Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

4MA1/2HR June 2022 Worked Solutions

June 2022 paper rebuilt with the worked solution after each question.

25	2020–2026	Single-paper solutions
paper questions	52 past papers	checked by Dr Eslam



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PAST PAPER

4MA1/2HR June 2022

June 2022

25	100	2 h
questions	marks	exam time

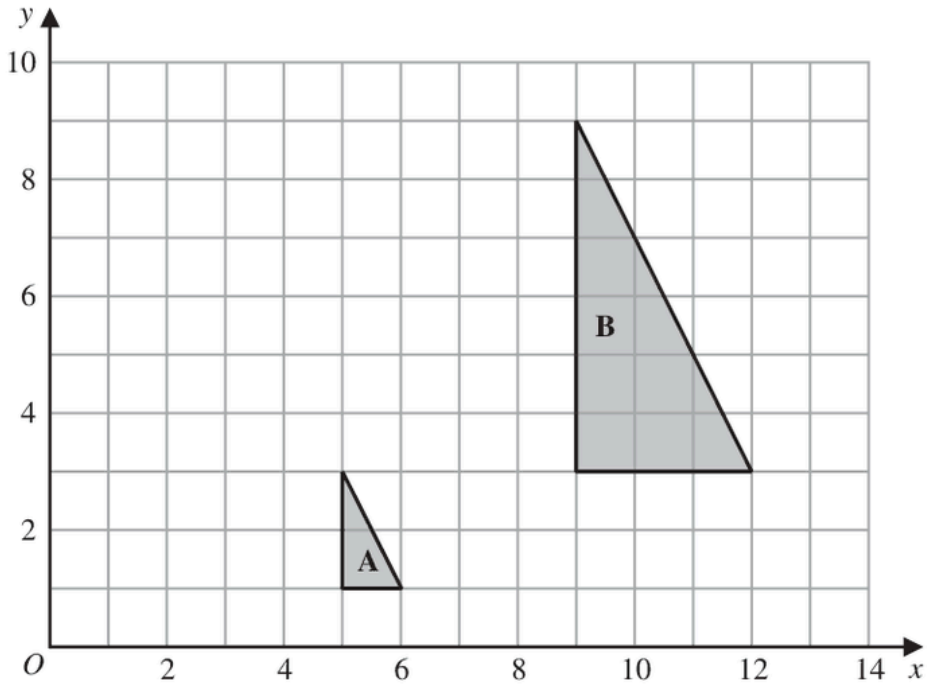
Each question is followed by its worked solution and final answer.

4MA1/2HR JUNE 2022

Question 1

4 marks | Transformations

1



(a) Describe fully the single transformation that maps triangle **A** onto triangle **B**

.....

.....

(3)

(b) On the grid above, translate triangle **A** by the vector $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

Label your triangle **C**

(1)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR June 2022

1 **Use trigonometry**

(a) Triangle A is enlarged to triangle B.

2 **The scale factor is**

The scale factor is

3 **Use Transformations tag correct**

Using corresponding vertices gives centre $(3, 0)$.

4 **Use Transformations tag correct**

So the transformation is an enlargement, scale factor 3, centre $(3, 0)$.

5 **(b) Translating triangle by**

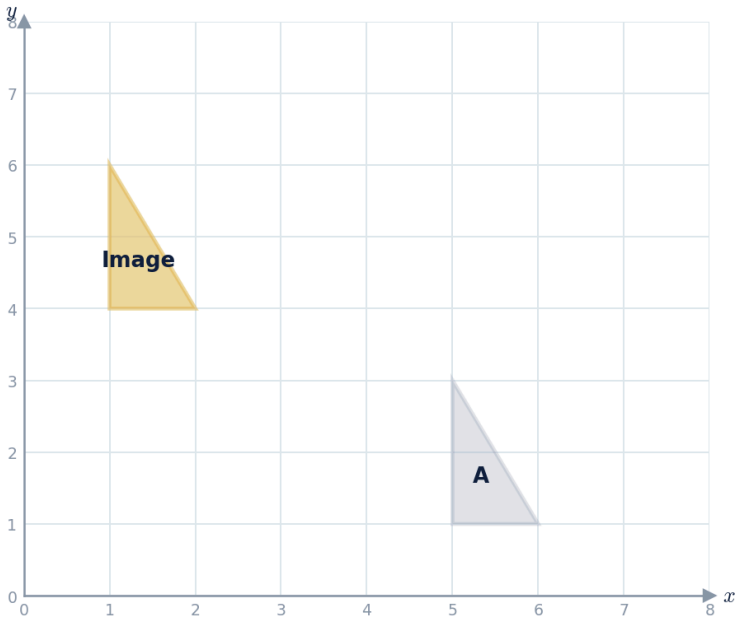
(b) Translating triangle A by

$$\begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

6 **gives vertices**

gives vertices

$$(1, 4),\ (2, 4),\ (1, 6)$$



FINAL ANSWER

(a) enlargement, scale factor 3, centre $(3, 0)$; (b) vertices $(1, 4), (2, 4), (1, 6)$.

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Question 2

3 marks | Prime Factors, HCF & LCM

2 Write 1200 as a product of powers of its prime factors.
Show your working clearly.

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2HR June 2022

1 Find the LCM

$$1200 = 12 \times 100$$

$$1200 = (2^2 \times 3)(2^2 \times 5^2)$$

$$1200 = 2^4 \times 3 \times 5^2$$

FINAL ANSWER

$2^4 \times 3 \times 5^2$

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Question 3

3 marks | Statistics Toolkit

3 Alberto, Bill, Candela and Diana are four friends.

Here is some information about the height of each of these friends.

Alberto's height is 158 cm.
Bill's height is 175 cm.
Candela's height is greater than Diana's height.

The median height of these four friends is 160 cm.
The range of the heights of these four friends is 21 cm.

Work out Candela's height and Diana's height.

Candela ..... cm

Diana ..... cm

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2HR June 2022

1 The median is . Since

The median is 160. Since the ordered heights are

$$\text{Diana},\ 158,\ \text{Candela},\ 175$$

$$\frac{158 + \text{Candela}}{2} = 160$$

$$\text{Candela} = 162$$

2 The range is

The range is 21:

$$175 - \text{Diana} = 21$$

$$\text{Diana} = 154$$

FINAL ANSWER

Candela $= 162$ cm, Diana $= 154$ cm.

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Question 4

5 marks | Set Notation & Venn Diagrams

4 $\mathscr{E} = \{9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$
$A = \{\text{multiples of } 3\}$
$B = \{\text{odd numbers}\}$

(a) List the members of the set

(i) $A \cap B$

.....

(1)

(ii) $A \cup B$

.....

(1)

(b) Is it true that $24 \in A$?

Tick one of the boxes below.

Yes	No
☐	☐

Give a reason for your answer.

.....

(1)

Set C has 4 members such that $C \cap B' = \{10, 18\}$

(c) List the members of one possible set C

.....

(2)

(Total for Question 4 is 5 marks)

WORKED SOLUTION

Question 4 · 4MA1/2HR June 2022

1 **Calculate value**

$$\mathcal{E} = \{9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$$

2 **The multiples of in are**

The multiples of 3 in $\mathcal{E}$ are

$$A = \{9, 12, 15, 18\}$$

3 **The odd numbers in are**

The odd numbers in $\mathcal{E}$ are

$$B = \{9, 11, 13, 15, 17, 19\}$$

4 **Calculate value**

So

$$A \cap B = \{9, 15\}$$

5 **Calculate value**

and

$$A \cup B = \{9, 11, 12, 13, 15, 17, 18, 19\}$$

6 **The statement is false because**

The statement $24 \in A$ is false because 24 is not in the universal set $\mathcal{E}$.

7 **Calculate value**

For C, we need $C \cap B' = \{10, 18\}$ and C has 4 members. One valid choice is

$$C = \{9, 10, 11, 18\}$$

✓ **FINAL ANSWER**

(a)(i) $\{9, 15\}$, (a)(ii) $\{9, 11, 12, 13, 15, 17, 18, 19\}$, (b) No, (c) one possible set is $\{9, 10, 11, 18\}$

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Question 5

4 marks | Perimeter & Area

5 The diagram shows a shape made from a square $ABCD$ and 4 identical semicircles.

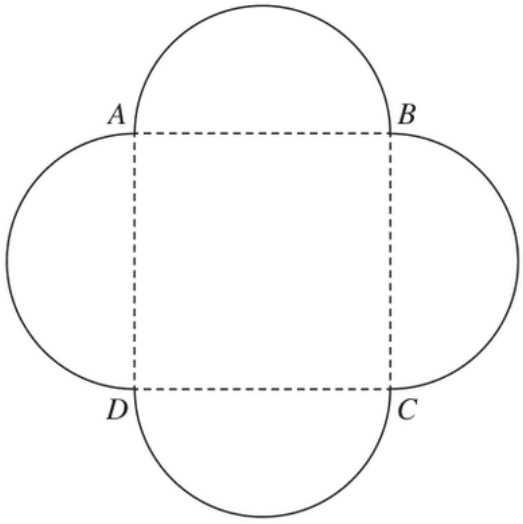


Diagram **NOT** accurately drawn

As shown in the diagram, the semicircles have AB, BC, CD and DA as diameters.

The area of the square is $36\,\text{cm}^2$

Calculate the total area of the shape.
Give your answer correct to one decimal place.

..... cm^2

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2HR June 2022

1 **Calculate area**

The square has area $36\ \text{cm}^2$, so its side length is

$$\sqrt{36} = 6\ \text{cm}$$

2 **Calculate area**

Each semicircle has diameter $6\ \text{cm}$, so radius

$$r = 3\ \text{cm}$$

3 Area of 4 semicircles

Area of 4 semicircles:

$$4 \times \frac{1}{2}\pi(3)^2 = 18\pi$$

4 Total area

Total area:

$$36 + 18\pi = 92.548\ldots$$

FINAL ANSWER

$92.5\ \text{cm}^2$.

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Question 6

8 marks | Linear Equations

6 (a) Solve $p = \dfrac{3p - 5}{10}$

Show clear algebraic working.

$p =$ .....

(3)

(b) Simplify a^0 where $a > 0$

.....

(1)

(c) Simplify fully $\dfrac{3xy^3}{6x^2y}$

.....

(2)

(d) Factorise fully $10c^3d^2 + 15cd^4$

.....

(2)

(Total for Question 6 is 8 marks)

WORKED SOLUTION

Question 6 · 4MA1/2HR June 2022

1 **(a)**

(a)

$$p = \frac{3p - 5}{10}$$

$$10p = 3p - 5$$

$$7p = -5$$

$$p = -\frac{5}{7}$$

2 **(b)**

(b) $a^0 = 1$.

---

3 **(c)**

(c)

$$\frac{3xy^3}{6x^2y} = \frac{y^2}{2x}$$

---

4 **(d)**

(d)

$$10c^3d^2 + 15cd^4 = 5cd^2(2c^2 + 3d^2)$$

✓ **FINAL ANSWER**

(a) $p = -\frac{5}{7}$, (b) 1, (c) $\frac{y^2}{2x}$, (d) $5cd^2(2c^2 + 3d^2)$

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Question 7

2 marks | Algebraic Roots & Indices

7 $\dfrac{2^k}{4^n} = 2^x$

Find an expression for x in terms of k and n

$x =$ .....

(Total for Question 7 is 2 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR June 2022

1 Use index laws

$$\frac{2^k}{4^n} = 2^x$$

2 Use index laws

Since $4 = 2^2$,

$$4^n = (2^2)^n = 2^{2n}$$

3 Use index laws

So

$$\frac{2^k}{2^{2n}} = 2^{k-2n}$$

4 Use index laws

Therefore

$$x = k - 2n$$

✓ **FINAL ANSWER**

$x = k - 2n$.

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Question 8

3 marks | Reverse Percentages

8 A cinema increased the cost of an adult ticket by 12%

After the increase, the cost of an adult ticket was £18.20

Work out the cost of an adult ticket before the increase.

£.....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR June 2022

1 Calculate percentage

After a 12% increase, the ticket price is 112% of the original price.

$$18.20 = 1.12 \times \text{original price}$$

$$\text{original price} = \frac{18.20}{1.12} = 16.25$$

FINAL ANSWER

$\pounds 16.25$

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Question 9

4 marks | Standard Form

9 The table gives information about the population, correct to 2 significant figures, of each of five cities in 2018

City	Population (2018)
Ahmedabad	7.7×10^6
Barcelona	5.5×10^6
Chicago	8.8×10^6
Lagos	1.3×10^7
Tokyo	3.7×10^7

(a) Write 8.8×10^6 as an ordinary number.

.....

(1)

(b) Which of these cities had the least population in 2018?

.....

(1)

(c) Work out the difference between the population of Tokyo and the population of Ahmedabad in 2018
Give your answer in standard form correct to 2 significant figures.

.....

(2)

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR June 2022

1 Convert standard form

$$8.8 \times 10^6 = 8800000$$

2 The least population in the

The least population in the table is Barcelona, with

$$5.5 \times 10^6$$

3 Convert standard form

For the difference between Tokyo and Ahmedabad:

$$3.7 \times 10^7 - 7.7 \times 10^6$$

$$37000000 - 7700000 = 29300000$$

$$29300000 = 2.93 \times 10^7$$

---

4 Correct to 2 significant figures

Correct to 2 significant figures:

$$2.9 \times 10^7$$

FINAL ANSWER

(a) 8800000, (b) Barcelona, (c) 2.9×10^7

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Question 10

5 marks | Right-Angled Triangles - Pythagoras & Trigonometry

10 The diagram shows triangle ABP inside the regular hexagon $ABCDEF$

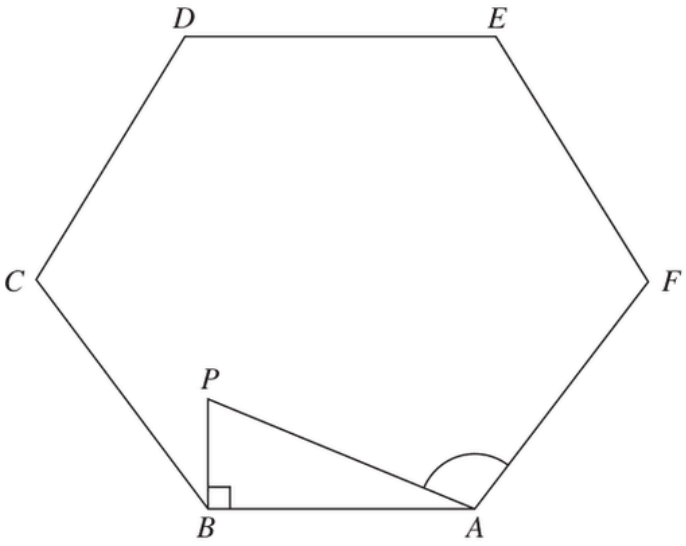


Diagram **NOT** accurately drawn

$AB = 5\,\text{cm}$ $BP = 2\,\text{cm}$ Angle $ABP = 90°$

Work out the size of angle PAF
Give your answer correct to 3 significant figures.

.....°

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/2HR June 2022

1 **A regular hexagon has interior**

A regular hexagon has interior angle

$$120°$$

2 **Use trigonometry**

In right-angled triangle ABP,

$$\tan(\angle PAB) = \frac{BP}{AB} = \frac{2}{5}$$

$$\angle PAB = \tan^{-1}\left(\frac{2}{5}\right) = 21.801\ldots°$$

3 Use trigonometry

Therefore

$$\angle PAF = 120° - 21.801\ldots° = 98.198\ldots°$$

FINAL ANSWER

$98.2°$.

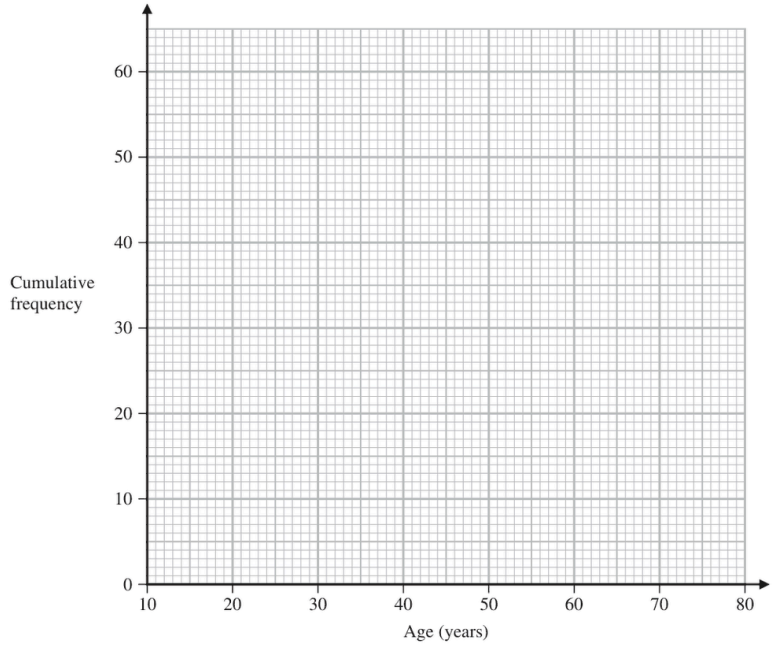
Question 11

7 marks | Cumulative Frequency Diagrams

11 The cumulative frequency table shows information about the ages of 60 people who went to a gym on Saturday.

Age (a years)	Cumulative frequency
$10 < a \leqslant 20$	13
$10 < a \leqslant 30$	36
$10 < a \leqslant 40$	42
$10 < a \leqslant 50$	47
$10 < a \leqslant 60$	52
$10 < a \leqslant 70$	56
$10 < a \leqslant 80$	60

(a) On the grid, draw a cumulative frequency graph for the information in the table.



(2)

Question 11 continued

(b) Use your graph to find an estimate for the median of the ages of these people.

..... years

(1)

(c) Use your graph to find an estimate for the interquartile range of the ages of these people.

..... years

(2)

(d) Use your graph to find an estimate for the number of these people who are older than 55 years.

.....

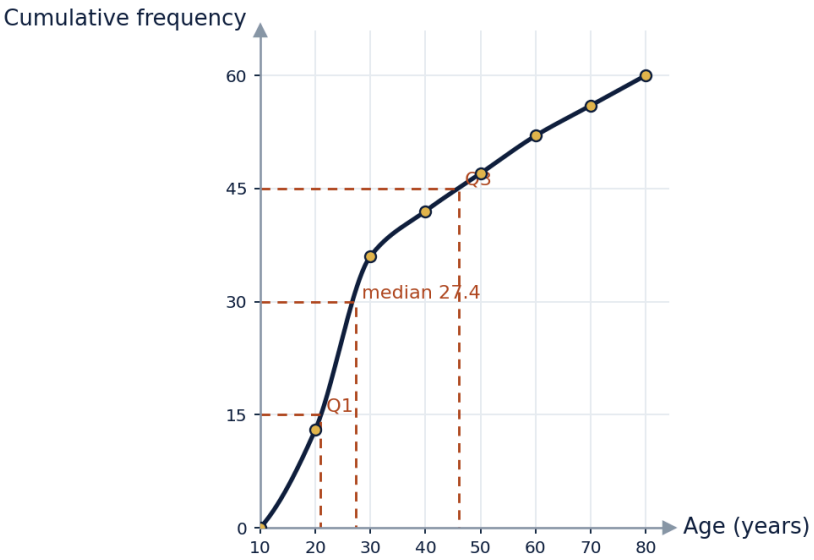
(2)

(Total for Question 11 is 7 marks)

1 Plot the points

Plot the points

$$(10, 0), (20, 13), (30, 36), (40, 42), (50, 47), (60, 52), (70, 56), (80, 60)$$



2 The median is at cumulative

The median is at cumulative frequency 30.

3 Convert standard form

This is between $(20, 13)$ and $(30, 36)$:

$$20 + \frac{30 - 13}{36 - 13} \times 10 = 27.39\ldots$$

4 Median age is about years

Median age is about 27.4 years.

5 Convert standard form

For the interquartile range:

$$Q_1 \text{ is at } 15, \qquad Q_3 \text{ is at } 45$$

$$Q_1 \approx 20 + \frac{15 - 13}{36 - 13} \times 10 = 20.9$$

$$Q_3 \approx 40 + \frac{45 - 42}{47 - 42} \times 10 = 46$$

$$\text{IQR} \approx 46 - 20.9 = 25.1$$

6 Use cumulative frequency

For people older than 55:

$$F(55) \approx 47 + \frac{5}{10}(52 - 47) = 49.5$$

$$60 - 49.5 = 10.5$$

FINAL ANSWER

median about 27.4 years, IQR about 25.1 years, about 11 people older than 55.

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Question 12

3 marks | Similar Shapes

12

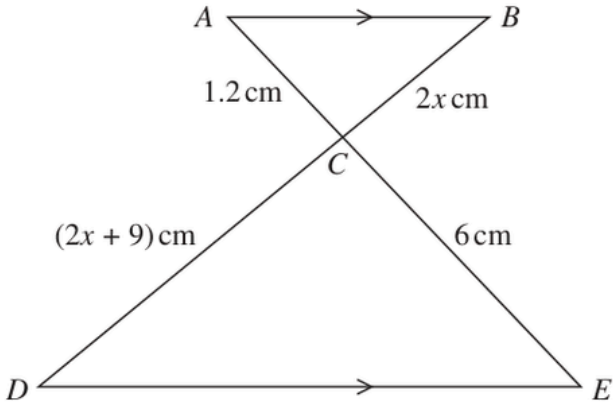


Diagram **NOT** accurately drawn

ACE and *BCD* are straight lines.
AB is parallel to *DE*

Work out the value of x

$x =$ .....

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2HR June 2022

1 Find the gradient

Since AB is parallel to DE, triangles ACB and ECD are similar.

2 Split the ratio

So corresponding sides are in the same ratio:

$$\frac{AC}{CE} = \frac{BC}{CD}$$

3 Substitute the lengths

Substitute the lengths:

$$\frac{1.2}{6} = \frac{2x}{2x+9}$$

$$\frac{1}{5} = \frac{2x}{2x+9}$$

$$2x + 9 = 10x$$

$$8x = 9$$

$$x = \frac{9}{8}$$

FINAL ANSWER

$x = \frac{9}{8}$.

4MA1/2HR JUNE 2022

Question 13

3 marks | Circles, Arcs & Sectors

13 The diagram shows a sector AOB of a circle with centre O

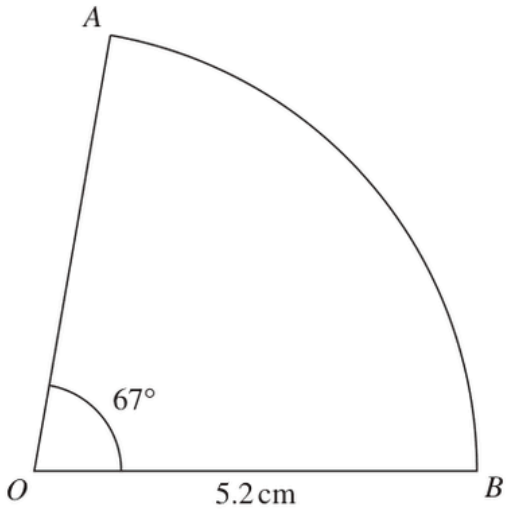


Diagram **NOT** accurately drawn

Angle $AOB = 67°$
$OA = OB = 5.2\,\text{cm}$

Calculate the perimeter of the sector.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2HR June 2022

1 Arc length

Arc length:

$$\frac{67}{360} \times 2\pi(5.2) = 6.080\ldots$$

2 Perimeter

Perimeter:

$$5.2 + 5.2 + 6.080\ldots = 16.480\ldots$$

FINAL ANSWER

$16.5\ \text{cm}$.

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Question 14

3 marks | Tree Diagrams & Conditional Probability

14 Ciara throws **four** fair six-sided dice.

The faces of each dice are labelled with the numbers 1, 2, 3, 4, 5, 6

Work out the probability that at least one of the dice lands on an even number.

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/2HR June 2022

1 Use the complement

"At least one even" is the opposite of "all four dice are odd". Each die is odd with probability $\frac{1}{2}$.

2 All four dice odd

$$\left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

3 At least one even

$$1 - \frac{1}{16} = \frac{15}{16}$$

FINAL ANSWER

$$\frac{15}{16}$$

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Question 15

3 marks | Sine & Cosine Rules

15 The diagram shows a kite $ABCD$

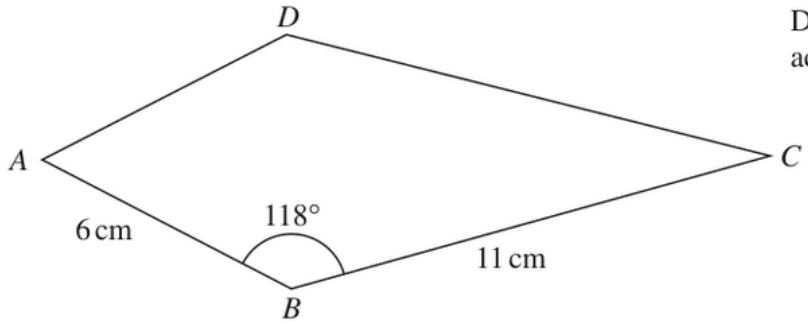


Diagram **NOT** accurately drawn

$AB = 6\,\text{cm}$ $BC = 11\,\text{cm}$ Angle $ABC = 118°$

Calculate the area of the kite.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/2HR June 2022

1 A kite can be split

A kite can be split into two congruent triangles.

2 Area of triangle

Area of triangle ABC:

$$\frac{1}{2} \times 6 \times 11 \times \sin 118°$$

3 Use trigonometry

So the area of the kite is

$$2 \times \frac{1}{2} \times 6 \times 11 \times \sin 118°$$

$$= 6 \times 11 \times \sin 118° = 58.274\ldots$$

FINAL ANSWER

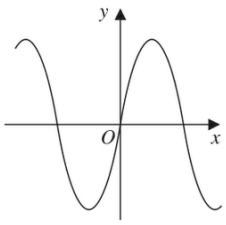
$58.3\ \text{cm}^2$.

Question 16

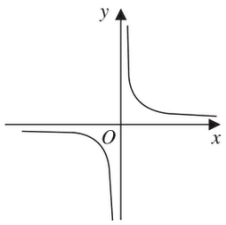
3 marks | Graphs of Functions

16 Here are nine graphs.

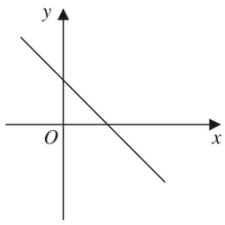
Graph A



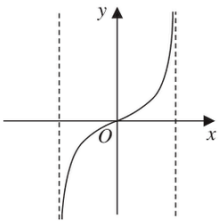
Graph B



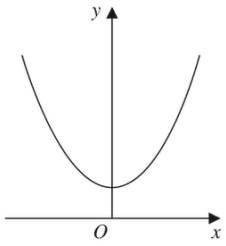
Graph C



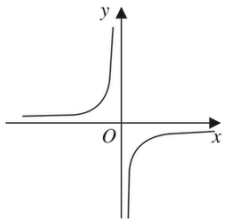
Graph D



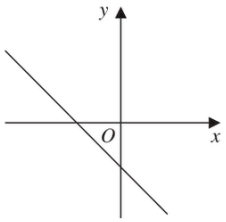
Graph E



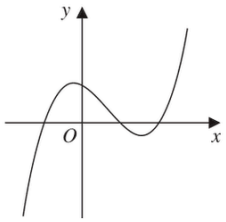
Graph F



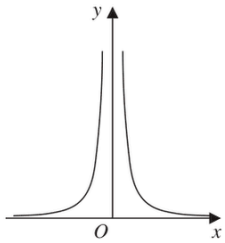
Graph G



Graph H



Graph I



Complete the table below with the letter of the graph that could represent each given equation.
Write each answer on the dotted line.

Equation	Graph
$y = -2x + 3$	.....
$y = -\frac{1}{x}$	.....
$y = \tan x°$	.....
$y = (x + 1)(x - 1)(x - 2)$	.....

(Total for Question 16 is 3 marks)

1 **is a negative straight line**

$$y = -2x + 3$$

is a negative straight line with positive y-intercept, so it is **Graph C**.

2 **Read the graph**

$$y = \frac{1}{x}$$

is the reciprocal graph in quadrants I and III, so it is **Graph B**.

3 **has repeating branches with vertical**

$$y = \tan x$$

has repeating branches with vertical asymptotes and passes through the origin, so it is **Graph D**.

4 **Read the graph**

$$y = (x + 1)(x - 1)(x - 2)$$

is a positive cubic with three roots, so it is **Graph H**.

✓ **FINAL ANSWER**

C, B, D, H.

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Question 17

2 marks | Algebraic Proof

17 Use algebra to show that $0.3\dot{4}\dot{5} = \frac{19}{55}$

(Total for Question 17 is 2 marks)

WORKED SOLUTION

Question 17 · 4MA1/2HR June 2022

1 Let

Let

$$x = 0.3\dot{4}\dot{5}$$

2 Calculate value

Then

$$10x = 3.\dot{4}\dot{5}$$

3 Calculate value

and

$$1000x = 345.\dot{4}\dot{5}$$

4 Subtract

Subtract:

$$1000x - 10x = 345.\dot{4}\dot{5} - 3.\dot{4}\dot{5}$$

$$990x = 342$$

$$x = \frac{342}{990} = \frac{19}{55}$$

FINAL ANSWER

$0.3\dot{4}\dot{5} = \frac{19}{55}$.

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Question 18

4 marks | Rounding, Estimation & Bounds

18 Kaidan and Sonja went on two different car journeys.

For Kaidan's journey
distance = 80 km correct to the nearest 5 km
time = 2.7 hours correct to 1 decimal place

For Sonja's journey
distance = 33 km correct to 2 significant figures
time = 1 hour correct to the nearest 0.1 hour

Kaidan says,
"My average speed could have been greater than Sonja's average speed."

By considering bounds, show that Kaidan is correct.
Show your working clearly.

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR June 2022

1 To show Kaidan s average

To show Kaidan's average speed could be greater, compare Kaidan's greatest possible speed with Sonja's smallest possible speed.

2 Estimate the value

For Kaidan:

$$d < 82.5, \quad t \geq 2.65$$

$$\text{greatest speed} = \frac{82.5}{2.65} = 31.13\ldots \text{ km/h}$$

3 Estimate the value

For Sonja:

$$d \geq 32.5, \quad t < 1.05$$

$$\text{smallest speed} = \frac{32.5}{1.05} = 30.95\ldots \text{ km/h}$$

4 **Estimate the value**

Since

$$31.13\ldots > 30.95\ldots$$

5 **Kaidan s average speed could**

Kaidan's average speed could have been greater.

✓ **FINAL ANSWER**

Kaidan is correct.

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Question 19

4 marks | Functions

19 $f(x) = x^2 - 4$

$g(x) = 2x + 1$

Solve $fg(x) > 0$

Show clear algebraic working.

(Total for Question 19 is 4 marks)

WORKED SOLUTION

Question 19 · 4MA1/2HR June 2022

1 Solve quadratic equation

$$f(x) = x^2 - 4, \qquad g(x) = 2x + 1$$

$$fg(x) = f(g(x)) = (2x + 1)^2 - 4$$

$$= 4x^2 + 4x + 1 - 4 = 4x^2 + 4x - 3$$

2 Solve

Solve

$$4x^2 + 4x - 3 > 0$$

3 Factorise

Factorise:

$$(2x + 3)(2x - 1) > 0$$

4 The critical values are

The critical values are

$$x = -\frac{3}{2}, \qquad x = \frac{1}{2}$$

5 The product is positive outside

The product is positive outside the roots.

FINAL ANSWER

$x < -\frac{3}{2}$ or $x > \frac{1}{2}$.

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Question 20

4 marks | Linear Graphs

20 The centre O of a circle has coordinates $(4, 7)$

The point A, on the circle, has coordinates $(6, 11)$ and AOP is a diameter of the circle.

Find an equation of the tangent to the circle at the point P

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR June 2022

1 The centre of the circle

The centre of the circle is

$$O = (4, 7)$$

2 The point is

The point A is

$$A = (6, 11)$$

3 Find the midpoint

Since AOP is a diameter, O is the midpoint of AP.

4 Let

Let $P = (x, y)$.

$$\left(\frac{6+x}{2}, \frac{11+y}{2}\right) = (4, 7)$$

$$6 + x = 8, \qquad 11 + y = 14$$

$$P = (2, 3)$$

5 The gradient of is

The gradient of OP is

$$\frac{3-7}{2-4} = 2$$

6 **Use circle theorem**

The tangent is perpendicular to the radius, so its gradient is

$$-\frac{1}{2}$$

---

7 **Evaluate fraction**

Using point $P = (2, 3)$,

$$y - 3 = -\frac{1}{2}(x - 2)$$

$$y = -\frac{1}{2}x + 4$$

✓ **FINAL ANSWER**

$$y = -\frac{1}{2}x + 4$$

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Question 21

5 marks | Simultaneous Equations

21 Solve the simultaneous equations

$$\begin{aligned} x - 2y &= 3 \\ x^2 - y^2 + 2x &= 10 \end{aligned}$$

Show clear algebraic working.

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR June 2022

1 **Calculate value**

$$x - 2y = 3$$

2 **Calculate value**

So

$$x = 2y + 3$$

3 **Calculate value**

Substitute into

$$x^2 - y^2 + 2x = 10$$

$$(2y + 3)^2 - y^2 + 2(2y + 3) = 10$$

$$4y^2 + 12y + 9 - y^2 + 4y + 6 = 10$$

$$3y^2 + 16y + 5 = 0$$

$$(3y + 1)(y + 5) = 0$$

4 **Evaluate fraction**

So

$$y = -\frac{1}{3} \quad \text{or} \quad y = -5$$

5 **Find**

Find x:

$$y = -\frac{1}{3} \Rightarrow x = \frac{7}{3}$$

$$y = -5 \Rightarrow x = -7$$

✓ **FINAL ANSWER**

$(x, y) = \left(\frac{7}{3}, -\frac{1}{3}\right)$ or $(-7, -5)$

Question 22

4 marks | Transformations of Graphs

22 The point A with coordinates $(-3, 2)$ lies on the straight line with equation $y = f(x)$

(a) Find the coordinates of the image of the point A on the straight line with equation

(i) $y = f(x) - 3$

(..... , ...)

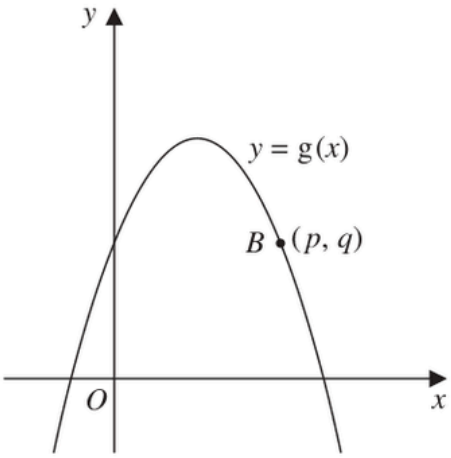
(1)

(ii) $y = f\left(\frac{x}{2}\right)$

(..... , ...)

(1)

Here is a sketch of part of the curve with equation $y = g(x)$



The point B with coordinates (p, q) lies on the curve.

(b) Find the coordinates of the image of the point B on the curve with equation

$$y = -g(x - c)$$

where c is a constant.

(..... , ...)

(2)

(Total for Question 22 is 4 marks)

1 **Point lies on**

Point $A = (-3, 2)$ lies on $y = f(x)$.

2 **Calculate value**

For

$$y = f(x) - 3$$

3 **the graph moves 3 units**

the graph moves 3 units down:

$$(-3, 2) \to (-3, -1)$$

4 **Evaluate fraction**

For

$$y = f\left(\frac{x}{2}\right)$$

5 **the coordinate is multiplied by**

the x-coordinate is multiplied by 2:

$$(-3, 2) \to (-6, 2)$$

6 **Calculate value**

For part (b), point $B = (p, q)$ lies on $y = g(x)$.

7 **Calculate value**

For

$$y = -g(x - c)$$

8 **Read the graph**

$x - c$ moves the graph c units right, and the minus sign reflects the graph in the x-axis.

9 **Apply transformation**

So

$$(p, q) \to (p + c, -q)$$

✓ **FINAL ANSWER**

$(-3, -1)$, $(-6, 2)$, and $(p + c, -q)$

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Question 23

3 marks | Algebraic Fractions

23 Express $\left(\frac{20}{x^2-36}-\frac{2}{x-6}\right)\times\frac{1}{4-x}$ as a single fraction in its simplest form.

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR June 2022

1 Use trigonometry

$$\left(\frac{20}{x^2-36}-\frac{2}{x-6}\right)\times\frac{1}{4-x}$$

2 Factor

Factor:

$$x^2-36=(x-6)(x+6)$$

3 Use trigonometry

So

$$\frac{20}{(x-6)(x+6)}-\frac{2}{x-6}$$

4 Use the common denominator

Use the common denominator $(x-6)(x+6)$:

$$\frac{20-2(x+6)}{(x-6)(x+6)}$$

$$=\frac{20-2x-12}{(x-6)(x+6)}$$

$$=\frac{8-2x}{(x-6)(x+6)}$$

$$=\frac{-2(x-4)}{(x-6)(x+6)}$$

5 **Use trigonometry**

Now multiply by

$$\frac{1}{4-x} = -\frac{1}{x-4}$$

$$\frac{-2(x-4)}{(x-6)(x+6)} \times \left(-\frac{1}{x-4}\right)$$

---

6 **Cancel**

Cancel $x-4$:

$$\frac{2}{(x-6)(x+6)}$$

$$= \frac{2}{x^2-36}$$

✓ **FINAL ANSWER**

$$\boxed{\frac{2}{x^2-36}}$$

Question 24

6 marks | Area & Volume of Similar Shapes

24 The diagram shows a frustum of a cone, and a sphere.

The frustum, shown shaded in the diagram, is made by removing the small cone from the large cone.
The small cone and the large cone are similar.

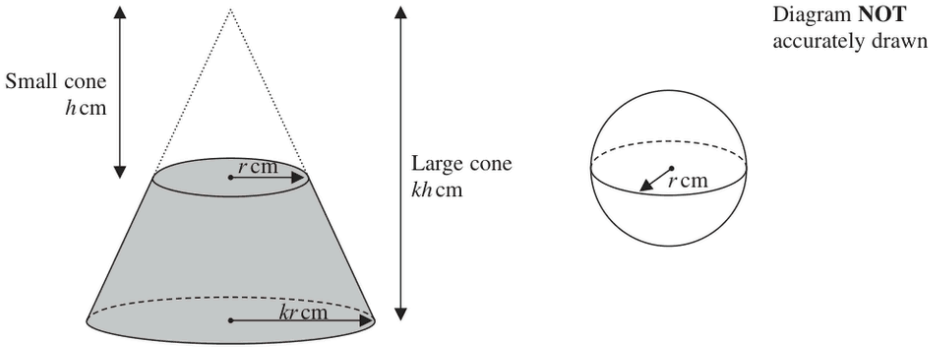


Diagram **NOT** accurately drawn

The height of the small cone is h cm and the radius of the base of the small cone is r cm.
The height of the large cone is kh cm and the radius of the base of the large cone is kr cm.
The radius of the sphere is r cm.

The sphere is divided into two hemispheres, each of radius r cm.

Solid **A** is formed by joining one of the hemispheres to the frustum.
The plane face of the hemisphere coincides with the upper plane face of the frustum, as shown in the diagram below.

Solid **B** is formed by joining the other hemisphere to the small cone that was removed from the large cone.
The plane face of the hemisphere coincides with the plane face of the base of the small cone, as shown in the diagram below.

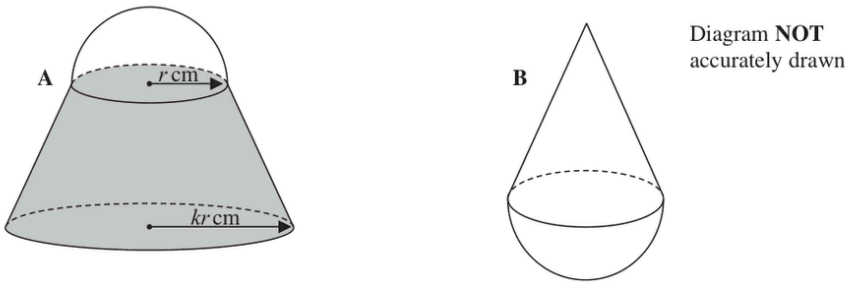


Diagram **NOT** accurately drawn

The volume of solid **A** is 6 times the volume of solid **B**.

Given that $k > \sqrt[3]{7}$

find an expression for h in terms of k and r

$h =$ .....

(Total for Question 24 is 6 marks)

1 Small cone volume

Small cone volume:

$$V_s = \frac{1}{3}\pi r^2 h$$

2 The larger similar cone has

The larger similar cone has radius kr and height kh, so its volume is

$$V_L = \frac{1}{3}\pi (kr)^2 (kh) = \frac{1}{3}\pi k^3 r^2 h$$

3 Calculate volume

So the frustum volume is

$$V_L - V_s = \frac{1}{3}\pi r^2 h(k^3 - 1)$$

4 The volume of one hemisphere

The volume of one hemisphere of radius r is

$$\frac{2}{3}\pi r^3$$

5 Solid A is the frustum

Solid A is the frustum plus one hemisphere:

$$V_A = \frac{1}{3}\pi r^2 h(k^3 - 1) + \frac{2}{3}\pi r^3$$

6 Solid B is the small

Solid B is the small cone plus one hemisphere:

$$V_B = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

7 Given

Given $V_A = 6V_B$:

$$\frac{1}{3}\pi r^2 h(k^3 - 1) + \frac{2}{3}\pi r^3 = 6\left(\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right)$$

8 **Divide by**

Divide by $\frac{1}{3}\pi r^2$:

$$h(k^3-1)+2r=6h+12r$$

$$h(k^3-7)=10r$$

$$h=\frac{10r}{k^3-7}$$

✓ **FINAL ANSWER**

$$h=\frac{10r}{k^3-7}$$

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Question 25

5 marks | Vectors

25 $ABCD$ is a parallelogram and ADM is a straight line.

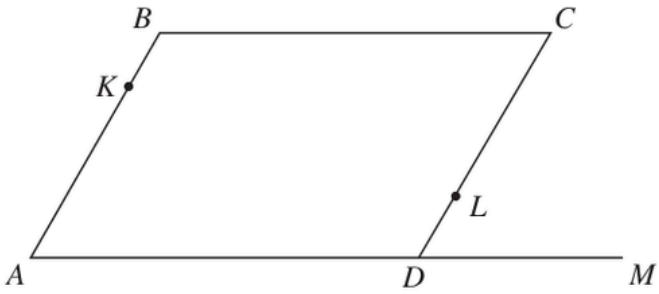


Diagram **NOT** accurately drawn

$\overrightarrow{AB} = \mathbf{a}$ $\overrightarrow{BC} = \mathbf{b}$ $\overrightarrow{DM} = \frac{1}{2}\mathbf{b}$

K is the point on AB such that $AK : AB = \lambda : 1$
L is the point on CD such that $CL : CD = \mu : 1$
KLM is a straight line.

Given that $\lambda : \mu = 1 : 2$

use a vector method to find the value of λ and the value of μ

$\lambda =$ .....

$\mu =$ .....

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/2HR June 2022

1 Let be the origin

Let A be the origin.

$$\vec{AB} = \mathbf{a}, \qquad \vec{BC} = \mathbf{b}$$

2 Read the graph

So

$$B = \mathbf{a}, \qquad D = \mathbf{b}, \qquad C = \mathbf{a} + \mathbf{b}$$

3 **Read the graph**

Also

$$\vec{DM} = \frac{1}{2}\mathbf{b}$$

---

4 **Read the graph**

So

$$M = \frac{3}{2}\mathbf{b}$$

---

5 **Point is on**

Point K is on AB:

$$K = \lambda\mathbf{a}$$

---

6 **Read the graph**

Point L is on CD, and $\vec{CD} = -\mathbf{a}$:

$$L = C + \mu(D - C)$$

$$L = \mathbf{a} + \mathbf{b} + \mu(-\mathbf{a})$$

$$L = (1 - \mu)\mathbf{a} + \mathbf{b}$$

---

7 **Given**

Given

$$\lambda : \mu = 1 : 2$$

$$\mu = 2\lambda$$

---

8 **Read the graph**

Since K, L, M are collinear, $\vec{KM}$ is a multiple of $\vec{KL}$.

$$\vec{KM} = -\lambda\mathbf{a} + \frac{3}{2}\mathbf{b}$$

$$\vec{KL} = (1 - \mu - \lambda)\mathbf{a} + \mathbf{b}$$

$$\vec{KL} = (1 - 3\lambda)\mathbf{a} + \mathbf{b}$$

---

9 **Compare coefficients using**

Compare coefficients using

$$\vec{KM} = \frac{3}{2}\vec{KL}$$

$$-\lambda = \frac{3}{2}(1 - 3\lambda)$$

$$-2\lambda = 3 - 9\lambda$$

$$7\lambda = 3$$

$$\lambda = \frac{3}{7}$$

---

10 **Read the graph**

Then

$$\mu = 2\lambda = \frac{6}{7}$$

✓ **FINAL ANSWER**

$$\lambda = \frac{3}{7}, \quad \mu = \frac{6}{7}$$