Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

4MA1/2H January 2022 Worked Solutions

January 2022 paper rebuilt with the worked solution after each question.

26	2020–2026	Single-paper solutions
paper questions	52 past papers	checked by Dr Eslam



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PAST PAPER

4MA1/2H January 2022

January 2022

26	100	2 h
questions	marks	exam time

Each question is followed by its worked solution and final answer.

4MA1/2H JANUARY 2022

Question 1

4 marks | Expanding Brackets

1 (a) Expand and simplify $(y + 4)(2 - y)$

.....

(2)

(b) Factorise fully $15b^5c - 35b^3c^9$

.....

(2)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/2H January 2022

1 **(a)**

(a)

$$(y+4)(2-y) = 2y - y^2 + 8 - 4y = -y^2 - 2y + 8$$

2 **(b)**

(b)

$$15b^5c - 35b^3c^9 = 5b^3c(3b^2 - 7c^8)$$

✓ **FINAL ANSWER**

(a) $-y^2 - 2y + 8$, (b) $5b^3c(3b^2 - 7c^8)$

4MA1/2H JANUARY 2022

Question 2

3 marks | Fractions

2 Show that $6\frac{3}{4} \div 2\frac{4}{7} = 2\frac{5}{8}$

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2H January 2022

1 Convert to improper fractions

Convert to improper fractions:

$$6\frac{3}{4} = \frac{27}{4}$$

$$2\frac{4}{7} = \frac{18}{7}$$

2 Divide by multiplying by the

Divide by multiplying by the reciprocal:

$$\frac{27}{4} \div \frac{18}{7} = \frac{27}{4} \times \frac{7}{18}$$

$$= \frac{21}{8} = 2\frac{5}{8}$$

FINAL ANSWER

$2\frac{5}{8}$

4MA1/2H JANUARY 2022

Question 3

3 marks | Similar Shapes

3

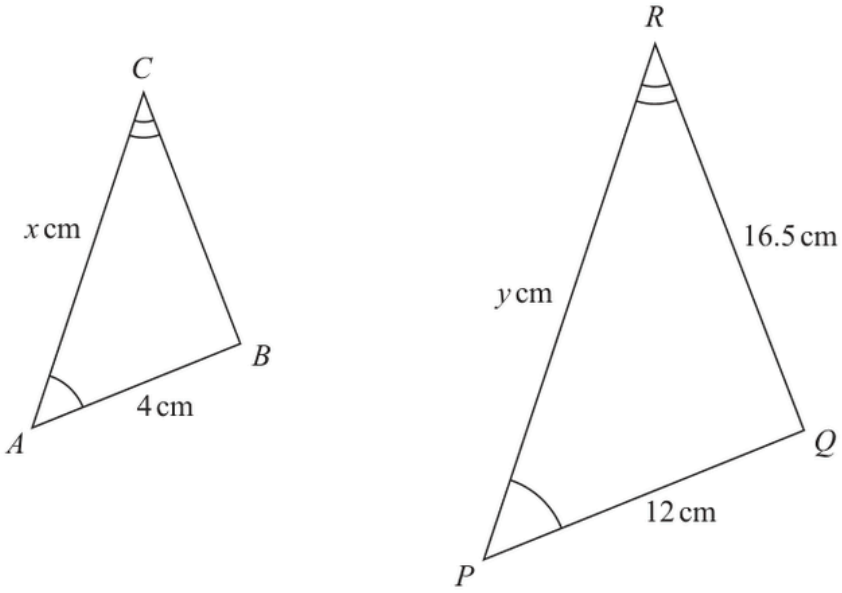


Diagram **NOT** accurately drawn

Triangle ABC is similar to triangle PQR

$AB = 4\,\text{cm}$ $PQ = 12\,\text{cm}$ $RQ = 16.5\,\text{cm}$ $AC = x\,\text{cm}$ $PR = y\,\text{cm}$

(a) Calculate the length of BC

..... cm

(2)

(b) Write down an expression for y in terms of x

$y =$ .....

(1)

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/2H January 2022

1 **The scale factor from triangle**

The scale factor from triangle ABC to triangle PQR is

$$\frac{PQ}{AB} = \frac{12}{4} = 3$$

2 **Evaluate fraction**

So

$$BC = \frac{RQ}{3} = \frac{16.5}{3} = 5.5$$

3 **Calculate value**

Also PR corresponds to AC, so

$$y = 3x$$

✓ **FINAL ANSWER**

$BC = 5.5\ \text{cm}$, and $y = 3x$.

4MA1/2H JANUARY 2022

Question 4

2 marks | Rounding, Estimation & Bounds

4 Each side of a regular octagon has a length of 18 mm, correct to the nearest 0.5 mm

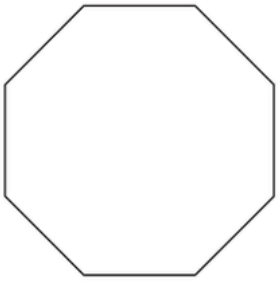


Diagram **NOT** accurately drawn

(a) Write down the lower bound of the length of each side of the octagon.

..... mm

(1)

(b) Write down the upper bound of the length of each side of the octagon.

..... mm

(1)

(Total for Question 4 is 2 marks)

WORKED SOLUTION

Question 4 · 4MA1/2H January 2022

1 The side length is mm

The side length is 18 mm correct to the nearest 0.5 mm.

2 Half of mm is mm

Half of 0.5 mm is 0.25 mm.

$$18 - 0.25 = 17.75$$

$$18 + 0.25 = 18.25$$

FINAL ANSWER

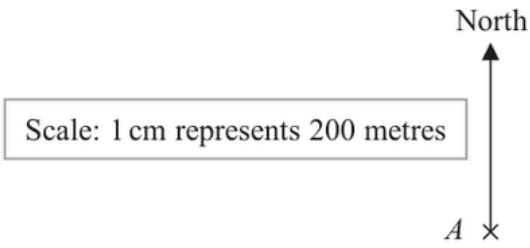
(a) 17.75 mm, (b) 18.25 mm

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Question 5

4 marks | Constructions & Loci

5 The scale diagram shows the position on a map of a house, A



House C is on a bearing of 110° from A
The distance from A to C is 700 m

(a) Mark the position of C on the diagram with a cross (×)
Label your cross C

(3)

(b) Write the scale of the map in the form $1 : n$

1 : .....

(1)

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/2H January 2022

1 The distance from to is

The distance from A to C is $700\ \text{m}$.

2 The scale is

The scale is

$$1\ \text{cm represents } 200\ \text{m}$$

3 Use bearings

So the map distance is

$$700 \div 200 = 3.5\ \text{cm}$$

4 **Use bearings**

Part (a): from A, draw a bearing of $110°$ and mark C at $3.5\ \text{cm}$ from A.

5 **Use bearings**

For part (b),

$$200\ \text{m} = 20000\ \text{cm}$$

6 **Use bearings**

So the scale is

$$1 : 20000$$

FINAL ANSWER

Draw C 3.5 cm from A on a bearing of $110°$, and the scale is $1 : 20000$.

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Question 6

5 marks | Probability Toolkit

6 A bag contains only pink sweets, white sweets, green sweets and red sweets.

The table gives each of the probabilities that, when a sweet is taken at random from the bag, the sweet will be green or the sweet will be red.

Sweet	pink	white	green	red
Probability			0.2	0.35

The ratio
number of pink sweets : number of white sweets = 2 : 1

There are 28 red sweets in the bag.

Work out the number of white sweets in the bag.

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/2H January 2022

1 **Split the ratio**

The probabilities for green and red are

$$0.20 + 0.35 = 0.55$$

2 **Split the ratio**

So

$$P(\text{pink or white}) = 1 - 0.55 = 0.45$$

3 **Split the ratio**

Pink : white $= 2 : 1$, so white is $\frac{1}{3}$ of 0.45.

$$P(\text{white}) = \frac{1}{3} \times 0.45 = 0.15$$

4 **Split the ratio**

The red probability is 0.35, and there are 28 red sweets.

$$\text{total sweets} = \frac{28}{0.35} = 80$$

5 **Number of white sweets**

Number of white sweets:

$$0.15 \times 80 = 12$$

✓ **FINAL ANSWER**

12

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Question 7

3 marks | Prime Factors, HCF & LCM

7 Find the lowest common multiple (LCM) of 28, 42 and 63
Show your working clearly.

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2H January 2022

1 Find the LCM

$$28 = 2^2 \times 7$$

$$42 = 2 \times 3 \times 7$$

$$63 = 3^2 \times 7$$

2 Find the LCM

For the LCM, take the highest power of each prime:

$$\text{LCM} = 2^2 \times 3^2 \times 7 = 252$$

FINAL ANSWER

252

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Question 8

5 marks | Reverse Percentages

8 The table gives information about the average house price in England in 2018 and in 2019

Year	2017	2018	2019
Average house price (£)		228 314	231 776

(a) Work out the percentage increase in the average house price from 2018 to 2019
Give your answer correct to one decimal place.

..... %

(2)

The average house price in 2019 was 7.7% greater than the average house price in 2017

(b) Work out the average house price in 2017
Give your answer correct to 3 significant figures.

£ .....

(3)

(Total for Question 8 is 5 marks)

WORKED SOLUTION

Question 8 · 4MA1/2H January 2022

1 Increase from 2018 to 2019

Increase from 2018 to 2019:

$$231776 - 228314 = 3462$$

2 Percentage increase

Percentage increase:

$$\frac{3462}{228314} \times 100 = 1.516\ldots$$

3 Correct to one decimal place

Correct to one decimal place:

$$1.5\%$$

4 **Calculate percentage**

For part (b), the 2019 price was 7.7% greater than the 2017 price.

$$231776 = 1.077 \times 2017\ \text{price}$$

$$2017\ \text{price} = \frac{231776}{1.077} = 215205.19\ldots$$

5 **Correct to 3 significant figures**

Correct to 3 significant figures:

$$215000$$

FINAL ANSWER

(a) 1.5%, (b) $\pounds 215000$

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Question 9

4 marks | Averages from Frequency Tables

9 The frequency table gives information about the number of points scored by a player.

Number of points	Frequency
0	13
1	17
2	8
3	x
4	11

The mean number of points scored is 2

Work out the value of x

$x =$ .....

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4MA1/2H January 2022

1 **The total number of points**

The total number of points is

$$0(13) + 1(17) + 2(8) + 3x + 4(11) = 77 + 3x$$

2 **The total frequency is**

The total frequency is

$$13 + 17 + 8 + x + 11 = 49 + x$$

3 **Calculate statistic**

The mean is 2, so

$$\frac{77+3x}{49+x}=2$$

$$77+3x=98+2x$$

$$x=21$$

FINAL ANSWER

$x=21$.

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Question 10

3 marks | Simultaneous Equations

10 Solve the simultaneous equations

$$3x + 5y = 3.1$$
$$6x + 3y = 3.75$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2H January 2022

1 Solve simultaneous equations

$$3x + 5y = 3.1$$

$$6x + 3y = 3.75$$

2 Double the first equation

Double the first equation:

$$6x + 10y = 6.2$$

3 Subtract the second equation

Subtract the second equation:

$$7y = 2.45$$

$$y = 0.35$$

4 **Solve simultaneous equations**

Substitute into $3x + 5y = 3.1$:

$$3x + 5(0.35) = 3.1$$

$$3x + 1.75 = 3.1$$

$$3x = 1.35$$

$$x = 0.45$$

FINAL ANSWER

$x = 0.45$, $y = 0.35$.

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Question 11

4 marks | Angles in Polygons & Parallel Lines

11 The diagram shows a regular 10-sided polygon, *ABCDEFGHIJ*

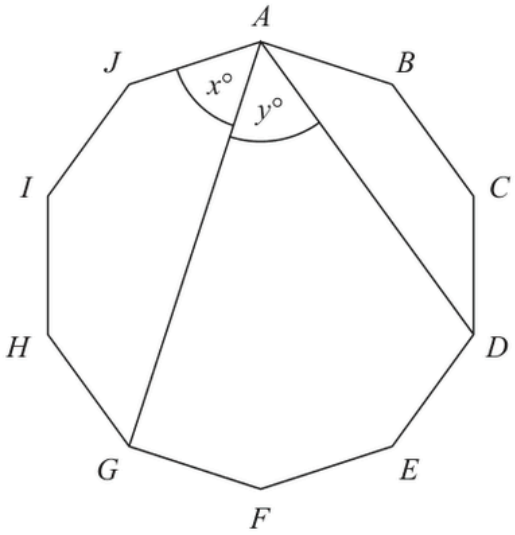


Diagram **NOT** accurately drawn

Show that $x = y$

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/2H January 2022

1 **A regular 10 sided polygon**

A regular 10-sided polygon can be divided into 10 equal arcs around its circumcircle.

2 **The angle subtends the arc**

The angle x subtends the arc from J to G, which covers 3 equal sides of the decagon.

3 **The angle subtends the arc**

The angle y subtends the arc from G to D, which also covers 3 equal sides of the decagon.

4 **Equal arcs subtend equal angles**

Equal arcs subtend equal angles at the circumference, so

$$x = y$$

✓ **FINAL ANSWER**

$x = y$.

4MA1/2H JANUARY 2022

Question 12

3 marks | Standard Form

12 $a = 6 \times 10^{40}$

Work out the value of a^3
Give your answer in standard form.

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2H January 2022

1 Convert standard form

$$a = 6 \times 10^{40}$$

$$a^3 = (6 \times 10^{40})^3$$

$$a^3 = 6^3 \times 10^{120}$$

$$a^3 = 216 \times 10^{120}$$

2 Write this in standard form

Write this in standard form:

$$216 \times 10^{120} = 2.16 \times 10^{122}$$

FINAL ANSWER

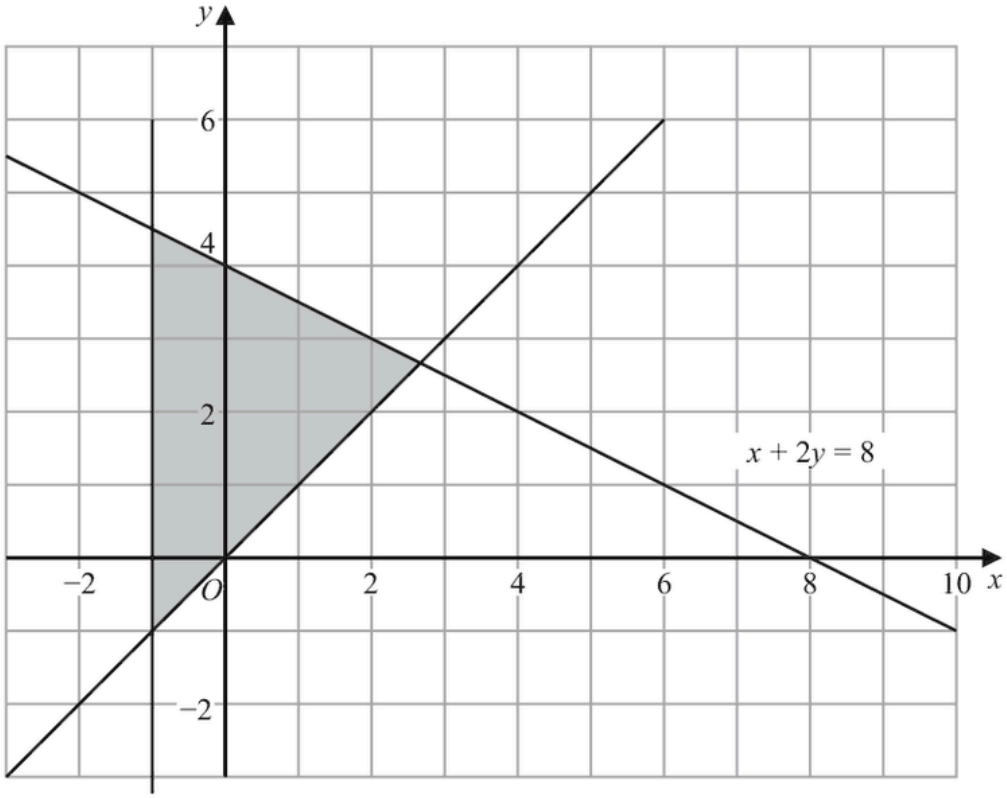
2.16×10^{122}

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Question 13

3 marks | Inequalities (Solving & Graphing)

13 The shaded region in the diagram is bounded by three lines.
The equation of one of the lines is given.



Write down three inequalities that define the shaded region.

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2H January 2022

1 **The region is to the**

The region is to the right of the y-axis:

$$x \geq 0$$

2 **It is above the line**

It is above the line $y = x$:

$$y \geq x$$

3 It is below the line

It is below the line

$$x + 2y = 8$$

so

$$x + 2y \leq 8$$

FINAL ANSWER

$x \geq 0$, $y \geq x$, $x + 2y \leq 8$.

4MA1/2H JANUARY 2022

Question 14

3 marks

Right-Angled Triangles - Pythagoras & Trigonometry

14 A zip wire is shown as the dashed line AC in the diagram.

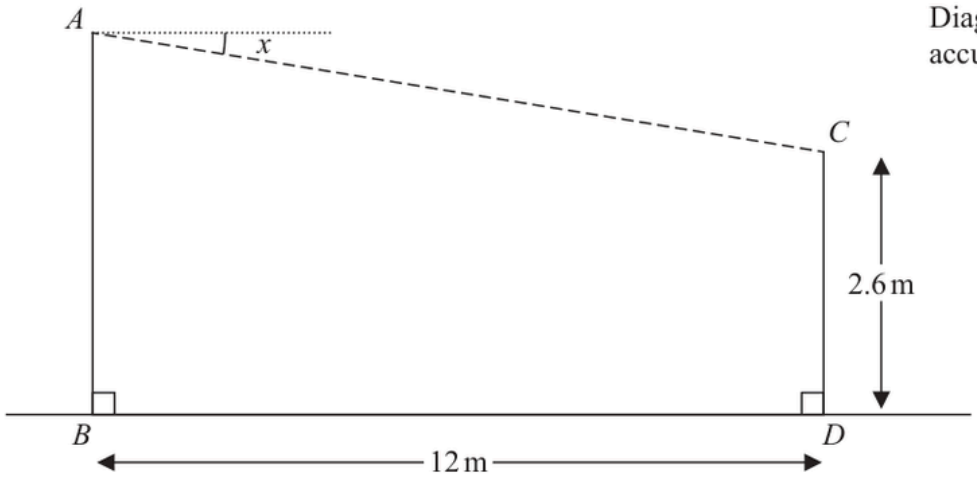


Diagram **NOT** accurately drawn

The zip wire is supported by two vertical posts AB and CD standing on horizontal ground.

$CD = 2.6\,\text{m}$ $BD = 12\,\text{m}$

The zip wire makes an angle x with the horizontal, as shown in the diagram.
The design of the zip wire requires the angle x to be at least $5°$

Work out the least possible height of the post AB
Give your answer correct to 3 significant figures.

..... m

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/2H January 2022

1 **Use trigonometry**

The horizontal distance is $\mathbf{12\ m}$ and the zip wire is $\mathbf{13\ m}$.

2 **Vertical difference**

Vertical difference:

$$\sqrt{13^2 - 12^2} = 5$$

3 **Use trigonometry**

Since the shorter post is $2.6\ \text{m}$,

$$AB = 2.6 + 5 = 7.6$$

✓ **FINAL ANSWER**

$7.60\ \text{m}$.

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Question 15

3 marks | Statistics Toolkit

15 Diyar recorded the distance, in kilometres, that he cycled each day for 11 days.
Here are his results.

8 10 12 13 5 23 21 7 5 16 14

Find the interquartile range of his results.

..... km

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/2H January 2022

1 Put the distances in order

Put the distances in order:

$$5,\ 5,\ 7,\ 8,\ 10,\ 12,\ 13,\ 14,\ 16,\ 21,\ 23$$

2 Calculate statistic

There are 11 values, so the median is the 6th value.

3 The lower half is

The lower half is

$$5,\ 5,\ 7,\ 8,\ 10$$

4 Calculate value

so

$$Q_1 = 7$$

5 The upper half is

The upper half is

$$13,\ 14,\ 16,\ 21,\ 23$$

6 **Calculate value**

so

$$Q_3 = 16$$

$$\text{IQR} = 16 - 7 = 9$$

✓ **FINAL ANSWER**

$9\ \text{km}$.

4MA1/2H JANUARY 2022

Question 16

4 marks | Circle Theorems

16 D, E, F and G are points on a circle, centre O

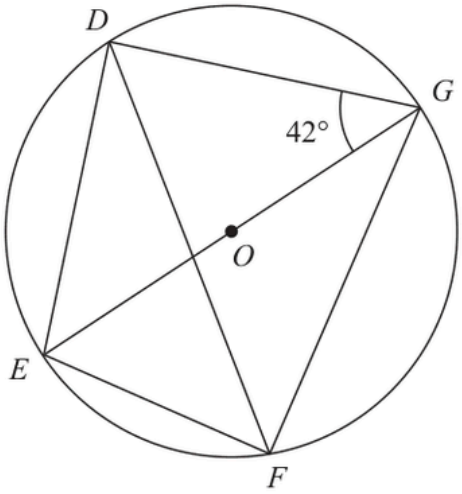


Diagram **NOT** accurately drawn

EOG is a diameter of the circle.

Angle $EGD = 42°$

Calculate the size of angle DFG
Give a reason for each stage of your working.

Angle DFG = ... °

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/2H January 2022

1 Use trigonometry

Since EOG is a diameter,

$$\angle EDG = 90°$$

2 Use trigonometry

In triangle EDG,

$$\angle DEG = 180° - 90° - 42° = 48°$$

3 Angles and stand on the

Angles DEG and DFG stand on the same chord DG, so they are equal.

$$\angle DFG = 48°$$

FINAL ANSWER

$48°$.

4MA1/2H JANUARY 2022

Question 17

3 marks | Algebraic Proof

17 Show that $\dfrac{\sqrt{12}}{\sqrt{3}+2}$

can be written in the form $a+\sqrt{b}$ where a and b are integers.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2H January 2022

1 Simplify surd

$$\frac{\sqrt{12}}{\sqrt{3}+2}=\frac{2\sqrt{3}}{\sqrt{3}+2}$$

2 Multiply by the conjugate

Multiply by the conjugate:

$$\frac{2\sqrt{3}}{\sqrt{3}+2}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=\frac{2\sqrt{3}(2-\sqrt{3})}{4-3}$$

$$=4\sqrt{3}-6$$

3 Simplify surd

Since $4\sqrt{3}=\sqrt{48}$,

$$4\sqrt{3}-6=-6+\sqrt{48}$$

FINAL ANSWER

$-6+\sqrt{48}$, so $a=-6$ and $b=48$.

4MA1/2H JANUARY 2022

Question 18

3 marks | Algebraic Proof

18 Prove that when the sum of the squares of any two consecutive odd numbers is divided by 8, the remainder is always 2
Show clear algebraic working.

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/2H January 2022

1 Let the two consecutive odd

Let the two consecutive odd numbers be

$$2n+1 \quad \text{and} \quad 2n+3$$

2 The sum of their squares

The sum of their squares is

$$(2n+1)^2+(2n+3)^2$$

$$=4n^2+4n+1+4n^2+12n+9$$

$$=8n^2+16n+10$$

$$=8(n^2+2n+1)+2=8(n+1)^2+2$$

3 Use Algebraic proof correct

This is a multiple of 8 plus 2, so the remainder is always 2.

FINAL ANSWER

The remainder is always 2.

4MA1/2H JANUARY 2022

Question 19

3 marks | Circle Theorems

19

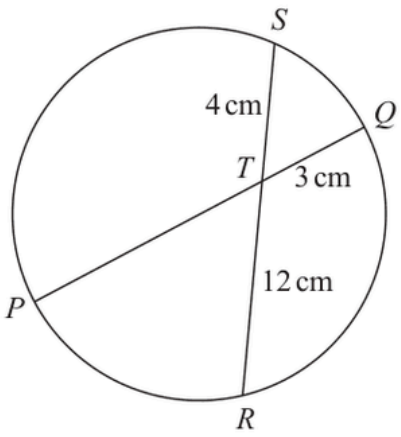


Diagram **NOT** accurately drawn

PTQ is a diameter of a circle.
RTS is a chord of the circle.

$TQ = 3\,\text{cm}$ $ST = 4\,\text{cm}$ $TR = 12\,\text{cm}$

Calculate the radius of the circle.

..... cm

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2H January 2022

1 Use the intersecting chords theorem

Use the intersecting chords theorem:

$$PT \times TQ = ST \times TR$$

$$PT \times 3 = 4 \times 12$$

$$PT = 16$$

2 Use circle theorem

Since PTQ is a diameter,

$$PQ = PT + TQ = 16 + 3 = 19$$

3 **Use circle theorem**

So the radius is

$$19 \div 2 = 9.5$$

✓ **FINAL ANSWER**

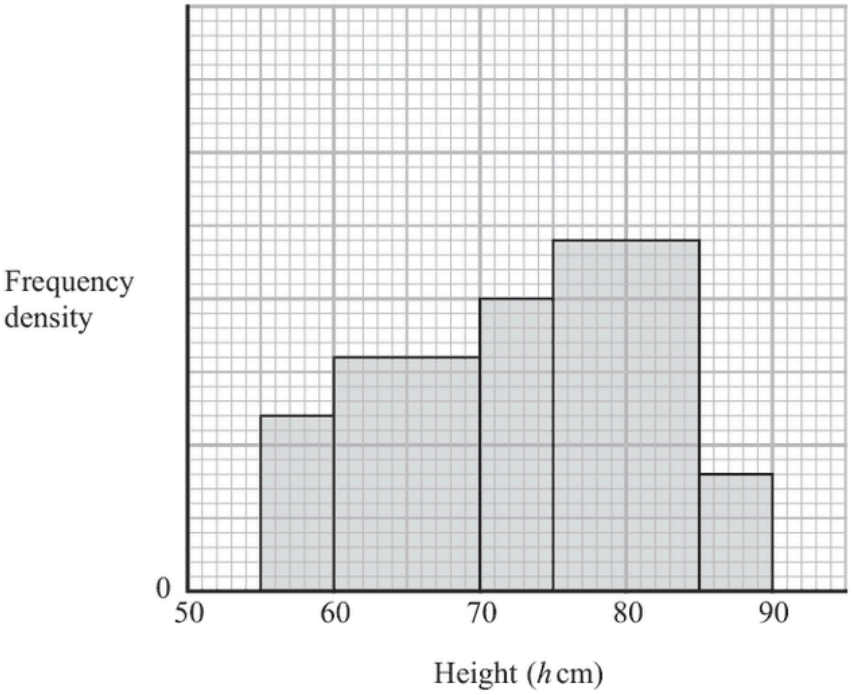
$9.5\ \text{cm}$.

4MA1/2H JANUARY 2022

Question 20

4 marks | Histograms

20 The histogram gives information about the heights, h cm, of some tomato plants.



There are 12 tomato plants for which $75 < h \leqslant 85$
One of the tomato plants is selected at random.

Find an estimate for the probability that this tomato plant has a height greater than 82.5 cm

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2H January 2022

1 Use histogram

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 Use frequency table

For $75 < h \leq 85$, the frequency is 12 and the class width is 10, so

$$\text{frequency density} = \frac{12}{10} = 1.2$$

3 Use histogram

Using this scale from the histogram, the frequency densities are:

$$0.6,\ 0.8,\ 1.0,\ 1.2,\ 0.4$$

4 The total number of tomato

The total number of tomato plants is

$$(5)(0.6) + (10)(0.8) + (5)(1.0) + (10)(1.2) + (5)(0.4)$$

$$= 3 + 8 + 5 + 12 + 2 = 30$$

5 Solve inequality

For $h > 82.5$:

$$(2.5)(1.2) + (5)(0.4) = 3 + 2 = 5$$

6 Calculate probability

So the probability is

$$\frac{5}{30} = \frac{1}{6}$$

FINAL ANSWER

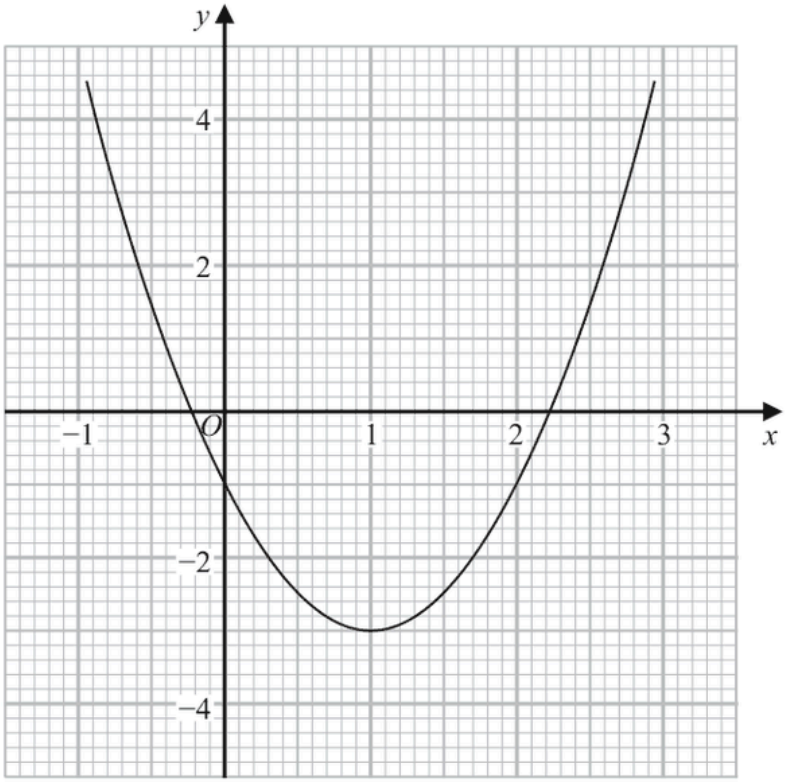
$\frac{1}{6}$

4MA1/2H JANUARY 2022

Question 21

5 marks | Graphs of Functions

21 Part of the graph of $y = 2x^2 - 4x - 1$ is shown on the grid.



(a) Use the graph to find estimates for the solutions of the equation $2x^2 - 4x - 1 = 0$
Give your solutions correct to one decimal place.

.....

(2)

(b) By drawing a suitable straight line on the grid, find estimates for the solutions of the equation $x^2 - x - 1 = 0$
Show your working clearly.
Give your solutions correct to one decimal place.

.....

(3)

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/2H January 2022

1 **Read the graph**

For part (a), solve

$$2x^2 - 4x - 1 = 0$$

2 **Read the graph**

Using the graph gives approximately

$$x = -0.2, \quad x = 2.2$$

3 **Read the graph**

For part (b), we need to solve

$$x^2 - x - 1 = 0$$

4 **Multiply by 2**

Multiply by 2:

$$2x^2 - 2x - 2 = 0$$

5 **The given graph is**

The given graph is

$$y = 2x^2 - 4x - 1$$

6 **Read the graph**

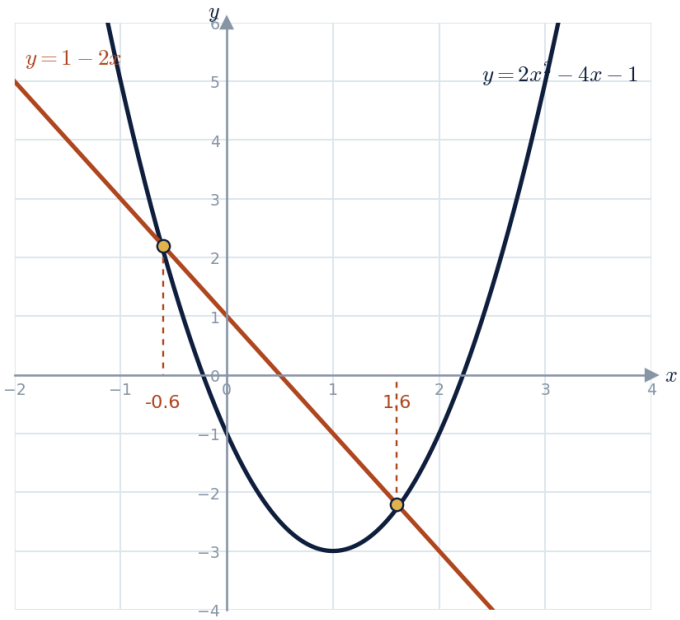
Now

$$2x^2 - 2x - 2 = (2x^2 - 4x - 1) + 2x - 1$$

7 **Read the graph**

So draw the straight line

$$y = 1 - 2x$$



8 **The intersections of this line**

The intersections of this line with the curve give approximately

$$x = -0.6, \quad x = 1.6$$

✓ **FINAL ANSWER**

(a) $-0.2,\ 2.2$; (b) draw $y = 1 - 2x$, giving $-0.6,\ 1.6$

4MA1/2H JANUARY 2022

Question 22

5 marks | Inequalities (Solving & Graphing)

22 Here is a rectangle.

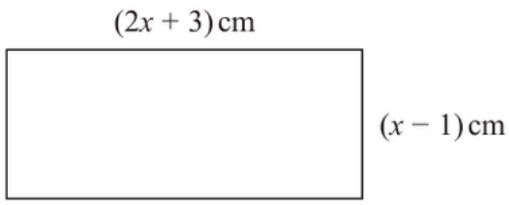


Diagram **NOT** accurately drawn

Given that the area of the rectangle is less than $75\,\text{cm}^2$

find the range of possible values of x

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/2H January 2022

1 The rectangle has side lengths

The rectangle has side lengths

$$2x + 3$$

2 Use Functions Solving Inequalities

and

$$x - 1$$

3 Solve inequality

For the side lengths to be possible,

$$x > 1$$

4 The area is less than

The area is less than 75:

$$(2x + 3)(x - 1) < 75$$

$$2x^2 + x - 3 < 75$$

$$2x^2 + x - 78 < 0$$

5 **Factorise**

Factorise:

$$2x^2 + x - 78 = (2x + 13)(x - 6)$$

---

6 **Evaluate fraction**

So

$$-\frac{13}{2} < x < 6$$

---

7 **Solve inequality**

Combining with $x > 1$:

$$1 < x < 6$$

✓ **FINAL ANSWER**

$1 < x < 6$

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Question 23

6 marks | Sine & Cosine Rules

23 The diagram shows triangle PQR

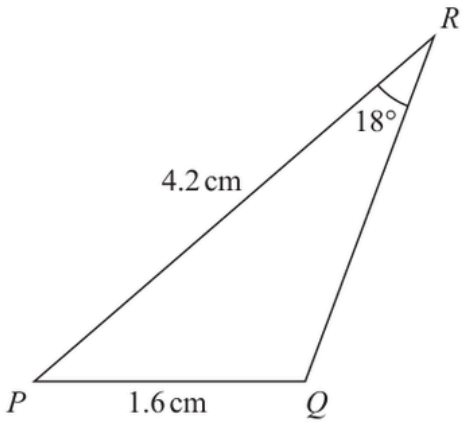


Diagram **NOT** accurately drawn

$PQ = 1.6\,\text{cm}$ $PR = 4.2\,\text{cm}$ Angle $PRQ = 18°$

Given that angle PQR is obtuse,

work out the area of triangle PQR
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 23 is 6 marks)

WORKED SOLUTION

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1 **Use trigonometry**

In triangle PQR,

$$PQ = 1.6, \quad PR = 4.2, \quad \angle PRQ = 18°$$

2 **Use the sine rule**

Use the sine rule:

$$\frac{\sin Q}{PR} = \frac{\sin R}{PQ}$$

$$\frac{\sin Q}{4.2} = \frac{\sin 18^\circ}{1.6}$$

$$\sin Q = \frac{4.2 \sin 18^\circ}{1.6}$$

$$\sin Q = 0.811169\ldots$$

---

3 **Use trigonometry**

Since angle PQR is obtuse,

$$Q = 180^\circ - \sin^{-1}(0.811169\ldots)$$

$$Q = 125.7896\ldots^\circ$$

---

4 **Use trigonometry**

Now find angle P:

$$P = 180^\circ - 18^\circ - 125.7896\ldots^\circ$$

$$P = 36.2103\ldots^\circ$$

---

5 **Area of triangle**

Area of triangle PQR:

$$\frac{1}{2}(PQ)(PR)\sin P$$

$$= \frac{1}{2}(1.6)(4.2)\sin(36.2103\ldots^\circ)$$

$$= 1.984925\ldots$$

---

6 **Correct to 3 significant figures**

Correct to 3 significant figures:

$$1.98$$

FINAL ANSWER

$\boxed{1.98\ \text{cm}^2}$

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Question 24

5 marks | Differentiation & Turning Points

24 A particle P moves along a straight line that passes through the fixed point O

The displacement, x metres, of P from O at time t seconds, where $t \geqslant 0$, is given by

$$x = 4t^3 - 27t + 8$$

The direction of motion of P reverses when P is at the point A on the line.

The acceleration of P at the instant when P is at A is $a\ \text{m/s}^2$

Find the value of a

$a =$ .....

(Total for Question 24 is 5 marks)

WORKED SOLUTION

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1 Split the ratio

$$x = 4t^3 - 27t + 8$$

2 Velocity is

Velocity is

$$\frac{dx}{dt} = 12t^2 - 27$$

3 The direction of motion reverses

The direction of motion reverses when velocity is zero:

$$12t^2 - 27 = 0$$

$$12t^2 = 27$$

$$t^2 = \frac{9}{4}$$

4 Split the ratio

Since $t \geq 0$, $t = \frac{3}{2}$.

5 **Acceleration is**

Acceleration is

$$\frac{d^2x}{dt^2} = 24t$$

---

6 **Split the ratio**

At $t = \frac{3}{2}$,

$$a = 24\left(\frac{3}{2}\right) = 36$$

✓ **FINAL ANSWER**

$a = 36$

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Question 25

5 marks | Functions

25 The function g is defined as

$$g: x \mapsto 5 + 6x - x^2 \quad \text{with domain} \quad \{x : x \geqslant 3\}$$

(a) Express the inverse function g^{-1} in the form $g^{-1}: x \mapsto \ldots$

$g^{-1}: x \mapsto$ .....

(4)

(b) State the domain of g^{-1}

.....

(1)

(Total for Question 25 is 5 marks)

WORKED SOLUTION

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1 Calculate value

$$g(x) = 5 + 6x - x^2$$

2 Complete the square

Complete the square:

$$g(x) = 14 - (x - 3)^2$$

3 Let

Let

$$y = 14 - (x - 3)^2$$

$$(x - 3)^2 = 14 - y$$

4 Find inverse function

The domain of g is $x \geq 3$, so take the positive square root:

$$x - 3 = \sqrt{14 - y}$$

$$x = 3 + \sqrt{14 - y}$$

5 Simplify surd

Therefore

$$g^{-1} : x \mapsto 3 + \sqrt{14 - x}$$

6 Calculate statistic

The range of g is $x \leq 14$, so this is the domain of g^{-1}.

FINAL ANSWER

(a) $g^{-1} : x \mapsto 3 + \sqrt{14 - x}$, (b) domain $x \leq 14$

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Question 26

5 marks | Sequences

26 An arithmetic series has first term a and common difference d, where d is a prime number.

The sum of the first n terms of the series is S_n and

$S_m = 39$

$S_{2m} = 320$

Find the value of d and the value of m
Show clear algebraic working.

$d =$ .....

$m =$ .....

(Total for Question 26 is 5 marks)

WORKED SOLUTION

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1 Calculate value

$$S_m = 39$$

$$S_{2m} = 320$$

2 Use

Use

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

3 Calculate value

Then

$$2S_m = m(2a + (m-1)d) = 78$$

4 Calculate value

and

$$S_{2m} = m(2a + (2m-1)d) = 320$$

5 **Subtract**

Subtract:

$$S_{2m} - 2S_m = m[(2m-1)d - (m-1)d]$$

$$320 - 78 = m(md)$$

$$242 = m^2 d$$

$$242 = 2 \times 11^2$$

---

6 **Square factor**

Since d is prime and m^2 is a square factor,

$$d = 2, \quad m^2 = 121$$

$$m = 11$$

✓ **FINAL ANSWER**

$d = 2,\ m = 11$