Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

4MA1/1H January 2022 Worked Solutions

January 2022 paper rebuilt with the worked solution after each question.

24	2020–2026	Single-paper solutions
paper questions	52 past papers	checked by Dr Eslam



Dr Eslam Ahmed
Assistant Lecturer, Faculty of Engineering, Cairo University
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PAST PAPER

4MA1/1H January 2022

January 2022

24	100	2 h
questions	marks	exam time

Each question is followed by its worked solution and final answer.

4MA1/1H JANUARY 2022

Question 1

6 marks | Algebraic Roots & Indices

1 (a) Simplify $a^7 \times a^4$

.....

(1)

(b) Simplify $w^{15} \div w^3$

.....

(1)

(c) Simplify $(8x^5y^3)^2$

.....

(2)

(d) Make t the subject of $c = t^3 - 8v$

.....

(2)

(Total for Question 1 is 6 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H January 2022

1 **(a)**

(a)

$$a^7 \times a^4 = a^{11}$$

2 **(b)**

(b)

$$w^{15} \div w^3 = w^{12}$$

3 **(c)**

(c)

$$(8x^5y^3)^2 = 64x^{10}y^6$$

4 **(d)**

(d)

$$c = t^3 - 8v$$

$$t^3 = c + 8v$$

✓ **FINAL ANSWER**

a^{11}, w^{12}, $64x^{10}y^6$, $t = \sqrt[3]{c + 8v}$

4MA1/1H JANUARY 2022

Question 2

3 marks | Ratio Toolkit

2 Danil, Gabriel and Hadley share some money in the ratios $3:5:9$
The difference between the amount of money that Gabriel receives and the amount of money that Hadley receives is 196 euros.

Work out the amount of money that Danil receives.

..... euros

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H January 2022

1 The ratio is

The ratio is

$$\text{Danil} : \text{Gabriel} : \text{Hadley} = 3 : 5 : 9$$

2 Split the ratio

The difference between Gabriel and Hadley is

$$9 - 5 = 4 \text{ parts}$$

3 Split the ratio

These 4 parts are 196 euros, so one part is

$$196 \div 4 = 49$$

4 Danil has 3 parts

Danil has 3 parts:

$$3 \times 49 = 147$$

FINAL ANSWER

Danil receives 147 euros.

4MA1/1H JANUARY 2022

Question 3

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

3 The diagram shows triangle ABC

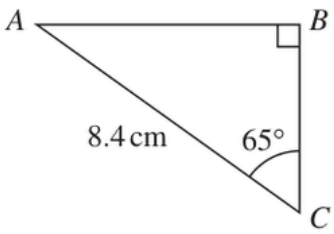


Diagram **NOT** accurately drawn

Work out the length of the side AB
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 3 is 3 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H January 2022

1 **Use trigonometry**

Side AB is opposite the $65°$ angle, and $AC = 8.4$ cm is the hypotenuse.

$$\sin 65° = \frac{AB}{8.4}$$

$$AB = 8.4 \sin 65° = 7.613\ldots$$

✓ **FINAL ANSWER**

7.61 cm.

4MA1/1H JANUARY 2022

Question 4

5 marks | Reverse Percentages

4 Sarah makes and sells mugs.

One day she makes 150 mugs.
Her total cost for making these mugs is £1140

Of these mugs

$\frac{2}{5}$ are small mugs

32% are medium mugs

and the rest are large mugs

Here is Sarah's price list for selling each mug.

MUGS	
Small	**£8.50**
Medium	**£11.20**
Large	**£14.20**

Sarah sells all 150 mugs.

Work out her percentage profit.
Give your answer correct to the nearest whole number.

..... %

(Total for Question 4 is 5 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H January 2022

1 **Small mugs**

Small mugs:

$$\frac{2}{5} \times 150 = 60$$

2 **Medium mugs**

Medium mugs:

$$32\% \times 150 = 48$$

3 **Large mugs**

Large mugs:

$$150 - 60 - 48 = 42$$

---

4 **Revenue**

Revenue:

$$60(8.50) + 48(11.20) + 42(14.20)$$

$$= 510 + 537.60 + 596.40 = 1644$$

---

5 **Profit**

Profit:

$$1644 - 1140 = 504$$

---

6 **Percentage profit**

Percentage profit:

$$\frac{504}{1140} \times 100 = 44.21\ldots$$

---

7 **Correct to the nearest whole**

Correct to the nearest whole number:

$$44\%$$

✓ **FINAL ANSWER**

44%

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Question 5

6 marks | Statistics Toolkit

5 Jenny has six cards.

Each card has a whole number written on it so that

the smallest number is 5
the largest number is 24
the median of the six numbers is 14
the mode of the six numbers is 8

Jenny arranges her cards so that the numbers are in order of size.



(a) For the remaining four cards, write on each dotted line a number that could be on the card.

(3)

A basketball team plays 6 games.
After playing 5 games, the team has a mean score of 21 points per game.
After playing 6 games, the team has a mean score of 23 points per game.

(b) Work out the number of points the team scored in its 6th game.

.....

(3)

(Total for Question 5 is 6 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H January 2022

1 **One possible ordered set is**

One possible ordered set is

$$5,\ 8,\ 8,\ 20,\ 21,\ 24$$

2 **Check**

Check:

$$\text{median} = \frac{8 + 20}{2} = 14$$

3 **Calculate statistic**

The mode is 8, the smallest number is 5, and the largest number is 24.

4 **Calculate value**

For the basketball scores:

$$5 \times 21 = 105$$

$$6 \times 23 = 138$$

$$138 - 105 = 33$$

✓ **FINAL ANSWER**

$5, 8, 8, 20, 21, 24$ and 33.

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Question 6

5 marks | Inequalities (Solving & Graphing)

6 (a) Solve the inequality $5x - 7 \leqslant 2$

.....

(2)

(b) (i) Factorise $y^2 - 2y - 35$

.....

(2)

(ii) Hence, solve $y^2 - 2y - 35 = 0$

.....

(1)

(Total for Question 6 is 5 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H January 2022

1 **(a)**

(a)

$$5x - 7 < 2$$

$$5x < 9$$

$$x < \frac{9}{5}$$

2 **(b)(i)**

(b)(i)

$$y^2 - 2y - 35 = (y - 7)(y + 5)$$

3 **Factorise expression**

(b)(ii) $y = 7$ or $y = -5$.

FINAL ANSWER

(a) $x < \frac{9}{5}$, (b)(i) $(y - 7)(y + 5)$, (b)(ii) $y = 7$ or $y = -5$

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Question 7

3 marks | Set Notation & Venn Diagrams

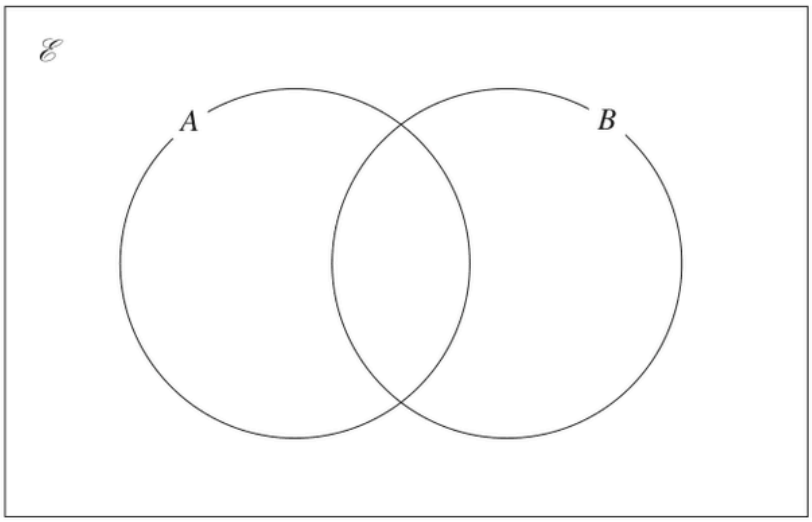
7 $\mathscr{E} = \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

$A \cap B = \{5, 10, 15\}$

$B' = \{7, 8, 9, 11, 12, 13, 14\}$

$A' = \{4, 6, 7, 8, 14\}$

Complete the Venn diagram for this information.



(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H January 2022

1 Calculate value

$$\mathcal{E} = \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$$

2 We are given

We are given

$$A \cap B = \{5, 10, 15\}$$

3 Calculate value

Also,

$$B' = \{7, 8, 9, 11, 12, 13, 14\}$$

4 **Calculate value**

so

$$B = \{4, 5, 6, 10, 15\}$$

5 **Calculate value**

And

$$A' = \{4, 6, 7, 8, 14\}$$

6 **Calculate value**

so

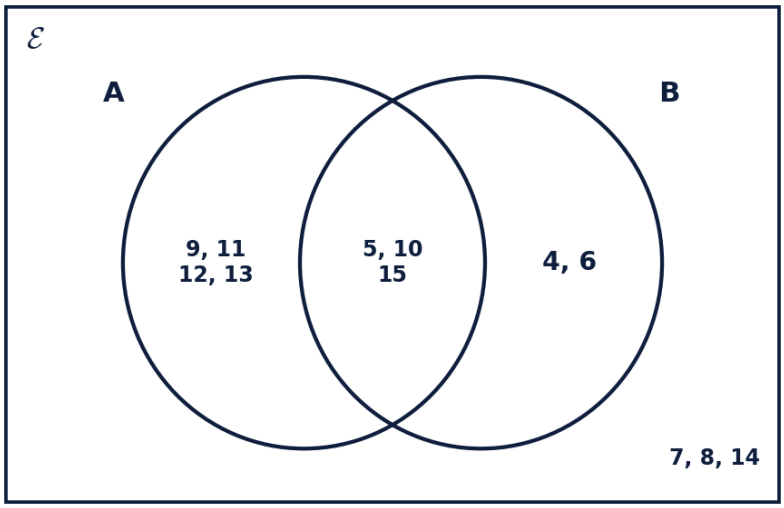
$$A = \{5, 9, 10, 11, 12, 13, 15\}$$

7 **Use Set notation Venn diagrams**

Therefore the Venn diagram regions are:

8 **only only outside both sets**

- $A \cap B$: $\{5, 10, 15\}$
- A only: $\{9, 11, 12, 13\}$
- B only: $\{4, 6\}$
- outside both sets: $\{7, 8, 14\}$



✓ **FINAL ANSWER**

Put $\{5, 10, 15\}$ in the overlap, $\{9, 11, 12, 13\}$ in A only, $\{4, 6\}$ in B only, and $\{7, 8, 14\}$ outside.

4MA1/1H JANUARY 2022

Question 8

2 marks | Standard Form

8 $a = 4.2 \times 10^{-24}$ $b = 3 \times 10^{145}$

Work out the value of $a \times b$
Give your answer in standard form.

(Total for Question 8 is 2 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H January 2022

1 Convert standard form

$$a = 4.2 \times 10^{-24}$$

$$b = 3 \times 10^{145}$$

2 Convert standard form

So

$$ab = (4.2 \times 3) \times 10^{-24+145}$$

$$ab = 12.6 \times 10^{121}$$

3 Write this in standard form

Write this in standard form:

$$12.6 \times 10^{121} = 1.26 \times 10^{122}$$

FINAL ANSWER

1.26×10^{122}

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Question 9

4 marks | Sine & Cosine Rules

9 The diagram shows isosceles triangle ABC

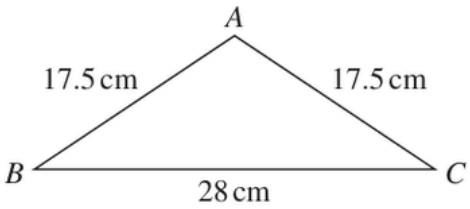


Diagram **NOT** accurately drawn

$AB = AC = 17.5$ cm $\quad\quad$ $BC = 28$ cm

Calculate the area of triangle ABC

..... cm^2

(Total for Question 9 is 4 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H January 2022

1 **Use cosine rule**

Split the isosceles triangle into two right-angled triangles.

$$\frac{BC}{2} = 14$$

$$h^2 = 17.5^2 - 14^2 = 110.25$$

$$h = 10.5$$

$$\text{area} = \frac{1}{2} \times 28 \times 10.5 = 147$$

FINAL ANSWER

$147\ \text{cm}^2$.

4MA1/1H JANUARY 2022

Question 10

3 marks | Linear Graphs

10 The straight line **L** has equation $2y + 7x = 10$

(a) Find the gradient of **L**

.....

(2)

(b) Find the coordinates of the point where **L** crosses the y-axis.

(..... , ...)

(1)

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H January 2022

1 **The line is**

The line is

$$2y + 7x = 10$$

$$2y = -7x + 10$$

$$y = -\frac{7}{2}x + 5$$

2 **Find the gradient**

So the gradient is

$$-\frac{7}{2}$$

3 **Evaluate fraction**

For the x-axis, $y = 0$:

$$2(0) + 7x = 10$$

$$x = \frac{10}{7}$$

FINAL ANSWER

Gradient $-\frac{7}{2}$, and x-axis intersection $\left(\frac{10}{7}, 0\right)$.

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Question 11

3 marks | Compound Interest & Depreciation

11 Himari invests 200000 yen for 3 years in a savings account paying compound interest.

The rate of interest is 1.8% for the first year and x% for each of the second year and the third year.

The value of the investment at the end of the third year is 209754 yen.

Work out the value of x
Give your answer correct to one decimal place.

$x =$ .....

(Total for Question 11 is 3 marks)

WORKED SOLUTION

Question 11 · 4MA1/1H January 2022

1 The first year multiplier is

The first-year multiplier is

$$1.018$$

2 The second and third years

The second and third years both have multiplier $1 + \frac{x}{100}$, so

$$200000(1.018)\left(1 + \frac{x}{100}\right)^2 = 209754$$

$$\left(1 + \frac{x}{100}\right)^2 = \frac{209754}{200000(1.018)}$$

$$1 + \frac{x}{100} = 1.015000\ldots$$

$$x = 1.5000\ldots$$

FINAL ANSWER

$x = 1.5$

Question 12

6 marks | Cumulative Frequency Diagrams

12 The table gives information about the times, in minutes, taken by 80 customers to do their shopping in a supermarket.

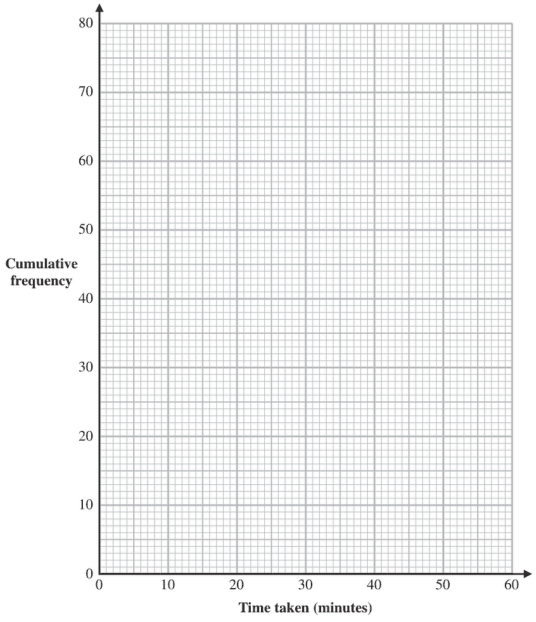
Time taken (t minutes)	Frequency
$0 < t \leqslant 10$	7
$10 < t \leqslant 20$	26
$20 < t \leqslant 30$	24
$30 < t \leqslant 40$	14
$40 < t \leqslant 50$	7
$50 < t \leqslant 60$	2

(a) Complete the cumulative frequency table.

Time taken (t minutes)	Cumulative frequency
$0 < t \leqslant 10$	
$0 < t \leqslant 20$	
$0 < t \leqslant 30$	
$0 < t \leqslant 40$	
$0 < t \leqslant 50$	
$0 < t \leqslant 60$	

(1)

(b) On the grid opposite, draw a cumulative frequency graph for your table.



(2)

(c) Use your graph to find an estimate for the median time taken.

..... minutes

(1)

One of the 80 customers is chosen at random.

(d) Use your graph to find an estimate for the probability that the time taken by this customer was more than 42 minutes.

.....

(2)

(Total for Question 12 is 6 marks)

1 Complete the cumulative frequencies

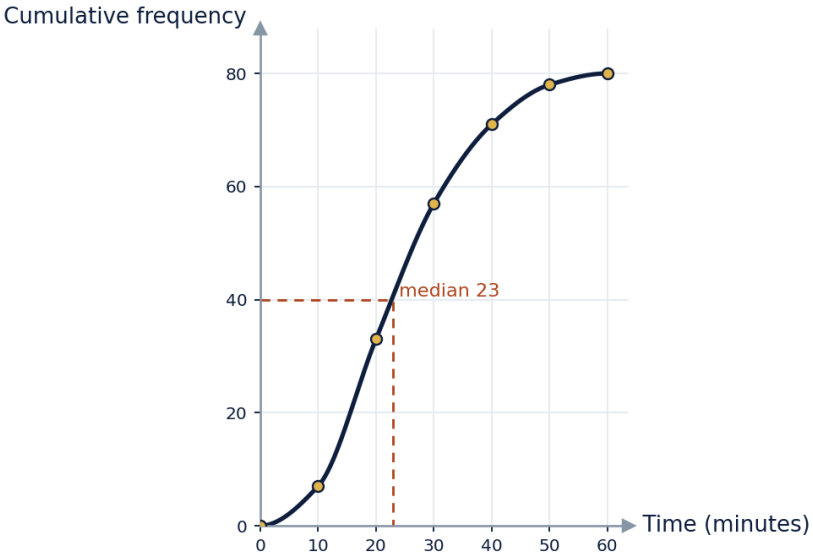
Complete the cumulative frequencies:

$$7,\ 33,\ 57,\ 71,\ 78,\ 80$$

2 Plot the points

Plot the points

$$(0, 0), (10, 7), (20, 33), (30, 57), (40, 71), (50, 78), (60, 80)$$



3 The median is at cumulative

The median is at cumulative frequency 40.

4 Convert standard form

This is between $(20, 33)$ and $(30, 57)$:

$$20 + \frac{40 - 33}{57 - 33} \times 10 = 22.9$$

5 Use cumulative frequency

So the median is about 23 minutes.

6 At minutes

At 42 minutes:

$$71 + \frac{2}{10}(78 - 71) = 72.4$$

7 Number taking more than minutes

Number taking more than 42 minutes:

$$80 - 72.4 = 7.6$$

$$P(t > 42) \approx \frac{7.6}{80} = 0.095$$

FINAL ANSWER

cumulative frequencies $7, 33, 57, 71, 78, 80$, median about 23 minutes, probability about 0.095.

4MA1/1H JANUARY 2022

Question 13

6 marks | Expanding Brackets

13 (a) Expand and simplify $5x(x + 2)(3x - 4)$

.....

(3)

(b) Simplify completely $\left(\frac{16w^8}{y^{20}}\right)^{-\frac{3}{4}}$

.....

(3)

(Total for Question 13 is 6 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H January 2022

1 **(a)**

(a)

$$5x(x+2)(3x-4) = 5x(3x^2+2x-8)$$

$$= 15x^3 + 10x^2 - 40x$$

2 **(b)**

(b)

$$\left(\frac{16w^8}{y^{20}}\right)^{-3/4} = 16^{-3/4}w^{-6}y^{15}$$

3 **Expand brackets**

Since $16^{3/4} = 8$,

$$= \frac{y^{15}}{8w^6}$$

✓ **FINAL ANSWER**

(a) $15x^3 + 10x^2 - 40x$, (b) $\frac{y^{15}}{8w^6}$

4MA1/1H JANUARY 2022

Question 14

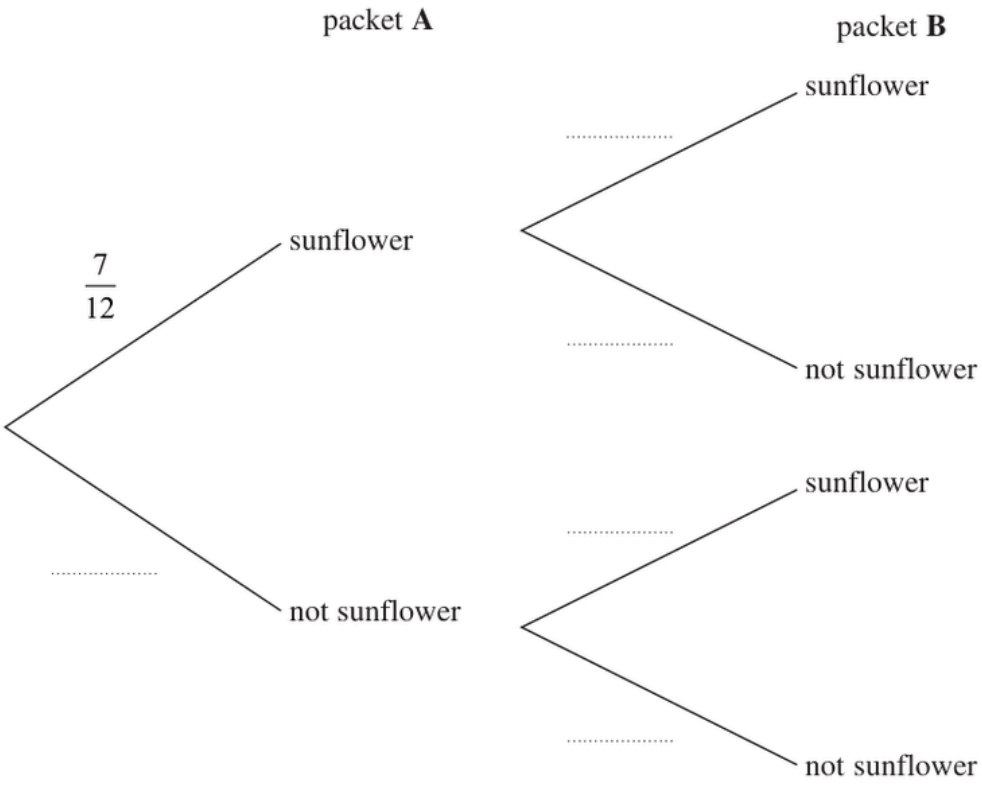
4 marks | Tree Diagrams & Conditional Probability

14 Aika has 2 packets of seeds, packet **A** and packet **B**

There are 12 seeds in packet **A** and 7 of these are sunflower seeds.
There are 15 seeds in packet **B** and 8 of these are sunflower seeds.

Aika is going to take at random a seed from packet **A** and a seed from packet **B**

(a) Complete the probability tree diagram.



(2)

(b) Calculate the probability that Aika will take two sunflower seeds.

.....

(2)

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H January 2022

1 **(a) Complete the tree**

Packet A: not sunflower $1 - \frac{7}{12} = \frac{5}{12}$.

Packet B (on both branches): sunflower $\frac{8}{15}$, not sunflower $\frac{7}{15}$.

2 **(b) Two sunflower seeds**

$$\frac{7}{12} \times \frac{8}{15} = \frac{56}{180} = \frac{14}{45}$$

✓ **FINAL ANSWER**

(a) $\frac{5}{12}$; $\frac{8}{15}$, $\frac{7}{15}$ on both branches (b) $\frac{14}{45}$

4MA1/1H JANUARY 2022

Question 15

3 marks | Direct & Inverse Proportion

15 A is inversely proportional to C^2

$A = 40$ when $C = 1.5$

Calculate the value of C when $A = 1000$

$C =$ .....

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H January 2022

1 Find inverse function

A is inversely proportional to C^2, so

$$A = \frac{k}{C^2}$$

2 Use when

Use $A = 40$ when $C = 1.5$:

$$40 = \frac{k}{1.5^2}$$

$$k = 40 \times 1.5^2 = 90$$

3 Find inverse function

So

$$A = \frac{90}{C^2}$$

4 Find inverse function

When $A = 1000$,

$$1000 = \frac{90}{C^2}$$

$$C^2 = 0.09$$

$$C = 0.3$$

FINAL ANSWER

$C = 0.3$.

4MA1/1H JANUARY 2022

Question 16

4 marks | Circles, Arcs & Sectors

16 The diagram shows a circle with centre O

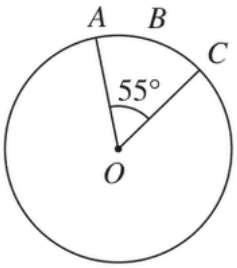


Diagram **NOT** accurately drawn

A, B and C are points on the circle so that the length of the arc ABC is 5 cm.

Given that angle $AOC = 55°$

work out the area of the circle.
Give your answer correct to one decimal place.

..... cm^2

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H January 2022

1 Use trigonometry

The arc length is $5\ \text{cm}$, and the angle is $55°$.

$$5 = \frac{55}{360} \times 2\pi r$$

$$r = 5.208\ldots$$

2 Area of the circle

Area of the circle:

$$\pi r^2 = \pi(5.208\ldots)^2 = 85.233\ldots$$

FINAL ANSWER

$85.2\ \text{cm}^2$.

4MA1/1H JANUARY 2022

Question 17

4 marks | Area & Volume of Similar Shapes

17 **A** and **B** are two similar vases.

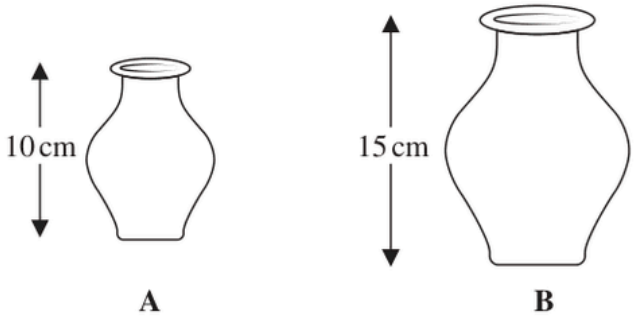


Diagram **NOT** accurately drawn

Vase **A** has height 10 cm.
Vase **B** has height 15 cm.

The difference between the volume of vase **A** and the volume of vase **B** is 1197 cm^3

Calculate the volume of vase **A**

..... cm^3

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/1H January 2022

1 **Calculate area**

The linear scale factor from vase A to vase B is

$$\frac{15}{10} = 1.5$$

2 **Calculate area**

So the volume scale factor is

$$1.5^3 = 3.375$$

3 **Let the volume of vase**

Let the volume of vase A be V.

$$3.375V - V = 1197$$

$$2.375V = 1197$$

$$V = 504$$

✓ **FINAL ANSWER**

$504\ \text{cm}^3$.

4MA1/1H JANUARY 2022

Question 18

3 marks | Rounding, Estimation & Bounds

18 $A = w - \dfrac{x^2}{y}$

$w = 3.45$ correct to 2 decimal places.
$x = 1.9$ correct to 1 decimal place.
$y = 5$ correct to the nearest whole number.

Work out the lower bound of the value of A
Show your working clearly.

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H January 2022

1 Estimate the value

$$A = w - \frac{x^2}{y}$$

2 Find lower bound

For the lower bound, use the lower bound of w, and make $\frac{x^2}{y}$ as large as possible.

$$w \geq 3.445, \quad x < 1.95, \quad y \geq 4.5$$

$$A_{\text{lower}} = 3.445 - \frac{1.95^2}{4.5}$$

$$A_{\text{lower}} = 2.6$$

FINAL ANSWER

2.6

4MA1/1H JANUARY 2022

Question 19

5 marks | Simultaneous Equations

19 Solve the simultaneous equations

$$3x^2 + y^2 - xy = 5$$
$$y = 2x - 3$$

Show clear algebraic working.

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H January 2022

1 Solve simultaneous equations

$$3x^2 + y^2 - xy = 5$$

$$y = 2x - 3$$

2 Substitute

Substitute $y = 2x - 3$:

$$3x^2 + (2x - 3)^2 - x(2x - 3) = 5$$

$$3x^2 + 4x^2 - 12x + 9 - 2x^2 + 3x = 5$$

$$5x^2 - 9x + 9 = 5$$

$$5x^2 - 9x + 4 = 0$$

3 Factorise

Factorise:

$$(5x - 4)(x - 1) = 0$$

4 Solve simultaneous equations

So

$$x = \frac{4}{5} \quad \text{or} \quad x = 1$$

5 **Solve simultaneous equations**

If $x = \frac{4}{5}$:

$$y = 2\left(\frac{4}{5}\right) - 3 = -\frac{7}{5}$$

---

6 **Solve simultaneous equations**

If $x = 1$:

$$y = 2(1) - 3 = -1$$

✓ **FINAL ANSWER**

$(x, y) = \left(\frac{4}{5}, -\frac{7}{5}\right)$ or $(1, -1)$.

4MA1/1H JANUARY 2022

Question 20

4 marks | Completing the Square

20 (a) Express $7 + 12x - 3x^2$ in the form $a + b(x + c)^2$ where a, b and c are integers.

.....

(3)

C is the curve with equation $y = 7 + 12x - 3x^2$
The point A is the maximum point on **C**

(b) Use your answer to part (a) to write down the coordinates of A

(..... , ...)

(1)

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H January 2022

1 **(a)**

(a)

$$7 + 12x - 3x^2 = -3x^2 + 12x + 7$$

$$= -3(x^2 - 4x) + 7$$

$$= -3\left((x - 2)^2 - 4\right) + 7$$

$$= 19 - 3(x - 2)^2$$

2 **This is the same as**

This is the same as

$$19 - 3(x + (-2))^2$$

3 **(b) The maximum point is**

(b) The maximum point is read from the completed square form.

$$y = 19 - 3(x - 2)^2$$

4

Calculate value

The maximum occurs when $x = 2$, giving $y = 19$.

✓ **FINAL ANSWER**

(a) $19 - 3(x - 2)^2$, (b) $A = (2, 19)$

4MA1/1H JANUARY 2022

Question 21

5 marks | 3D Pythagoras & Trigonometry

21 The diagram shows the prism $ABCDEFGHJK$ with horizontal base $AEFG$

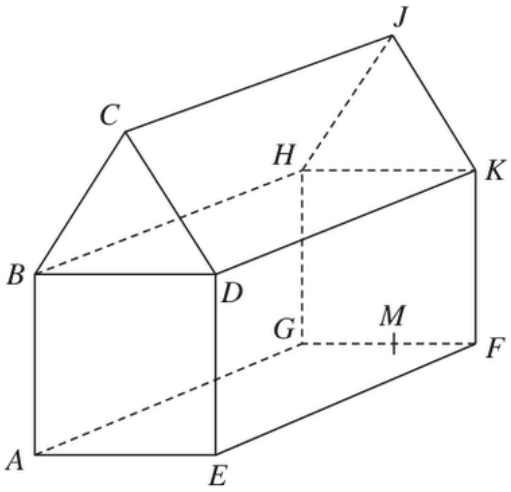


Diagram **NOT** accurately drawn

$ABCDE$ is a cross section of the prism where
$ABDE$ is a square
BCD is an equilateral triangle

$EF = 2 \times AE$
M is the midpoint of GF so that JM is vertical.

Angle $MAJ = y°$

Given that $\tan y° = T$

find the value of T, giving your answer in the form $\dfrac{\sqrt{p} + \sqrt{q}}{17}$ where p and q are integers.

$T =$ .....

(Total for Question 21 is 5 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H January 2022

1 **Let**

Let

$$AE = x$$

2 **Use trigonometry**

Since $ABDE$ is a square,

$$AB = BD = DE = AE = x$$

3 **Use trigonometry**

Since BCD is an equilateral triangle, the height from C to BD is

$$\frac{\sqrt{3}}{2}x$$

4 **Use trigonometry**

Therefore the vertical height from the base to J is

$$JM = x + \frac{\sqrt{3}}{2}x$$

$$JM = \frac{x(2+\sqrt{3})}{2}$$

5 **Use trigonometry**

Also,

$$EF = 2AE = 2x$$

6 **Find the midpoint**

Since M is the midpoint of GF, the horizontal distance from A to M is

$$AM = \sqrt{\left(\frac{x}{2}\right)^2 + (2x)^2}$$

$$AM = \sqrt{\frac{x^2}{4} + 4x^2}$$

$$AM = \frac{x\sqrt{17}}{2}$$

7 **Use trigonometry**

In right triangle AMJ,

$$\tan y^\circ = \frac{JM}{AM}$$

$$T = \frac{\frac{x(2+\sqrt{3})}{2}}{\frac{x\sqrt{17}}{2}}$$

$$T = \frac{2+\sqrt{3}}{\sqrt{17}}$$

---

8 **Rationalise the denominator**

Rationalise the denominator:

$$T = \frac{(2+\sqrt{3})\sqrt{17}}{17}$$

$$T = \frac{2\sqrt{17}+\sqrt{51}}{17}$$

$$T = \frac{\sqrt{68}+\sqrt{51}}{17}$$

✓ **FINAL ANSWER**

$$T = \frac{\sqrt{68}+\sqrt{51}}{17}$$

4MA1/1H JANUARY 2022

Question 22

5 marks | Vectors

22 The diagram shows triangle OAB

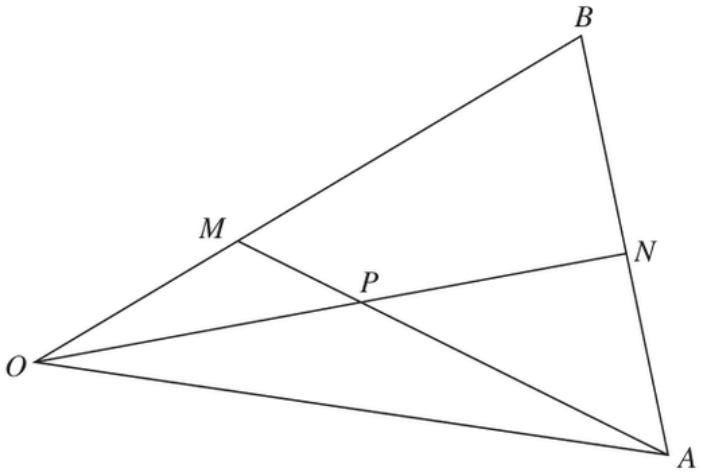


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 8\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$

M is the point on OB such that $OM : MB = 1 : 2$
N is the midpoint of AB
P is the point of intersection of ON and AM

Using a vector method, find $\overrightarrow{OP}$ as a simplified expression in terms of **a** and **b**
Show your working clearly.

$\overrightarrow{OP} =$ .....

(Total for Question 22 is 5 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H January 2022

1 **Use vectors**

$$\vec{OA} = 8\mathbf{a}, \qquad \vec{OB} = 6\mathbf{b}$$

2 **Use vectors**

Since $OM : MB = 1 : 2$,

$$\vec{OM} = \frac{1}{3}\vec{OB}$$

$$\vec{OM} = 2\mathbf{b}$$

---

3 **Find the midpoint**

Since N is the midpoint of AB,

$$\vec{ON} = \frac{8\mathbf{a} + 6\mathbf{b}}{2}$$

$$\vec{ON} = 4\mathbf{a} + 3\mathbf{b}$$

---

4 **Use vectors**

Point P lies on ON, so

$$\vec{OP} = \lambda(4\mathbf{a} + 3\mathbf{b})$$

---

5 **Use vectors**

Point P also lies on AM, so

$$\vec{OP} = 8\mathbf{a} + \mu(2\mathbf{b} - 8\mathbf{a})$$

$$\vec{OP} = (8 - 8\mu)\mathbf{a} + 2\mu\mathbf{b}$$

---

6 **Equate coefficients**

Equate coefficients:

$$4\lambda = 8 - 8\mu$$

$$3\lambda = 2\mu$$

---

7 **Use vectors**

From $3\lambda = 2\mu$,

$$\mu = \frac{3}{2}\lambda$$

---

8 **Substitute**

Substitute:

$$4\lambda = 8 - 8\left(\frac{3}{2}\lambda\right)$$

$$4\lambda = 8 - 12\lambda$$

$$16\lambda = 8$$

$$\lambda = \frac{1}{2}$$

---

9 **Use vectors**

Therefore

$$\vec{OP} = \frac{1}{2}(4\mathbf{a} + 3\mathbf{b})$$

$$\vec{OP} = 2\mathbf{a} + \frac{3}{2}\mathbf{b}$$

✓ **FINAL ANSWER**

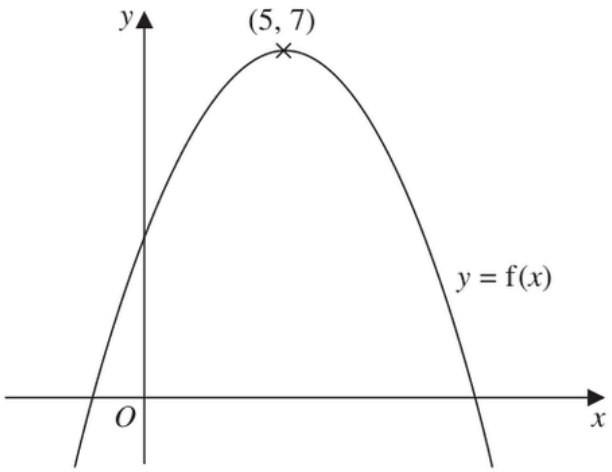
$$\vec{OP} = 2\mathbf{a} + \frac{3}{2}\mathbf{b}$$

4MA1/1H JANUARY 2022

Question 23

2 marks | Transformations of Graphs

23 The diagram shows a sketch of the curve with equation $y = f(x)$



There is only one maximum point on the curve.
The coordinates of this maximum point are (5, 7)

Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x + 9)$

(..... , ...)

(ii) $y = f(x) + 3$

(..... , ...)

(Total for Question 23 is 2 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H January 2022

1 **The maximum point of is**

The maximum point of $y = f(x)$ is $(5, 7)$.

2 **Calculate value**

For

$$y = f(x + 9)$$

3 **the graph moves 9 units**

the graph moves 9 units left:

$$(5, 7) \rightarrow (-4, 7)$$

4 **Calculate value**

For

$$y = f(x) + 3$$

5 **the graph moves 3 units**

the graph moves 3 units up:

$$(5, 7) \to (5, 10)$$

✓ **FINAL ANSWER**

$(-4, 7)$ and $(5, 10)$

4MA1/1H JANUARY 2022

Question 24

6 marks | Differentiation & Turning Points

24 The curve **C** has equation $y = ax^3 + bx^2 - 12x + 6$ where a and b are constants.

The point A with coordinates $(2, -6)$ lies on **C**
The gradient of the curve at A is 16

Find the y coordinate of the point on the curve whose x coordinate is 3
Show clear algebraic working.

$y =$ .....

(Total for Question 24 is 6 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H January 2022

1 Calculate value

$$y = ax^3 + bx^2 - 12x + 6$$

2 Point lies on the curve

Point $A = (2, -6)$ lies on the curve:

$$-6 = 8a + 4b - 24 + 6$$

$$8a + 4b = 12$$

$$2a + b = 3$$

3 Differentiate

Differentiate:

$$\frac{dy}{dx} = 3ax^2 + 2bx - 12$$

4 **The gradient at is 16**

The gradient at $x = 2$ is 16:

$$16 = 12a + 4b - 12$$

$$12a + 4b = 28$$

$$3a + b = 7$$

---

5 **Solve**

Now solve

$$2a + b = 3$$

$$3a + b = 7$$

---

6 **Subtracting gives**

Subtracting gives

$$a = 4$$

---

7 **Calculate value**

Then

$$2(4) + b = 3$$

$$b = -5$$

---

8 **Calculate value**

For $x = 3$,

$$y = 4(3)^3 - 5(3)^2 - 12(3) + 6$$

$$y = 108 - 45 - 36 + 6$$

$$y = 33$$

✓ **FINAL ANSWER**

$y = 33$