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Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

4MA1/1HR June 2022 Worked Solutions

June 2022 paper rebuilt with the worked solution after each question.

25	2020–2026	Single-paper solutions
paper questions	52 past papers	checked by Dr Eslam



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PAST PAPER

4MA1/1HR June 2022

June 2022

25	100	2 h
questions	marks	exam time

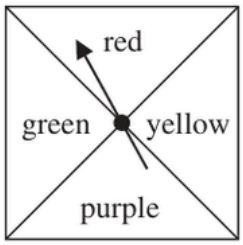
Each question is followed by its worked solution and final answer.

4MA1/1HR JUNE 2022

Question 1

4 marks | Probability Toolkit

1 Here is a biased spinner.



When the spinner is spun once, the probabilities that it lands on red or on yellow or on green are given in the table.

Colour	red	yellow	purple	green
Probability	0.25	0.2		0.2

(a) Work out the probability that the spinner lands on red or on yellow.

.....

(1)

Yang is going to spin the spinner 300 times.

(b) Work out an estimate for the number of times the spinner will land on purple.

.....

(3)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR June 2022

1 **(a) Red or yellow**

$$0.25 + 0.2 = 0.45$$

2 **(b) Probability of purple**

$$1 - (0.25 + 0.2 + 0.2) = 0.35$$

3 **(b) Estimate for 300 spins**

$$300 \times 0.35 = 105$$

FINAL ANSWER

(a) 0.45 (b) 105

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Question 2

4 marks | Sequences

2 In a warehouse there are two types of shelves, type **R** and type **S**.

These two types of shelves are arranged into shelving units that form a sequence of patterns.

Here are the first three terms in the sequence.

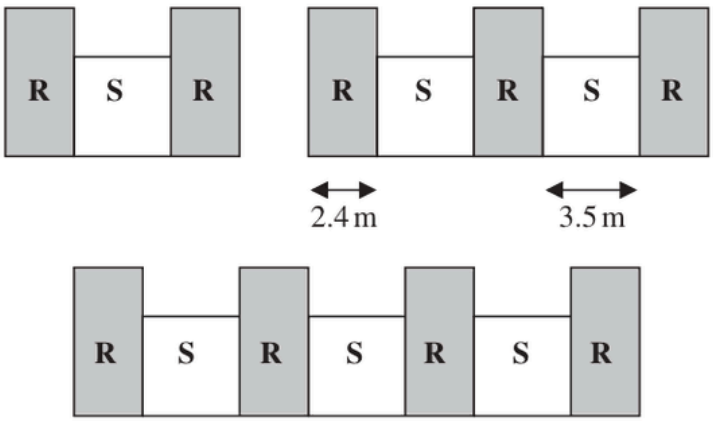


Diagram **NOT** accurately drawn

The width of each type **R** shelf is 2.4 m and the width of each type **S** shelf is 3.5 m

(a) Work out the total width of a shelving unit that has 6 type **R** shelves.

..... m

(2)

A shelving unit has n type **R** shelves.
The total width of this shelving unit is W metres.

(b) Find an expression for W in terms of n
Give your answer in its simplest form.

$W =$ .....

(2)

(Total for Question 2 is 4 marks)

WORKED SOLUTION

Question 2 · 4MA1/1HR June 2022

1 **Find the nth term**

For a shelving unit with 6 type R shelves, there are 5 type S shelves.

$$6(2.4) + 5(3.5) = 14.4 + 17.5 = 31.9$$

2 **Find the nth term**

For n type R shelves, there are $n-1$ type S shelves.

$$W = 2.4n + 3.5(n-1)$$

$$W = 2.4n + 3.5n - 3.5 = 5.9n - 3.5$$

✓ **FINAL ANSWER**

$31.9\ \text{m}$, and $W = 5.9n - 3.5$.

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Question 3

3 marks | Statistics Toolkit

3 Here are five cards.

Each card has a number written on it.



The mean of the five numbers is 12

Work out the value of x

$x =$ .....

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR June 2022

1 The mean of the five

The mean of the five cards is 12, so the total is

$$5 \times 12 = 60$$

2 The known total is

The known total is

$$15 + 7 - 2 + 23 = 43$$

$$x = 60 - 43 = 17$$

FINAL ANSWER

$x = 17$.

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Question 4

4 marks | Percentages

4 The language department of a college has 180 students.
Each student studies exactly one of French, German, Italian or Spanish.

15 students study French.
45% of the students study German.

Express the percentage of students studying Italian or Spanish as a percentage of those studying French or German.

.....%

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/1HR June 2022

1 Total students

Total students:

$$180$$

2 French

French:

$$15$$

3 German

German:

$$45\% \times 180 = 81$$

4 French or German

French or German:

$$15 + 81 = 96$$

5 Italian or Spanish

Italian or Spanish:

$$180 - 96 = 84$$

6 **Convert standard form**

Write 84 as a percentage of 96:

$$\frac{84}{96} \times 100 = 87.5$$

✓ **FINAL ANSWER**

87.5%

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Question 5

5 marks | Factorising

5 (a) Expand $3c^3(c + 4)$

.....

(2)

(b) (i) Factorise $x^2 + 8x - 9$

.....

(2)

(ii) Hence, solve $x^2 + 8x - 9 = 0$

.....

(1)

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4MA1/1HR June 2022

1 (a)

(a) $3c^3(c+4) = 3c^4 + 12c^3$.

2 (b)(i)

(b)(i)

$$x^2 + 8x - 9 = (x+9)(x-1)$$

3 (b)(ii)

(b)(ii)

$$(x+9)(x-1) = 0$$

FINAL ANSWER

(a) $3c^4 + 12c^3$, (b)(i) $(x+9)(x-1)$, (b)(ii) $x = -9$ or $x = 1$

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Question 6

3 marks | Fractions

6 Show that $2\frac{2}{3} + 3\frac{3}{4} = 6\frac{5}{12}$

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR June 2022

1 Simplify fraction

$$2\frac{2}{3} = \frac{8}{3}$$

$$3\frac{3}{4} = \frac{15}{4}$$

2 Use a common denominator

Use a common denominator:

$$\frac{8}{3} + \frac{15}{4} = \frac{32}{12} + \frac{45}{12}$$

$$= \frac{77}{12} = 6\frac{5}{12}$$

FINAL ANSWER

$6\frac{5}{12}$

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Question 7

3 marks | Standard & Compound Units

7 The diagram shows a solid cylinder made from iron.

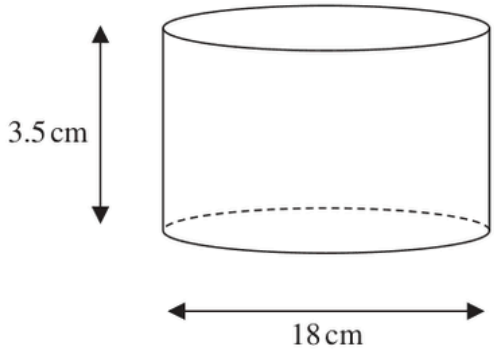


Diagram **NOT** accurately drawn

The cylinder has diameter 18 cm and height 3.5 cm
The mass of the cylinder is 7.04 kg

Work out the density of the iron.
Give your answer in g/cm^3 correct to 2 significant figures.

..... g/cm^3

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1HR June 2022

1 Radius of the cylinder is

Radius of the cylinder is

$$\frac{18}{2} = 9 \text{ cm}$$

2 Volume is

Volume is

$$\pi r^2 h = \pi(9^2)(3.5) = 283.5\pi$$

$$= 890.64\ldots \text{ cm}^3$$

3 Mass is

Mass is

$$7.04 \text{ kg} = 7040 \text{ g}$$

4 **Density is**

Density is

$$\frac{7040}{890.64\ldots} = 7.904\ldots$$

✓ **FINAL ANSWER**

$7.9\ \mathrm{g/cm^3}$ to 2 significant figures.

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Question 8

3 marks | Compound Interest & Depreciation

8 Jane bought a new car for $18 000
The car depreciates in value by 15% each year.

Work out the value of the car at the end of 4 years.
Give your answer correct to the nearest $

$.....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR June 2022

1 A depreciation of gives multiplier

A depreciation of 15% gives multiplier

$$0.85$$

2 Use compound interest

After 4 years,

$$18000(0.85)^4 = 9396.1125$$

3 Use compound interest

Correct to the nearest dollar,

$$9396.1125 \approx 9396$$

✓ **FINAL ANSWER**

$9396

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Question 9

2 marks | Inequalities (Solving & Graphing)

9 Solve the inequality $3 - 4x \leqslant 11$

(Total for Question 9 is 2 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR June 2022

1 Solve inequality

$$3 - 4x \leq 11$$

$$-4x \leq 8$$

2 Solve inequality

Divide by -4, reversing the inequality:

$$x \geq -2$$

FINAL ANSWER

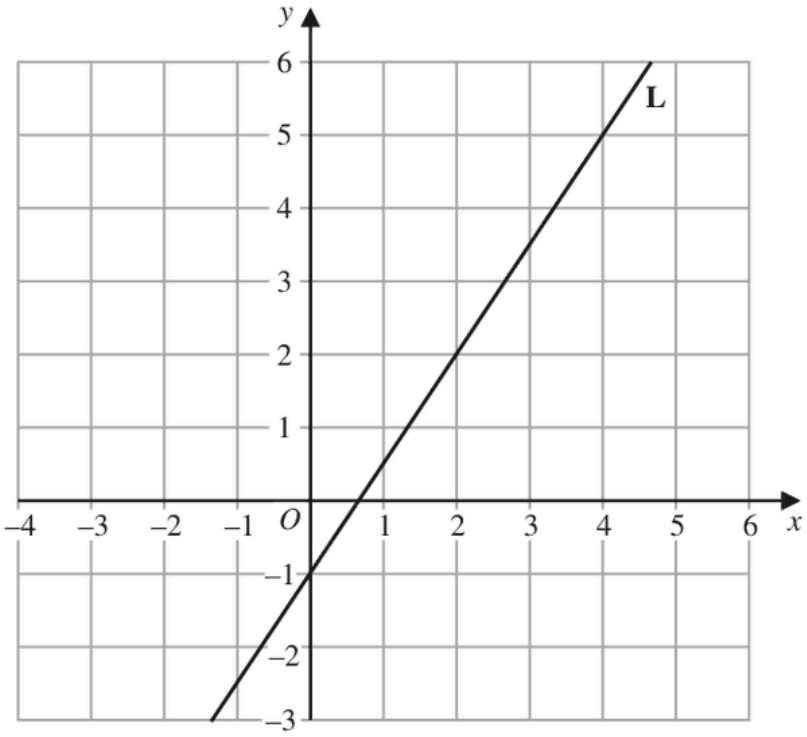
$x \geq -2$.

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Question 10

3 marks | Linear Graphs

10 Line **L** is drawn on the grid.



Find an equation for **L**
Give your answer in the form $y = mx + c$

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/1HR June 2022

1 **Solve equation**

From the graph, the line crosses the y-axis at -1, so

$$c = -1$$

2 **Find the gradient**

Using two points on the line, for example $(0, -1)$ and $(2, 2)$, the gradient is

$$\frac{2-(-1)}{2-0} = \frac{3}{2}$$

3 **Solve equation**

So the equation is

$$y = \frac{3}{2}x - 1$$

✓ **FINAL ANSWER**

$y = \frac{3}{2}x - 1$.

Question 11

6 marks | Perimeter & Area

11 The diagram shows a quadrilateral $ABCD$

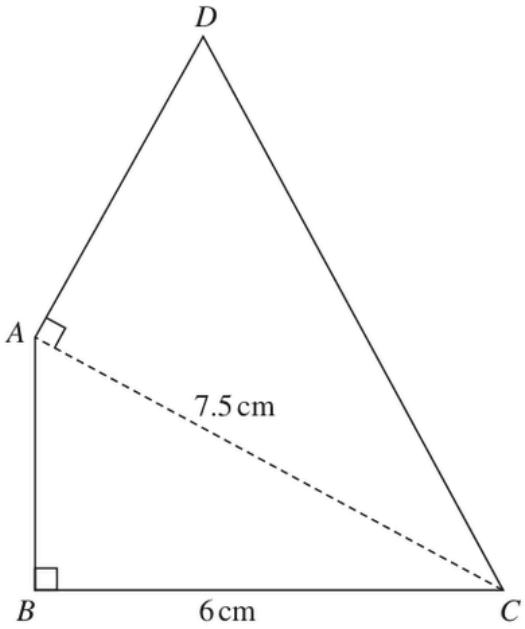


Diagram **NOT** accurately drawn

In the diagram, ABC and DAC are right-angled triangles.

$BC = 6\,\text{cm}$ $AC = 7.5\,\text{cm}$

The area of quadrilateral $ABCD$ is $31.5\,\text{cm}^2$

Work out the length of AD

Question 11 continued.

..... cm

(Total for Question 11 is 6 marks)

1 **Use trigonometry**

In right-angled triangle ABC,

$$AB^2 + BC^2 = AC^2$$

$$AB^2 + 6^2 = 7.5^2$$

$$AB^2 = 56.25 - 36 = 20.25$$

$$AB = 4.5 \text{ cm}$$

2 **Area of triangle**

Area of triangle ABC:

$$\frac{1}{2} \times 6 \times 4.5 = 13.5 \text{ cm}^2$$

3 **Area of triangle**

Area of triangle ADC:

$$31.5 - 13.5 = 18 \text{ cm}^2$$

4 **Use trigonometry**

Triangle ADC is right-angled, so

$$\frac{1}{2} \times AD \times 7.5 = 18$$

$$AD = \frac{36}{7.5} = 4.8$$

✓ **FINAL ANSWER**

4.8 cm.

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Question 12

3 marks | Prime Factors, HCF & LCM

12 $P = 3^3 \times 5^2 \times 7$
$Q = 3^2 \times 5 \times 7^2$

(a) Write down the highest common factor (HCF) of P and Q

.....

(1)

$P = 3^3 \times 5^2 \times 7$
$Q = 3^2 \times 5 \times 7^2$

(b) Work out the value of $P^3 \times Q$
Give your answer in the form $3^x \times 5^y \times 7^z$ where x, y and z are positive integers.

.....

(2)

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/1HR June 2022

1 Find the LCM

$$P = 3^3 \times 5^2 \times 7$$

$$Q = 3^2 \times 5 \times 7^2$$

2 Find the LCM

For the HCF, take the smaller power of each common prime:

$$\text{HCF} = 3^2 \times 5 \times 7$$

3 Find the LCM

For $P^3 \times Q$:

$$P^3 = (3^3 \times 5^2 \times 7)^3 = 3^9 \times 5^6 \times 7^3$$

4 **Find the LCM**

Now multiply by Q:

$$P^3 \times Q = (3^9 \times 5^6 \times 7^3)(3^2 \times 5 \times 7^2)$$

$$P^3 \times Q = 3^{11} \times 5^7 \times 7^5$$

✓ **FINAL ANSWER**

(a) $3^2 \times 5 \times 7$, (b) $3^{11} \times 5^7 \times 7^5$

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Question 13

2 marks | Statistics Toolkit

13 Here is the number of runs scored by a baseball team in each of its 15 games this season.

The number of runs have been arranged in order of size.

0 1 1 3 5 6 7 7 8 9 9 12 12 15 16

Work out the interquartile range of the number of runs.

(Total for Question 13 is 2 marks)

WORKED SOLUTION

Question 13 · 4MA1/1HR June 2022

1 Use Statistics Toolkit correct

The runs are already in order:

$$0,\ 1,\ 1,\ 3,\ 5,\ 6,\ 7,\ 7,\ 8,\ 9,\ 9,\ 12,\ 12,\ 15,\ 16$$

2 Calculate statistic

There are 15 values, so ignore the median when finding the quartiles.

$$Q_1 = 3, \qquad Q_3 = 12$$

$$\text{IQR} = 12 - 3 = 9$$

FINAL ANSWER

9.

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Question 14

4 marks | Simultaneous Equations

14 Solve the simultaneous equations

$$3x - 5y = 25$$
$$4x + 3y = 14$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/1HR June 2022

1 Solve simultaneous equations

$$3x - 5y = 25$$

$$4x + 3y = 14$$

2 Multiply the first equation by

Multiply the first equation by 3:

$$9x - 15y = 75$$

3 Multiply the second equation by

Multiply the second equation by 5:

$$20x + 15y = 70$$

4 Add

Add:

$$29x = 145$$

$$x = 5$$

5 **Solve simultaneous equations**

Substitute into $3x - 5y = 25$:

$$15 - 5y = 25$$

$$-5y = 10$$

$$y = -2$$

✓ **FINAL ANSWER**

$x = 5$, $y = -2$.

Question 15

3 marks | Circle Theorems

15

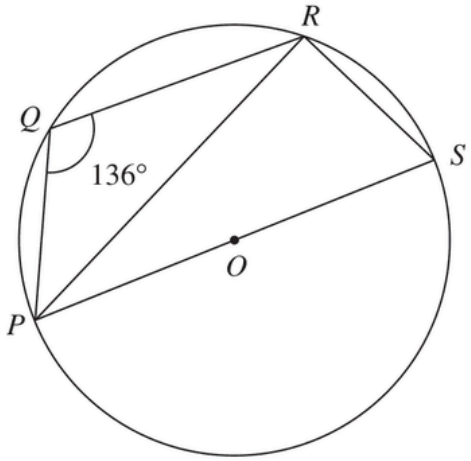


Diagram **NOT** accurately drawn

P, Q, R and S are points on a circle with centre O

PS is a diameter of the circle.

Angle $PQR = 136°$

Work out the size of angle RPS

.....°

(Total for Question 15 is 3 marks)

1 **Opposite angles in a cyclic**

Opposite angles in a cyclic quadrilateral add to $180°$:

$$\angle PSR = 180° - 136° = 44°$$

2 **Use trigonometry**

Since PS is a diameter,

$$\angle PRS = 90°$$

3 **Use trigonometry**

In triangle PRS,

$$\angle RPS = 180° - 90° - 44° = 46°$$

✓ **FINAL ANSWER**

$46°$.

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Question 16

6 marks | Expanding Brackets

16 (a) Expand and simplify $(3x - 1)(x + 2)(3x + 1)$

.....

(3)

(b) Simplify fully $\left(\frac{2x^5}{8xy^2}\right)^{-2}$

.....

(3)

(Total for Question 16 is 6 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR June 2022

1 **(a)**

(a)

$$(3x - 1)(x + 2)(3x + 1) = (9x^2 - 1)(x + 2)$$

$$= 9x^3 + 18x^2 - x - 2$$

2 **(b)**

(b)

$$\left(\frac{2x^5}{8xy^2}\right)^{-2} = \left(\frac{x^4}{4y^2}\right)^{-2}$$

$$= \left(\frac{4y^2}{x^4}\right)^{2} = \frac{16y^4}{x^8}$$

✓ **FINAL ANSWER**

(a) $9x^3 + 18x^2 - x - 2$, (b) $\frac{16y^4}{x^8}$

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Question 17

5 marks | Sine & Cosine Rules

17 Here is a parallelogram $PQRS$, in which angle SPQ is acute.

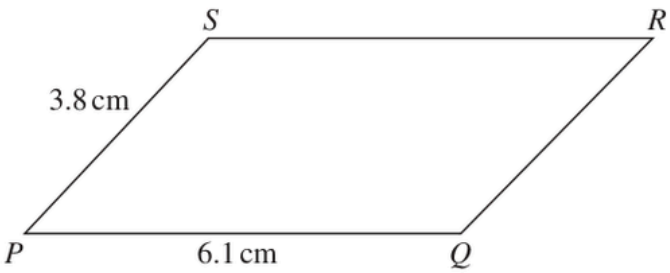


Diagram **NOT** accurately drawn

$PQ = 6.1\,\text{cm}$ $PS = 3.8\,\text{cm}$

The area of the parallelogram is $18\,\text{cm}^2$

Work out the length of QS
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 17 is 5 marks)

WORKED SOLUTION

Question 17 · 4MA1/1HR June 2022

1 Height of the parallelogram

Height of the parallelogram:

$$h = \frac{18}{6.1} = 2.9508\ldots$$

2 Use cosine rule

The horizontal offset from P to S is

$$\sqrt{3.8^2 - h^2} = \sqrt{3.8^2 - 2.9508\ldots^2} = 2.394\ldots$$

3 Use trigonometry

So the horizontal distance from S to Q is

$$6.1 - 2.394\ldots = 3.705\ldots$$

$$QS = \sqrt{3.705\ldots^2 + 2.9508\ldots^2} = 4.737\ldots$$

FINAL ANSWER

4.74 cm.

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Question 18

4 marks | 3D Pythagoras & Trigonometry

18 The diagram shows a cube $ABCDEFGH$ with sides of length 6 cm.

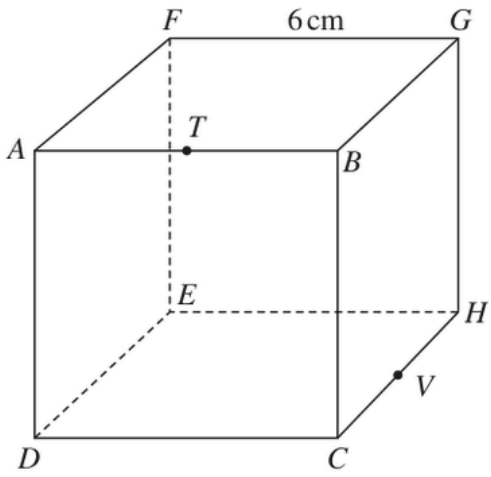


Diagram **NOT** accurately drawn

T is the midpoint of AB and V is the midpoint of CH

Work out the distance from T to V in a straight line through the cube.
Give your answer in the form $\sqrt{a}$ cm where a is an integer.

..... cm

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/1HR June 2022

1 The cube has side length

The cube has side length $\mathbf{6}$ cm.

2 Find the midpoint

Let T be the midpoint of AB and V be the midpoint of CH.

3 The changes in the three

The changes in the three perpendicular directions from T to V are

$$3, \quad 3, \quad 6$$

4 **Use trigonometry**

So by 3D Pythagoras,

$$TV = \sqrt{3^2 + 3^2 + 6^2}$$

$$TV = \sqrt{9 + 9 + 36} = \sqrt{54}$$

✓ **FINAL ANSWER**

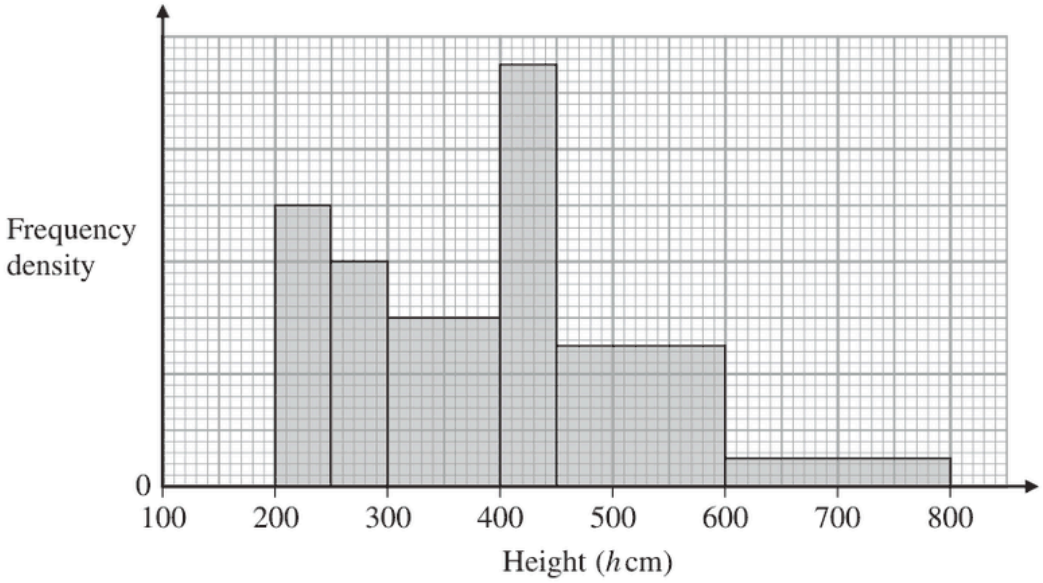
$\sqrt{54}\ \text{cm}$.

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Question 19

3 marks | Histograms

19 The histogram gives information about the height, h cm, of each tree in part of a forest.



There are no trees for which $h \leqslant 200$ and for which $h > 800$

The number of trees for which $300 < h \leqslant 400$ is 8 fewer than the number of trees for which $400 < h \leqslant 500$

Work out an estimate for the number of trees in this part of the forest that have a height greater than 500 cm.

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR June 2022

1 **Use relative densities from the**

Use relative densities from the histogram:

$$10,\ 8,\ 6,\ 15,\ 5,\ 1$$

2 **Use histogram**

The number of trees with $300 < h \leq 400$ is

$$100(6s) = 600s$$

3 **Use histogram**

The number of trees with $400 < h \leq 500$ is

$$50(15s) + 50(5s) = 1000s$$

4 **The difference is**

The difference is 8:

$$1000s - 600s = 8$$

$$400s = 8$$

$$s = 0.02$$

5 **Use histogram**

For height greater than 500:

$$100(5s) + 200(1s) = 700s$$

$$700(0.02) = 14$$

✓ **FINAL ANSWER**

14 trees.

4MA1/1HR JUNE 2022

Question 20

4 marks | Area & Volume of Similar Shapes

20 The diagram shows two similar metal statues.



A



B

Diagram **NOT** accurately drawn

The volume of statue **B** is 20% less than the volume of statue **A**

The surface area of statue **B** is $k\%$ less than the surface area of statue **A**

Work out the value of k
Give your answer correct to 3 significant figures.

$k =$ .....

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR June 2022

1 The volume of statue is

The volume of statue B is 20% less than statue A, so the volume scale factor is 0.8.

2 Calculate area

For similar solids, the linear scale factor is the cube root of the volume scale factor:

$$\text{linear scale factor} = \sqrt[3]{0.8}$$

3 Surface area scale factor is

Surface area scale factor is the square of the linear scale factor:

$$\text{surface area scale factor} = (\sqrt[3]{0.8})^2 = 0.8^{2/3} = 0.861773\ldots$$

4 **Calculate area**

So statue B has $86.1773...\%$ of the surface area of statue A.

$k = 100 - 86.1773... = 13.8226...$

✓ **FINAL ANSWER**

$k = 13.8$ to 3 significant figures.

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Question 21

4 marks | Surds

21 Express $\dfrac{3+\sqrt{8}}{\left(\sqrt{2}-1\right)^2}$ in the form $p+\sqrt{q}$ where p and q are integers.

Show each stage of your working clearly.

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR June 2022

1 **Simplify surd**

$$\frac{3+\sqrt{8}}{(\sqrt{2}-1)^2}$$

2 **Simplify surd**

Since $\sqrt{8}=2\sqrt{2}$,

$$\frac{3+2\sqrt{2}}{(\sqrt{2}-1)^2}$$

3 **Simplify surd**

Also

$$(\sqrt{2}-1)^2=3-2\sqrt{2}$$

4 **Simplify surd**

So

$$\frac{3+2\sqrt{2}}{3-2\sqrt{2}}\times\frac{3+2\sqrt{2}}{3+2\sqrt{2}}=\frac{(3+2\sqrt{2})^2}{9-8}$$

$$=(3+2\sqrt{2})^2=9+12\sqrt{2}+8$$

$$=17+12\sqrt{2}=17+\sqrt{288}$$

FINAL ANSWER

$17+\sqrt{288}$

Question 22

7 marks | Differentiation & Turning Points

22 The diagram shows a sketch of part of the curve with equation $y = x^2 - \frac{p}{x}$ where p is a positive constant.

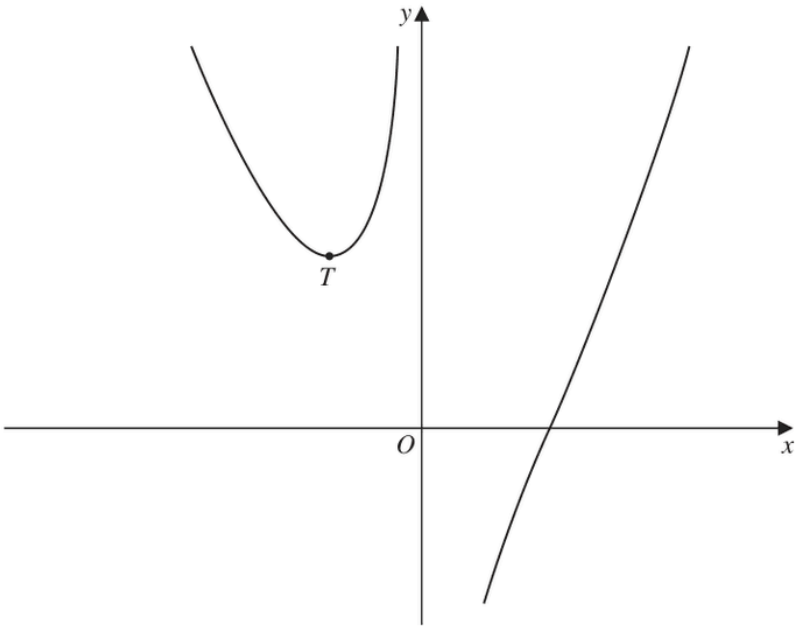


Diagram **NOT** accurately drawn

For all values of p, the curve has exactly one turning point and this turning point is a minimum shown as the point T in the sketch.

For the curve where the x coordinate of T is -3

(a) find the value of p

$p =$ .....

(4)

The line with equation $y = k$ is a tangent to the curve with equation $y = x^2 - \frac{16}{x}$

(b) Find the value of k

$k =$ .....

(3)

(Total for Question 22 is 7 marks)

1 **Find the gradient**

For part (a),

$$y = x^2 - \frac{p}{x}$$

$$\frac{dy}{dx} = 2x + \frac{p}{x^2}$$

2 **Find the gradient**

At the minimum point, $x = -3$ and the gradient is 0.

$$2(-3) + \frac{p}{(-3)^2} = 0$$

$$-6 + \frac{p}{9} = 0$$

$$p = 54$$

3 **Find the gradient**

For part (b),

$$y = x^2 - \frac{16}{x}$$

$$\frac{dy}{dx} = 2x + \frac{16}{x^2}$$

4 **Find the gradient**

At a horizontal tangent,

$$2x + \frac{16}{x^2} = 0$$

$$2x^3 + 16 = 0$$

$$x^3 = -8$$

$$x = -2$$

5 **Find the gradient**

Now substitute $x = -2$ into the curve:

$$k = (-2)^2 - \frac{16}{-2}$$

$$k = 4 + 8 = 12$$

✓ **FINAL ANSWER**

$p = 54,\ k = 12$

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Question 23

5 marks | Completing the Square

23 (a) Express $2x^2 - 12x + 3$ in the form $a(x + b)^2 + c$ where a, b and c are integers.

.....

(3)

The curve **C** has equation $y = 2(x + 4)^2 - 12(x + 4) + 3$

The point M is the minimum point on **C**

(b) Find the coordinates of M

(....., ...)

(2)

(Total for Question 23 is 5 marks)

WORKED SOLUTION

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1 **(a)**

(a)

$$2x^2 - 12x + 3 = 2(x^2 - 6x) + 3$$

$$= 2\left((x - 3)^2 - 9\right) + 3$$

$$= 2(x - 3)^2 - 18 + 3$$

$$= 2(x - 3)^2 - 15$$

2 **(b)**

(b)

$$y = 2(x + 4)^2 - 12(x + 4) + 3$$

3 **Let**

Let $u = x + 4$. From part (a),

$$2u^2 - 12u + 3 = 2(u - 3)^2 - 15$$

4 **Calculate value**

So

$$y = 2((x + 4) - 3)^2 - 15$$

$$y = 2(x + 1)^2 - 15$$

---

5 **The minimum point is when**

The minimum point is when $x = -1$, giving $y = -15$.

✓ **FINAL ANSWER**

(a) $2(x - 3)^2 - 15$, (b) $M = (-1, -15)$

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Question 24

5 marks | Probability Toolkit

24 Elliot has x counters.

Each counter has one red face and one green face.

Elliot spreads all the counters out on a table and sees that the number of counters showing a red face is 5

Elliot then picks at random one of the counters and turns the counter over.
He then picks at random a second counter and turns the counter over.

The probability that there are still 5 counters showing a red face is $\frac{19}{32}$

Work out the value of x
Show clear algebraic working.

$x =$ .....

(Total for Question 24 is 5 marks)

WORKED SOLUTION

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1 **There are counters. Initially 5**

There are x counters. Initially 5 show red, so $x - 5$ show green.

2 **List probability cases**

After two turns, there are still 5 red faces if:

3 **the same counter is picked**

- the same counter is picked twice, or
- one red counter and one green counter are picked.

4 **The probability of picking the**

The probability of picking the same counter twice is

$$\frac{1}{x}$$

5 **The probability of picking one**

The probability of picking one red and one green in either order is

$$\frac{5}{x} \cdot \frac{x-5}{x} + \frac{x-5}{x} \cdot \frac{5}{x}$$

$$= \frac{10(x-5)}{x^2}$$

---

6 **Evaluate fraction**

So

$$\frac{1}{x} + \frac{10(x-5)}{x^2} = \frac{19}{32}$$

$$\frac{11x-50}{x^2} = \frac{19}{32}$$

$$32(11x-50) = 19x^2$$

$$19x^2 - 352x + 1600 = 0$$

$$(x-8)(19x-200) = 0$$

---

7 **Calculate value**

Since x must be a whole number,

$$x = 8$$

✓ **FINAL ANSWER**

$x = 8$

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Question 25

5 marks | Sequences

25 The sum of the first 10 terms of an arithmetic series is 4 times the sum of the first 5 terms of the same series.

The 8th term of this series is 45

Find the first term of this series.
Show clear algebraic working.

(Total for Question 25 is 5 marks)

WORKED SOLUTION

Question 25 · 4MA1/1HR June 2022

1 **Calculate value**

$$S_{10} = 4S_5$$

2 **Use**

Use

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{10} = 5(2a + 9d)$$

$$S_5 = \frac{5}{2}(2a + 4d) = 5(a + 2d)$$

3 **Calculate value**

So

$$5(2a + 9d) = 4 \cdot 5(a + 2d)$$

$$2a + 9d = 4a + 8d$$

$$d = 2a$$

4 **The 8th term is 45**

The 8th term is 45:

$$u_8 = a + 7d = 45$$

$$a + 7(2a) = 45$$

$$15a = 45$$

$$a = 3$$

✓ **FINAL ANSWER**

The first term is $\mathbf{3}$