Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

4MA1/1HR January 2022 Worked Solutions

January 2022 paper rebuilt with the worked solution after each question.

24	2020–2026	Single-paper solutions
paper questions	52 past papers	checked by Dr Eslam



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PAST PAPER

4MA1/1HR January 2022

January 2022

24	100	2 h
questions	marks	exam time

Each question is followed by its worked solution and final answer.

4MA1/1HR JANUARY 2022

Question 1

3 marks | Inequalities (Solving & Graphing)

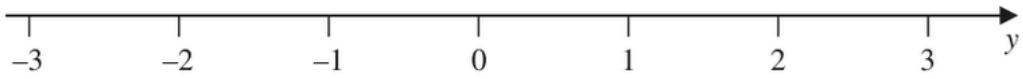
1 n is an integer.

(a) Write down all the values of n such that $-2 \leqslant n < 3$

.....

(2)

(b) On the number line, represent the inequality $y \leqslant 1$



(1)

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR January 2022

1 **(a)**

(a)

$$-2 \leq n < 3$$

2 **Calculate value**

Since n is an integer,

$$n = -2, -1, 0, 1, 2$$

3 **Solve inequality**

(b) For

$$y \leq 1$$

4 **use a closed circle at**

use a closed circle at 1 and shade to the left.

✓ **FINAL ANSWER**

$n = -2, -1, 0, 1, 2$; closed at 1, shaded left.

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Question 2

2 marks | Probability Toolkit

2 Each time John plays a game, the probability that he wins the game is 0.65

John is going to play the game 300 times.

Work out an estimate for the number of games that John wins.

(Total for Question 2 is 2 marks)

WORKED SOLUTION

Question 2 · 4MA1/1HR January 2022

1 **Expected number = probability × number of games**

$$0.65 \times 300 = 195$$

FINAL ANSWER

195

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Question 3

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

3 The shaded shape is made using three identical right-angled triangles and a square.

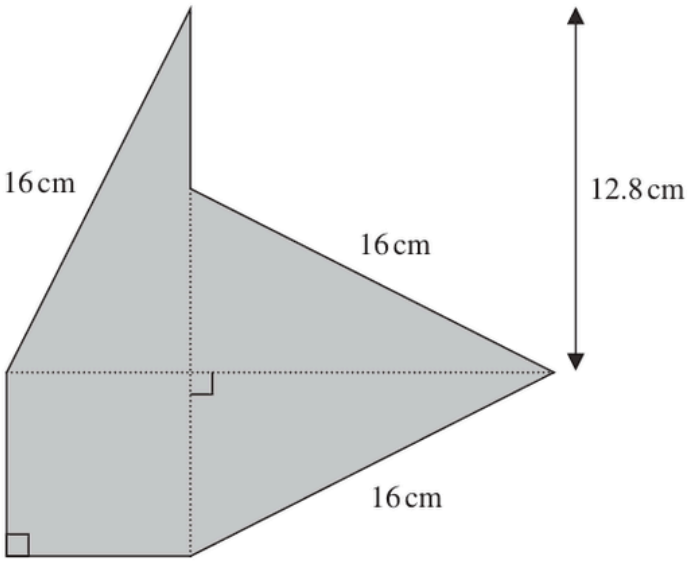


Diagram **NOT** accurately drawn

Work out the perimeter of the shaded shape.

..... cm

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR January 2022

1 **Use trigonometry**

Each triangle has hypotenuse $16\ \text{cm}$ and one side $12.8\ \text{cm}$.

2 **Find the other side**

Find the other side:

$$\sqrt{16^2 - 12.8^2} = 9.6\ \text{cm}$$

3 **The exposed short vertical side**

The exposed short vertical side is

$$12.8 - 9.6 = 3.2\ \text{cm}$$

4 **Perimeter**

Perimeter:

$$3(16) + 9.6 + 9.6 + 3.2 = 70.4$$

✓ **FINAL ANSWER**

$70.4\ \text{cm}$.

Question 4

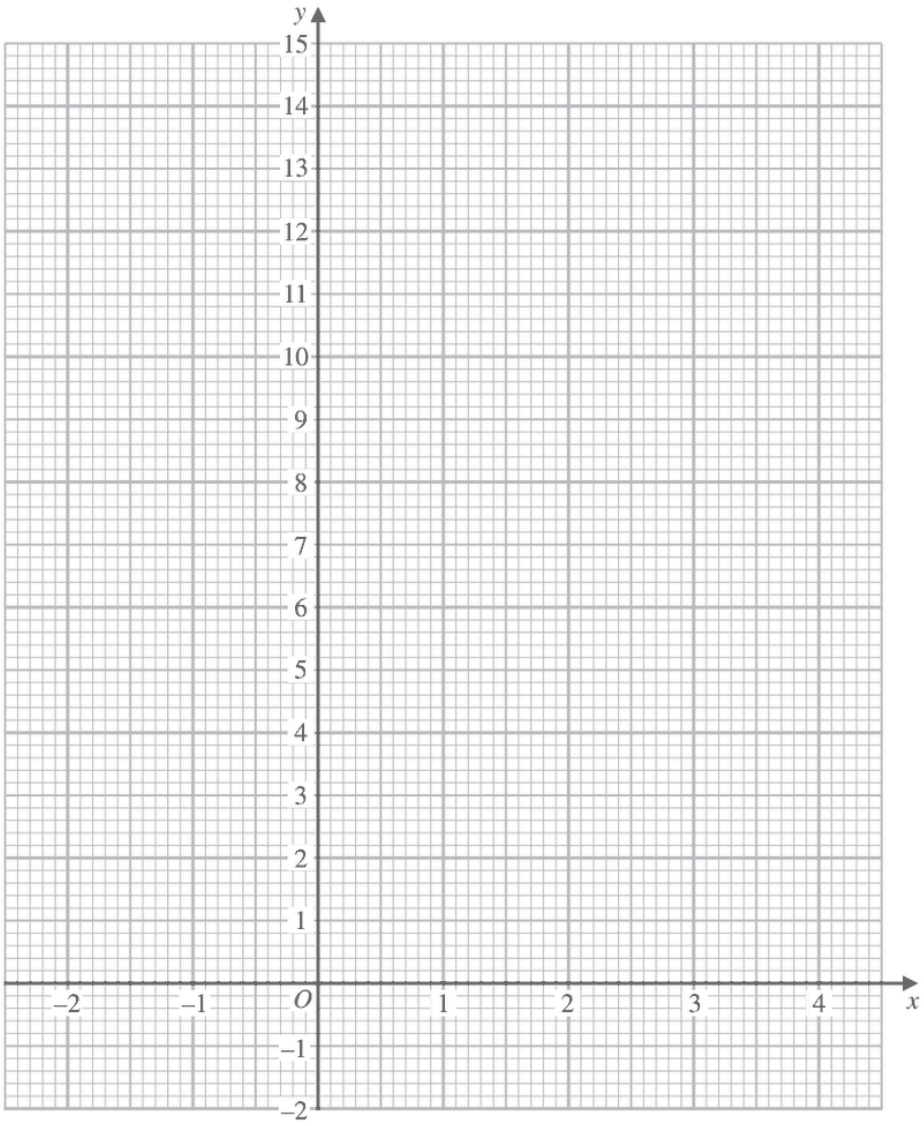
4 marks | Graphs of Functions

4 (a) Complete the table of values for $y = x^2 - 4x + 3$

x	–2	–1	0	1	2	3	4
y		8	3			0	

(2)

(b) On the grid, draw the graph of $y = x^2 - 4x + 3$ for values of x from –2 to 4



(2)

(Total for Question 4 is 4 marks)

1 Solve quadratic equation

$$y = x^2 - 4x + 3$$

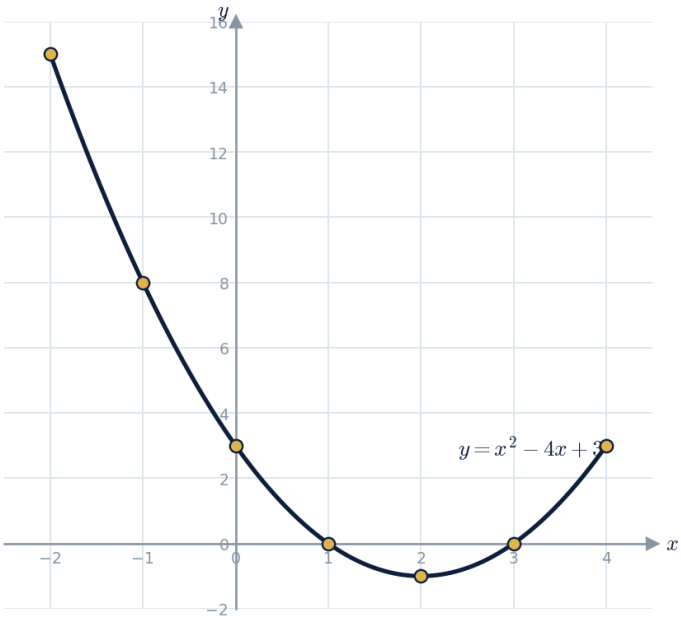
2 The completed table is

The completed table is:

x	-2	-1	0	1	2	3	4
y	15	8	3	0	-1	0	3

3 Plot the points and join

Plot the points and join them with a smooth U-shaped curve.



FINAL ANSWER

Missing values are $15, 0, -1, 3$.

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Question 5

4 marks | Statistics Toolkit

5 Yusuf sat 8 examinations.

Here are his marks for 5 of the examinations.

68 72 75 77 80

For his results in all 8 examinations

the mode of his marks is 80
the median of his marks is 74
the range of his marks is 16

Find Yusuf's marks for each of the other 3 examinations.

(Total for Question 5 is 4 marks)

WORKED SOLUTION

Question 5 · 4MA1/1HR January 2022

1 The range is . Since

The range is 16. Since the greatest known mark is 80,

$$80 - 16 = 64$$

2 Use Statistics Toolkit tag correct

So one missing mark is 64.

3 Calculate statistic

The mode is 80, so another missing mark is 80.

4 Use Statistics Toolkit tag correct

Now the marks in order are

$$64,\ 68,\ 72,\ x,\ 75,\ 77,\ 80,\ 80$$

5 Calculate statistic

The median is 74, so

$$\frac{x + 75}{2} = 74$$

$$x = 73$$

FINAL ANSWER

$64,\ 73,\ 80.$

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Question 6

4 marks | Prime Factors, HCF & LCM

6 (a) Work out the lowest common multiple (LCM) of 36 and 120

.....

(2)

$A = 5^2 \times 7^4 \times 11^p$
$B = 5^m \times 7^{n-5} \times 11$

m, n and p are integers such that
$m > 2$
$n > 10$
$p > 1$

(b) Find the highest common factor (HCF) of A and B
Give your answer as a product of powers of its prime factors.

.....

(2)

(Total for Question 6 is 4 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR January 2022

1 **Find the LCM**

$$36 = 2^2 \times 3^2$$

$$120 = 2^3 \times 3 \times 5$$

2 **Find the LCM**

For the LCM, take the highest power of each prime:

$$\text{LCM} = 2^3 \times 3^2 \times 5 = 360$$

3 **Find the LCM**

For part (b),

$$A = 5^2 \times 7^4 \times 11^p$$

$$B = 5^m \times 7^{n-5} \times 11$$

4 **Find the LCM**

Since $m > 2$, the smaller power of 5 is 5^2.

5 **Find the LCM**

Since $n > 10$, $n - 5 > 5$, so the smaller power of 7 is 7^4.

6 **Find the LCM**

Since $p > 1$, the smaller power of 11 is 11^1.

$$\text{HCF} = 5^2 \times 7^4 \times 11$$

✓ **FINAL ANSWER**

(a) 360, (b) $5^2 \times 7^4 \times 11$

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Question 7

4 marks | Standard & Compound Units

7 Milly went on a car journey.
She travelled from Anesey to Breigh to Clando and then to Duckbridge.

For Anesey to Breigh, Milly drove the 245 km in 2.5 hours.
For Breigh to Clando, Milly drove the 220 km at an average speed of 80 km/h
For Clando to Duckbridge, Milly drove at an average speed of 72 km/h in 50 minutes.

Work out Milly's average speed, in km/h, for the journey from Anesey to Duckbridge.
Give your answer correct to one decimal place.

..... km/h

(Total for Question 7 is 4 marks)

WORKED SOLUTION

Question 7 · 4MA1/1HR January 2022

1 Calculate value

First part:

$$\text{distance} = 245 \text{ km}, \qquad \text{time} = 2.5 \text{ h}$$

2 Second part

Second part:

$$\text{time} = \frac{220}{80} = 2.75 \text{ h}$$

3 Third part

Third part:

$$50 \text{ minutes} = \frac{5}{6} \text{ h}$$

$$\text{distance} = 72 \times \frac{5}{6} = 60 \text{ km}$$

4 Total distance

Total distance:

$$245 + 220 + 60 = 525 \text{ km}$$

5 **Total time**

Total time:

$$2.5 + 2.75 + \frac{5}{6} = 6.0833\ldots\ \text{h}$$

6 **Average speed**

Average speed:

$$\frac{525}{6.0833\ldots} = 86.301\ldots$$

✓ **FINAL ANSWER**

$86.3\ \text{km/h}$.

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Question 8

4 marks | Standard Form

8 (a) Write 5×10^4 as an ordinary number.

.....

(1)

(b) Write 0.000 06 in standard form.

.....

(1)

(c) Work out $(4 \times 10^{512}) \div (1.6 \times 10^{700})$
Give your answer in standard form.

.....

(2)

(Total for Question 8 is 4 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR January 2022

1 Convert standard form

$$5 \times 10^4 = 50000$$

$$0.00006 = 6 \times 10^{-5}$$

2 Convert standard form

For part (c),

$$\frac{4 \times 10^{512}}{1.6 \times 10^{700}} = \frac{4}{1.6} \times 10^{512-700}$$

$$= 2.5 \times 10^{-188}$$

FINAL ANSWER

(a) 50000, (b) 6×10^{-5}, (c) 2.5×10^{-188}

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Question 9

5 marks | Factorising

9 (a) Simplify $x^4 \times x^5$

.....

(1)

(b) Simplify $(4y^2)^3$

.....

(2)

(c) Factorise $n^2 - 7n + 12$

.....

(2)

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR January 2022

1 **(a)**

(a) $x^4 \times x^5 = x^9$.

2 **(b)**

(b) $(4y^2)^3 = 64y^6$.

3 **(c)**

(c)

$$n^2 - 7n + 12 = (n - 3)(n - 4)$$

✓ **FINAL ANSWER**

(a) x^9, (b) $64y^6$, (c) $(n - 3)(n - 4)$

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Question 10

6 marks | Volume & Surface Area

10 Jonty has a storage container in the shape of a cuboid, as shown in the diagram.

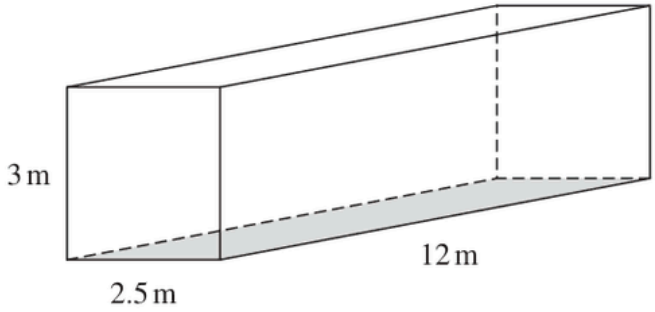


Diagram **NOT** accurately drawn

Jonty is going to paint the outside of his storage container, apart from the base which is shown shaded in the diagram.
He needs enough paint to cover the four sides and the top.

Each tin of paint covers an area of $15\,m^2$

The cost of each tin of paint recently increased by 10%
After the increase, the cost of each tin of paint is £26.95

Jonty says

"**Before** the increase, I could have bought enough paint for less than £200"

Show that Jonty is correct.
Show your working clearly.

(Total for Question 10 is 6 marks)

WORKED SOLUTION

Question 10 · 4MA1/1HR January 2022

1 **Jonty paints the four sides**

Jonty paints the four sides and the top, but not the base.

2 **Top**

Top:

$$12 \times 2.5 = 30$$

3 **Two long sides**

Two long sides:

$$2(12 \times 3) = 72$$

4 **Two short sides**

Two short sides:

$$2(2.5 \times 3) = 15$$

5 **Total area**

Total area:

$$30 + 72 + 15 = 117\ \text{m}^2$$

6 **Calculate area**

Each tin covers $15\ \text{m}^2$, so the number of tins needed is

$$\frac{117}{15} = 7.8$$

7 **He must buy tins**

He must buy 8 tins.

8 **Calculate area**

After the 10% increase, each tin costs $\pounds 26.95$. Before the increase:

$$\frac{26.95}{1.10} = 24.50$$

9 **Calculate area**

Before the increase, 8 tins would cost

$$8 \times 24.50 = 196$$

10 **Calculate area**

Since $\pounds 196 < \pounds 200$, Jonty is correct.

✓ **FINAL ANSWER**

Jonty is correct; before the increase the paint would have cost $\pounds 196$.

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Question 11

2 marks | Circles, Arcs & Sectors

11 The diagram shows sector OPQ of a circle, centre O

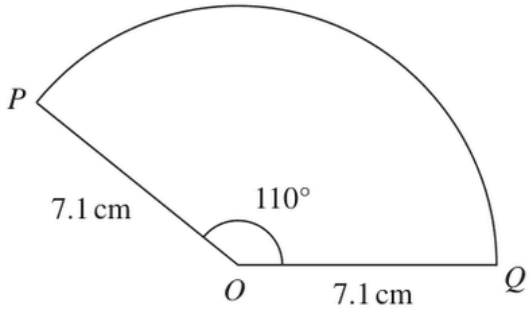


Diagram **NOT** accurately drawn

$OP = OQ = 7.1$ cm
Angle $POQ = 110°$

Calculate the area of sector OPQ
Give your answer correct to one decimal place.

..... cm²

(Total for Question 11 is 2 marks)

WORKED SOLUTION

Question 11 · 4MA1/1HR January 2022

1 Calculate area

$$\text{area} = \frac{110}{360} \times \pi(7.1)^2$$

$$= 48.390\ldots$$

FINAL ANSWER

$48.4\ \text{cm}^2$.

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Question 12

5 marks | Algebraic Fractions

12 (a) Expand and simplify $n(n-4)(3n+5)$

.....

(2)

(b) Express

$$\frac{3}{x}+\frac{x+2}{2x}+\frac{1}{4}$$

as a single fraction in its simplest form.

.....

(3)

(Total for Question 12 is 5 marks)

WORKED SOLUTION

Question 12 · 4MA1/1HR January 2022

1 **(a)**

(a)

$$n(n-4)(3n+5) = n(3n^2-7n-20) = 3n^3-7n^2-20n$$

2 **(b)**

(b)

$$\frac{3}{x}+\frac{x+2}{2x}+\frac{1}{4}$$

3 **Use denominator**

Use denominator $4x$:

$$\frac{12}{4x}+\frac{2(x+2)}{4x}+\frac{x}{4x}=\frac{12+2x+4+x}{4x}$$

$$=\frac{3x+16}{4x}$$

FINAL ANSWER

(a) $3n^3-7n^2-20n$, (b) $\frac{3x+16}{4x}$

Question 13

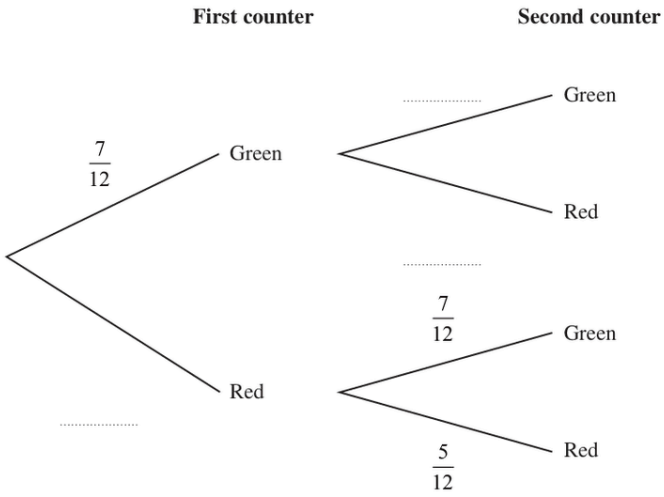
7 marks | Tree Diagrams & Conditional Probability

13 Hector has a bag that contains 12 counters.
There are 7 green counters and 5 red counters in the bag.

Hector takes at random a counter from the bag.
He looks at the counter and puts the counter back into the bag.

Hector then takes at random a second counter from the bag.
He looks at the counter and puts the counter back into the bag.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that both counters are red.

.....

(2)

Meghan has a jar containing 15 counters.
There are only blue counters, green counters and red counters in the jar.

Hector is going to take at random one of the counters from his bag of 12 counters.
He will look at the counter and put the counter back into the bag.

Hector is then going to take at random a second counter from his bag.
He will look at the counter and put the counter back into the bag.

Meghan is then going to take at random one of the counters from her jar of counters.
She will look at the counter and put the counter back into the jar.

The probability that the 3 counters each have a different colour is $\frac{7}{24}$

(c) Work out how many blue counters there are in the jar.

.....

(3)

(Total for Question 13 is 7 marks)

1

(a) Complete the tree (counter is replaced)

First counter red: $\frac{5}{12}$.

After green, second counter: green $\frac{7}{12}$, red $\frac{5}{12}$.

2

(b) Both red

$$\frac{5}{12} \times \frac{5}{12} = \frac{25}{144}$$

3

(c) What "three different colours" needs

Hector only has green and red, so he must take one green and one red (either order), and Meghan must take blue.

$$P(\text{one green, one red}) = 2 \times \frac{7}{12} \times \frac{5}{12} = \frac{35}{72}$$

4

(c) Solve for the number of blue counters

Let b be the number of blue counters out of 15.

$$\frac{35}{72} \times \frac{b}{15} = \frac{7}{24} \Rightarrow \frac{b}{15} = \frac{7}{24} \times \frac{72}{35} = \frac{3}{5} \Rightarrow b = 9$$

✓ **FINAL ANSWER**

(a) $\frac{5}{12}$; $\frac{7}{12}$, $\frac{5}{12}$ (b) $\frac{25}{144}$ (c) 9

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Question 14

4 marks | Circle Theorems

14

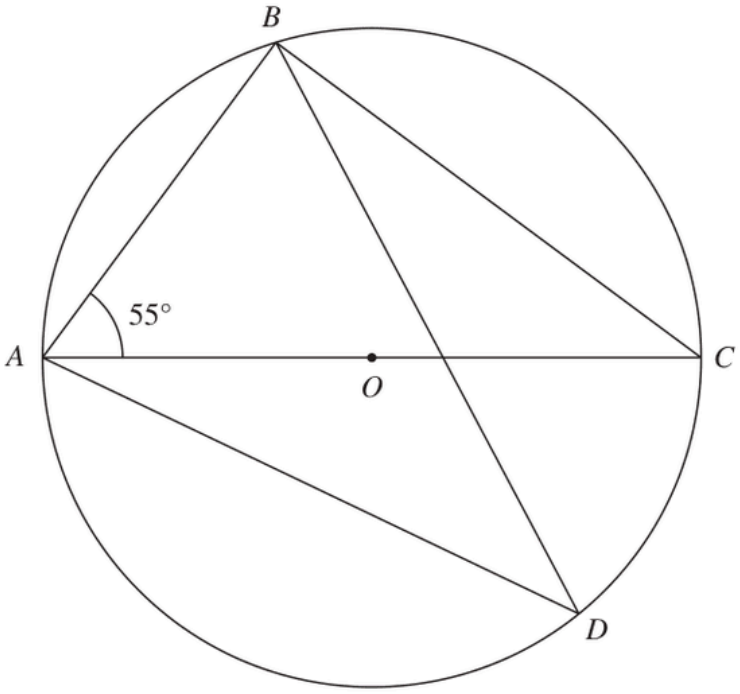


Diagram **NOT** accurately drawn

A, B, C and D are points on a circle, centre O
AOC is a diameter of the circle.

Angle $BAC = 55°$

Work out the size of angle ADB
Give a reason for each stage of your working.

.....°

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/1HR January 2022

1 **Use trigonometry**

Since AC is a diameter,

$$\angle ABC = 90°$$

2 **Use trigonometry**

In triangle ABC,

$$\angle ACB = 180° - 90° - 55° = 35°$$

3 **Angles in the same segment**

Angles in the same segment are equal, so

$$\angle ADB = \angle ACB = 35^\circ$$

✓ **FINAL ANSWER**

35°.

4MA1/1HR JANUARY 2022

Question 15

3 marks | Algebraic Proof

15 Using algebra, prove that, given any 3 consecutive whole numbers, the sum of the square of the smallest number and the square of the largest number is always 2 more than twice the square of the middle number.

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/1HR January 2022

1 Let the three consecutive whole

Let the three consecutive whole numbers be

$$n, \quad n+1, \quad n+2$$

2 The sum of the squares

The sum of the squares of the smallest and largest is

$$n^2 + (n+2)^2 = n^2 + n^2 + 4n + 4 = 2n^2 + 4n + 4$$

3 Twice the square of the

Twice the square of the middle number is

$$2(n+1)^2 = 2(n^2 + 2n + 1) = 2n^2 + 4n + 2$$

4 Calculate value

So

$$2n^2 + 4n + 4 = (2n^2 + 4n + 2) + 2$$

FINAL ANSWER

The sum is always 2 more than twice the square of the middle number.

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Question 16

4 marks | Sequences

16 An arithmetic series has first term 1 and common difference 4

Find the sum of all terms of the series from the 41st term to the 100th term inclusive.

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR January 2022

1 Find the nth term

The first term is 1 and the common difference is 4.

2 The 41st term is

The 41st term is

$$1 + 40(4) = 161$$

3 The 100th term is

The 100th term is

$$1 + 99(4) = 397$$

4 There are

There are

$$100 - 41 + 1 = 60$$

5 Find the nth term

terms from the 41st to the 100th inclusive.

$$S = \frac{60}{2}(161 + 397)$$

$$S = 30 \times 558 = 16740$$

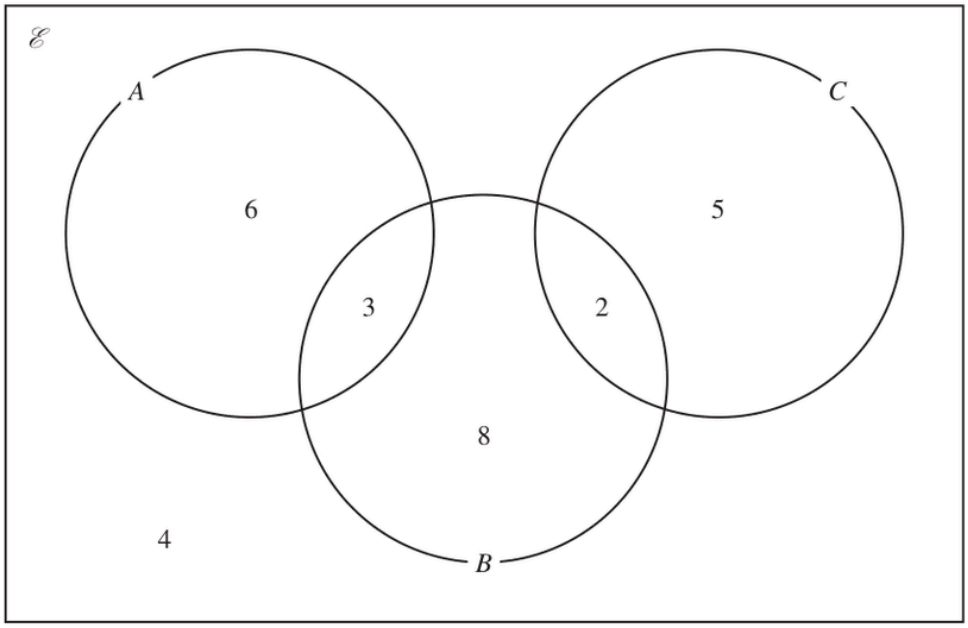
FINAL ANSWER

16740.

Question 17

4 marks | Set Notation & Venn Diagrams

17 The Venn diagram shows a universal set $\mathscr{E}$ and three sets A, B and C.



6, 3, 8, 2, 5 and 4 represent the **numbers** of elements.

Find

(i) $n(A \cup B)$

.....

(1)

(ii) $n(A \cap C)$

.....

(1)

(iii) $n(B \cap C')$

.....

(1)

(iv) $n(A' \cup B' \cup C')$

.....

(1)

(Total for Question 17 is 4 marks)

1 **From the Venn diagram**

From the Venn diagram:

$$A \text{ only} = 6, \quad A \cap B = 3, \quad B \text{ only} = 8, \quad B \cap C = 2, \quad C \text{ only} = 5, \quad \text{outside} = 4$$

2 **Calculate value**

There is no region in both A and C, and there is no triple intersection.

$$n(A \cup B) = 6 + 3 + 8 + 2 = 19$$

$$n(A \cap C) = 0$$

$$n(B \cap C') = 3 + 8 = 11$$

3 **Use**

For $A' \cup B' \cup C'$, use

$$A' \cup B' \cup C' = (A \cap B \cap C)'$$

4 **Set**

Since $A \cap B \cap C$ is empty, this includes everyone in the universal set.

$$6 + 3 + 8 + 2 + 5 + 4 = 28$$

✓ **FINAL ANSWER**

(i) 19, (ii) 0, (iii) 11, (iv) 28

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Question 18

4 marks | Area & Volume of Similar Shapes

18 The three solids **A**, **B** and **C** are similar such that

the surface area of **A** : the surface area of **B** = 4 : 9

and

the volume of **B** : the volume of **C** = 125 : 343

Work out the ratio

the height of **A** : the height of **C**

Give your ratio in its simplest form.

(Total for Question 18 is 4 marks)

WORKED SOLUTION

Question 18 · 4MA1/1HR January 2022

1 Split the ratio

Surface area ratio $A : B = 4 : 9$, so the length ratio is

$$\sqrt{4} : \sqrt{9} = 2 : 3$$

2 Split the ratio

Volume ratio $B : C = 125 : 343$, so the length ratio is

$$\sqrt[3]{125} : \sqrt[3]{343} = 5 : 7$$

3 Split the ratio

Therefore

$$\frac{A}{C} = \frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$$

FINAL ANSWER

$10 : 21$

4MA1/1HR JANUARY 2022

Question 19

3 marks | Completing the Square

19 Given that $\left(\sqrt[3]{\frac{1}{x}}\right)^4 = x^m$

(a) find the value of m

$m =$ .....

(1)

Given that a, b and c are integers,

(b) express $3x^2 + 12x + 19$ in the form $a(x + b)^2 + c$

.....

(2)

(Total for Question 19 is 3 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR January 2022

1 **(a)**

(a)

$$\sqrt[3]{\frac{1}{x}} = x^{-1/3}$$

2 **Evaluate fraction**

So $m = -\frac{1}{3}$.

3 **(b)**

(b)

$$3x^2 + 12x + 19 = 3(x^2 + 4x) + 19$$

$$= 3\left((x + 2)^2 - 4\right) + 19$$

$$= 3(x + 2)^2 + 7$$

FINAL ANSWER

$m = -\frac{1}{3}$, and $3(x + 2)^2 + 7$

Question 20

6 marks | Transformations of Graphs

20 The curve with equation $y = f(x)$ has one turning point.

The coordinates of this turning point are $(-6, -4)$

(a) Write down the coordinates of the turning point on the curve with equation

(i) $y = f(x) + 5$

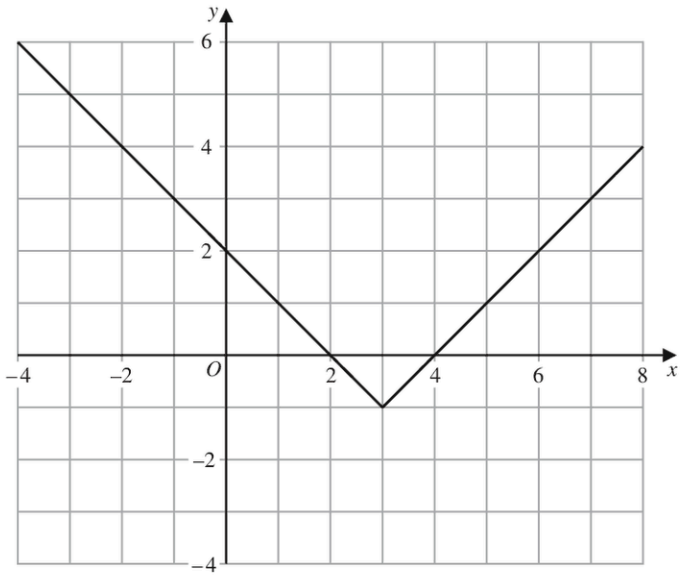
(.....,.....)

(ii) $y = f(3x)$

(.....,.....)

(2)

The graph of $y = g(x)$ is shown on the grid below.



(b) On the grid, sketch the graph of $y = 2g(x)$ for $-1 \leqslant x \leqslant 7$

(2)

The graph of $y = h(x)$ intersects the x-axis at two points.
The coordinates of the two points are $(-1, 0)$ and $(6, 0)$

The graph of $y = h(x + a)$ passes through the point with coordinates $(2, 0)$, where a is a constant.

(c) Find the two possible values of a

....., ...

(2)

(Total for Question 20 is 6 marks)

1 **The turning point of is**

The turning point of $y = f(x)$ is $(-6, -4)$.

2 **Calculate value**

For

$$y = f(x) + 5$$

3 **the graph moves 5 units**

the graph moves 5 units up:

$$(-6, -4) \rightarrow (-6, 1)$$

4 **Calculate value**

For

$$y = f(3x)$$

5 **the coordinate is divided by**

the x-coordinate is divided by 3:

$$(-6, -4) \rightarrow (-2, -4)$$

6 **Read the graph**

For the sketch of $y = 2g(x)$, keep every x-coordinate the same and double every y-coordinate.

7 **Useful points are**

Useful points are

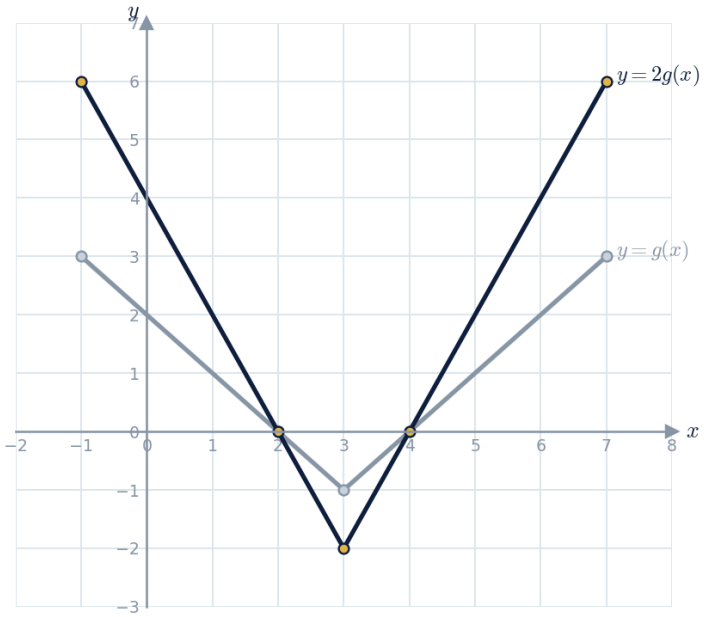
$$(-1,3) \to (-1,6)$$

$$(2,0) \to (2,0)$$

$$(3,-1) \to (3,-2)$$

$$(4,0) \to (4,0)$$

$$(7,3) \to (7,6)$$



8 **Solve inequality**

Draw straight line segments through these points for $-1 \leq x \leq 7$.

9 **Calculate value**

For part (c), $h(x)$ has roots at $x = -1$ and $x = 6$.

10 **Calculate value**

For $h(x + a)$ to pass through $(2,0)$,

$$2 + a = -1 \quad \text{or} \quad 2 + a = 6$$

$$a = -3 \quad \text{or} \quad a = 4$$

FINAL ANSWER

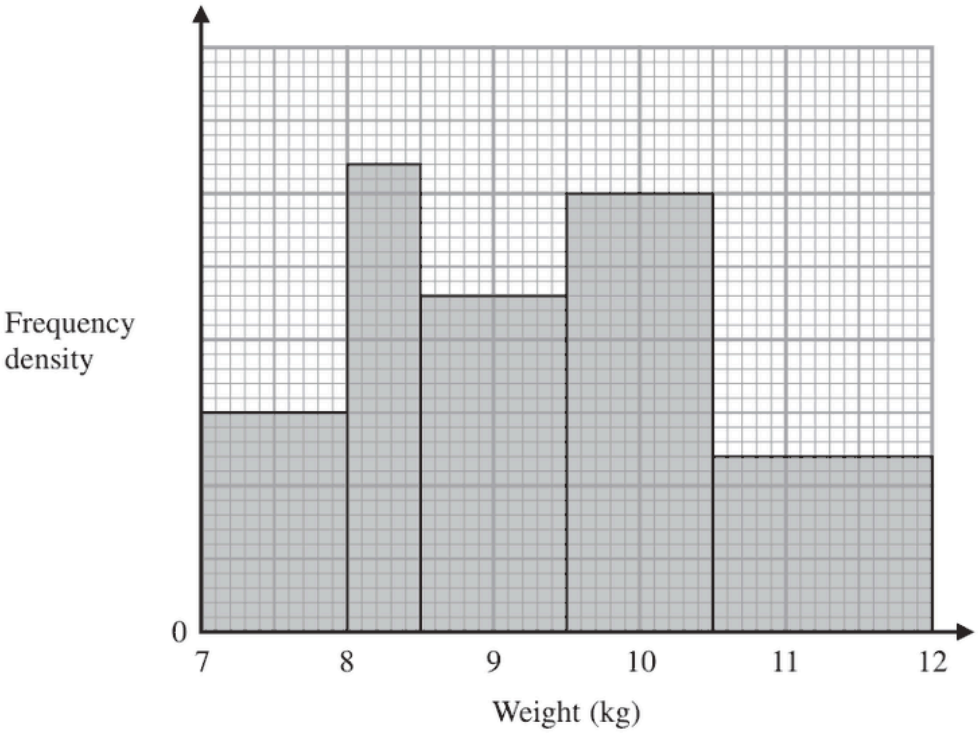
$(-6, 1)$, $(-2, -4)$, sketch through the doubled points, and $a = -3$ or $a = 4$

4MA1/1HR JANUARY 2022

Question 21

3 marks | Histograms

21



The histogram gives information about the weights, in kg, of all the watermelons in a field.

There are 16 watermelons with a weight between 8 kg and 8.5 kg

Work out the total number of watermelons in the field.

(Total for Question 21 is 3 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR January 2022

1 **Use histogram**

For a histogram,

$$\text{frequency} = \text{class width} \times \text{frequency density}$$

2 **Use frequency table**

For $8 \leq w < 8.5$, the frequency is 16 and the class width is 0.5, so

$$\text{frequency density} = \frac{16}{0.5} = 32$$

3 Use histogram

Using this scale from the histogram, the frequency densities are:

$$15,\ 32,\ 23,\ 30,\ 12$$

4 The total frequency is

The total frequency is

$$(1)(15) + (0.5)(32) + (1)(23) + (1)(30) + (1.5)(12)$$

$$= 15 + 16 + 23 + 30 + 18$$

$$= 102$$

FINAL ANSWER

102 watermelons

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Question 22

4 marks | Rounding, Estimation & Bounds

22 The diagram shows triangle ABC

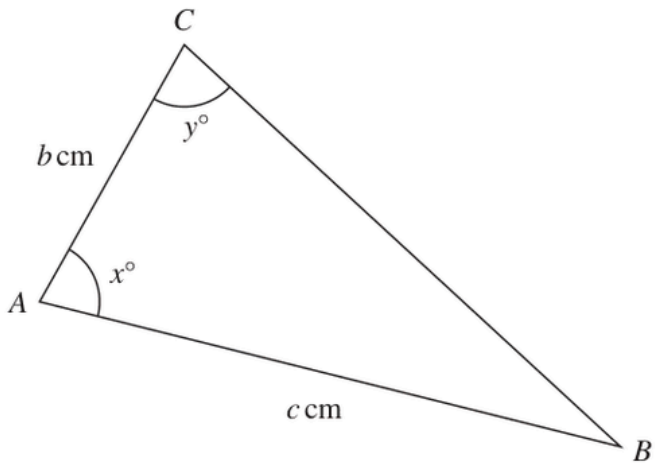


Diagram **NOT** accurately drawn

$c = 11.5$ correct to one decimal place
$x = 80$ correct to the nearest whole number
$y = 75$ correct to the nearest whole number

Calculate the upper bound for the value of b
Show your working clearly.
Give your answer correct to 3 significant figures.

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/1HR January 2022

1 Find upper bound

For an upper bound for b, use the upper bound of c:

$c = 11.55$

2 Use sine rule

Use the lower bounds of x and y, because this makes angle B as large as possible and makes $\sin y$ as small as possible.

$x = 79.5°, \quad y = 74.5°$

3 Use trigonometry

So

$\angle B = 180° - 79.5° - 74.5° = 26°$

4 **Use trigonometry**

Using the sine rule,

$$\frac{b}{\sin 26^\circ} = \frac{11.55}{\sin 74.5^\circ}$$

$$b = \frac{11.55 \sin 26^\circ}{\sin 74.5^\circ} = 5.254...$$

✓ **FINAL ANSWER**

5.25 cm to 3 significant figures.

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Question 23

6 marks | Differentiation & Turning Points

23 Two particles, P and Q, move along a straight line.
The fixed point O lies on this line.

The displacement of P from O at time t seconds is s metres, where

$$s = t^3 - 4t^2 + 5t \quad \text{for } t > 1$$

The displacement of Q from O at time t seconds is x metres, where

$$x = t^2 - 4t + 4 \quad \text{for } t > 1$$

Find the range of values of t where $t > 1$ for which both particles are moving in the same direction along the straight line.

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR January 2022

1 **Particle**

Particle P:

$$s = t^3 - 4t^2 + 5t$$

$$\frac{ds}{dt} = 3t^2 - 8t + 5$$

$$\frac{ds}{dt} = (3t - 5)(t - 1)$$

2 **Particle**

Particle Q:

$$x = t^2 - 4t + 4$$

$$\frac{dx}{dt} = 2t - 4 = 2(t - 2)$$

3 **Differentiate**

For $t > 1$:

---

4 **moves in the negative direction**

- P moves in the negative direction when $1 < t < \frac{5}{3}$.
- P moves in the positive direction when $t > \frac{5}{3}$.
- Q moves in the negative direction when $1 < t < 2$.
- Q moves in the positive direction when $t > 2$.

---

5 **Differentiate**

So both particles move in the same direction when both velocities have the same sign.

✓ **FINAL ANSWER**

$1 < t < \frac{5}{3}$ or $t > 2$

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Question 24

5 marks | Vectors

24 $OXYZ$ is a trapezium.

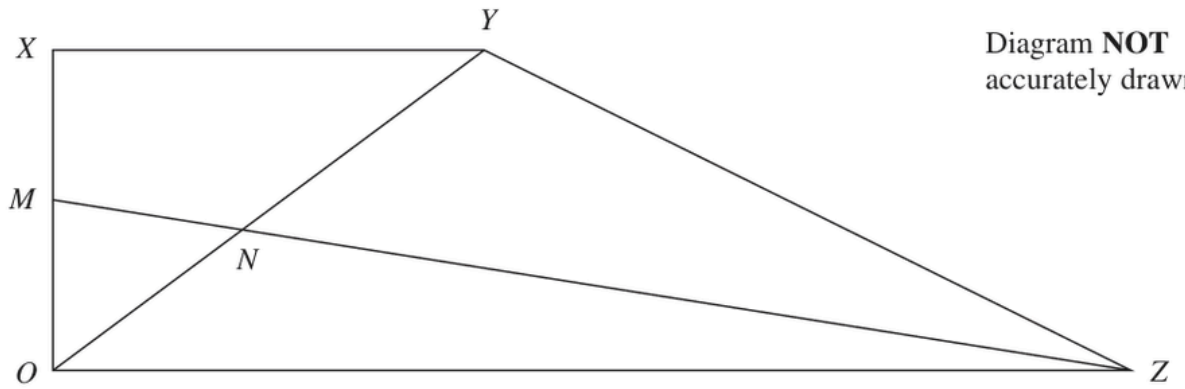


Diagram **NOT** accurately drawn

$\overrightarrow{OX} = \mathbf{a}$

$\overrightarrow{XY} = \mathbf{b}$

$\overrightarrow{OZ} = 3\mathbf{b}$

M is the midpoint of OX
N is the point such that MNZ and ONY are straight lines.

Given that $ON : OY = \lambda : 1$

use a vector method to find the value of λ

$\lambda =$ .....

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/1HR January 2022

1 **Take as the origin**

Take O as the origin.

$$\vec{OX} = \mathbf{a}, \qquad \vec{XY} = \mathbf{b}, \qquad \vec{OZ} = 3\mathbf{b}$$

2 **Use vectors**

So

$$\vec{OY} = \mathbf{a} + \mathbf{b}$$

---

3 **Find the midpoint**

Since M is the midpoint of OX,

$$\vec{OM} = \frac{1}{2}\mathbf{a}$$

---

4 **Use vectors**

Given $ON : OY = \lambda : 1$,

$$\vec{ON} = \lambda(\mathbf{a} + \mathbf{b})$$

---

5 **Use vectors**

Point N also lies on MZ, so

$$\vec{ON} = \frac{1}{2}\mathbf{a} + t\left(3\mathbf{b} - \frac{1}{2}\mathbf{a}\right)$$

$$\vec{ON} = \left(\frac{1}{2} - \frac{t}{2}\right)\mathbf{a} + 3t\mathbf{b}$$

---

6 **Use vectors**

Equate coefficients with $\lambda\mathbf{a} + \lambda\mathbf{b}$:

$$\lambda = \frac{1}{2} - \frac{t}{2}$$

$$\lambda = 3t$$

---

7 **Use vectors**

So

$$t = \frac{\lambda}{3}$$

---

8 **Substitute**

Substitute:

$$\lambda = \frac{1}{2} - \frac{1}{2}\left(\frac{\lambda}{3}\right)$$

$$\lambda = \frac{1}{2} - \frac{\lambda}{6}$$

$$\frac{7}{6}\lambda = \frac{1}{2}$$

$$\lambda = \frac{3}{7}$$

✓ **FINAL ANSWER**

$$\lambda = \frac{3}{7}$$