Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

4MA1/1H January 2021 Worked Solutions

January 2021 paper rebuilt with the worked solution after each question.

24	2020–2026	Single-paper solutions
paper questions	52 past papers	checked by Dr Eslam



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PAST PAPER

4MA1/1H January 2021

January 2021

24	100	2 h
questions	marks	exam time

Each question is followed by its worked solution and final answer.

4MA1/1H JANUARY 2021

Question 1

4 marks | Ratio Toolkit

1 Pieter owns a currency conversion shop.

Last Monday, Pieter changed a total of 20 160 rand into a number of different currencies.

He changed $\frac{3}{10}$ of the 20 160 rand into euros.

He changed the rest of the rands into dollars, rupees and francs in the ratios 9 : 5 : 2

Pieter changed more rands into dollars than he changed into francs.

Work out how many more.

..... rand

(Total for Question 1 is 4 marks)

WORKED SOLUTION

Question 1 · 4MA1/1H January 2021

1 of the money is changed

$\frac{3}{10}$ of the money is changed to euros, so the rest is

$$\frac{7}{10} \times 20160 = 14112$$

2 Split the ratio

This rest is divided in the ratio

$$\text{dollars} : \text{rupees} : \text{francs} = 9 : 5 : 2$$

3 There are

There are

$$9 + 5 + 2 = 16 \text{ parts}$$

4 Split the ratio

The difference between dollars and francs is

$$9 - 2 = 7 \text{ parts}$$

5 **Split the ratio**

So the difference is

$$\frac{7}{16} \times 14112 = 6174$$

✓ **FINAL ANSWER**

6174 rand more is changed into dollars than francs.

4MA1/1H JANUARY 2021

Question 2

5 marks | Averages from Frequency Tables

2 The table gives information about the speeds, in kilometres per hour, of 80 motorbikes as each pass under a bridge.

Speed (s kilometres per hour)	Frequency
$40 < s \leqslant 50$	10
$50 < s \leqslant 60$	16
$60 < s \leqslant 70$	19
$70 < s \leqslant 80$	23
$80 < s \leqslant 90$	12

(a) Write down the modal class.

.....

(1)

(b) Work out an estimate for the mean speed of the motorbikes as they pass under the bridge. Give your answer correct to 3 significant figures.

..... kilometres per hour

(4)

(Total for Question 2 is 5 marks)

WORKED SOLUTION

Question 2 · 4MA1/1H January 2021

1 **(a) The greatest frequency is**

(a) The greatest frequency is $\mathbf{23}$, so the modal class is

$$70 < s \leq 80$$

2 **(b) Use midpoints**

(b) Use midpoints $45, 55, 65, 75, 85$:

$$\frac{45(10) + 55(16) + 65(19) + 75(23) + 85(12)}{80} = 66.375$$

3 **To 3 significant figures**

To 3 significant figures:

$$66.4$$

✓ **FINAL ANSWER**

(a) $70 < s \leq 80$, (b) $66.4\ \text{km/h}$.

4MA1/1H JANUARY 2021

Question 3

4 marks | Volume & Surface Area

3 The diagram shows a container for water in the shape of a prism.

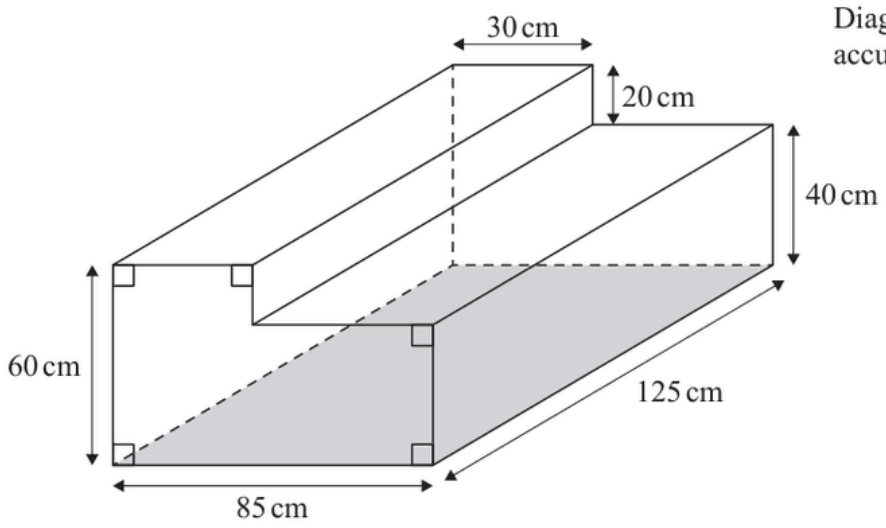


Diagram **NOT** accurately drawn

The rectangular base of the prism, shown shaded in the diagram, is horizontal.
The container is completely full of water.

Tuah is going to use a pump to empty the water from the container so that the volume of water in the container decreases at a constant rate.

The pump starts to empty water from the container at 10 30 and at 12 00 the water level in the container has dropped by 20 cm.

Find the time at which all the water has been pumped out of the container.

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/1H January 2021

1 The cross section is an

The cross-section is an L-shape.

2 Area of the lower rectangle

Area of the lower rectangle:

$$85 \times 40 = 3400$$

3 **Area of the upper rectangle**

Area of the upper rectangle:

$$30 \times 20 = 600$$

---

4 **Calculate area**

So the cross-sectional area is

$$3400 + 600 = 4000 \text{ cm}^2$$

---

5 **The full volume is**

The full volume is

$$4000 \times 125 = 500000 \text{ cm}^3$$

---

6 **Calculate area**

In the first 1.5 hours, the water level drops by 20 cm. This is the top rectangular part:

$$30 \times 20 \times 125 = 75000 \text{ cm}^3$$

---

7 **Calculate area**

So the pumping rate is

$$\frac{75000}{1.5} = 50000 \text{ cm}^3/\text{hour}$$

---

8 **Time to empty the whole**

Time to empty the whole container:

$$\frac{500000}{50000} = 10 \text{ hours}$$

---

9 **Calculate area**

Starting at $10{:}30$, ten hours later is

$$20{:}30$$

✓ **FINAL ANSWER**

$20{:}30$

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Question 4

2 marks | Set Notation & Venn Diagrams

4 $\mathscr{E} = \{20, 21, 22, 23, 24, 25, 26, 27, 28, 29\}$

$A = \{\text{odd numbers}\}$
$B = \{\text{multiples of } 3\}$

List the members of the set

(i) $A \cap B$

.....

(1)

(ii) $A \cup B$

.....

(1)

(Total for Question 4 is 2 marks)

WORKED SOLUTION

Question 4 · 4MA1/1H January 2021

1 Calculate value

$$\mathcal{E} = \{20, 21, 22, 23, 24, 25, 26, 27, 28, 29\}$$

2 The odd numbers are

The odd numbers are

$$A = \{21, 23, 25, 27, 29\}$$

3 The multiples of are

The multiples of 3 are

$$B = \{21, 24, 27\}$$

4 The numbers in both sets

The numbers in both sets are

$$A \cap B = \{21, 27\}$$

5 The numbers in either set

The numbers in either set are

$$A \cup B = \{21, 23, 24, 25, 27, 29\}$$

FINAL ANSWER

(i) $\{21, 27\}$, (ii) $\{21, 23, 24, 25, 27, 29\}$

4MA1/1H JANUARY 2021

Question 5

5 marks | Linear Equations

5 (a) Factorise fully $15y^4 + 20uy^3$

.....

(2)

(b) Solve $4 - 3x = \dfrac{5 - 8x}{4}$

Show clear algebraic working.

$x =$ .....

(3)

(Total for Question 5 is 5 marks)

WORKED SOLUTION

Question 5 · 4MA1/1H January 2021

1 **(a)**

(a)

$$15y^4 + 20uy^3 = 5y^3(3y + 4u)$$

2 **(b)**

(b)

$$4 - 3x = \frac{5 - 8x}{4}$$

$$16 - 12x = 5 - 8x$$

$$11 = 4x$$

FINAL ANSWER

(a) $5y^3(3y + 4u)$, (b) $x = \frac{11}{4}$

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Question 6

2 marks | Standard Form

6 (a) Write 2 840 000 000 in standard form.

.....

(1)

(b) Write 2.5×10^{-4} as an ordinary number.

.....

(1)

(Total for Question 6 is 2 marks)

WORKED SOLUTION

Question 6 · 4MA1/1H January 2021

1 Convert standard form

$$2840000000 = 2.84 \times 10^{9}$$

2 Convert standard form

Also,

$$2.5 \times 10^{-4} = 0.00025$$

FINAL ANSWER

(a) 2.84×10^{9}, (b) 0.00025

4MA1/1H JANUARY 2021

Question 7

6 marks | Compound Interest & Depreciation

7 Chen invests 40 000 yuan in a fixed-term bond for 3 years.

The fixed-term bond pays compound interest at a rate of 3.5% each year.

(a) Work out the value of Chen's investment at the end of 3 years.
Give your answer to the nearest yuan.

..... yuan

(3)

Wang invested P yuan.
The value of his investment decreased by 6.5% each year.

At the end of the first year, the value of Wang's investment was 30 481 yuan.

(b) Work out the value of P.

$P =$ .....

(3)

(Total for Question 7 is 6 marks)

WORKED SOLUTION

Question 7 · 4MA1/1H January 2021

1 Use compound interest

For part (a),

$$40000(1.035)^3 = 44348.715$$

2 Use compound interest

Correct to the nearest yuan,

$$44348.715 \approx 44349$$

3 Use compound interest

For part (b), after one year the value is 93.5% of P.

$$0.935P = 30481$$

$$P = \frac{30481}{0.935} = 32600$$

FINAL ANSWER

(a) $\mathbf{44349}$ yuan, (b) $\boldsymbol{P = 32600}$

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Question 8

3 marks | Circles, Arcs & Sectors

8 The region, shown shaded in the diagram, is a path.

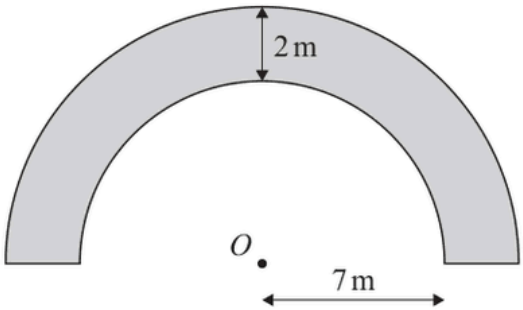


Diagram **NOT** accurately drawn

The boundary of the path is formed by two semicircles, with the same centre O, and two straight lines.

The inner semicircle has a radius of 7 metres.
The path has a width of 2 metres.

Work out the perimeter of the path.
Give your answer correct to one decimal place.

..... m

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1H January 2021

1 **The inner radius is m**

The inner radius is 7 m.

2 **Calculate value**

The path is 2 m wide, so the outer radius is

$$7 + 2 = 9 \text{ m}$$

3 Perimeter of the path

Perimeter of the path:

$$\text{inner semicircle} + \text{outer semicircle} + 2 \text{ straight ends}$$

$$= 7\pi + 9\pi + 2 + 2$$

$$= 16\pi + 4$$

$$= 54.265\ldots$$

FINAL ANSWER

$54.3\ \text{m}$.

4MA1/1H JANUARY 2021

Question 9

5 marks | Factorising

9 (a) Simplify $(2x^3y^5)^4$

.....

(2)

(b) (i) Factorise $x^2 + 5x - 36$

.....

(2)

(ii) Hence, solve $x^2 + 5x - 36 = 0$

.....

(1)

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/1H January 2021

1 **(a)**

(a)

$$(2x^3y^5)^4 = 16x^{12}y^{20}$$

2 **(b)(i)**

(b)(i)

$$x^2 + 5x - 36 = (x + 9)(x - 4)$$

3 **Factorise expression**

(b)(ii) $x = -9$ or $x = 4$.

✓ **FINAL ANSWER**

(a) $16x^{12}y^{20}$, (b)(i) $(x + 9)(x - 4)$, (b)(ii) $x = -9$ or $x = 4$

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Question 10

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

10 Here is isosceles triangle ABC.

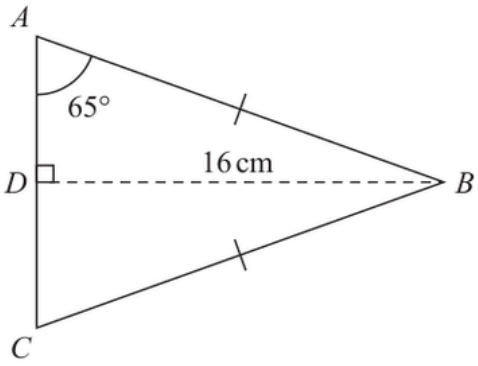


Diagram **NOT** accurately drawn

D is the midpoint of AC and $DB = 16$ cm.

Angle $DAB = 65°$

Work out the perimeter of triangle ABC.
Give your answer correct to one decimal place.

..... cm

(Total for Question 10 is 4 marks)

WORKED SOLUTION

Question 10 · 4MA1/1H January 2021

1 **Find the gradient**

Since ABC is isosceles and D is the midpoint of AC, BD is perpendicular to AC.

2 **Use trigonometry**

In right triangle ABD,

$$BD = 16, \qquad \angle DAB = 65°$$

$$\sin 65° = \frac{16}{AB}$$

$$AB = \frac{16}{\sin 65°}$$

3 **Use trigonometry**

Also,

$$\tan 65^\circ = \frac{16}{AD}$$

$$AD = \frac{16}{\tan 65^\circ}$$

---

4 **The perimeter is**

The perimeter is

$$AB + BC + AC = 2AB + 2AD$$

$$= 2\left(\frac{16}{\sin 65^\circ}\right) + 2\left(\frac{16}{\tan 65^\circ}\right)$$

$$= 50.2299\ldots$$

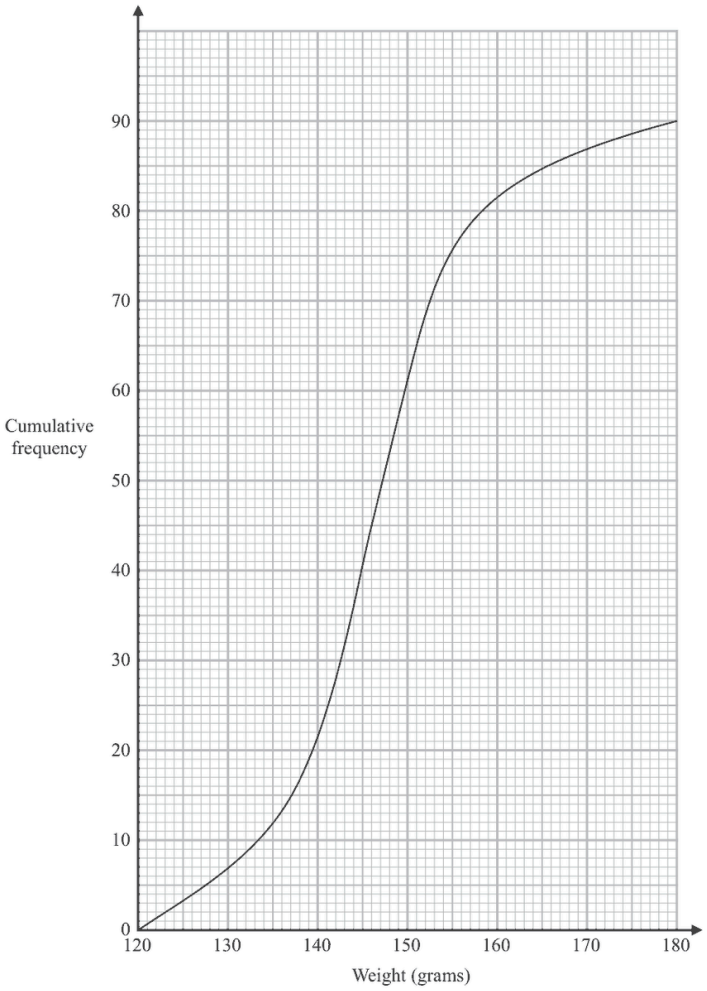
✓ **FINAL ANSWER**

$50.2\ \text{cm}$.

Question 11

5 marks | Cumulative Frequency Diagrams

11 The cumulative frequency graph gives information about the weights, in grams, of 90 bags of sweets.



(a) Find an estimate for the median of the weights of these bags of sweets.

..... grams

(2)

Roberto sells the bags of sweets to raise money for charity.
Bags with a weight greater than d grams are labelled large bags and sold for 3.75 euros each bag.

The total amount of money he receives by selling all the large bags is 93.75 euros.

(b) Find the value of d.

$d =$ .....

(3)

(Total for Question 11 is 5 marks)

1 **Use cumulative frequency**

There are 90 bags, so the median is at cumulative frequency 45.

2 **Use cumulative frequency**

From the graph,

$$\text{median} \approx 146 \text{ g}$$

3 **Large bags sold**

Large bags sold:

$$\frac{93.75}{3.75} = 25$$

4 **Use cumulative frequency**

So 25 bags have weight greater than d.

5 **That means**

That means

$$90 - 25 = 65$$

6 **Use cumulative frequency**

bags have weight d grams or less.

7 **Use cumulative frequency**

From the graph, cumulative frequency 65 gives

$$d \approx 150$$

✓ **FINAL ANSWER**

median about 146 g, and $d \approx 150$.

4MA1/1H JANUARY 2021

Question 12

6 marks | Expanding Brackets

12 (a) Express $\frac{4}{x-2} - \frac{3}{x+1}$ as a single fraction.
Give your answer in its simplest form.

.....

(3)

(b) Expand and simplify $2x(x-5)(x-3)$

.....

(3)

(Total for Question 12 is 6 marks)

WORKED SOLUTION

Question 12 · 4MA1/1H January 2021

1 **(a)**

(a)

$$\frac{4}{x-2} - \frac{3}{x+1} = \frac{4(x+1) - 3(x-2)}{(x-2)(x+1)}$$

$$= \frac{4x + 4 - 3x + 6}{(x-2)(x+1)} = \frac{x+10}{(x-2)(x+1)}$$

2 **(b)**

(b)

$$2x(x-5)(x-3) = 2x(x^2 - 8x + 15)$$

$$= 2x^3 - 16x^2 + 30x$$

✓ **FINAL ANSWER**

(a) $\frac{x+10}{(x-2)(x+1)}$, (b) $2x^3 - 16x^2 + 30x$

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Question 13

3 marks | Linear Graphs

13 Point A has coordinates (5, 8)
Point B has coordinates (9, –4)

(a) Work out the gradient of AB.

.....

(2)

The straight line **L** has equation $y = -4x + 5$

(b) Write down the gradient of a straight line that is perpendicular to **L**.

.....

(1)

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/1H January 2021

1 **Find the gradient**

(a) The gradient of AB, where $A(5, 8)$ and $B(9, -4)$, is

$$\frac{-4-8}{9-5} = \frac{-12}{4} = -3$$

2 **Find the gradient**

(b) A line perpendicular to a line with gradient -3 has gradient

$$\frac{1}{3}$$

FINAL ANSWER

Gradient of $AB = -3$, perpendicular gradient $\frac{1}{3}$.

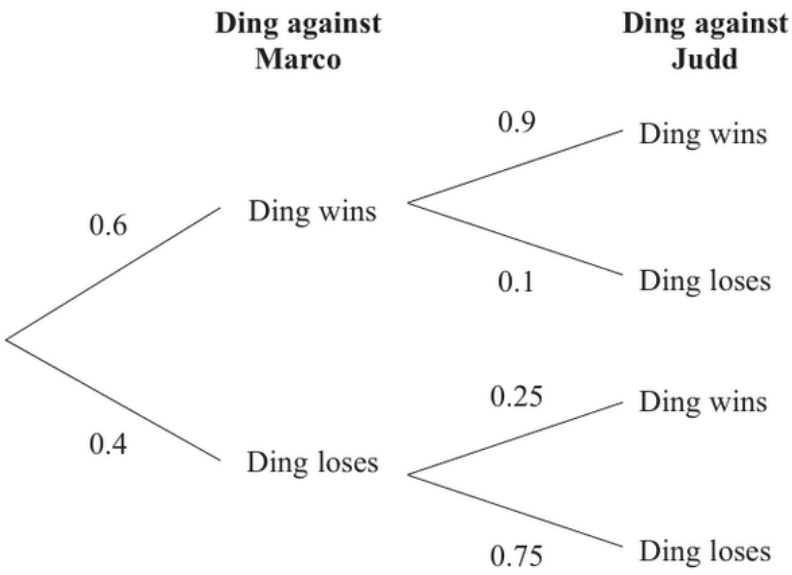
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Question 14

5 marks | Tree Diagrams & Conditional Probability

14 Ding is going to play one game of snooker against each of two of his friends, Marco and Judd.

The probability tree diagram gives information about the probabilities that Ding will win or lose each of these two games.



(a) Work out the probability that Ding will win both games.

.....

(2)

(b) Work out the probability that Ding will win exactly one of the games.

.....

(3)

(Total for Question 14 is 5 marks)

WORKED SOLUTION

Question 14 · 4MA1/1H January 2021

1 **(a) Win both games**

Multiply along the "win, win" branch:

$$0.6 \times 0.9 = 0.54$$

2

(b) Win exactly one game

Win then lose, or lose then win:

$$0.6 \times 0.1 + 0.4 \times 0.25 = 0.06 + 0.10 = 0.16$$

✓ **FINAL ANSWER**

(a) 0.54 (b) 0.16

4MA1/1H JANUARY 2021

Question 15

3 marks | Rounding, Estimation & Bounds

15

$$a = \frac{v - u}{t}$$

$v = 9.6$ correct to 1 decimal place
$u = 3.8$ correct to 1 decimal place
$t = 1.84$ correct to 2 decimal places

Calculate the upper bound for the value of a.
Give your answer as a decimal correct to 2 decimal places.
Show your working clearly.

(Total for Question 15 is 3 marks)

WORKED SOLUTION

Question 15 · 4MA1/1H January 2021

1 Estimate the value

$$a = \frac{v - u}{t}$$

2 Find upper bound

For the upper bound, use the upper bound of v, the lower bound of u, and the lower bound of t.

$$v < 9.65, \quad u \geq 3.75, \quad t \geq 1.835$$

$$a_{\text{upper}} = \frac{9.65 - 3.75}{1.835} = 3.2152\ldots$$

FINAL ANSWER

3.22

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Question 16

5 marks | Bearings

16 The diagram shows the positions of three ships, A, B and C.

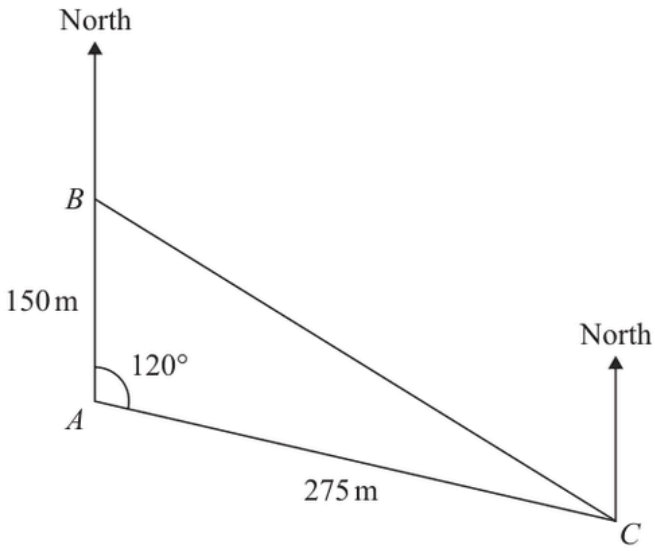


Diagram **NOT** accurately drawn

Ship B is due north of ship A.

The bearing of ship C from ship A is 120°

Calculate the bearing of ship C from ship B.
Give your answer correct to the nearest degree.

.....°

(Total for Question 16 is 5 marks)

WORKED SOLUTION

Question 16 · 4MA1/1H January 2021

1 **Use bearings**

From A, ship C is on a bearing of $120°$, and $AC = 275\ \text{m}$.

2 **Resolve**

Resolve AC:

$$\text{east component} = 275\sin 120^\circ = 238.16\ldots$$

$$\text{north component} = 275\cos 120^\circ = -137.5$$

---

3 **Use bearings**

Ship B is $150\ \text{m}$ north of A, so from B to C:

$$\text{east} = 238.16\ldots, \qquad \text{north} = -137.5 - 150 = -287.5$$

---

4 **Use bearings**

The bearing is therefore in the south-east quadrant:

$$180^\circ - \tan^{-1}\left(\frac{238.16\ldots}{287.5}\right) = 140.36\ldots^\circ$$

✓ **FINAL ANSWER**

140°.

Question 17

5 marks | Differentiation & Turning Points

17 A solid, **S**, is made from a hemisphere and a cylinder.

The centre of the circular face of the hemisphere and the centre of the top face of the cylinder are at the same point.

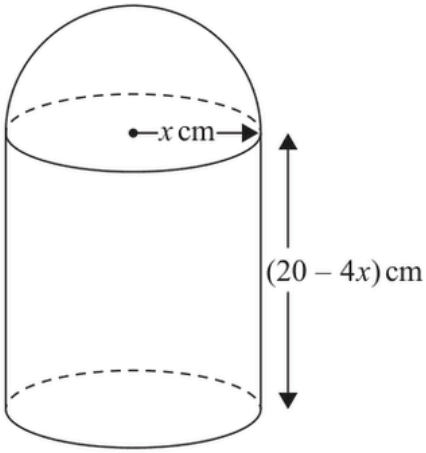


Diagram **NOT** accurately drawn

The radius of the cylinder and the radius of the hemisphere are both x cm.
The height of the cylinder is $(20 - 4x)$ cm.

The volume of **S** is $V\,\text{cm}^3$ where $V = \frac{1}{3}\pi y$

Find the maximum value of y.
Show clear algebraic working.

(Total for Question 17 is 5 marks)

1 **Volume of the solid is**

Volume of the solid is cylinder plus hemisphere:

$$V = \pi x^2(20 - 4x) + \frac{2}{3}\pi x^3$$

$$V = 20\pi x^2 - 4\pi x^3 + \frac{2}{3}\pi x^3$$

$$V = 20\pi x^2 - \frac{10}{3}\pi x^3$$

2 **The question gives**

The question gives

$$V = \frac{1}{3}\pi y$$

3 **Differentiate**

So

$$y = 60x^2 - 10x^3$$

4 **Differentiate**

Differentiate:

$$\frac{dy}{dx} = 120x - 30x^2$$

5 **Differentiate**

For a maximum,

$$120x - 30x^2 = 0$$

$$30x(4 - x) = 0$$

6 **Differentiate**

So the valid value is

$$x = 4$$

$$y = 60(4^2) - 10(4^3) = 960 - 640 = 320$$

FINAL ANSWER

The maximum value of y is 320.

4MA1/1H JANUARY 2021

Question 18

3 marks | Surds

18 Given that $(8 - \sqrt{x})(5 + \sqrt{x}) = y\sqrt{x} + 21$ where x is a prime number and y is an integer,

find the value of x and the value of y.
Show each stage of your working clearly.

$x =$ .....

$y =$ .....

(Total for Question 18 is 3 marks)

WORKED SOLUTION

Question 18 · 4MA1/1H January 2021

1 Simplify surd

$$(8 - \sqrt{x})(5 + \sqrt{x}) = 40 + 8\sqrt{x} - 5\sqrt{x} - x$$

$$= 40 + 3\sqrt{x} - x$$

2 Simplify surd

This is equal to

$$y\sqrt{x} + 21$$

3 Compare the rational parts

Compare the rational parts:

$$40 - x = 21$$

$$x = 19$$

4 Compare the surd coefficients

Compare the surd coefficients:

$$y = 3$$

FINAL ANSWER

$x = 19, y = 3$

4MA1/1H JANUARY 2021

Question 19

5 marks | Simultaneous Equations

19 Solve the simultaneous equations

$$x^2 - 9y - x = 2y^2 - 12$$
$$x + 2y - 1 = 0$$

Show clear algebraic working.

(Total for Question 19 is 5 marks)

WORKED SOLUTION

Question 19 · 4MA1/1H January 2021

1 Solve simultaneous equations

$$x^2 - 9y - x = 2y^2 - 12$$

$$x + 2y - 1 = 0$$

2 From the second equation

From the second equation:

$$x = 1 - 2y$$

3 Substitute into the first equation

Substitute into the first equation:

$$(1 - 2y)^2 - 9y - (1 - 2y) = 2y^2 - 12$$

$$4y^2 - 11y = 2y^2 - 12$$

$$2y^2 - 11y + 12 = 0$$

4 Factorise

Factorise:

$$(2y - 3)(y - 4) = 0$$

5 **Solve simultaneous equations**

So

$$y = \frac{3}{2} \quad \text{or} \quad y = 4$$

---

6 **Solve simultaneous equations**

If $y = \frac{3}{2}$:

$$x = 1 - 2\left(\frac{3}{2}\right) = -2$$

---

7 **Solve simultaneous equations**

If $y = 4$:

$$x = 1 - 8 = -7$$

✓ **FINAL ANSWER**

$(x, y) = \left(-2, \frac{3}{2}\right)$ or $(-7, 4)$.

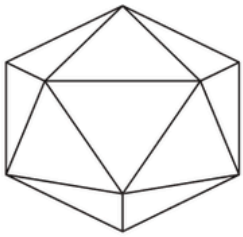
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Question 20

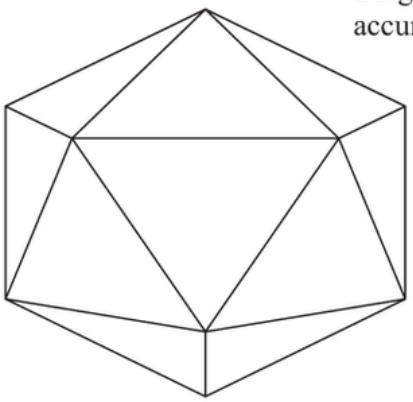
3 marks | Area & Volume of Similar Shapes

20 **A** and **B** are two similar solids.

Diagram **NOT** accurately drawn



A



B

A has a volume of $1836\,\text{cm}^3$
B has a volume of $4352\,\text{cm}^3$

B has a total surface area of $1120\,\text{cm}^2$

Work out the total surface area of **A**.

..... cm^2

(Total for Question 20 is 3 marks)

WORKED SOLUTION

Question 20 · 4MA1/1H January 2021

1 The volume ratio of solid

The volume ratio of solid A to solid B is

$$1836 : 4352$$

2 Simplify

Simplify:

$$\frac{1836}{4352} = \frac{27}{64}$$

3 Split the ratio

So the linear ratio is

$$\sqrt[3]{27} : \sqrt[3]{64} = 3 : 4$$

4 **Surface area ratio**

Surface area ratio:

$$3^2 : 4^2 = 9 : 16$$

---

5 **Solid B has surface area**

Solid B has surface area $1120\ \text{cm}^2$, so solid A has surface area

$$1120 \times \frac{9}{16} = 630$$

✓ **FINAL ANSWER**

$\boxed{630\ \text{cm}^2}$

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Question 21

4 marks | Transformations of Graphs

21 A curve has equation $y = f(x)$

The coordinates of the minimum point on this curve are $(-9, 15)$

(a) Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x + 3)$

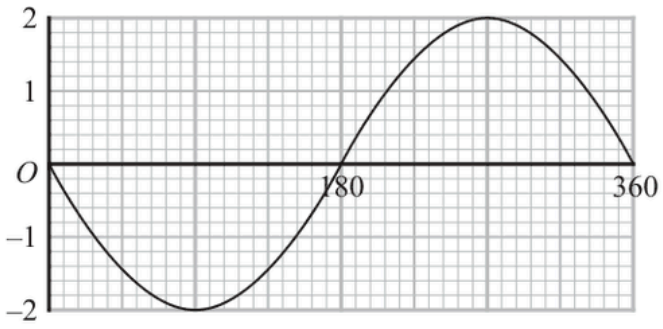
(..... , ...)

(ii) $y = \frac{1}{3} f(x)$

(..... , ...)

(2)

The graph of $y = a\cos(x + b)°$ for $0 \leqslant x \leqslant 360$ is drawn on the grid below.



Given that $a > 0$ and that $0 < b < 360$

(b) find the value of a and the value of b.

$a =$ .....

$b =$ .....

(2)

(Total for Question 21 is 4 marks)

WORKED SOLUTION

Question 21 · 4MA1/1H January 2021

1 **The minimum point of is**

The minimum point of $y = f(x)$ is $(-9, 15)$.

2 **Calculate value**

For

$$y = f(x + 3)$$

3 **the graph moves 3 units**

the graph moves 3 units left:

$$(-9, 15) \to (-12, 15)$$

4 **Evaluate fraction**

For

$$y = \frac{1}{3} f(x)$$

5 **the coordinate is divided by**

the y-coordinate is divided by 3:

$$(-9, 15) \to (-9, 5)$$

6 **Use trigonometry**

For part (b), the cosine graph has maximum 2 and minimum -2, so

$$a = 2$$

7 **The graph has a minimum**

The graph has a minimum at $x = 90°$. For $a\cos(x + b)°$, a minimum occurs when

$$x + b = 180$$

8 **Calculate value**

So

$$90 + b = 180$$

$$b = 90$$

✓ **FINAL ANSWER**

$(-12, 15)$, $(-9, 5)$, $a = 2,\ b = 90$

4MA1/1H JANUARY 2021

Question 22

3 marks | Functions

22 The function f is such that $f(x) = x^2 - 8x + 5$ where $x \leqslant 4$

Express the inverse function f^{-1} in the form $f^{-1}(x) = ...$

$f^{-1}(x) =$ .....

(Total for Question 22 is 3 marks)

WORKED SOLUTION

Question 22 · 4MA1/1H January 2021

1 Calculate value

$$f(x) = x^2 - 8x + 5$$

$$f(x) = (x-4)^2 - 11$$

2 Let

Let

$$y = (x-4)^2 - 11$$

$$y + 11 = (x-4)^2$$

3 Find inverse function

Since the domain is $x < 4$, take the negative square root:

$$x - 4 = -\sqrt{y+11}$$

$$x = 4 - \sqrt{y+11}$$

FINAL ANSWER

$$f^{-1}(x) = 4 - \sqrt{x+11}$$

4MA1/1H JANUARY 2021

Question 23

6 marks | Vectors

23 OAB is a triangle.

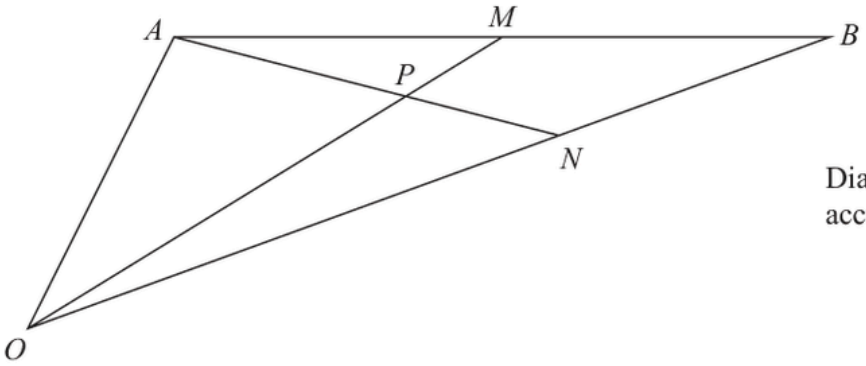


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 2\mathbf{a}$ and $\overrightarrow{OB} = 2\mathbf{b}$

M is the midpoint of AB.
N is the point on OB such that $ON : NB = 2 : 1$

P is the point on AN such that OPM is a straight line.

Use a vector method to find $OP : PM$
Show your working clearly.

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4MA1/1H January 2021

1 **Use vectors**

$$\vec{OA} = 2\mathbf{a}, \qquad \vec{OB} = 2\mathbf{b}$$

2 **Find the midpoint**

Since M is the midpoint of AB,

$$\vec{OM} = \frac{2\mathbf{a} + 2\mathbf{b}}{2}$$

$$\vec{OM} = \mathbf{a} + \mathbf{b}$$

3 **Use vectors**

Since $ON : NB = 2 : 1$,

$$\vec{ON} = \frac{2}{3}\vec{OB}$$

$$\vec{ON} = \frac{2}{3}(2\mathbf{b}) = \frac{4}{3}\mathbf{b}$$

---

4 **Use vectors**

Point P lies on OM, so

$$\vec{OP} = t(\mathbf{a} + \mathbf{b})$$

---

5 **Use vectors**

Point P also lies on AN, so

$$\vec{OP} = 2\mathbf{a} + s\left(\frac{4}{3}\mathbf{b} - 2\mathbf{a}\right)$$

$$\vec{OP} = (2 - 2s)\mathbf{a} + \frac{4s}{3}\mathbf{b}$$

---

6 **Use vectors**

Equate coefficients of $\mathbf{a}$ and $\mathbf{b}$:

$$t = 2 - 2s$$

$$t = \frac{4s}{3}$$

---

7 **Use vectors**

So

$$\frac{4s}{3} = 2 - 2s$$

$$4s = 6 - 6s$$

$$10s = 6$$

$$s = \frac{3}{5}$$

---

8 **Use vectors**

Then

$$t = \frac{4}{5}$$

9 **Use vectors**

So P is $\frac{4}{5}$ of the way from O to M.

✓ **FINAL ANSWER**

$OP : PM = 4 : 1$

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Question 24

4 marks | Sequences

24 An arithmetic series has first term a and common difference d.

The sum of the first $2n$ terms of the series is four times the sum of the first n terms of the series.

Find an expression for a in terms of d.
Show your working clearly.

$a =$ .....

(Total for Question 24 is 4 marks)

WORKED SOLUTION

Question 24 · 4MA1/1H January 2021

1 Evaluate fraction

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{2n} = n(2a + (2n-1)d)$$

2 Given

Given

$$S_{2n} = 4S_n$$

$$n(2a + (2n-1)d) = 4 \cdot \frac{n}{2}(2a + (n-1)d)$$

3 Divide by

Divide by n:

$$2a + (2n-1)d = 2(2a + (n-1)d)$$

$$2a + 2nd - d = 4a + 2nd - 2d$$

$$d = 2a$$

$$a = \frac{d}{2}$$

FINAL ANSWER

$$a = \frac{d}{2}$$