frac{c+d}{2}=8$$

$$\frac{c+9}{2}=8$$

$$c=7$$

4 **The integers are in order**

The integers are in order and $a<b<c$, so

$$b=6$$

FINAL ANSWER

$a=5,\ b=6,\ c=7,\ d=9$.

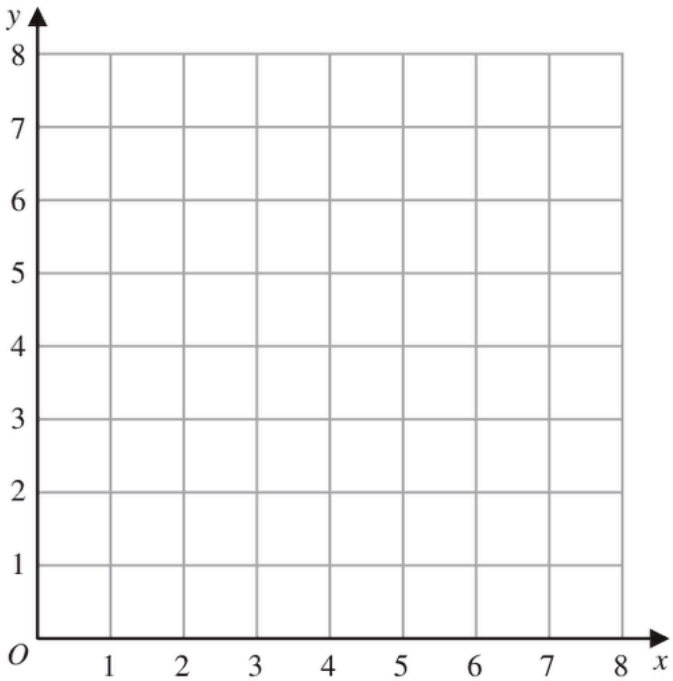
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Question 2

4 marks | Inequalities (Solving & Graphing)

2 (a) On the grid, draw and label with its equation the straight line with equation

(i) $y = 1$ (ii) $x = 2$ (iii) $x + y = 7$



(3)

(b) Show, by shading on the grid, the region that satisfies **all three** of the inequalities

$y \geqslant 1$ $x \geqslant 2$ $x + y \leqslant 7$

Label the region **R**.

(1)

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H June 2022

1 **Draw the boundary lines**

Draw the boundary lines:

$$y = 1, \quad x = 2, \quad x + y = 7$$

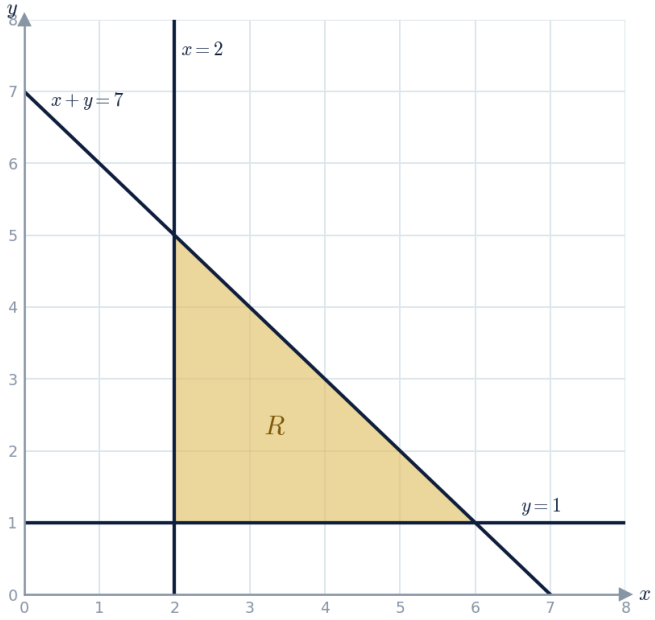
2 **The required region is**

The required region is

$$y \geq 1$$

$$x \geq 2$$

$$x + y \leq 7$$



FINAL ANSWER

Shade $y \geq 1$, $x \geq 2$, $x + y \leq 7$.

4MA1/2H JUNE 2022

Question 3

3 marks | Standard & Compound Units

3 An aeroplane travelled from New York City to Los Angeles.

The aeroplane travelled a distance of 3980 kilometres in 5 hours 24 minutes.

Work out the average speed of the aeroplane.
Give your answer in kilometres per hour correct to the nearest whole number.

..... kilometres per hour

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H June 2022

1 The time is

The time is

$$5\text{ h }24\text{ min} = 5.4\text{ h}$$

2 Average speed

Average speed:

$$3980 \div 5.4 = 737.037\ldots$$

✓ FINAL ANSWER

737 kilometres per hour.

4MA1/2H JUNE 2022

Question 4

3 marks | Fractions

4 Show that $5\frac{1}{3} - 2\frac{6}{7} = 2\frac{10}{21}$

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H June 2022

1 Simplify fraction

$$5\frac{1}{3} = \frac{16}{3}$$

$$2\frac{6}{7} = \frac{20}{7}$$

2 Use a common denominator

Use a common denominator:

$$\frac{16}{3} - \frac{20}{7} = \frac{112}{21} - \frac{60}{21}$$

$$= \frac{52}{21} = 2\frac{10}{21}$$

FINAL ANSWER

$2\frac{10}{21}$

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Question 5

4 marks | Perimeter & Area

5 The diagram shows an 8-sided shape *ABCDEFGH*.

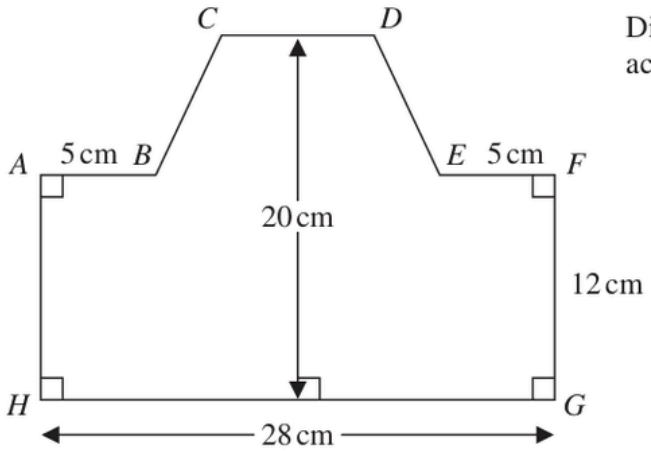


Diagram **NOT** accurately drawn

$HG = 28\,\text{cm}$ $FG = 12\,\text{cm}$ $AB = EF = 5\,\text{cm}$
The height of the shape is 20 cm
CD is parallel to *HG*

The area of shape *ABCDEFGH* is $434\,\text{cm}^2$

Find the length of *CD*.

..... cm

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H June 2022

1 **The lower rectangle has width**

The lower rectangle has width $28\ \text{cm}$ and height $12\ \text{cm}$:

$$28 \times 12 = 336\ \text{cm}^2$$

2 **The top part is a**

The top part is a trapezium of height

$$20 - 12 = 8\ \text{cm}$$

3 **Find the gradient**

The lower parallel side of this trapezium is

$$28 - 5 - 5 = 18\ \text{cm}$$

4 **Area of the trapezium**

Area of the trapezium:

$$434 - 336 = 98 \text{ cm}^2$$

---

5 **Use**

Use

$$\text{area} = \frac{1}{2}(a+b)h$$

$$98 = \frac{1}{2}(18+CD)(8)$$

$$98 = 4(18+CD)$$

$$18+CD = 24.5$$

$$CD = 6.5$$

✓ **FINAL ANSWER**

6.5 cm.

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Question 6

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

6 The diagram shows triangle PQR.

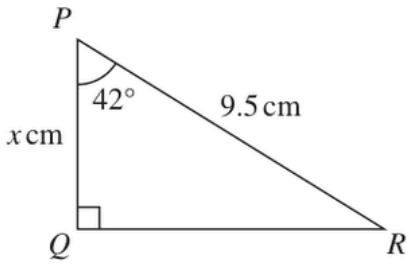


Diagram **NOT** accurately drawn

Work out the value of x
Give your answer correct to one decimal place.

$x =$ .....

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H June 2022

1 **Use trigonometry**

In the right-angled triangle, x is adjacent to the $42°$ angle and 9.5 cm is the hypotenuse.

$$\cos 42° = \frac{x}{9.5}$$

$$x = 9.5 \cos 42° = 7.059\ldots$$

✓ **FINAL ANSWER**

7.1 cm.

4MA1/2H JUNE 2022

Question 7

3 marks | Standard & Compound Units

7 Change a speed of 81 kilometres per hour to a speed in metres per second.

..... metres per second

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H June 2022

1 Convert km/h to m/s divide

To convert km/h to m/s, divide by $\mathbf{3.6}$.

$$81 \div 3.6 = 22.5$$

FINAL ANSWER

$22.5\ \text{m/s}$.

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Question 8

5 marks | Percentages

8 Behnaz makes 300 celebration cards so that

$$\text{number of birthday cards} : \text{number of anniversary cards} : \text{number of congratulations cards} = 7:5:3$$

$\frac{2}{5}$ of the birthday cards have numbers on them.
36% of the anniversary cards have numbers on them.
None of the congratulations cards have numbers on them.

Work out what fraction of the 300 cards have numbers on them.
Give your answer in its simplest form.

(Total for Question 8 is 5 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H June 2022

1 The ratio has parts

The ratio $7 : 5 : 3$ has 15 parts.

2 Birthday cards

Birthday cards:

$$\frac{7}{15} \times 300 = 140$$

3 Anniversary cards

Anniversary cards:

$$\frac{5}{15} \times 300 = 100$$

4 Congratulations cards

Congratulations cards:

$$\frac{3}{15} \times 300 = 60$$

5 Birthday cards with numbers

Birthday cards with numbers:

$$\frac{2}{5} \times 140 = 56$$

6 Anniversary cards with numbers

Anniversary cards with numbers:

$$36\% \times 100 = 36$$

7 Congratulations cards with numbers

Congratulations cards with numbers:

$$0$$

8 Total with numbers

Total with numbers:

$$56 + 36 = 92$$

9 Fraction of all cards

Fraction of all cards:

$$\frac{92}{300} = \frac{23}{75}$$

✓ **FINAL ANSWER**

$$\frac{23}{75}$$

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Question 9

3 marks | Compound Interest & Depreciation

9 Pasha invests 50 000 dollars in a savings account for 4 years.
He gets 1.3% per year compound interest.

Work out how much money Pasha will have in his savings account at the end of 4 years.
Give your answer correct to the nearest dollar.

..... dollars

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H June 2022

1 The multiplier for compound interest

The multiplier for 1.3% compound interest is

$$1.013$$

2 Use compound interest

After 4 years,

$$50000(1.013)^4 = 52651.1408\ldots$$

3 Use compound interest

Correct to the nearest dollar,

$$52651.1408\ldots \approx 52651$$

✓ **FINAL ANSWER**

52651 dollars

4MA1/2H JUNE 2022

Question 10

3 marks | Simultaneous Equations

10 Solve the simultaneous equations

$$7x + 3y = 3$$
$$3x - y = 7$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H June 2022

1 Solve simultaneous equations

$$7x + 3y = 3$$

$$3x - y = 7$$

2 From the second equation

From the second equation:

$$y = 3x - 7$$

3 Substitute into the first equation

Substitute into the first equation:

$$7x + 3(3x - 7) = 3$$

$$7x + 9x - 21 = 3$$

$$16x = 24$$

$$x = \frac{3}{2}$$

4 **Solve simultaneous equations**

Then

$$y = 3\left(\frac{3}{2}\right) - 7 = \frac{9}{2} - 7 = -\frac{5}{2}$$

✓ **FINAL ANSWER**

$x = \frac{3}{2}$, $y = -\frac{5}{2}$.

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Question 11

3 marks | Factorising

11 (i) Factorise $x^2 + 5x - 24$

.....

(2)

(ii) Hence, solve $x^2 + 5x - 24 = 0$

.....

(1)

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H June 2022

1 **Factorise expression**

$$x^2 + 5x - 24 = (x + 8)(x - 3)$$

2 **Factorise expression**

So

$$(x + 8)(x - 3) = 0$$

✓ **FINAL ANSWER**

$x = -8$ or $x = 3$

4MA1/2H JUNE 2022

Question 12

3 marks | Statistics Toolkit

12 Larry is a delivery man.

He has 7 parcels to deliver.
The mean weight of the 7 parcels is 2.7 kg

Larry delivers 3 of the parcels.
Each of these 3 parcels has a weight of W kg

The mean weight of the other 4 parcels is 3.3 kg

Work out the value of W

$W =$ .....

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H June 2022

1 Total mass of the 7

Total mass of the 7 parcels:

$$7 \times 2.7 = 18.9$$

2 Total mass of the 4

Total mass of the 4 remaining parcels:

$$4 \times 3.3 = 13.2$$

3 Total mass of the 3

Total mass of the 3 delivered parcels:

$$18.9 - 13.2 = 5.7$$

4 Each delivered parcel has mass

Each delivered parcel has mass W:

$$3W = 5.7$$

$$W = 1.9$$

FINAL ANSWER

1.9 kg.

Question 13

7 marks | Cumulative Frequency Diagrams

13 The table gives information about the ages, in years, of 80 people in a train carriage.

Age (a years)	Frequency
$0 < a \leqslant 20$	7
$20 < a \leqslant 30$	25
$30 < a \leqslant 40$	20
$40 < a \leqslant 50$	14
$50 < a \leqslant 60$	8
$60 < a \leqslant 70$	6

(a) Complete the cumulative frequency table.

Age (a years)	Cumulative frequency
$0 < a \leqslant 20$	
$0 < a \leqslant 30$	
$0 < a \leqslant 40$	
$0 < a \leqslant 50$	
$0 < a \leqslant 60$	
$0 < a \leqslant 70$	

(1)

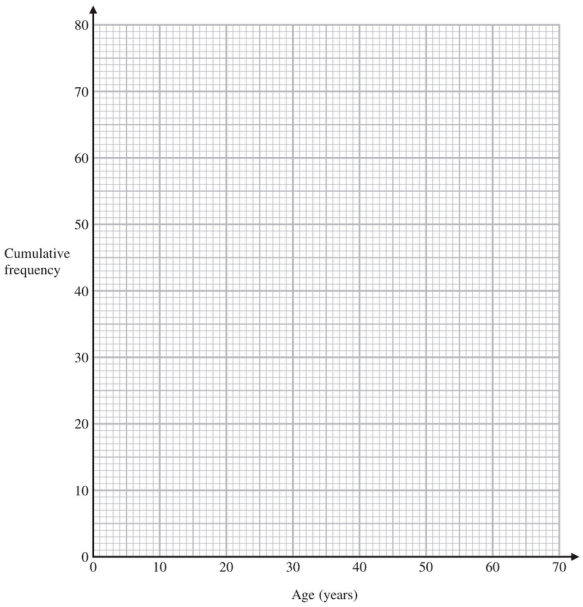
(b) On the grid opposite, draw a cumulative frequency graph for your table.

(2)

(c) Use your graph to find an estimate for the median age of the 80 people.

..... years

(1)



Of the people in the train carriage, 60% of those who are aged between 18 and 65 are going to work. None of the other people in the train carriage are going to work.

(d) Use your graph to find an estimate for the number of people in the train carriage who are going to work.

.....

(3)

(Total for Question 13 is 7 marks)

1 Complete the cumulative frequencies

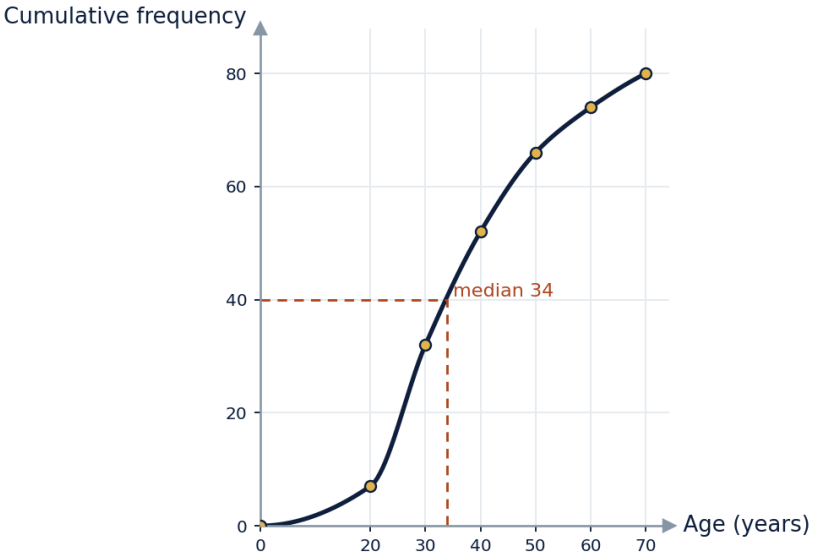
Complete the cumulative frequencies:

$$7,\ 32,\ 52,\ 66,\ 74,\ 80$$

2 Plot the points

Plot the points

$$(0, 0), (20, 7), (30, 32), (40, 52), (50, 66), (60, 74), (70, 80)$$



3 The median is at cumulative

The median is at cumulative frequency 40.

4 Convert standard form

This is between $(30, 32)$ and $(40, 52)$:

$$30 + \frac{40 - 32}{52 - 32} \times 10 = 34$$

5 Use cumulative frequency

So the median age is about 34 years.

6 Estimate the number aged between

Estimate the number aged between 18 and 65:

$$F(18) \approx \frac{18}{20} \times 7 = 6.3$$

$$F(65) \approx 74 + \frac{5}{10}(80 - 74) = 77$$

$$77 - 6.3 = 70.7$$

---

7 60% are going to work

60% are going to work:

$$0.6(70.7) = 42.42$$

FINAL ANSWER

cumulative frequencies $7, 32, 52, 66, 74, 80$, median about 34 years, about 42 people going to work.

4MA1/2H JUNE 2022

Question 14

7 marks | Rearranging Formulae

14 (a) Expand and simplify $(5 - x)(2x + 3)(x + 4)$
Show your working clearly.

.....

(3)

(b) Make c the subject of $g = \dfrac{c + 3}{4 + c} - 7$

.....

(4)

(Total for Question 14 is 7 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H June 2022

1 **(a)**

(a)

$$(5 - x)(2x + 3)(x + 4)$$

$$(2x + 3)(x + 4) = 2x^2 + 11x + 12$$

$$(5 - x)(2x^2 + 11x + 12) = -2x^3 - x^2 + 43x + 60$$

2 **(b)**

(b)

$$g = \frac{c + 3}{4 + c} - 7$$

$$g + 7 = \frac{c + 3}{c + 4}$$

$$(g + 7)(c + 4) = c + 3$$

$$c(g + 6) = -4g - 25$$

FINAL ANSWER

(a) $-2x^3 - x^2 + 43x + 60$, (b) $c = \frac{-4g-25}{g+6}$

4MA1/2H JUNE 2022

Question 15

7 marks | Inequalities (Solving & Graphing)

15 (a) Solve $\frac{4x+5}{3} - \frac{3-2x}{2} = 13$

Show clear algebraic working.

$x =$ .....

(4)

(b) Solve the inequality $2y^2 - 7y - 30 \leqslant 0$

Show your working clearly.

.....

(3)

(Total for Question 15 is 7 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H June 2022

1 (a)

(a)

$$\frac{4x+5}{3} - \frac{3-2x}{2} = 13$$

2 Multiply by 6

Multiply by 6:

$$2(4x+5) - 3(3-2x) = 78$$

$$8x + 10 - 9 + 6x = 78$$

$$14x + 1 = 78$$

$$14x = 77$$

$$x = \frac{77}{14} = \frac{11}{2}$$

3 **(b)**

(b)

$$2y^2 - 7y - 30 \leq 0$$

---

4 **Factorise**

Factorise:

$$(2y + 5)(y - 6) \leq 0$$

---

5 **Solve quadratic equation**

The roots are $-\frac{5}{2}$ and 6. The quadratic is ≤ 0 between the roots.

✓ **FINAL ANSWER**

$x = \frac{11}{2}$, and $-\frac{5}{2} \leq y \leq 6$.

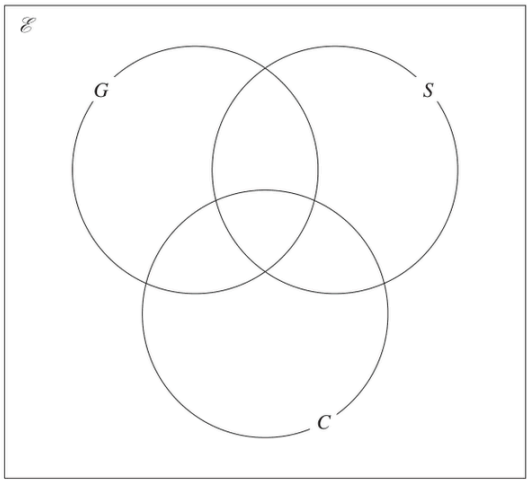
Question 16

8 marks | Set Notation & Venn Diagrams

16 100 farmers are asked if they have goats (G), sheep (S) or chickens (C) on their farms.

Of these farmers
- 31 have sheep
- 53 have chickens
- 6 have goats, sheep and chickens
- 11 have sheep and goats
- 17 have sheep and chickens
- 18 have goats and chickens
- 20 do not have any goats, sheep or chickens

(a) Using this information, complete the Venn diagram to show the number of farmers in each appropriate subset.



(3)

(b) Find

(i) n(G)

.....

(1)

(ii) n($[G \cup S]'$)

.....

(1)

(iii) n($G' \cap C$)

.....

(1)

One of the farmers who has chickens is chosen at random.

(c) Find the probability that this farmer also has goats.

.....

(2)

(Total for Question 16 is 8 marks)

1 **(a) Start from the centre**

All three: 6. Then the "two only" regions:

$$G \cap S \text{ only} = 11 - 6 = 5, \quad S \cap C \text{ only} = 17 - 6 = 11, \quad G \cap C \text{ only} = 18 - 6 = 12$$

2 **(a) Single-animal regions**

$$S \text{ only} = 31 - 5 - 11 - 6 = 9, \qquad C \text{ only} = 53 - 11 - 12 - 6 = 24$$

80 farmers are inside the circles ($100 - 20$), so

$$G \text{ only} = 80 - (9 + 24 + 5 + 11 + 12 + 6) = 13$$

Outside the circles: 20.

3 **(b)(i) n(G)**

$$13 + 5 + 12 + 6 = 36$$

4 **(b)(ii) Not in G or S**

Only "C only" and the outside region: $24 + 20 = 44$.

5 **(b)(iii) In C but not in G**

"C only" and "$S \cap C$ only": $24 + 11 = 35$.

6 **(c) Chickens farmers who also have goats**

53 farmers have chickens; $12 + 6 = 18$ of them also have goats.

$$\frac{18}{53}$$

FINAL ANSWER

(a) G only 13, S only 9, C only 24, $G \cap S$ only 5, $S \cap C$ only 11, $G \cap C$ only 12, centre 6, outside 20
(b)(i) 36 (ii) 44 (iii) 35 (c) $\frac{18}{53}$

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Question 17

4 marks | Direct & Inverse Proportion

17 M varies directly as the cube of h
$M = 4$ when $h = 0.5$

Find the value of h when $M = 500$

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H June 2022

1 Find inverse function

M varies directly as h^3, so

$$M = kh^3$$

2 Use when

Use $M = 4$ when $h = 0.5$:

$$4 = k(0.5)^3$$

$$k = \frac{4}{0.125} = 32$$

3 Find inverse function

So

$$M = 32h^3$$

4 Find inverse function

When $M = 500$,

$$500 = 32h^3$$

$$h^3 = 15.625$$

$$h = 2.5$$

FINAL ANSWER

$h = 2.5$.

4MA1/2H JUNE 2022

Question 18

3 marks | Rounding, Estimation & Bounds

18 $X = \dfrac{2a - b}{f}$

$a = 7.5$ correct to 1 decimal place.
$b = 3.42$ correct to 2 decimal places.
$f = 2$ correct to the nearest whole number.

Work out the upper bound of the value of X
Show your working clearly.

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H June 2022

1 Estimate the value

$$X = \frac{2a - b}{f}$$

2 Find upper bound

For the upper bound, make the numerator large and the denominator small.

$$a < 7.55, \quad b \geq 3.415, \quad f \geq 1.5$$

$$X_{\text{upper}} = \frac{2(7.55) - 3.415}{1.5} = 7.79$$

FINAL ANSWER

7.79

4MA1/2H JUNE 2022

Question 19

3 marks | Algebraic Fractions

19 $a = \dfrac{14}{3x - 7}$ $\quad x = \dfrac{7}{4y - 3}$

Express a in the form $\dfrac{py + q}{ry + s}$ where p, q, r and s are integers.

Give your answer in its simplest form.

$a =$ .....

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H June 2022

1 Simplify fraction

$$a = \frac{14}{3x - 7}, \qquad x = \frac{7}{4y - 3}$$

2 Simplify fraction

Substitute for x:

$$3x - 7 = \frac{21}{4y - 3} - 7$$

$$= \frac{21 - 7(4y - 3)}{4y - 3} = \frac{42 - 28y}{4y - 3}$$

3 Simplify fraction

So

$$a = \frac{14}{\frac{42 - 28y}{4y - 3}} = \frac{4y - 3}{3 - 2y}$$

FINAL ANSWER

$a = \frac{4y - 3}{3 - 2y}$

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Question 20

4 marks | Circles, Arcs & Sectors

20 The diagram shows four identical circles drawn inside a square.

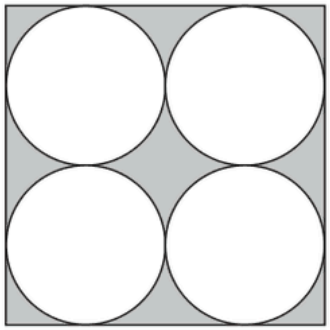


Diagram **NOT** accurately drawn

Each circle touches two other circles and two sides of the square.

The region inside the square that is outside the circles, shown shaded in the diagram, has a total area of $40\,\text{cm}^2$

Work out the perimeter of the square.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H June 2022

1 Let the radius of each

Let the radius of each circle be r.

2 There are two circles across

There are two circles across the width of the square, so the side length of the square is

$$4r$$

3 Area of the square

Area of the square:

$$(4r)^2 = 16r^2$$

4 Area of the four circles

Area of the four circles:

$$4\pi r^2$$

5 **Calculate area**

The shaded area is $40\ \text{cm}^2$, so

$$16r^2 - 4\pi r^2 = 40$$

$$r^2(16 - 4\pi) = 40$$

$$r = \sqrt{\frac{40}{16 - 4\pi}} = 3.4131\ldots$$

---

6 **The perimeter of the square**

The perimeter of the square is

$$4(4r) = 16r$$

$$16r = 54.6101\ldots$$

✓ **FINAL ANSWER**

$\boxed{54.6\ \text{cm}}$

4MA1/2H JUNE 2022

Question 21

5 marks | Vectors

21

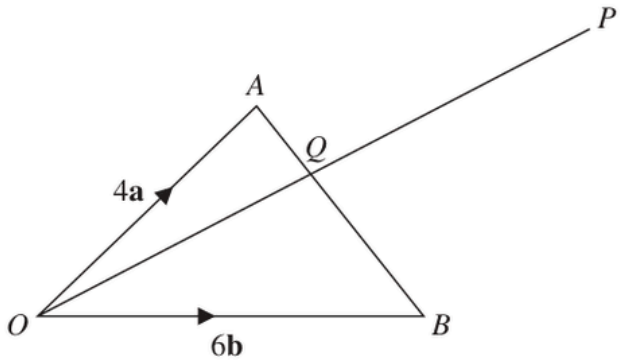


Diagram **NOT** accurately drawn

OAB is a triangle.
Q is the point on AB such that OQP is a straight line.

$\overrightarrow{OA} = 4\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$ $\overrightarrow{AP} = 2\mathbf{a} + 8\mathbf{b}$

Using a vector method, find the ratio $AQ : QB$

$AQ : QB =$ .....

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H June 2022

1 Split the ratio

$$\overrightarrow{OA} = 4a, \quad \overrightarrow{OB} = 6b, \quad \overrightarrow{AP} = 2a + 8b$$

2 Split the ratio

So

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = 4a + (2a + 8b) = 6a + 8b$$

3 Split the ratio

Since Q lies on OP, let

$$\overrightarrow{OQ} = \lambda(6a + 8b)$$

4 **Split the ratio**

Since Q lies on AB, let

$$\overrightarrow{OQ} = \overrightarrow{OA} + \mu\overrightarrow{AB}$$

$$\overrightarrow{AB} = 6b - 4a$$

5 **Split the ratio**

So

$$\overrightarrow{OQ} = 4a + \mu(6b - 4a) = (4 - 4\mu)a + 6\mu b$$

6 **Equating coefficients**

Equating coefficients:

$$6\lambda = 4 - 4\mu, \quad 8\lambda = 6\mu$$

7 **Split the ratio**

From $8\lambda = 6\mu$, $\mu = \frac{4}{3}\lambda$.

$$6\lambda = 4 - \frac{16}{3}\lambda$$

$$34\lambda = 12, \quad \lambda = \frac{6}{17}$$

$$\mu = \frac{8}{17}$$

8 **Split the ratio**

So $AQ : QB = \frac{8}{17} : \frac{9}{17}$.

✓ **FINAL ANSWER**

$AQ : QB = 8 : 9$

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Question 22

7 marks | Linear Graphs

22 $ABCD$ is a kite, with diagonals AC and BD, drawn on a centimetre square grid, with a scale of 1 cm for 1 unit on each axis.

A is the point with coordinates $(-3, 4)$

The diagonals of the kite intersect at the point M with coordinates $(0, 2)$

Given that $AB = AD = 6.5$ cm and the x coordinate of B is positive,

find the coordinates of the points B and D.

(..... , ...)

(..... , ...)

(Total for Question 22 is 7 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H June 2022

1 The midpoint of is

The midpoint of BD is $M = (0, 2)$.

2 Find the gradient

Since $A = (-3, 4)$, the gradient of AM is

$$\frac{2-4}{0-(-3)} = -\frac{2}{3}$$

3 Find the gradient

For the kite, diagonal AC is perpendicular to BD, so the gradient of BD is

$$\frac{3}{2}$$

4 Use direction vector along

Use direction vector $(2, 3)$ along BD.

5 **Let**

Let

$$B = (2t, 2 + 3t)$$

6 **Calculate value**

and

$$D = (-2t, 2 - 3t)$$

7 **Calculate value**

Since $AB = 6.5$,

$$(2t - (-3))^2 + (2 + 3t - 4)^2 = 6.5^2$$

$$(2t + 3)^2 + (3t - 2)^2 = 42.25$$

$$4t^2 + 12t + 9 + 9t^2 - 12t + 4 = 42.25$$

$$13t^2 + 13 = 42.25$$

$$13t^2 = 29.25$$

$$t^2 = 2.25$$

$$t = 1.5$$

8 **Use**

The x-coordinate of B is positive, so use $t = 1.5$.

$$B = (3, 6.5)$$

$$D = (-3, -2.5)$$

✓ **FINAL ANSWER**

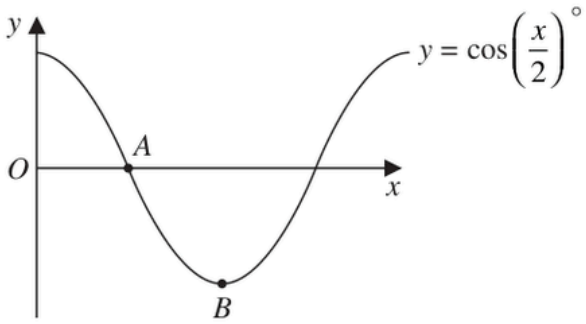
$B = (3, 6.5),\ D = (-3, -2.5)$

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Question 23

2 marks | Graphs of Functions

23 The diagram shows a sketch of the graph of $y = \cos\left(\frac{x}{2}\right)^\circ$



(i) Find the coordinates of the point A

(.....,.....)

(1)

(ii) Find the coordinates of the point B

(.....,.....)

(1)

(Total for Question 23 is 2 marks)

WORKED SOLUTION

Question 23 · 4MA1/2H June 2022

1 **The graph is**

The graph is

$$y = \cos\left(\frac{x}{2}\right)^\circ$$

2 **Point is the first positive**

Point A is the first positive x-intercept, so $y = 0$.

$$\cos\left(\frac{x}{2}\right) = 0$$

$$\frac{x}{2} = 90^\circ$$

$$x = 180$$

3 **Read the graph**

So

$$A = (180, 0)$$

---

4 **Point is the minimum point**

Point B is the minimum point.

---

5 **The minimum value of cosine**

The minimum value of cosine is -1, which occurs when

$$\frac{x}{2} = 180°$$

$$x = 360$$

---

6 **Read the graph**

So

$$B = (360, -1)$$

✓ **FINAL ANSWER**

$A = (180, 0),\ B = (360, -1)$

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Question 24

3 marks | Algebraic Roots & Indices

24 $$\frac{18 \times (\sqrt{27})^{4n+6}}{6 \times 9^{2n+8}} = 3^x$$

Express x in terms of n
Show your working clearly and simplify your expression.

$x =$ .....

(Total for Question 24 is 3 marks)

WORKED SOLUTION

Question 24 · 4MA1/2H June 2022

1 Simplify surd

$$\frac{18(\sqrt{27})^{4n+6}}{6 \times 9^{2n+8}} = 3^x$$

2 Use index laws

First,

$$\frac{18}{6} = 3 = 3^1$$

3 Simplify surd

Also,

$$\sqrt{27} = \sqrt{3^3} = 3^{3/2}$$

4 Simplify surd

So

$$(\sqrt{27})^{4n+6} = 3^{\frac{3}{2}(4n+6)} = 3^{6n+9}$$

5 Use index laws

And

$$9^{2n+8} = (3^2)^{2n+8} = 3^{4n+16}$$

6 Use index laws

Therefore the whole expression is

$$3^{1+6n+9-(4n+16)} = 3^{2n-6}$$

7 **Use index laws**

So

$$x = 2n - 6$$

✓ **FINAL ANSWER**

$x = 2n - 6$.