Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

4MA1/2HR January 2022 Worked Solutions

January 2022 paper rebuilt with the worked solution after each question.

23	2020–2026	Single-paper solutions
paper questions	52 past papers	checked by Dr Eslam



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PAST PAPER

4MA1/2HR January 2022

January 2022

23	100	2 h
questions	marks	exam time

Each question is followed by its worked solution and final answer.

4MA1/2HR JANUARY 2022

Question 1

4 marks | Probability Toolkit

1 A tin contains tea bags with a choice of four different flavours of tea.
The four flavours of tea are Assam or Darjeeling or Nilgiri or Rize.

Sara takes at random a tea bag from the tin.

The table shows each of the probabilities that the flavour of the tea Sara takes is Assam or Darjeeling or Rize.

Flavour of tea	Assam	Darjeeling	Nilgiri	Rize
Probability	0.38	0.24		0.16

(a) Work out the probability that the flavour of the tea Sara takes is Nilgiri.

.....

(2)

(b) Work out the probability that the flavour of the tea Sara takes is either Darjeeling or Rize.

.....

(2)

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR January 2022

1 **(a) Probabilities add to 1**

$$P(\text{Nilgiri}) = 1 - (0.38 + 0.24 + 0.16) = 1 - 0.78 = 0.22$$

2 **(b) Darjeeling or Rize**

$$0.24 + 0.16 = 0.40$$

✓ **FINAL ANSWER**

(a) 0.22 (b) 0.4

Question 2

7 marks | Reverse Percentages

2 Mary saves for a holiday each year.

In 2020 she saved a total of \$720
In 2021, each month she saved \$78

The total amount Mary saved in 2021 was P% more than the total she saved in 2020

(a) Work out the value of P

.....

(4)

Roberto is going to go on holiday.

He has two coupons that will save him money on his holiday.

Coupon A	**Coupon B**
18% off the cost of the accommodation	12.5% off the total cost of the accommodation **and** the flights

For Roberto's holiday
the cost of the accommodation is \$1600
the cost of the flights is \$800

Roberto can only use one of the coupons.
He wants to save as much money as he can.

(b) Which of the two coupons, **A** or **B**, should he use?
Show your working clearly.

(3)

(Total for Question 2 is 7 marks)

1 **Mary saved in 2021**

Mary saved in 2021:

$$78 \times 12 = 936$$

2 **Increase from 2020**

Increase from 2020:

$$936 - 720 = 216$$

3 **Percentage increase**

Percentage increase:

$$\frac{216}{720} \times 100 = 30$$

4 **Calculate percentage**

So $P = 30$.

5 **Coupon A saves**

Coupon A saves

$$18\% \times 1600 = 0.18 \times 1600 = 288$$

6 **Coupon B saves of the**

Coupon B saves 12.5% of the accommodation and flights together:

$$1600 + 800 = 2400$$

$$12.5\% \times 2400 = 0.125 \times 2400 = 300$$

7 **Coupon B saves more money**

Coupon B saves more money.

✓ **FINAL ANSWER**

(a) $P = 30$, (b) Coupon B

4MA1/2HR JANUARY 2022

Question 3

5 marks | Linear Equations

3 (a) Solve $4y + 5 > 12$

.....

(2)

(b) Solve $6x - 5 = \frac{4x - 7}{2}$

Show clear algebraic working.

$x =$ .....

(3)

(Total for Question 3 is 5 marks)

WORKED SOLUTION

Question 3 · 4MA1/2HR January 2022

1 **(a)**

(a)

$$4y + 5 > 12$$

$$4y > 7$$

$$y > \frac{7}{4}$$

2 **(b)**

(b)

$$6x - 5 = \frac{4x - 7}{2}$$

3 Multiply by 2

Multiply by 2:

$$12x - 10 = 4x - 7$$

$$8x = 3$$

$$x = \frac{3}{8}$$

FINAL ANSWER

$y > \frac{7}{4}$, and $x = \frac{3}{8}$.

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Question 4

4 marks | Angles in Polygons & Parallel Lines

4 The diagram shows a regular octagon $ABCDEFGH$ and a regular pentagon $ABIJK$

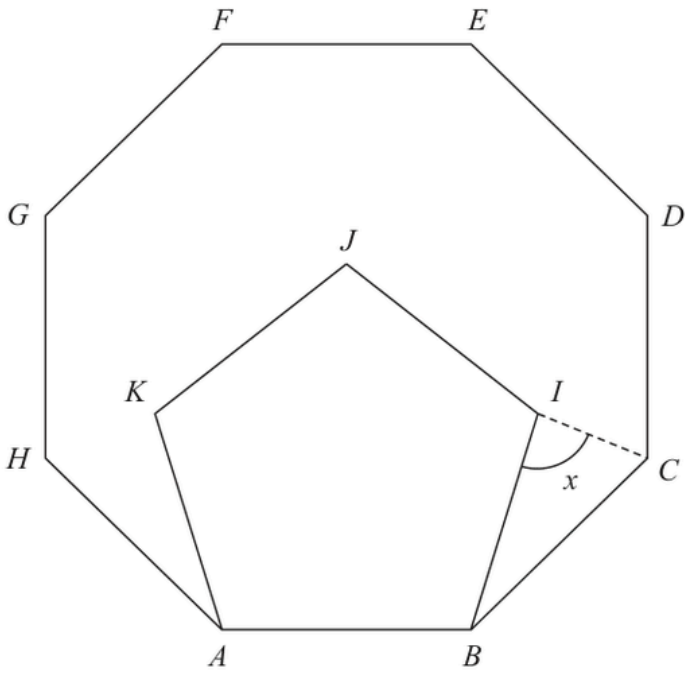


Diagram **NOT** accurately drawn

Work out the size of the angle x

.....°

(Total for Question 4 is 4 marks)

WORKED SOLUTION

Question 4 · 4MA1/2HR January 2022

1 The interior angle of a

The interior angle of a regular octagon is

$$\frac{(8-2)\times 180}{8} = 135°$$

2 The interior angle of a

The interior angle of a regular pentagon is

$$\frac{(5-2)\times 180}{5} = 108°$$

3 **Find the gradient**

At B,

$$\angle IBC = 135° - 108° = 27°$$

4 **Find the gradient**

Since both polygons share side AB,

$$BI = BC$$

5 **Find the gradient**

So triangle BIC is isosceles.

$$x = \frac{180° - 27°}{2} = 76.5°$$

✓ **FINAL ANSWER**

$76.5°$.

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Question 5

3 marks | Compound Interest & Depreciation

5 Shane invests 7200 dollars for 3 years in a savings account.
He gets 2.5% per year compound interest.

How much money will Shane have in his savings account at the end of 3 years?
Give your answer to the nearest dollar.

..... dollars

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/2HR January 2022

1 The multiplier for compound interest

The multiplier for 2.5% compound interest is

$$1.025$$

2 Use compound interest

After 3 years,

$$7200(1.025)^3 = 7753.6125$$

3 Use compound interest

Correct to the nearest dollar,

$$7753.6125 \approx 7754$$

✓ **FINAL ANSWER**

7754 dollars

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Question 6

4 marks | Algebraic Roots & Indices

6 (a) Write down the value of x^0

.....

(1)

Given that $2^{-3} \times 2^9 = 2^n$

(b) find the value of n

$n =$ .....

(1)

Given that $\dfrac{7^{206} \times 7^m}{7^{214}} = 7^{-3}$

(c) find the value of m

$m =$ .....

(2)

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/2HR January 2022

1 **Use index laws**

$$x^0 = 1$$

2 **Use index laws**

For part (b),

$$2^{-3} \times 2^9 = 2^{-3+9} = 2^6$$

3 **Use index laws**

So

$$n = 6$$

4 **Use index laws**

For part (c),

$$\frac{7^{206} \times 7^m}{7^{214}} = 7^{206+m-214}$$

5 **Use index laws**

This equals 7^{-3}, so

$$206 + m - 214 = -3$$

$$m = 5$$

✓ **FINAL ANSWER**

(a) 1, (b) $n = 6$, (c) $m = 5$.

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Question 7

5 marks | Inequalities (Solving & Graphing)

7 (a) Write down an equation of the straight line with gradient -3 and which passes through the point with coordinates $(0, 5)$

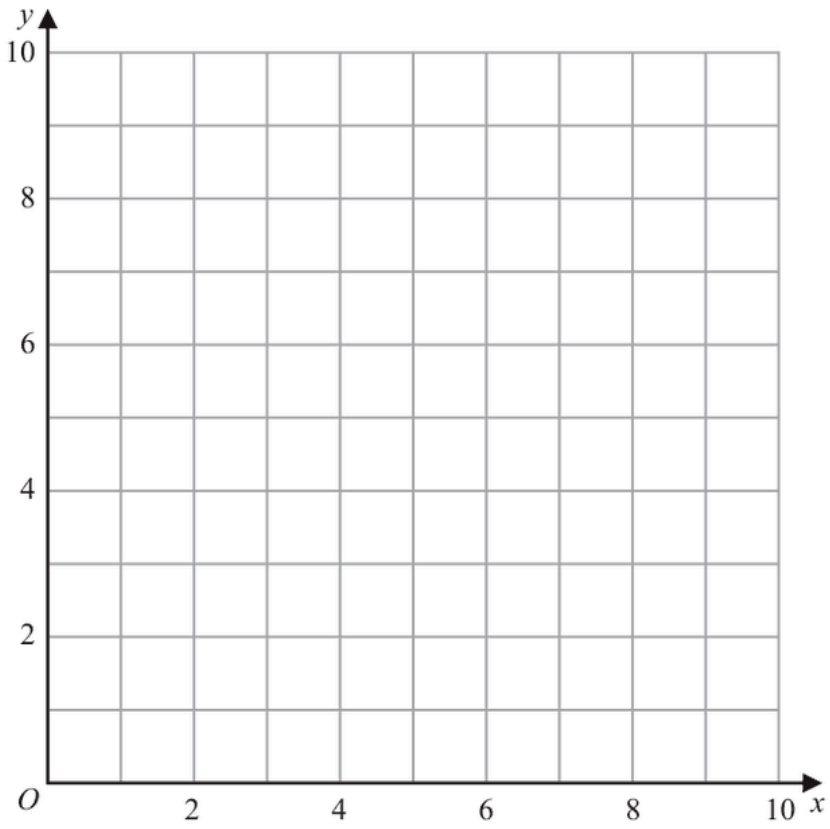
.....

(2)

(b) Show, by shading on the grid, the region defined by **all three** of the inequalities

$$x \leqslant 6 \qquad y \geqslant 2 \qquad y \leqslant x + 1$$

Label the region **R**



(3)

(Total for Question 7 is 5 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR January 2022

1 **A straight line with gradient**

A straight line with gradient -3 passing through $(0, 5)$ has equation

$$y = -3x + 5$$

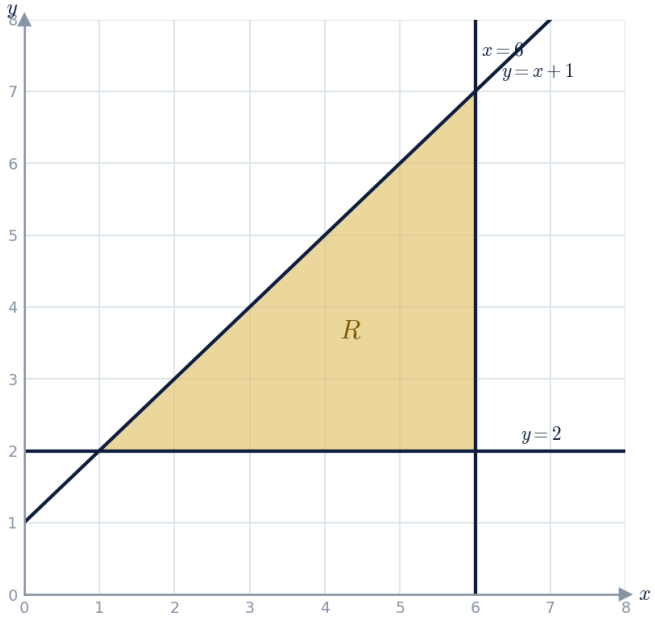
2 **Use the inequalities**

For the region, use the inequalities:

$$x < 6$$

$$y > 2$$

$$y \leq x + 1$$



FINAL ANSWER

(a) $y = -3x + 5$. (b) Shade $x < 6$, $y > 2$, $y \leq x + 1$.

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Question 8

3 marks | Statistics Toolkit

8 A scientist is investigating the weight of 50 tigers.

Here is some information about these tigers.

	Type of tiger	
	Siberian	**Bengal**
Number of tigers	22	28
Mean weight of tigers (kg)	260	

The mean weight of all 50 tigers is 218 kg

Work out the mean weight of the Bengal tigers.

..... kg

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR January 2022

1 Total mass of all 50

Total mass of all 50 tigers:

$$50 \times 218 = 10900$$

2 Total mass of the 22

Total mass of the 22 Siberian tigers:

$$22 \times 260 = 5720$$

3 Total mass of the 28

Total mass of the 28 Bengal tigers:

$$10900 - 5720 = 5180$$

4 Mean mass of the Bengal

Mean mass of the Bengal tigers:

$$\frac{5180}{28} = 185$$

FINAL ANSWER

185 kg.

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Question 9

5 marks | Circles, Arcs & Sectors

9 In the diagram, ABC is a right-angled triangle and DEF is a semicircular arc.

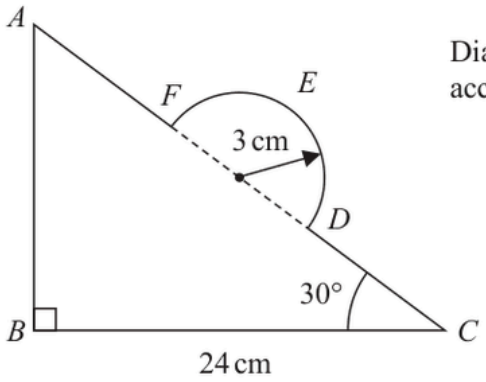


Diagram **NOT** accurately drawn

In triangle ABC

$BC = 24\,\text{cm}$ angle $ABC = 90°$ angle $BCA = 30°$

The points D and F lie on AC so that DF is the diameter of the semicircular arc DEF
The radius of the semicircular arc is 3 cm.

Work out the length of $AFEDC$
Give your answer correct to 2 significant figures.

..... cm

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR January 2022

1 **Use trigonometry**

In triangle ABC,

$$\cos 30° = \frac{24}{AC}$$

$$AC = \frac{24}{\cos 30°} = 27.712\ldots$$

2 **Calculate value**

The semicircle has radius $3\ \text{cm}$, so its diameter is $6\ \text{cm}$.

$$AF + DC = AC - DF = 27.712\ldots - 6$$

3 **Arc is a semicircle**

Arc FED is a semicircle:

$$\text{arc length} = 3\pi$$

4 **Calculate value**

Therefore

$$AFEDC = 27.712\ldots - 6 + 3\pi = 31.137\ldots$$

✓ **FINAL ANSWER**

$31\ \text{cm}$.

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Question 10

3 marks | Number Toolkit

10 The table gives information about the population and the total amount of money, in dollars, spent on healthcare for two countries in 2016

Country	Total population	Total spent on healthcare ($)
Austria	8.7×10^6	4.2×10^{10}
Luxembourg	6.3×10^5	3.7×10^9

Work out how much more was spent **per person** on healthcare in Luxembourg than in Austria.
Give your answer correct to the nearest whole number.

..... dollars

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2HR January 2022

1 Austria spending per person

Austria spending per person:

$$\frac{4.2 \times 10^{10}}{8.7 \times 10^6} = 4827.586\ldots$$

2 Luxembourg spending per person

Luxembourg spending per person:

$$\frac{3.7 \times 10^9}{6.3 \times 10^5} = 5873.015\ldots$$

3 Difference

Difference:

$$5873.015\ldots - 4827.586\ldots = 1045.429\ldots$$

4 Convert standard form

To the nearest whole number, this is

$$1045$$

FINAL ANSWER

1045 dollars

Question 11

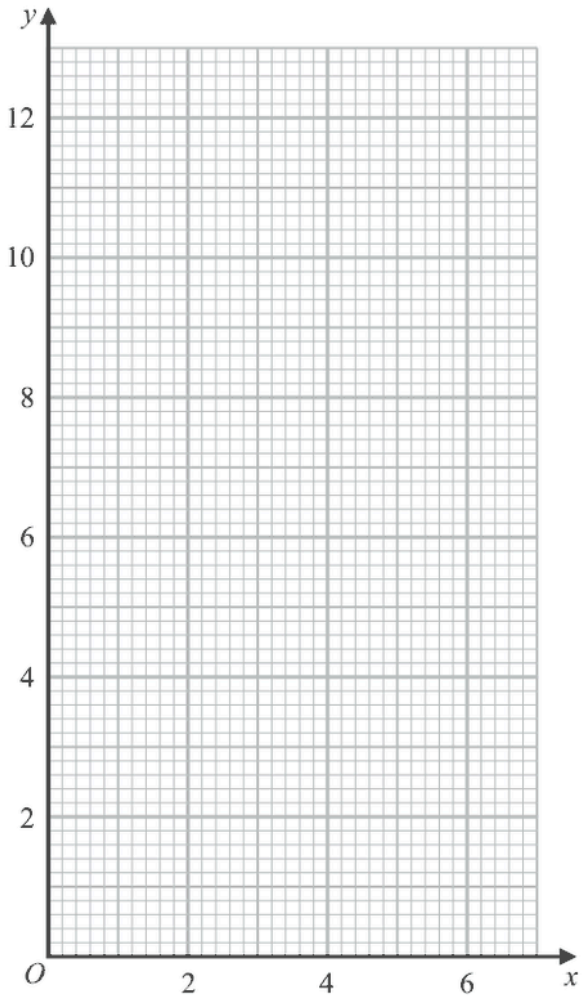
4 marks | Graphs of Functions

11 (a) Complete the table of values for $y = \frac{6}{x}$

x	0.5	1	2	3	4	5	6
y		6		2			1

(2)

(b) On the grid, draw the graph of $y = \frac{6}{x}$ for $0.5 \leqslant x \leqslant 6$



(2)

(Total for Question 11 is 4 marks)

1 Read the graph

$$y = \frac{6}{x}$$

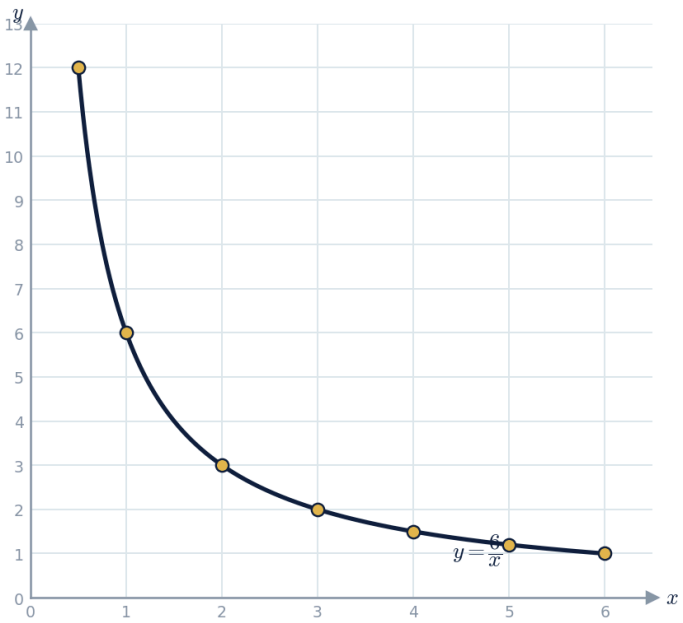
2 The completed table is

The completed table is:

x	0.5	1	2	3	4	5	6
y	12	6	3	2	1.5	1.2	1

3 Plot the points and join

Plot the points and join them with a smooth decreasing reciprocal curve.



FINAL ANSWER

Missing values are $12, 3, 1.5, 1.2$.

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Question 12

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

12 The diagram shows two vertical phone masts, AB and CD, on horizontal ground.

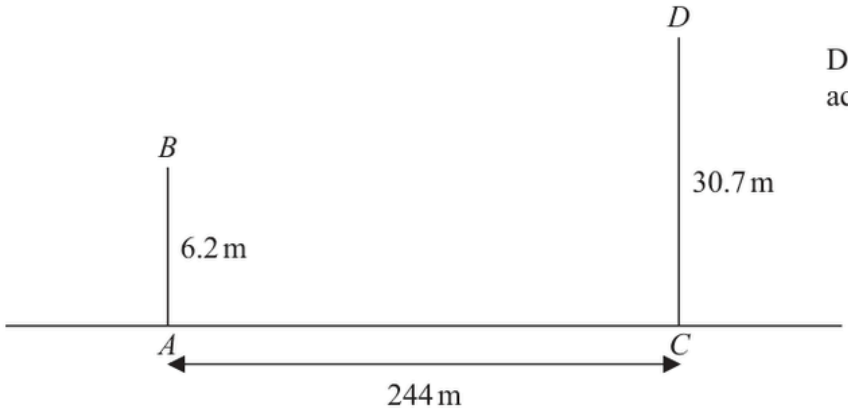


Diagram **NOT** accurately drawn

$AB = 6.2\,\text{m}$ $AC = 244\,\text{m}$ $CD = 30.7\,\text{m}$

Work out the size of the angle of depression of B from D
Give your answer correct to one decimal place.

.....°

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2HR January 2022

1 The vertical difference between the

The vertical difference between the tops of the masts is

$$30.7 - 6.2 = 24.5\ \text{m}$$

2 The horizontal distance is

The horizontal distance is $244\ \text{m}$.

$$\tan\theta = \frac{24.5}{244}$$

$$\theta = 5.733\ldots^\circ$$

FINAL ANSWER

5.7°.

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Question 13

3 marks | Algebraic Proof

13 $a = \sqrt{8} + 4$

$b = \sqrt{8} - 4$

$(a - b)(a + b)$ can be written in the form $y\sqrt{4y}$

Find the value of y
Show your working clearly.

$y =$ .....

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2HR January 2022

1 Simplify surd

$$a = \sqrt{8} + 4, \qquad b = \sqrt{8} - 4$$

$$a - b = (\sqrt{8} + 4) - (\sqrt{8} - 4) = 8$$

$$a + b = (\sqrt{8} + 4) + (\sqrt{8} - 4) = 2\sqrt{8} = 4\sqrt{2}$$

2 Simplify surd

So

$$(a - b)(a + b) = 8 \times 4\sqrt{2} = 32\sqrt{2}$$

3 The question says this equals

The question says this equals $y\sqrt{4y}$. Try $y = 8$:

$$8\sqrt{32} = 8 \times 4\sqrt{2} = 32\sqrt{2}$$

FINAL ANSWER

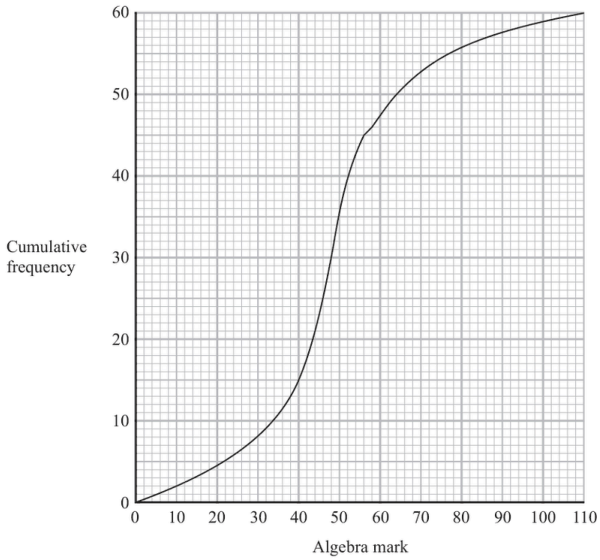
$y = 8$.

Question 14

8 marks | Cumulative Frequency Diagrams

14 A group of 60 students each sat an algebra test and a geometry test.
Each test was marked out of 110

The cumulative frequency graph gives information about the marks gained by the 60 students in the algebra test.



(a) Use the graph to find an estimate for the median mark in the algebra test.

.....

(1)

(b) Use the graph to find an estimate for the number of students who gained 58 marks or less in the algebra test.

.....

(1)

(c) Use the graph to find an estimate for the interquartile range of the marks gained in the algebra test.

.....

(2)

The interquartile range of the marks gained in the geometry test is 9

Luis says

"The students' marks are more spread out in the algebra test than in the geometry test."

(d) Is Luis correct?
Give a reason for your answer.

.....

.....

(1)

To be awarded a grade A in the algebra test, a student had to gain a mark greater than 64

Two students are to be selected at random from the 60 students in the group.

(e) Use the graph to find an estimate for the probability that both of these students were awarded a grade A in the algebra test.

.....

(3)

(Total for Question 14 is 8 marks)

1 **There are students**

There are 60 students.

2 **The median is at cumulative**

The median is at cumulative frequency 30. From the graph,

$$\text{median} \approx 49$$

3 **Use cumulative frequency**

At 58 marks, the cumulative frequency is about 45, so about 45 students gained 58 marks or less.

4 **Use cumulative frequency**

For the interquartile range:

$$Q_1 \text{ is at } 15, \qquad Q_3 \text{ is at } 45$$

5 **Use cumulative frequency**

From the graph,

$$Q_1 \approx 40, \qquad Q_3 \approx 57$$

$$\text{IQR} \approx 57 - 40 = 17$$

6 **Use cumulative frequency**

The geometry IQR is 9, so Luis is correct because the algebra IQR is larger.

7 **Use cumulative frequency**

For grade A, use mark 64. From the graph, about 50 students gained 64 or less, so about

$$60 - 50 = 10$$

8 **students gained more than**

students gained more than 64.

9 **Probability both selected students were**

Probability both selected students were grade A:

$$\frac{10}{60} \times \frac{9}{59} = 0.0254\ldots$$

FINAL ANSWER

median about 49, about 45 students, IQR about 17, Luis is correct, probability about 0.025.

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Question 15

4 marks | Rearranging Formulae

15 Make t the subject of $n^2 = \dfrac{4d + t^3}{t^3}$

(Total for Question 15 is 4 marks)

WORKED SOLUTION

Question 15 · 4MA1/2HR January 2022

1 Rearrange formula

$$n^2 = \frac{4d + t^3}{t^3}$$

$$n^2 t^3 = 4d + t^3$$

$$t^3(n^2 - 1) = 4d$$

$$t^3 = \frac{4d}{n^2 - 1}$$

FINAL ANSWER

$$t = \sqrt[3]{\frac{4d}{n^2 - 1}}$$

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Question 16

5 marks | Sine & Cosine Rules

16 The diagram shows quadrilateral $ABCD$

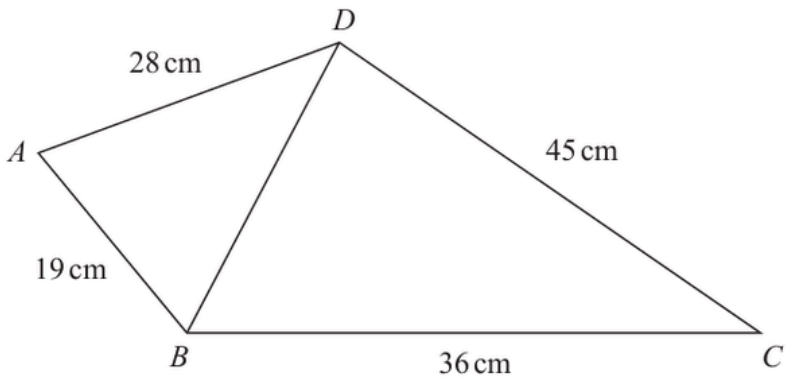


Diagram **NOT** accurately drawn

The angle BCD is acute.

Given that the area of triangle $BCD = 405\,\text{cm}^2$

work out the size of angle ABD
Give your answer correct to one decimal place.

.....°

(Total for Question 16 is 5 marks)

WORKED SOLUTION

Question 16 · 4MA1/2HR January 2022

1 **Use trigonometry**

For triangle BCD,

$$405 = \frac{1}{2}(36)(45)\sin BCD$$

$$\sin BCD = 0.5$$

2 **Use trigonometry**

The angle BCD is acute, so

$$BCD = 30°$$

3 **Find**

Find BD:

$$BD^2 = 36^2 + 45^2 - 2(36)(45)\cos 30°$$

$$BD = 22.695\ldots$$

---

4 **Use trigonometry**

Now use cosine rule in triangle ABD:

$$\cos ABD = \frac{19^2 + BD^2 - 28^2}{2(19)(BD)}$$

$$ABD = 83.871\ldots°$$

✓ **FINAL ANSWER**

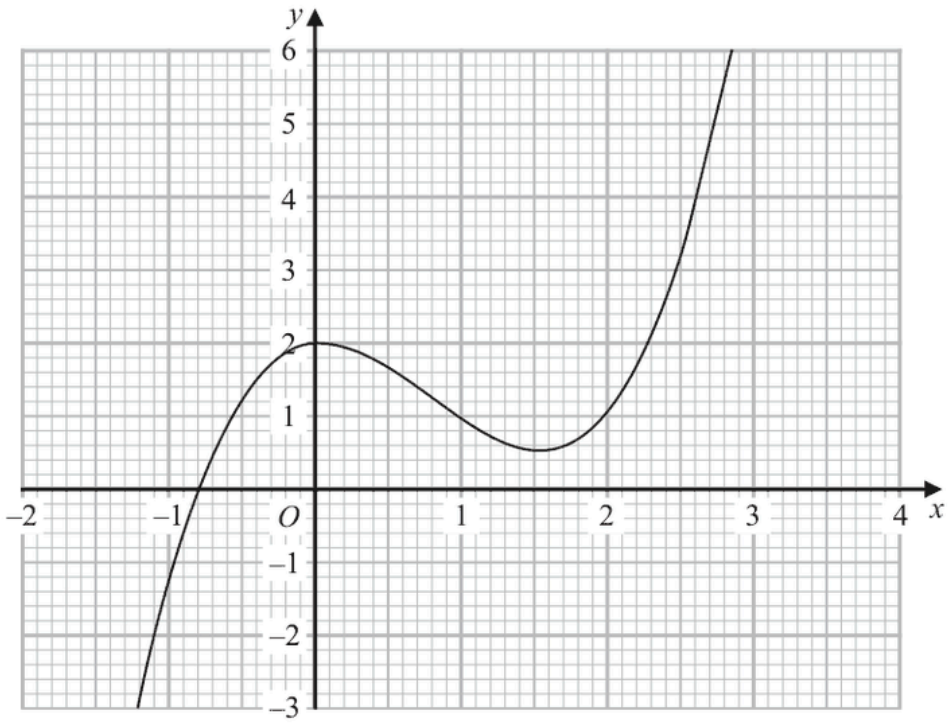
$83.9°$.

4MA1/2HR JANUARY 2022

Question 17

3 marks | Graphs of Functions

17 Part of the curve with equation $y = f(x)$ is shown on the grid.



Find an estimate for the gradient of the curve at the point where $x = 2$
Show your working clearly.

(Total for Question 17 is 3 marks)

WORKED SOLUTION

Question 17 · 4MA1/2HR January 2022

1 Draw a tangent to the

Draw a tangent to the curve at the point where

$$x = 2$$

2 Find the gradient

Using two clear points on the tangent, for example approximately $(1.5, 0.6)$ and $(2.3, 2.2)$,

$$\text{gradient} \approx \frac{2.2 - 0.6}{2.3 - 1.5}$$

$$\text{gradient} \approx 2.0$$

FINAL ANSWER

The gradient is approximately $\mathbf{2}$.

4MA1/2HR JANUARY 2022

Question 18

5 marks | Linear Graphs

18 The line with equation $2y = x + 1$ intersects the curve with equation $3y^2 + 7y + 16 = x^2 - x$ at the points A and B

Find the coordinates of A and the coordinates of B
Show clear algebraic working.

(..... , ...) and (..... , ...)

(Total for Question 18 is 5 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR January 2022

1 The line is

The line is

$$2y = x + 1$$

2 Calculate value

So

$$x = 2y - 1$$

3 Substitute into the curve

Substitute into the curve

$$3y^2 + 7y + 16 = x^2 - x$$

$$3y^2 + 7y + 16 = (2y - 1)^2 - (2y - 1)$$

$$3y^2 + 7y + 16 = 4y^2 - 6y + 2$$

$$y^2 - 13y - 14 = 0$$

$$(y - 14)(y + 1) = 0$$

4 **Calculate value**

So

$$y = 14 \quad \text{or} \quad y = -1$$

5 **Calculate value**

If $y = 14$, then $x = 27$.

6 **Calculate value**

If $y = -1$, then $x = -3$.

✓ **FINAL ANSWER**

$(27, 14)$ and $(-3, -1)$.

4MA1/2HR JANUARY 2022

Question 19

3 marks | 3D Pythagoras & Trigonometry

19 $ABCD$ is a horizontal rectangular field.

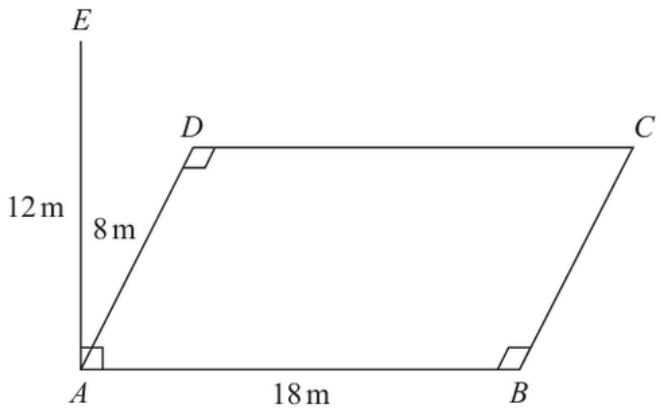


Diagram **NOT** accurately drawn

A vertical pole, AE, is placed at the corner A of the field.

$AE = 12\,\text{m}$ $AB = 18\,\text{m}$ $AD = 8\,\text{m}$

Calculate the size of the angle between EC and the plane $ABCD$
Give your answer correct to one decimal place.

.....°

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/2HR January 2022

1 The projection of onto the

The projection of EC onto the field is AC.

$$AC = \sqrt{18^2 + 8^2} = \sqrt{388}$$

2 Use trigonometry

The vertical height is $AE = 12\ \text{m}$, so

$$\tan\theta = \frac{12}{\sqrt{388}}$$

$$\theta = 31.350\ldots^\circ$$

FINAL ANSWER

$31.4°$.

4MA1/2HR JANUARY 2022

Question 20

4 marks | Direct & Inverse Proportion

20 y is inversely proportional to $\sqrt{x}$
x is directly proportional to T^3

Given that $y = 8$ when $T = 25$

find the exact value of T when $y = 27$

$T =$ .....

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR January 2022

1 Find inverse function

y is inversely proportional to $\sqrt{x}$, so

$$y \propto \frac{1}{\sqrt{x}}$$

2 Find inverse function

x is directly proportional to T^3, so $\sqrt{x} \propto T^{3/2}$.

3 Find inverse function

Therefore

$$y \propto \frac{1}{T^{3/2}}$$

4 Find inverse function

So

$$yT^{3/2} = k$$

5 Use when

Use $y = 8$ when $T = 25$:

$$k = 8 \times 25^{3/2} = 8 \times 125 = 1000$$

6 **Find inverse function**

When $y = 27$,

$$27T^{3/2} = 1000$$

$$T^{3/2} = \frac{1000}{27} = \left(\frac{10}{3}\right)^3$$

---

7 **Raise both sides to the**

Raise both sides to the power $\frac{2}{3}$:

$$T = \left(\frac{10}{3}\right)^2 = \frac{100}{9}$$

✓ **FINAL ANSWER**

$T = \frac{100}{9}$.

Question 21

6 marks | Standard & Compound Units

21 The diagram shows a solid made from a cylinder and a hemisphere.
The cylinder and the hemisphere are both made from the same metal.

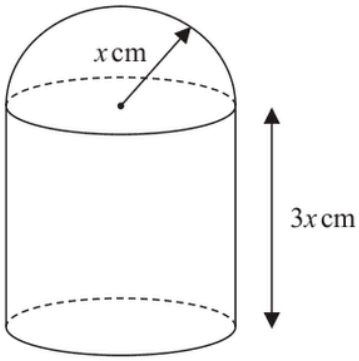


Diagram **NOT** accurately drawn

The plane face of the hemisphere coincides with the upper plane face of the cylinder.

The radius of the cylinder and the radius of the hemisphere are both x cm.
The height of the cylinder is $3x$ cm.

The total surface area of the solid is 81π cm^2
The mass of the solid is 840 grams.

The following table gives the density of each of four metals.

Metal	Density (g/cm^3)
Aluminium	2.7
Nickel	8.9
Gold	19.3
Silver	10.5

The metal used to make the solid is one of the metals in the table.

Determine the metal used to make the solid.
Show your working clearly.

(Total for Question 21 is 6 marks)

1 **The outside surface area is**

The outside surface area is:

2 **curved surface area of cylinder**

curved surface area of cylinder:

$$2\pi x(3x) = 6\pi x^2$$

3 **bottom of cylinder**

bottom of cylinder:

$$\pi x^2$$

4 **curved surface area of hemisphere**

curved surface area of hemisphere:

$$2\pi x^2$$

5 **Calculate value**

So

$$6\pi x^2 + \pi x^2 + 2\pi x^2 = 81\pi$$

$$9\pi x^2 = 81\pi$$

$$x^2 = 9$$

$$x = 3$$

6 **Volume of the cylinder**

Volume of the cylinder:

$$\pi x^2(3x) = 3\pi x^3$$

7 **Volume of the hemisphere**

Volume of the hemisphere:

$$\frac{2}{3}\pi x^3$$

8 **Total volume**

Total volume:

$$3\pi(3)^3 + \frac{2}{3}\pi(3)^3$$

$$= 81\pi + 18\pi = 99\pi$$

---

9 **Density**

Density:

$$\frac{840}{99\pi} = 2.70\ldots$$

---

10 **Compare density**

This matches aluminium, whose density is $2.7\ \text{g/cm}^3$.

✓ **FINAL ANSWER**

Aluminium

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Question 22

5 marks | Linear Graphs

22 ABC is a triangle in which angle $ABC = 90°$

p and q are integers such that

the coordinates of A are $(p, 10)$
the coordinates of B are $(-1, -5)$
the coordinates of C are $(8, q)$

Given that the gradient of AC is $-\frac{6}{7}$

work out the value of p and the value of q

$p =$ .....

$q =$ .....

(Total for Question 22 is 5 marks)

WORKED SOLUTION

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1 The gradient of is

The gradient of AC is $-\frac{6}{7}$.

$$\frac{q-10}{8-p} = -\frac{6}{7}$$

$$7(q-10) = -6(8-p)$$

$$7q - 70 = -48 + 6p$$

$$6p - 7q = -22$$

2 **Find the gradient**

Since $\angle ABC = 90°$, lines AB and BC are perpendicular.

$$\text{gradient of } AB = \frac{10-(-5)}{p-(-1)} = \frac{15}{p+1}$$

$$\text{gradient of } BC = \frac{q-(-5)}{8-(-1)} = \frac{q+5}{9}$$

---

3 **The product of perpendicular gradients**

The product of perpendicular gradients is -1:

$$\frac{15}{p+1} \times \frac{q+5}{9} = -1$$

$$15(q+5) = -9(p+1)$$

$$5(q+5) = -3(p+1)$$

$$3p + 5q = -28$$

---

4 **Solve**

Now solve

$$6p - 7q = -22$$

$$3p + 5q = -28$$

---

5 **Double the second equation**

Double the second equation:

$$6p + 10q = -56$$

---

6 **Subtract**

Subtract:

$$17q = -34$$

$$q = -2$$

---

7 **Calculate value**

Then

$$3p + 5(-2) = -28$$

$$3p = -18$$

$$p = -6$$

FINAL ANSWER

$p = -6,\ q = -2$

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Question 23

4 marks | Functions

23 The functions f and g are such that

$$f(x) = x + 25 \qquad g(x) = x^2 - 12x$$

The function h is such that $h(x) = fg(x)$

The domain of h is $\{x : x \leqslant 6\}$

Express the inverse function h^{-1} in the form $h^{-1}(x) = ...$

$h^{-1}(x) =$ .....

(Total for Question 23 is 4 marks)

WORKED SOLUTION

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1 **Calculate value**

$$f(x) = x + 25$$

$$g(x) = x^2 - 12x$$

$$h(x) = fg(x) = f(g(x))$$

$$h(x) = x^2 - 12x + 25$$

$$h(x) = (x - 6)^2 - 11$$

2 **Let**

Let

$$y = (x - 6)^2 - 11$$

$$y + 11 = (x - 6)^2$$

3 Find inverse function

Since the domain of h is $x < 6$, take the negative square root:

$$x - 6 = -\sqrt{y + 11}$$

$$x = 6 - \sqrt{y + 11}$$

FINAL ANSWER

$$h^{-1}(x) = 6 - \sqrt{x + 11}$$