Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

4MA1/2HR January 2020 Worked Solutions

January 2020 paper rebuilt with the worked solution after each question.

26	2020–2026	Single-paper solutions
paper questions	52 past papers	checked by Dr Eslam



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PAST PAPER

4MA1/2HR January 2020

January 2020

26	100	2 h
questions	marks	exam time

Each question is followed by its worked solution and final answer.

4MA1/2HR JANUARY 2020

Question 1

3 marks | Prime Factors, HCF & LCM

1 (a) Write $5^{17} \times 5^{2}$ as a single power of 5

.....

(1)

(b) Write 800 as a product of its prime factors.
Show your working clearly.

.....

(2)

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/2HR January 2020

1 Use the index law

Use the index law $a^m \times a^n = a^{m+n}$:

$$5^{17} \times 5^2 = 5^{19}$$

2 Find the LCM

Now write 800 as a product of prime factors:

$$800 = 8 \times 100$$

$$800 = 2^3 \times 2^2 \times 5^2$$

$$800 = 2^5 \times 5^2$$

FINAL ANSWER

(a) 5^{19}, (b) $2^5 \times 5^2$

4MA1/2HR JANUARY 2020

Question 2

3 marks | Statistics Toolkit

2 The table gives information about the amount of money, in £, that Fiona spent in a grocery store each week during 2019

Amount spent (£x)	Frequency
$0 \leqslant x < 20$	5
$20 \leqslant x < 40$	11
$40 \leqslant x < 60$	8
$60 \leqslant x < 80$	19
$80 \leqslant x < 100$	9

Work out an estimate for the total amount of money that Fiona spent in the grocery store during 2019

£.....

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/2HR January 2020

1 **Use midpoints**

Use midpoints $10, 30, 50, 70, 90$.

$$10(5) + 30(11) + 50(8) + 70(19) + 90(9)$$

$$= 50 + 330 + 400 + 1330 + 810 = 2920$$

FINAL ANSWER

$\pounds 2920$.

4MA1/2HR JANUARY 2020

Question 3

4 marks | Forming & Solving Equations

3 Three tins, A, B and C, each contain buttons.

Tin A contains x buttons.
Tin B contains 4 times the number of buttons that tin A contains.
Tin C contains 7 fewer buttons than tin A.

The total number of buttons in the three tins is 137

Work out the number of buttons in tin C.

(Total for Question 3 is 4 marks)

WORKED SOLUTION

Question 3 · 4MA1/2HR January 2020

1 Write each tin in terms of x

Tin A has x buttons, tin B has $4x$, tin C has $x - 7$.

2 Form and solve the equation

$$x + 4x + (x - 7) = 137 \Rightarrow 6x - 7 = 137 \Rightarrow 6x = 144 \Rightarrow x = 24$$

3 Buttons in tin C

$$24 - 7 = 17$$

FINAL ANSWER

17 buttons

4MA1/2HR JANUARY 2020

Question 4

3 marks | Right-Angled Triangles - Pythagoras & Trigonometry

4 The diagram shows a rectangle and a diagonal of the rectangle.

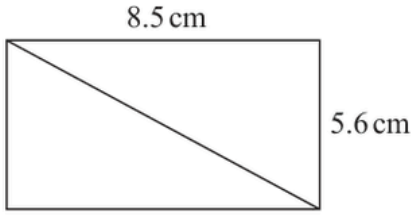


Diagram **NOT** accurately drawn

Work out the length of the diagonal of the rectangle.
Give your answer correct to 1 decimal place.

..... cm

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/2HR January 2020

1 Use trigonometry

$$d^2 = 8.5^2 + 5.6^2$$

$$d = \sqrt{103.61} = 10.178\ldots$$

FINAL ANSWER

10.2 cm.

4MA1/2HR JANUARY 2020

Question 5

3 marks | Standard & Compound Units

5 A plane takes 3 hours 36 minutes to fly from the Cayman Islands to New York.
The plane flies a distance of 2470 km.

Work out the average speed of the plane in km/h.
Give your answer correct to the nearest whole number.

..... km/h

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/2HR January 2020

1 The time is

The time is

$$3\text{ h }36\text{ min} = 3.6\text{ h}$$

2 Average speed

Average speed:

$$2470 \div 3.6 = 686.111\ldots$$

FINAL ANSWER

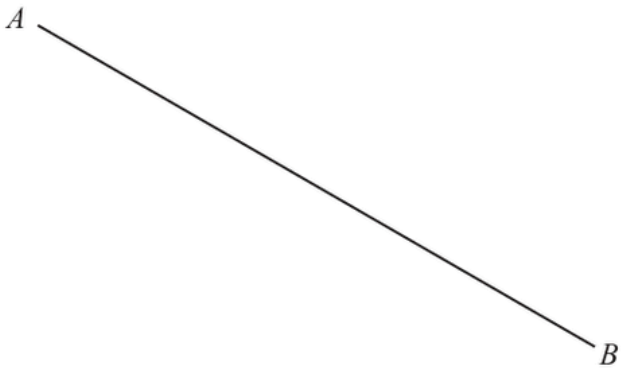
686 km/h.

4MA1/2HR JANUARY 2020

Question 6

2 marks | Constructions & Loci

6 Use ruler and compasses only to construct the perpendicular bisector of the line AB.
You must show all your construction lines.



(Total for Question 6 is 2 marks)

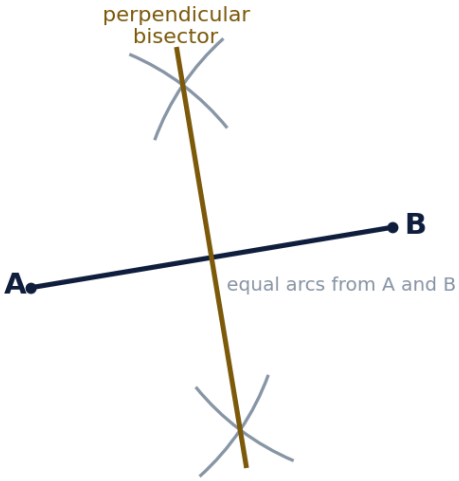
WORKED SOLUTION

Question 6 · 4MA1/2HR January 2020

1 Construction answer

To construct the perpendicular bisector of AB:

1. Set the compass radius to more than half of AB.
2. With centre A, draw arcs above and below the line.
3. With the same radius and centre B, draw arcs crossing the first pair.
4. Draw the straight line through the two arc intersections.



FINAL ANSWER

This line is the perpendicular bisector of AB.

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Question 7

3 marks | Simultaneous Equations

7 Solve the simultaneous equations

$$3x + 5y = 6$$
$$7x - 5y = -11$$

Show clear algebraic working.

$x =$ .....

$y =$ .....

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/2HR January 2020

1 Solve simultaneous equations

$$3x + 5y = 6$$

$$7x - 5y = -11$$

2 Add the equations

Add the equations:

$$10x = -5$$

$$x = -\frac{1}{2}$$

3 **Solve simultaneous equations**

Substitute into $3x + 5y = 6$:

$$3\left(-\frac{1}{2}\right) + 5y = 6$$

$$-\frac{3}{2} + 5y = 6$$

$$5y = \frac{15}{2}$$

$$y = \frac{3}{2}$$

✓ **FINAL ANSWER**

$x = -\frac{1}{2}$, $y = \frac{3}{2}$.

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Question 8

3 marks | Compound Interest & Depreciation

8 Hamish buys a new car for $20 000
The car depreciates in value by 19% each year.

Work out the value of the car at the end of 3 years.
Give your answer to the nearest $.

$.....

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/2HR January 2020

1 A depreciation of gives a

A depreciation of 19% gives a multiplier of

$$1 - 0.19 = 0.81$$

2 Use compound interest

After 3 years,

$$20000(0.81)^3 = 10628.82$$

3 Use compound interest

Correct to the nearest dollar,

$$10628.82 \approx 10629$$

✓ **FINAL ANSWER**

$10629

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Question 9

3 marks | Standard & Compound Units

9 The diagram shows a box in the shape of a cuboid.

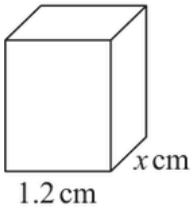


Diagram **NOT** accurately drawn

The box is put on a table.
The face of the box in contact with the table has length 1.2 metres and width x metres.

The force exerted by the box on the table is 27 newtons.
The pressure on the table due to the box is 30 newtons/m^2

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the value of x.

$x =$ .....

(Total for Question 9 is 3 marks)

WORKED SOLUTION

Question 9 · 4MA1/2HR January 2020

1 Calculate area

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

$$30 = \frac{27}{\text{area}}$$

$$\text{area} = \frac{27}{30} = 0.9 \text{ m}^2$$

2 **The base area is**

The base area is

$$1.2x = 0.9$$

$$x = \frac{0.9}{1.2} = 0.75$$

✓ **FINAL ANSWER**

$x = 0.75\ \text{m}$.

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Question 10

3 marks | Standard Form

10 The table shows information about the surface area of each of the world's oceans.

Ocean	Surface area in square kilometres
Pacific	1.56×10^8
Indian	6.86×10^7
Southern	2.03×10^7
Arctic	1.41×10^7
Atlantic	1.06×10^8

(a) Work out the difference, in square kilometres, between the surface area of the Atlantic Ocean and the surface area of the Indian Ocean.
Give your answer in standard form.

..... square kilometres
(2)

The surface area of the Pacific Ocean is k times the surface area of the Arctic Ocean.

(b) Work out the value of k.
Give your answer correct to the nearest whole number.

$k =$ .....
(1)

(Total for Question 10 is 3 marks)

WORKED SOLUTION

Question 10 · 4MA1/2HR January 2020

1 Atlantic surface area

Atlantic surface area:

$$1.06 \times 10^8 = 106000000$$

2 Indian surface area

Indian surface area:

$$6.86 \times 10^7 = 68600000$$

3 **Difference**

Difference:

$$106000000 - 68600000 = 37400000$$

$$37400000 = 3.74 \times 10^{7}$$

---

4 **Convert standard form**

For part (b),

$$k = \frac{1.56 \times 10^{8}}{1.41 \times 10^{7}}$$

$$k = 11.063\ldots$$

---

5 **Convert standard form**

To the nearest whole number,

$$k = 11$$

✓ **FINAL ANSWER**

(a) 3.74×10^{7}, (b) 11

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Question 11

4 marks | Inequalities (Solving & Graphing)

11 (a) Write down the integer values of x that satisfy the inequality $-2 < x \leqslant 4$

.....

(2)

The region **R**, shown shaded in the diagram, is bounded by three straight lines.

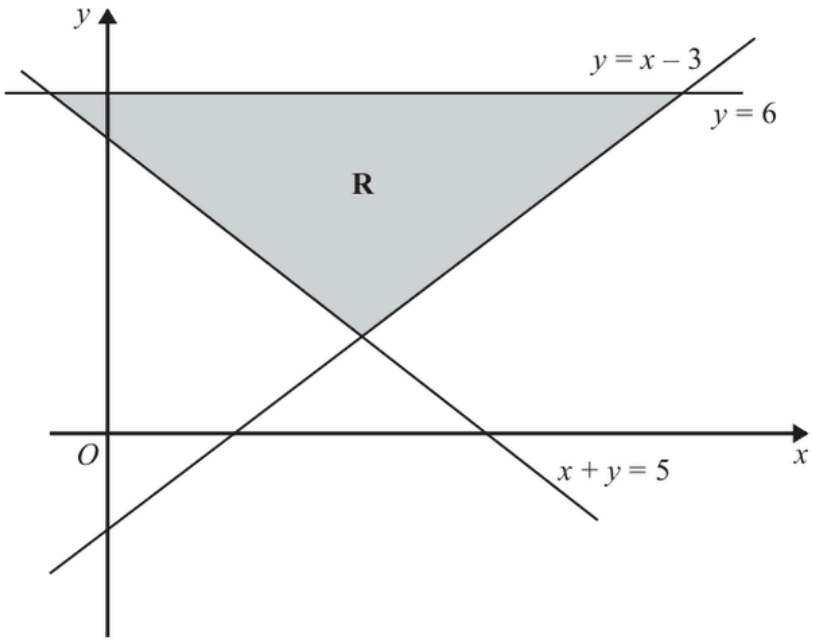


Diagram **NOT** accurately drawn

(b) Write down the three inequalities that define the region **R**.

.....

(2)

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/2HR January 2020

1 **Solve inequality**

For part (a),

$$2 < x \leq 4$$

2 **The integer values are**

The integer values are

$$3, 4$$

3 **Calculate value**

For the region R, the boundaries are

$$y = x - 3, \qquad x + y = 5, \qquad y = 6$$

4 **The shaded region is above**

The shaded region is above the two sloping lines and below the horizontal line:

$$y \geq x - 3$$

$$x + y \geq 5$$

$$y \leq 6$$

✓ **FINAL ANSWER**

(a) $3, 4$. (b) $y \geq x - 3$, $x + y \geq 5$, $y \leq 6$.

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Question 12

3 marks | Congruent Shapes

12 The diagram shows two congruent isosceles triangles and parts of two congruent regular polygons, **X** and **Y**.

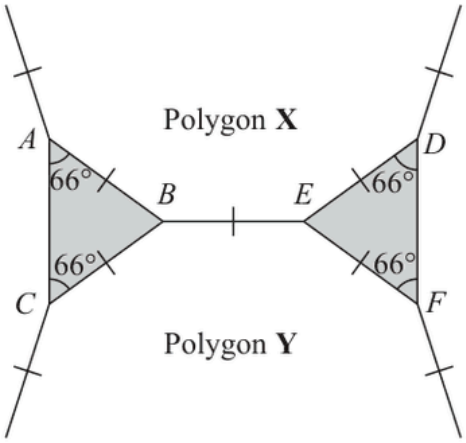


Diagram **NOT** accurately drawn

The two regular polygons each have n sides.

Work out the value of n.

$n =$ .....

(Total for Question 12 is 3 marks)

WORKED SOLUTION

Question 12 · 4MA1/2HR January 2020

1 **Each congruent isosceles triangle has**

Each congruent isosceles triangle has two angles of $66°$.

2 **Use trigonometry**

So the angle at the point between the two polygons is

$$180° - 66° - 66° = 48°$$

3 **The two regular polygons have**

The two regular polygons have equal interior angles. Around the point:

$$48^\circ + I + I = 360^\circ$$

$$2I = 312^\circ$$

$$I = 156^\circ$$

---

4 **The exterior angle is**

The exterior angle is

$$180^\circ - 156^\circ = 24^\circ$$

$$n = \frac{360}{24} = 15$$

✓ **FINAL ANSWER**

$n = 15$.

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Question 13

3 marks | Volume & Surface Area

13

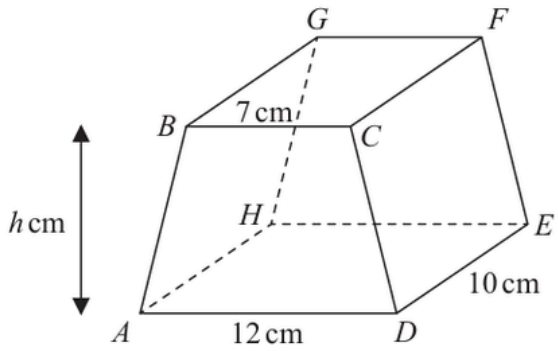


Diagram **NOT** accurately drawn

The diagram shows a prism $ABCDEFGH$ in which $ABCD$ is a trapezium with BC parallel to AD and $CDEF$ is a rectangle.

$BC = 7\,\text{cm}$ $AD = 12\,\text{cm}$ $DE = 10\,\text{cm}$

The height of trapezium $ABCD$ is $h\,\text{cm}$
The volume of the prism is $608\,\text{cm}^3$

Work out the value of h.

$h =$ .....

(Total for Question 13 is 3 marks)

WORKED SOLUTION

Question 13 · 4MA1/2HR January 2020

1 Area of the trapezium cross

Area of the trapezium cross-section:

$$\frac{1}{2}(7+12)h = 9.5h$$

2 Volume of the prism

Volume of the prism:

$$9.5h \times 10 = 608$$

$$95h = 608$$

$$h = 6.4$$

FINAL ANSWER

$h = 6.4\ \text{cm}$.

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Question 14

3 marks | Statistics Toolkit

14 Max kept a record of the marks he scored in each of the 11 spelling tests he took one term.

Here are his marks.

18 5 7 12 11 18 15 16 17 13 14

Find the interquartile range of the marks.

(Total for Question 14 is 3 marks)

WORKED SOLUTION

Question 14 · 4MA1/2HR January 2020

1 **Put the marks in order**

Put the marks in order:

$$5,\ 7,\ 11,\ 12,\ 13,\ 14,\ 15,\ 16,\ 17,\ 18,\ 18$$

$$Q_1 = 11, \qquad Q_3 = 17$$

$$\text{IQR} = 17 - 11 = 6$$

FINAL ANSWER

6.

Question 15

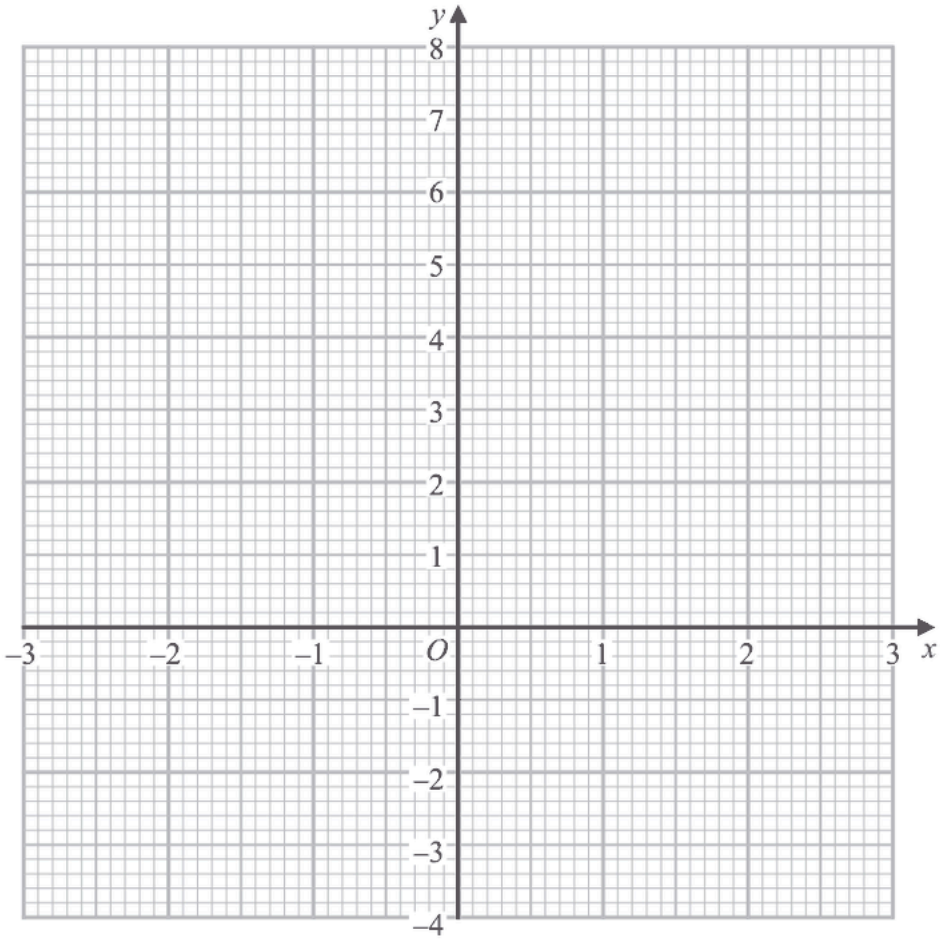
4 marks | Graphs of Functions

15 (a) Complete the table of values for $y = x^2 - \frac{x}{2} - 3$

x	–3	–2	–1	0	1	2	3
y	7.5				–2.5		4.5

(2)

(b) On the grid, draw the graph of $y = x^2 - \frac{x}{2} - 3$ for values of x from –3 to 3



(2)

(Total for Question 15 is 4 marks)

1 Solve quadratic equation

$$y = x^2 - \frac{x}{2} - 3$$

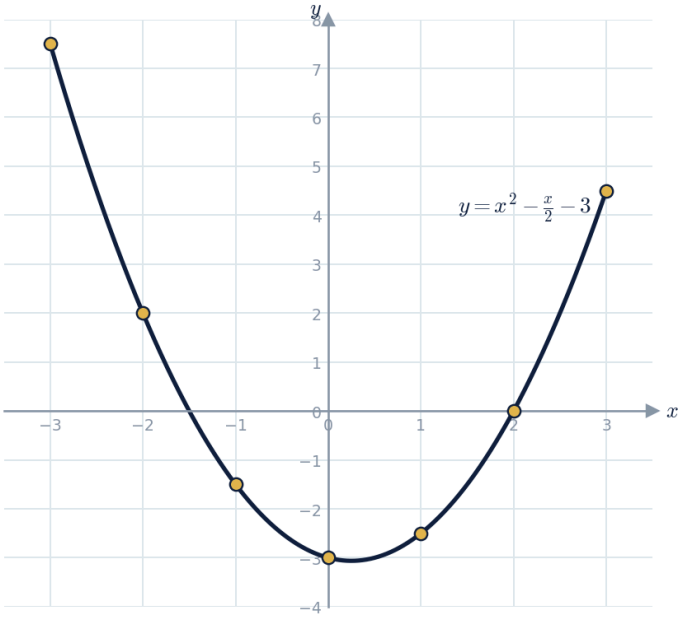
2 The completed table is

The completed table is:

x	-3	-2	-1	0	1	2	3
y	7.5	2	-1.5	-3	-2.5	0	4.5

3 Plot these points and join

Plot these points and join them with a smooth U-shaped curve.



FINAL ANSWER

Missing values are $2, -1.5, -3, 0$.

Question 16

8 marks | Tree Diagrams & Conditional Probability

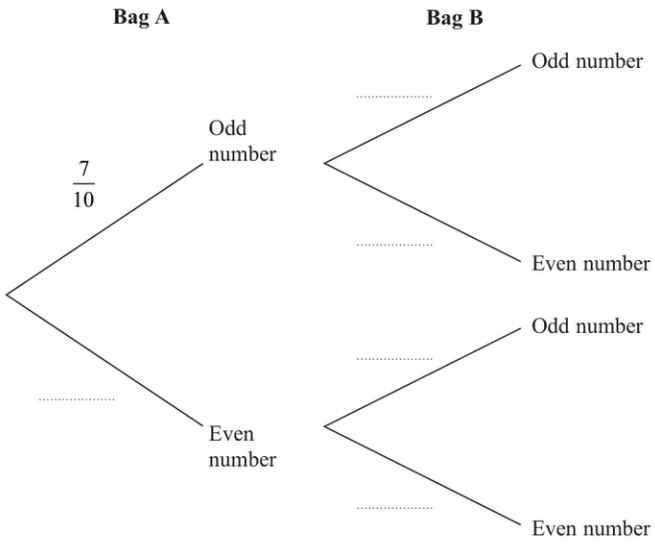
16 Cody has two bags of counters, bag **A** and bag **B**.

Each of the counters has either an odd number or an even number written on it.

There are 10 counters in bag **A** and 7 of these counters have an **odd** number written on them.
There are 12 counters in bag **B** and 7 of these counters have an **odd** number written on them.

Cody is going to take at random a counter from bag **A** and a counter from bag **B**.

(a) Complete the probability tree diagram.



(2)

(b) Calculate the probability that the total of the numbers on the two counters will be an odd number.

.....

(3)

Harriet also has a bag of counters.
Each of her counters also has either an odd number or an even number written on it.

Harriet is going to take at random a counter from her bag of counters.

The probability that the number on each of Cody's two counters **and** the number on Harriet's counter will all be even is $\frac{3}{100}$

(c) Find the least number of counters that Harriet has in her bag.
Show your working clearly.

.....

(3)

(Total for Question 16 is 8 marks)

1 **(a) Complete the tree**

Bag A: odd $\frac{7}{10}$, even $\frac{3}{10}$.

Bag B (on both branches): odd $\frac{7}{12}$, even $\frac{5}{12}$.

2 **(b) Odd total = odd + even or even + odd**

$$\frac{7}{10} \times \frac{5}{12} + \frac{3}{10} \times \frac{7}{12} = \frac{35}{120} + \frac{21}{120} = \frac{56}{120} = \frac{7}{15}$$

3 **(c) Probability Harriet takes an even counter**

Cody takes two even counters with probability $\frac{3}{10} \times \frac{5}{12} = \frac{1}{8}$.

$$\frac{1}{8} \times p = \frac{3}{100} \Rightarrow p = \frac{24}{100} = \frac{6}{25}$$

4 **(c) Least number of counters**

$\frac{6}{25}$ is in its simplest form, so the number of counters must be a multiple of 25. The least number is 25.

✓ **FINAL ANSWER**

(a) $\frac{3}{10}$; $\frac{7}{12}$, $\frac{5}{12}$ on both branches (b) $\frac{7}{15}$ (c) 25

Question 17

6 marks | Set Notation & Venn Diagrams

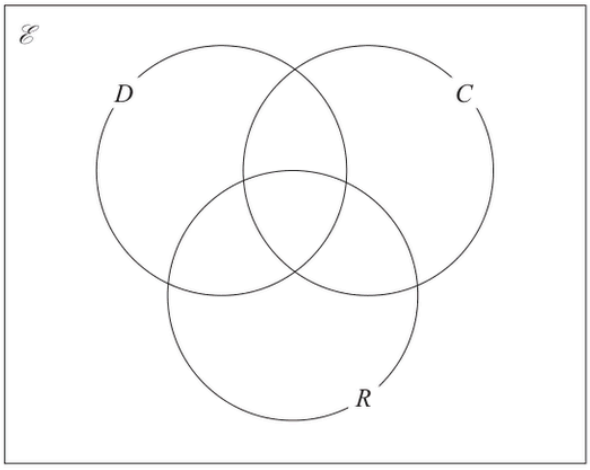
17 Some students in a school were asked the following question.

"Do you have a dog (D), a cat (C) or a rabbit (R)?"

Of these students

28 have a dog
18 have a cat
20 have a rabbit
8 have both a cat and a rabbit
9 have both a dog and a rabbit
x have both a dog and a cat
6 have a dog, a cat and a rabbit
5 have not got a dog or a cat or a rabbit

(a) Using this information, complete the Venn diagram to show the number of students in each appropriate subset.
Give the numbers in terms of x where necessary.



(3)

Given that a total of 50 students answered the question,

(b) work out the value of x.

$x =$ .....

(2)

(c) Find $n(C' \cap D')$

.....

(1)

(Total for Question 17 is 6 marks)

1 **Let the sets dogs**

Let the sets be D for dogs, C for cats, and R for rabbits.

2 **The number in all three**

The number in all three sets is 6.

$$D \cap R \text{ only} = 9 - 6 = 3$$

$$C \cap R \text{ only} = 8 - 6 = 2$$

$$D \cap C \text{ only} = x - 6$$

3 **Calculate value**

So the remaining one-set regions are

$$D \text{ only} = 25 - x, \quad C \text{ only} = 16 - x, \quad R \text{ only} = 9$$

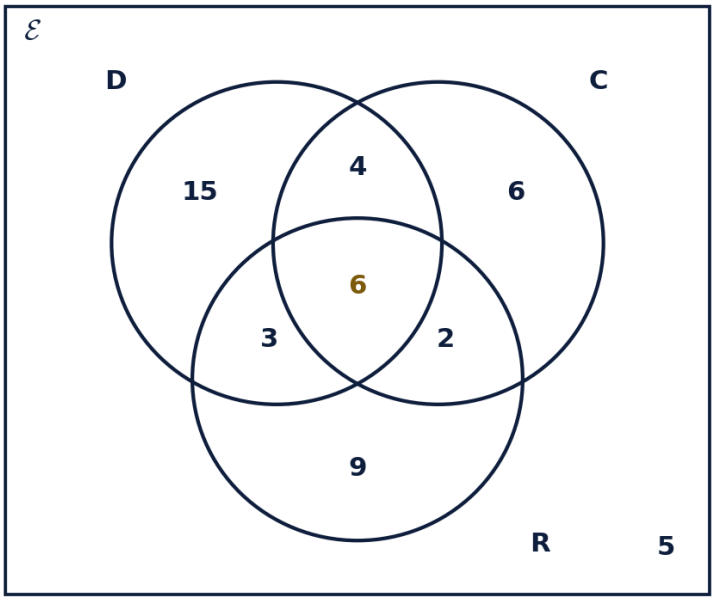
4 **There are students outside all**

There are 5 students outside all three sets. The total number of students is 50, so

$$(25 - x) + (16 - x) + 9 + (x - 6) + 3 + 2 + 6 + 5 = 50$$

$$60 - x = 50$$

$$x = 10$$



5 **Calculate value**

For $C' \cap D'$, we need students who are not in C and not in D. These are the rabbit-only students and the students outside all three sets.

$$9 + 5 = 14$$

✓ **FINAL ANSWER**

$x = 10$ and $n(C' \cap D') = 14$

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Question 18

2 marks | Circle Theorems

18

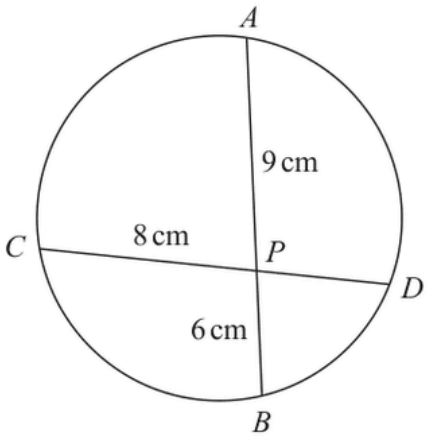


Diagram **NOT** accurately drawn

APB and *CPD* are chords of a circle.

$AP = 9\,\text{cm}$ $PB = 6\,\text{cm}$ $CP = 8\,\text{cm}$

Calculate the length of *PD*.

..... cm

(Total for Question 18 is 2 marks)

WORKED SOLUTION

Question 18 · 4MA1/2HR January 2020

1 Use the intersecting chords theorem

Use the intersecting chords theorem:

$$AP \times PB = CP \times PD$$

2 Substitute the values

Substitute the values:

$$9 \times 6 = 8 \times PD$$

$$54 = 8PD$$

$$PD = 6.75$$

FINAL ANSWER

$6.75\ \text{cm}$.

4MA1/2HR JANUARY 2020

Question 19

6 marks | Inequalities (Solving & Graphing)

19 (a) Solve $\frac{4-3x}{5} - \frac{3x-5}{2} = -3$

Show clear algebraic working.

$x =$ .....

(3)

(b) Solve the inequality $5y^2 - 17y \leqslant 40$

.....

(3)

(Total for Question 19 is 6 marks)

WORKED SOLUTION

Question 19 · 4MA1/2HR January 2020

1 **(a)**

(a)

$$\frac{4-3x}{5} - \frac{3x-5}{2} = -3$$

2 **Multiply by 10**

Multiply by 10:

$$2(4-3x) - 5(3x-5) = -30$$

$$8 - 6x - 15x + 25 = -30$$

$$33 - 21x = -30$$

$$-21x = -63$$

$$x = 3$$

3 **(b)**

(b)

$$5y^2 - 17y \leq 40$$

$$5y^2 - 17y - 40 \leq 0$$

---

4 **Factorise**

Factorise:

$$(5y + 8)(y - 5) \leq 0$$

---

5 **Solve quadratic equation**

The roots are $-\frac{8}{5}$ and 5. The quadratic is ≤ 0 between the roots.

✓ **FINAL ANSWER**

$x = 3$, and $-\frac{8}{5} \leq y \leq 5$.

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Question 20

4 marks | Area & Volume of Similar Shapes

20 The diagram shows two similar vases, **A** and **B**.

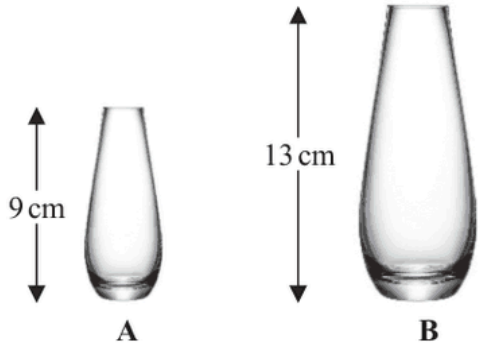


Diagram **NOT** accurately drawn

The height of vase **A** is 9 cm and the height of vase **B** is 13 cm.

Given that

surface area of vase **A** + surface area of vase **B** = 1800 cm^2

calculate the surface area of vase **A**.

..... cm^2

(Total for Question 20 is 4 marks)

WORKED SOLUTION

Question 20 · 4MA1/2HR January 2020

1 **The two vases are similar**

The two vases are similar.

2 **Their heights are in the**

Their heights are in the ratio

$$9 : 13$$

3 **Split the ratio**

So their surface areas are in the ratio

$$9^2 : 13^2 = 81 : 169$$

4 **The total surface area is**

The total surface area is $1800\ \text{cm}^2$, so there are

$$81 + 169 = 250$$

5 **parts altogether**

parts altogether.

6 **Surface area of vase A**

Surface area of vase A:

$$1800 \times \frac{81}{250} = 583.2$$

✓ **FINAL ANSWER**

$\boxed{583.2\ \text{cm}^2}$

4MA1/2HR JANUARY 2020

Question 21

7 marks | Algebraic Roots & Indices

21 (a) Simplify fully $\dfrac{10x^2 + 23x + 12}{4x^2 - 9}$

.....

(3)

$2^{2y} \times 2^{3y+2} = \dfrac{8^{5y}}{4^n}$

(b) Find an expression for n in terms of y.
Show clear algebraic working and simplify your expression.

.....

(4)

(Total for Question 21 is 7 marks)

WORKED SOLUTION

Question 21 · 4MA1/2HR January 2020

1 Factorise expression

For part (a), factorise the numerator and denominator:

$$10x^2 + 23x + 12 = (5x + 4)(2x + 3)$$

$$4x^2 - 9 = (2x - 3)(2x + 3)$$

2 Use index laws

So

$$\frac{10x^2 + 23x + 12}{4x^2 - 9} = \frac{(5x + 4)(2x + 3)}{(2x - 3)(2x + 3)}$$

$$= \frac{5x + 4}{2x - 3}$$

3 Use index laws

For part (b),

$$2^{2y} \times 2^{3y+2} = 2^{5y+2}$$

4 Use index laws

Also,

$$\frac{8^{5y}}{4^n} = \frac{(2^3)^{5y}}{(2^2)^n} = 2^{15y-2n}$$

5 Equate powers of 2

Equate powers of 2:

$$5y + 2 = 15y - 2n$$

$$2n = 10y - 2$$

$$n = 5y - 1$$

FINAL ANSWER

(a) $\frac{5x+4}{2x-3}$, (b) $n = 5y - 1$.

4MA1/2HR JANUARY 2020

Question 22

4 marks | Sequences

22 The first term of an arithmetic series S is -6
The sum of the first 30 terms of S is 2865

Find the 9th term of S.

(Total for Question 22 is 4 marks)

WORKED SOLUTION

Question 22 · 4MA1/2HR January 2020

1 Find the nth term

First term:

$$a = -6$$

$$S_{30} = 2865$$

2 Use

Use

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$2865 = \frac{30}{2}(2(-6) + 29d)$$

$$2865 = 15(-12 + 29d)$$

$$191 = -12 + 29d$$

$$29d = 203$$

$$d = 7$$

3 **The 9th term is**

The 9th term is

$$u_9 = a + 8d$$

$$u_9 = -6 + 8(7) = 50$$

✓ **FINAL ANSWER**

50

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Question 23

3 marks | Completing the Square

23 Express $7 - 12x - 2x^2$ in the form $a + b(x + c)^2$ where a, b and c are integers.

(Total for Question 23 is 3 marks)

WORKED SOLUTION

Question 23 · 4MA1/2HR January 2020

1 Solve inequality

$$7 - 12x - 2x^2 = -2x^2 - 12x + 7$$

$$= -2(x^2 + 6x) + 7$$

$$= -2\left((x + 3)^2 - 9\right) + 7$$

$$= -2(x + 3)^2 + 18 + 7$$

$$= 25 - 2(x + 3)^2$$

FINAL ANSWER

$25 - 2(x + 3)^2$

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Question 24

5 marks | Linear Graphs

24 $\mathbf{L}_1$ and $\mathbf{L}_2$ are two straight lines.
The origin of the coordinate axes is O.

$\mathbf{L}_1$ has equation $5x + 10y = 8$
$\mathbf{L}_2$ is perpendicular to $\mathbf{L}_1$ and passes through the point with coordinates (8, 6)

$\mathbf{L}_2$ crosses the x-axis at the point A.
$\mathbf{L}_2$ intersects the straight line with equation $x = -3$ at the point B.

Find the area of triangle AOB.
Show your working clearly.

(Total for Question 24 is 5 marks)

WORKED SOLUTION

Question 24 · 4MA1/2HR January 2020

1 Evaluate fraction

For L_1,

$$5x + 10y = 8$$

$$10y = -5x + 8$$

$$y = -\frac{1}{2}x + \frac{4}{5}$$

2 Find the gradient

So the gradient of L_1 is $-\frac{1}{2}$.

3 Find the gradient

Since L_2 is perpendicular to L_1, the gradient of L_2 is 2.

4 Calculate value

Using the point $(8, 6)$:

$$y - 6 = 2(x - 8)$$

$$y = 2x - 10$$

5 **Calculate value**

At A, the line crosses the x-axis, so $y = 0$:

$$0 = 2x - 10$$

$$x = 5$$

---

6 **Calculate value**

So $A = (5, 0)$.

---

7 **Calculate value**

At B, $x = -3$:

$$y = 2(-3) - 10 = -16$$

---

8 **Calculate value**

So $B = (-3, -16)$.

---

9 **Triangle has base and perpendicular**

Triangle AOB has base $OA = 5$ and perpendicular height 16.

$$\text{Area} = \frac{1}{2} \times 5 \times 16 = 40$$

✓ **FINAL ANSWER**

40

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Question 25

3 marks | Algebraic Proof

25 N is a multiple of 5

$A = N + 1$
$B = N - 1$

Prove, using algebra, that $A^2 - B^2$ is always a multiple of 20

(Total for Question 25 is 3 marks)

WORKED SOLUTION

Question 25 · 4MA1/2HR January 2020

1 Use trigonometry

Since N is a multiple of 5, let

$$N = 5k$$

2 where is an integer

where k is an integer.

3 Given

Given

$$A = N + 1, \qquad B = N - 1$$

4 Use trigonometry

So

$$A^2 - B^2 = (N+1)^2 - (N-1)^2$$

5 Expand

Expand:

$$(N+1)^2 = N^2 + 2N + 1$$

$$(N-1)^2 = N^2 - 2N + 1$$

6 **Use trigonometry**

Therefore

$$A^2 - B^2 = (N^2 + 2N + 1) - (N^2 - 2N + 1)$$

$$= 4N$$

---

7 **Substitute**

Substitute $N = 5k$:

$$4N = 4(5k) = 20k$$

---

8 **Use trigonometry**

Since k is an integer, $20k$ is a multiple of 20.

✓ **FINAL ANSWER**

$A^2 - B^2$ is always a multiple of 20.

4MA1/2HR JANUARY 2020

Question 26

5 marks | Vectors

26 The diagram shows trapezium $OACB$.

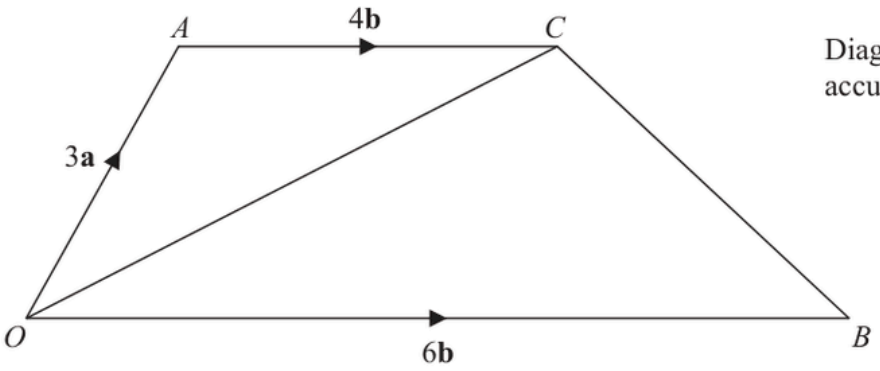


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = \mathbf{3a}$ $\overrightarrow{OB} = \mathbf{6b}$ $\overrightarrow{AC} = \mathbf{4b}$

N is the point on OC such that ANB is a straight line.

Find $\overrightarrow{ON}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$.

(Total for Question 26 is 5 marks)

WORKED SOLUTION

Question 26 · 4MA1/2HR January 2020

1 **Read the graph**

$$\vec{OA} = \mathbf{3a}, \quad \vec{OB} = \mathbf{6b}, \quad \vec{AC} = \mathbf{4b}$$

2 **Read the graph**

So

$$\vec{OC} = \mathbf{3a} + \mathbf{4b}$$

3 **Read the graph**

Point N lies on OC, so

$$\vec{ON} = t(\mathbf{3a} + \mathbf{4b})$$

4 **Read the graph**

Point N also lies on AB, so

$$\vec{ON} = 3\mathbf{a} + s(6\mathbf{b} - 3\mathbf{a})$$

$$\vec{ON} = (3 - 3s)\mathbf{a} + 6s\mathbf{b}$$

---

5 **Equate coefficients**

Equate coefficients:

$$3t = 3 - 3s$$

$$4t = 6s$$

---

6 **Read the graph**

So

$$t = 1 - s$$

---

7 **Read the graph**

and

$$t = \frac{3}{2}s$$

$$\frac{3}{2}s = 1 - s$$

$$5s = 2$$

$$s = \frac{2}{5}$$

---

8 **Read the graph**

Then

$$t = \frac{3}{5}$$

---

9 **Read the graph**

Therefore

$$\vec{ON} = \frac{3}{5}(3\mathbf{a} + 4\mathbf{b})$$

$$\vec{ON} = \frac{9}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}$$

✓ **FINAL ANSWER**

$$\vec{ON} = \frac{9}{5}\mathbf{a} + \frac{12}{5}\mathbf{b}$$