Elite Mathematics
Pearson Edexcel International GCSE

IGCSE LINEAR · 4MA1

4MA1/1HR January 2020 Worked Solutions

January 2020 paper rebuilt with the worked solution after each question.

23	2020–2026	Single-paper solutions
paper questions	52 past papers	checked by Dr Eslam



Dr Eslam Ahmed
Assistant Lecturer, Faculty of Engineering, Cairo University
eliteigcse.com · free for every student

PAST PAPER

4MA1/1HR January 2020

January 2020

23	100	2 h
questions	marks	exam time

Each question is followed by its worked solution and final answer.

4MA1/1HR JANUARY 2020

Question 1

3 marks | Ratio Toolkit

1 Brendon, Asha and Julie share some money in the ratios 3 : 2 : 6
The **total** amount of money that Asha and Julie receive is \$36

Work out the amount of money that Brendon receives.

\$.....

(Total for Question 1 is 3 marks)

WORKED SOLUTION

Question 1 · 4MA1/1HR January 2020

1 The ratio is

The ratio is

$$\text{Brendon} : \text{Asha} : \text{Julie} = 3 : 2 : 6$$

2 Asha and Julie together have

Asha and Julie together have

$$2 + 6 = 8 \text{ parts}$$

3 These 8 parts are worth

These 8 parts are worth $\$36$, so one part is

$$36 \div 8 = 4.50$$

4 Brendon has 3 parts

Brendon has 3 parts:

$$3 \times 4.50 = 13.50$$

FINAL ANSWER

Brendon receives $\$13.50$.

4MA1/1HR JANUARY 2020

Question 2

3 marks | Fractions

2 Show that $3\frac{1}{5} \times 2\frac{5}{8} = 8\frac{2}{5}$

(Total for Question 2 is 3 marks)

WORKED SOLUTION

Question 2 · 4MA1/1HR January 2020

1 Simplify fraction

Convert the mixed numbers to improper fractions:

$$3\frac{1}{5} = \frac{16}{5}$$

$$2\frac{5}{8} = \frac{21}{8}$$

2 Simplify fraction

Now multiply:

$$\frac{16}{5} \times \frac{21}{8} = \frac{2 \times 21}{5}$$

$$= \frac{42}{5} = 8\frac{2}{5}$$

FINAL ANSWER

$8\frac{2}{5}$

4MA1/1HR JANUARY 2020

Question 3

8 marks | Rearranging Formulae

3 (a) Make a the subject of $d = g + 2ac$

.....

(2)

(b) Factorise fully $9ef - 12f$

.....

(2)

(c) Expand and simplify $(x + 2)(x - 5)$

.....

(2)

(d) Simplify fully $\dfrac{n^4 \times n^7}{n^5}$

.....

(2)

(Total for Question 3 is 8 marks)

WORKED SOLUTION

Question 3 · 4MA1/1HR January 2020

1 **(a)**

(a)

$$d = g + 2ac$$

$$d - g = 2ac$$

$$a = \frac{d - g}{2c}$$

2 **(b)**

(b)

$$9ef - 12f = 3f(3e - 4)$$

3 **(c)**

(c)

$$(x+2)(x-5) = x^2 - 3x - 10$$

---

4 **(d)**

(d)

$$\frac{n^4 \times n^7}{n^5} = n^{4+7-5} = n^6$$

✓ **FINAL ANSWER**

$a = \frac{d-g}{2c}$, $3f(3e-4)$, $x^2 - 3x - 10$, n^6

4MA1/1HR JANUARY 2020

Question 4

3 marks | Set Notation & Venn Diagrams

4 $B = \{b, l, u, e\}$

$G = \{g, r, e, y\}$

$W = \{w, h, i, t, e\}$

(a) List all the members of the set

(i) $B \cup G$

(ii) $W \cap G'$

.....

(2)

Serena writes down the statement $B \cap G \cap W = \varnothing$

(b) Is Serena's statement correct?
You must give a reason for your answer.

.....

.....

(1)

(Total for Question 4 is 3 marks)

WORKED SOLUTION

Question 4 · 4MA1/1HR January 2020

1 Calculate value

$$B = \{b, l, u, e\}, \quad G = \{g, r, e, y\}, \quad W = \{w, h, i, t, e\}$$

2 means all the letters that

$B \cup G$ means all the letters that are in B or in G.

$$B \cup G = \{b, l, u, e, g, r, y\}$$

3 Calculate statistic

$W \cap G'$ means letters that are in W, but not in G. Since e is in G, it is not included.

$$W \cap G' = \{w, h, i, t\}$$

4 **Serena is not correct because**

Serena is not correct because e is in all three sets.

$$B \cap G \cap W = \{e\}$$

✓ **FINAL ANSWER**

(a)(i) $\{b, l, u, e, g, r, y\}$, (a)(ii) $\{w, h, i, t\}$. Serena is incorrect.

4MA1/1HR JANUARY 2020

Question 5

3 marks | Perimeter & Area

5 The diagram shows Yuen's garden.

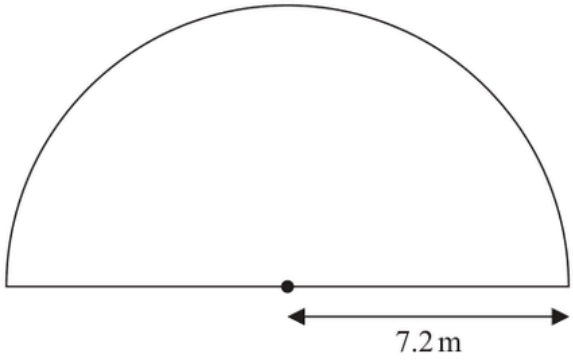


Diagram **NOT** accurately drawn

The garden is in the shape of a semicircle of radius 7.2 m.
Yuen is going to cover his garden with grass seed.

Yuen has 12 boxes of grass seed.
Each box of grass seed contains enough seed to cover 6 m² of the garden.

Has Yuen enough grass seed for his garden?
Show your working clearly.

(Total for Question 5 is 3 marks)

WORKED SOLUTION

Question 5 · 4MA1/1HR January 2020

1 The garden is a semicircle

The garden is a semicircle of radius $7.2\ \text{m}$.

$$\text{area} = \frac{1}{2}\pi r^2$$

$$\text{area} = \frac{1}{2}\pi(7.2)^2 = 81.43\ldots\ \text{m}^2$$

2 Calculate area

Yuen has 12 boxes, and each box covers $6\ \text{m}^2$:

$$12 \times 6 = 72\ \text{m}^2$$

3 Calculate area

Since

$$72 < 81.43\ldots$$

4

Yuen does not have enough

Yuen does not have enough grass seed.

✓ **FINAL ANSWER**

No, he does not have enough grass seed.

4MA1/1HR JANUARY 2020

Question 6

3 marks | Quadratic Equations

6 Solve $x^2 - 5x - 36 = 0$
Show clear algebraic working.

(Total for Question 6 is 3 marks)

WORKED SOLUTION

Question 6 · 4MA1/1HR January 2020

1 Solve quadratic equation

$$x^2 - 5x - 36 = 0$$

2 Factorise

Factorise:

$$(x - 9)(x + 4) = 0$$

3 Solve quadratic equation

So

$$x = 9 \quad \text{or} \quad x = -4$$

FINAL ANSWER

$x = 9$ or $x = -4$.

4MA1/1HR JANUARY 2020

Question 7

3 marks | Reverse Percentages

7 In a sale, the normal price of a hat is reduced by 15%
The sale price of the hat is 20.40 euros.

Work out the normal price of the hat.

..... euros

(Total for Question 7 is 3 marks)

WORKED SOLUTION

Question 7 · 4MA1/1HR January 2020

1 The hat is reduced by

The hat is reduced by 15%, so the sale price is

$$100\% - 15\% = 85\%$$

2 of the normal price

of the normal price.

$$0.85 \times \text{normal price} = 20.40$$

$$\text{normal price} = 20.40 \div 0.85 = 24$$

FINAL ANSWER

24 euros.

4MA1/1HR JANUARY 2020

Question 8

3 marks | Statistics Toolkit

8 5 children are playing on a trampoline.
The mean weight of the 5 children is 28 kg.

2 of the children get off the trampoline.
The mean weight of these 2 children is 26.5 kg.

Work out the mean weight of the 3 children who remain on the trampoline.

..... kg

(Total for Question 8 is 3 marks)

WORKED SOLUTION

Question 8 · 4MA1/1HR January 2020

1 Total weight of 5 children

Total weight of 5 children:

$$5 \times 28 = 140 \text{ kg}$$

2 Total weight of the 2

Total weight of the 2 children who get off:

$$2 \times 26.5 = 53 \text{ kg}$$

3 Total weight of the 3

Total weight of the 3 children left:

$$140 - 53 = 87 \text{ kg}$$

4 Mean weight

Mean weight:

$$\frac{87}{3} = 29$$

FINAL ANSWER

29 kg.

4MA1/1HR JANUARY 2020

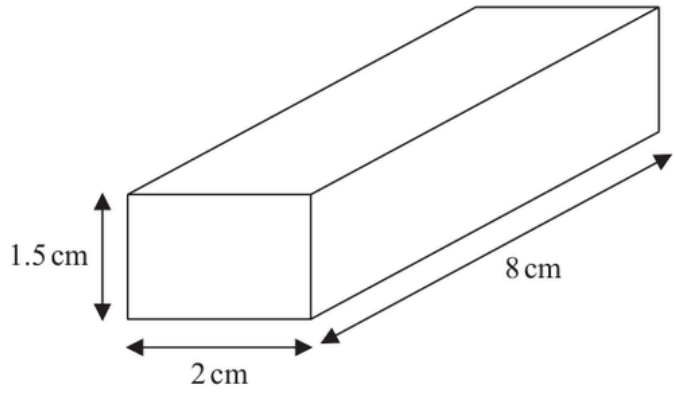
Question 9

5 marks | Standard & Compound Units

9 Pablo made a solid gold statue.

He melted down some gold blocks and used the gold to make the statue.
Each block of gold was a cuboid, as shown below.

Diagram **NOT** accurately drawn



The mass of the statue is 5.73 kg.
The density of gold is 19.32 g/cm^3

Work out the least number of gold blocks Pablo melted down in order to make the statue.
Show your working clearly.

(Total for Question 9 is 5 marks)

WORKED SOLUTION

Question 9 · 4MA1/1HR January 2020

1 Convert the mass to grams

Convert the mass to grams:

$$5.73 \text{ kg} = 5730 \text{ g}$$

2 Calculate volume

Using

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{volume of gold} = \frac{5730}{19.32} = 296.58\ldots \text{ cm}^3$$

3 **Volume of one cuboid block**

Volume of one cuboid block is

$$1.5 \times 2 \times 8 = 24 \text{ cm}^3$$

4 **Number of blocks needed is**

Number of blocks needed is

$$\frac{296.58\ldots}{24} = 12.35\ldots$$

5 **Pablo must have a whole**

Pablo must have a whole number of blocks, so he needs at least 13 blocks.

✓ **FINAL ANSWER**

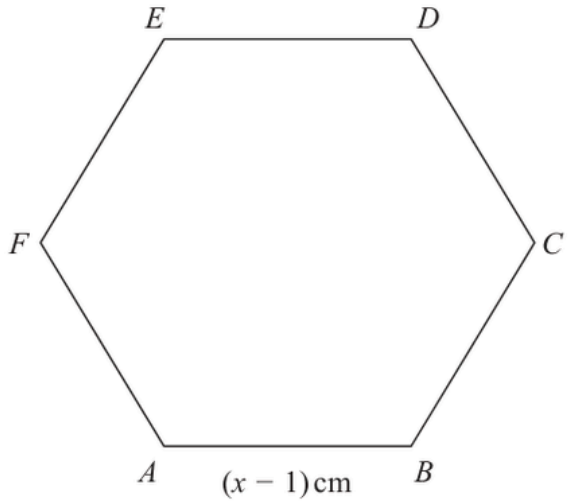
$\mathbf{13}$ blocks.

4MA1/1HR JANUARY 2020

Question 10

5 marks | Perimeter & Area

10 The diagram shows a regular hexagon, $ABCDEF$, and an isosceles triangle, GHJ.



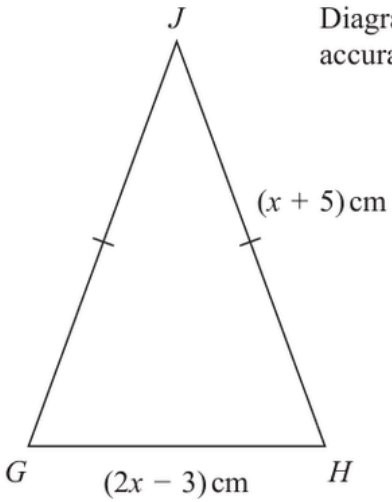


Diagram **NOT** accurately drawn

The perimeter of the hexagon is equal to the perimeter of the triangle.

Find the length of each side of the hexagon.
Show clear algebraic working.

..... cm

(Total for Question 10 is 5 marks)

WORKED SOLUTION

Question 10 · 4MA1/1HR January 2020

1 **The regular hexagon has side**

The regular hexagon has side length $(x - 1)$ cm, so its perimeter is

$$6(x - 1)$$

2 **The isosceles triangle has perimeter**

The isosceles triangle has perimeter

$$(x + 5) + (x + 5) + (2x - 3) = 4x + 7$$

3 The perimeters are equal

The perimeters are equal:

$$6(x-1) = 4x + 7$$

$$6x - 6 = 4x + 7$$

$$2x = 13$$

$$x = 6.5$$

---

4 Each side of the hexagon

Each side of the hexagon is

$$x - 1 = 6.5 - 1 = 5.5$$

FINAL ANSWER

$5.5\ \text{cm}$.

4MA1/1HR JANUARY 2020

Question 11

4 marks | Rounding, Estimation & Bounds

11 The weight of a cat is 4.3 kg correct to 2 significant figures.

(a) Write down the upper bound of the weight of the cat.

..... kg

(1)

(b) Write down the lower bound of the weight of the cat.

..... kg

(1)

$G = e - f$

e = 17 correct to the nearest integer
f = 9.4 correct to one decimal place

(c) Work out the upper bound for the value of G.

.....

(2)

(Total for Question 11 is 4 marks)

WORKED SOLUTION

Question 11 · 4MA1/1HR January 2020

1 **Estimate the value**

The weight 4.3 kg correct to 2 significant figures has bounds

$$4.25 \leq w < 4.35$$

2 **Find upper bound**

So the upper bound is 4.35 kg and the lower bound is 4.25 kg.

3 **Estimate the value**

For

$$G = e - f$$

4 **the upper bound uses the**

the upper bound uses the upper bound of e and the lower bound of f.

$$e < 17.5, \quad f \geq 9.35$$

$$G_{\text{upper}} = 17.5 - 9.35 = 8.15$$

✓ **FINAL ANSWER**

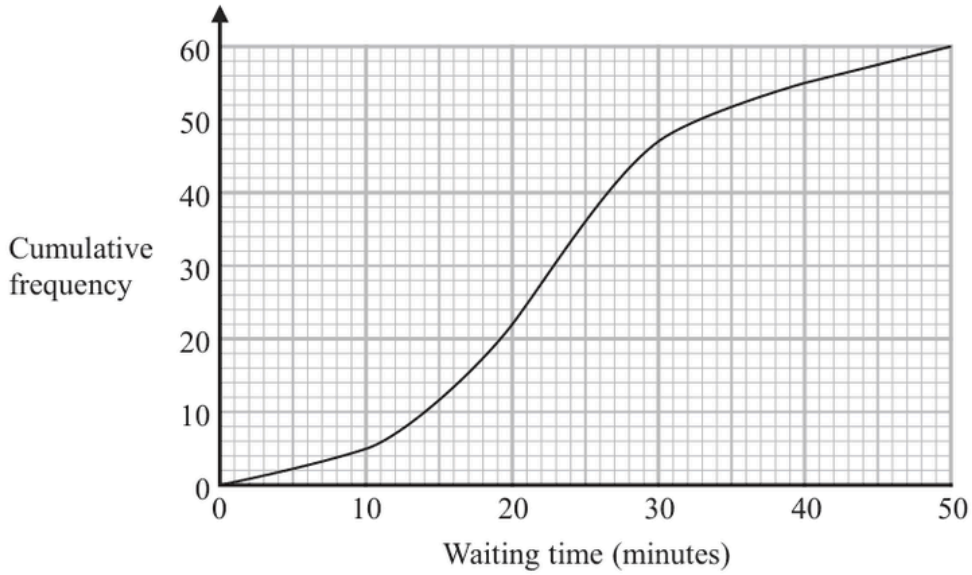
(a) 4.35 kg, (b) 4.25 kg, (c) 8.15

4MA1/1HR JANUARY 2020

Question 12

5 marks | Cumulative Frequency Diagrams

12 The cumulative frequency graph gives information about the waiting times, in minutes, of people with appointments at Hospital A.



(a) Use the graph to find an estimate of the median waiting time at Hospital A.

..... minutes

(1)

(b) Use the graph to find an estimate of the interquartile range of the waiting times at Hospital A.

..... minutes

(2)

At a different hospital, Hospital B, the median waiting time is 28 minutes and the interquartile range of the waiting times is 19 minutes.

(c) Compare the waiting times at Hospital A with the waiting times at Hospital B.

.....

.....

(2)

(Total for Question 12 is 5 marks)

WORKED SOLUTION

Question 12 · 4MA1/1HR January 2020

1 **There are people**

There are **60** people.

2 **The median is at cumulative**

The median is at cumulative frequency 30.

3 **Use cumulative frequency**

From the graph,

$$\text{median} \approx 22.5 \text{ minutes}$$

4 **Use cumulative frequency**

For the interquartile range:

$$Q_1 \text{ is at } 15, \qquad Q_3 \text{ is at } 45$$

5 **Use cumulative frequency**

From the graph,

$$Q_1 \approx 16.5, \qquad Q_3 \approx 28.5$$

$$\text{IQR} \approx 28.5 - 16.5 = 12$$

6 **Hospital B has median minutes**

Hospital B has median 28 minutes and IQR 19 minutes.

7 **Use cumulative frequency**

So Hospital A has a shorter median waiting time and a smaller spread.

✓ **FINAL ANSWER**

median about 22.5 minutes, IQR about 12 minutes. Hospital A is quicker and more consistent.

4MA1/1HR JANUARY 2020

Question 13

4 marks | Algebraic Proof

13 (a) Use algebra to show that $0.5\dot{7}\dot{2} = \frac{63}{110}$

(2)

Given that y is a prime number,

(b) express $\frac{3}{2-\sqrt{y}}$ in the form $\frac{a+b\sqrt{y}}{c-y}$ where a, b and c are integers.

.....

(2)

(Total for Question 13 is 4 marks)

WORKED SOLUTION

Question 13 · 4MA1/1HR January 2020

1 **(a) Let**

(a) Let

$$x = 0.5727272\ldots$$

2 **Calculate value**

Then

$$1000x = 572.727272\ldots$$

3 **Calculate value**

and

$$10x = 5.727272\ldots$$

4 **Subtract**

Subtract:

$$990x = 567$$

$$x = \frac{567}{990} = \frac{63}{110}$$

5 **Evaluate fraction**

So

$$0.5\dot{7}\dot{2} = \frac{63}{110}$$

---

6 **(b)**

(b)

$$\frac{3}{2-\sqrt{y}} \times \frac{2+\sqrt{y}}{2+\sqrt{y}} = \frac{3(2+\sqrt{y})}{4-y}$$

$$= \frac{6+3\sqrt{y}}{4-y}$$

✓ **FINAL ANSWER**

(a) shown, (b) $\frac{6+3\sqrt{y}}{4-y}$

4MA1/1HR JANUARY 2020

Question 14

4 marks | Circle Theorems

14

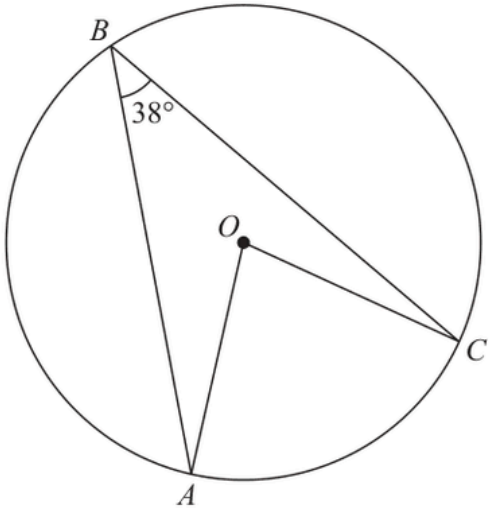


Diagram **NOT** accurately drawn

A, B and C are points on a circle, centre O.
Angle $ABC = 38°$

Work out the size of angle OAC.
Give a reason for each stage of your working.

.....°

(Total for Question 14 is 4 marks)

WORKED SOLUTION

Question 14 · 4MA1/1HR January 2020

1 Angle and central angle stand

Angle ABC and central angle AOC stand on the same arc AC.

2 The angle at the centre

The angle at the centre is twice the angle at the circumference:

$$\angle AOC = 2 \times 38° = 76°$$

3 Use trigonometry

Since $OA = OC$, triangle AOC is isosceles.

$$\angle OAC = \angle OCA = \frac{180° - 76°}{2} = 52°$$

FINAL ANSWER

$52°$.

4MA1/1HR JANUARY 2020

Question 15

4 marks | Right-Angled Triangles - Pythagoras & Trigonometry

15 The diagram shows two right-angled triangles, DEF and EFG.

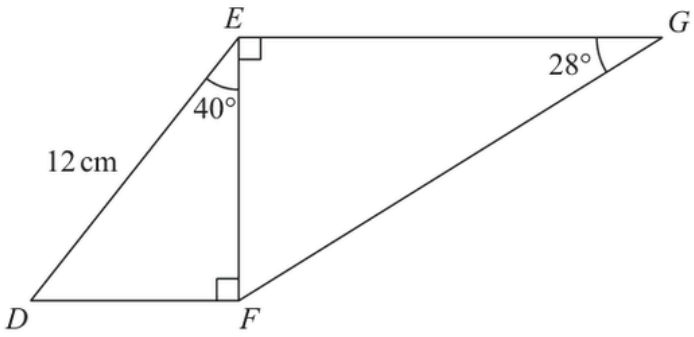


Diagram **NOT** accurately drawn

Work out the length of EG.
Give your answer correct to 3 significant figures.

..... cm

(Total for Question 15 is 4 marks)

WORKED SOLUTION

Question 15 · 4MA1/1HR January 2020

1 **Use trigonometry**

In triangle DEF,

$$EF = 12\cos 40^\circ = 9.192\ldots$$

2 **Use trigonometry**

In triangle EFG,

$$\sin 28^\circ = \frac{EF}{EG}$$

$$EG = \frac{9.192\ldots}{\sin 28^\circ} = 19.580\ldots$$

✓ **FINAL ANSWER**

19.6 cm.

4MA1/1HR JANUARY 2020

Question 16

4 marks | Tree Diagrams & Conditional Probability

16 Steffi is going to play one game of tennis and one game of chess.

The probability that she will win the game of tennis is 0.6
The probability that she will win **both** games is 0.42

Work out the probability that she will **not** win either game.

(Total for Question 16 is 4 marks)

WORKED SOLUTION

Question 16 · 4MA1/1HR January 2020

1 Find the probability of winning chess

Treat the two games as independent:

$$0.6 \times P(\text{win chess}) = 0.42 \Rightarrow P(\text{win chess}) = 0.42 \div 0.6 = 0.7$$

2 Probabilities of losing

$$P(\text{not win tennis}) = 1 - 0.6 = 0.4, \qquad P(\text{not win chess}) = 1 - 0.7 = 0.3$$

3 Win neither game

$$0.4 \times 0.3 = 0.12$$

FINAL ANSWER

0.12

4MA1/1HR JANUARY 2020

Question 17

4 marks | Functions

17 The function f is such that $f(x) = (x - 4)^2$ for all values of x.

(a) Find f(1)

.....

(1)

(b) State the range of the function f.

.....

(1)

The function g is such that $g(x) = \dfrac{4}{x+3}$ $\quad x \neq -3$

(c) Work out fg(2)

.....

(2)

(Total for Question 17 is 4 marks)

WORKED SOLUTION

Question 17 · 4MA1/1HR January 2020

1 **Calculate value**

$$f(x) = (x - 4)^2$$

2 **(a)**

(a)

$$f(1) = (1 - 4)^2 = 9$$

3 **(b) A square is always**

(b) A square is always greater than or equal to zero, so the range is

$$f(x) \geq 0$$

4 **(c)**

(c)

$$g(x) = \frac{4}{x+3}$$

$$g(2) = \frac{4}{2+3} = \frac{4}{5}$$

FINAL ANSWER

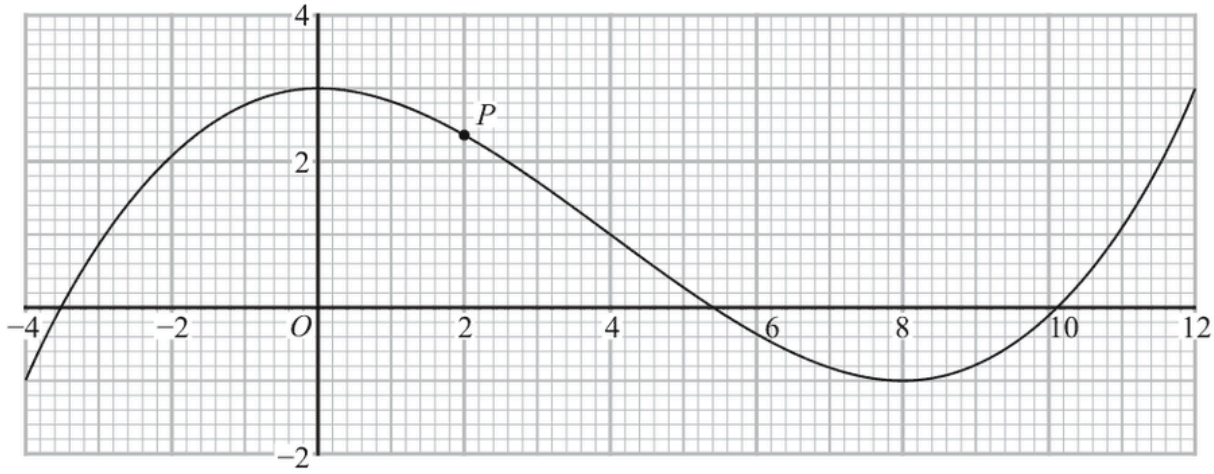
$f(1) = 9$, range $f(x) \geq 0$, and $g(2) = \frac{4}{5}$.

4MA1/1HR JANUARY 2020

Question 18

7 marks | Graphs of Functions

18 The diagram shows the graph of $y = f(x)$ for $-4 \leqslant x \leqslant 12$



The point P on the curve has x coordinate 2

(a) (i) Use the graph to find an estimate for the gradient of the curve at P.

.....

(3)

(ii) Hence find an equation of the tangent to the curve at P.
Give your answer in the form $y = mx + c$

.....

(2)

The equation $f(x) = k$ has exactly two different solutions for $-4 \leqslant x \leqslant 12$

(b) Use the graph to find the two possible values of k.

..... , ...

(2)

(Total for Question 18 is 7 marks)

WORKED SOLUTION

Question 18 · 4MA1/1HR January 2020

1 **Draw a tangent to the**

Draw a tangent to the curve at P.

2 **A reasonable tangent gives an**

A reasonable tangent gives an estimated gradient of about

$$m \approx -0.6$$

3 **The point is approximately**

The point P is approximately

$$(2, 2.4)$$

4 **Find the gradient**

Using $y = mx + c$,

$$2.4 \approx -0.6(2) + c$$

$$c \approx 3.6$$

5 **Find the gradient**

So an approximate tangent equation is

$$y \approx -0.6x + 3.6$$

6 **The equation has exactly two**

The equation $f(x) = k$ has exactly two different solutions when the horizontal line touches one turning level.

7 **Find the gradient**

From the graph, these levels are approximately

$$k = 3 \quad \text{and} \quad k = -1$$

✓ **FINAL ANSWER**

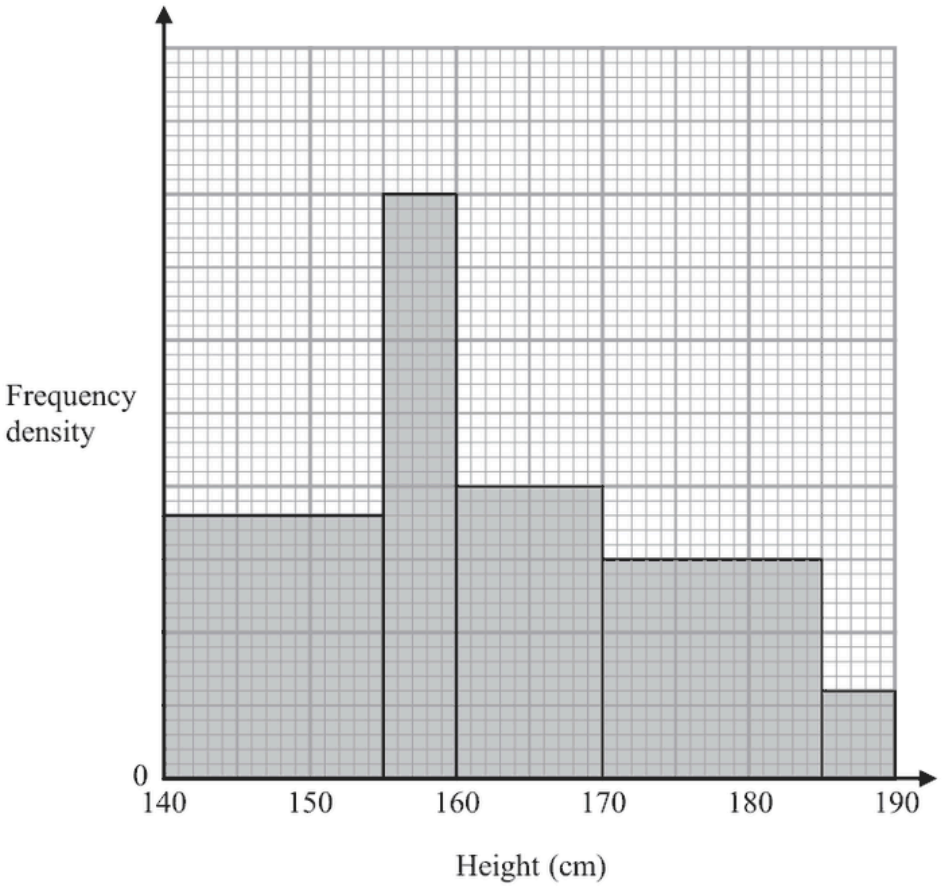
Gradient ≈ -0.6, tangent $y \approx -0.6x + 3.6$, and $k \approx 3, -1$.

4MA1/1HR JANUARY 2020

Question 19

4 marks | Histograms

19



The histogram gives information about the heights of all the Year 11 students at a school.

There are 160 students in Year 11 with a height between 155 cm and 170 cm.

Work out the total number of students in Year 11 at the school.

(Total for Question 19 is 4 marks)

WORKED SOLUTION

Question 19 · 4MA1/1HR January 2020

1 Use the relative histogram heights

Use the relative histogram heights.

2 Calculate area

For $155 < h \leq 170$, the relative area is

$$5(40) + 10(20) = 400$$

3 Calculate area

This represents 160 students, so each relative area unit represents

$$\frac{160}{400} = 0.4$$

4 Total relative area

Total relative area:

$$15(18) + 5(40) + 10(20) + 15(15) + 5(6) = 925$$

5 Estimated total

Estimated total:

$$925 \times 0.4 = 370$$

FINAL ANSWER

370 students.

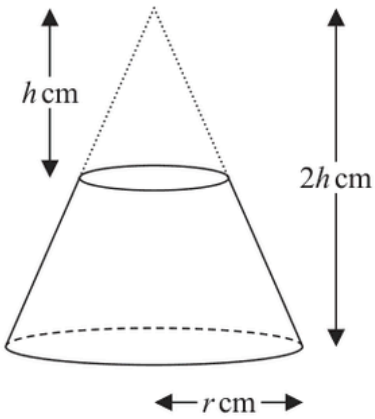
4MA1/1HR JANUARY 2020

Question 20

5 marks | Area & Volume of Similar Shapes

20 The diagram shows a frustum of a cone and a sphere.

The frustum is made by removing a small cone from a large cone.
The cones are similar.



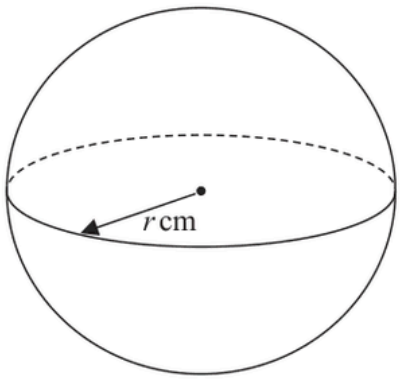


Diagram **NOT** accurately drawn

The height of the small cone is h cm.
The height of the large cone is $2h$ cm.
The radius of the base of the large cone is r cm.

The radius of the sphere is r cm.

Given that the volume of the frustum is equal to the volume of the sphere,

find an expression for r in terms of h.
Give your expression in its simplest form.

$r =$ .....

(Total for Question 20 is 5 marks)

WORKED SOLUTION

Question 20 · 4MA1/1HR January 2020

1 **The two cones are similar**

The two cones are similar.

2 **The large cone has height**

The large cone has height $2h$ and base radius r.

3 **The small cone has height**

The small cone has height h, so its linear scale factor compared with the large cone is

$$\frac{h}{2h} = \frac{1}{2}$$

4 **Define cone dimensions**

So the small cone has radius

$$\frac{r}{2}$$

5 **Volume of the large cone**

Volume of the large cone:

$$\frac{1}{3}\pi r^2(2h) = \frac{2}{3}\pi r^2 h$$

6 **Volume of the small cone**

Volume of the small cone:

$$\frac{1}{3}\pi\left(\frac{r}{2}\right)^2 h = \frac{1}{12}\pi r^2 h$$

7 **Calculate volume**

So the volume of the frustum is

$$\frac{2}{3}\pi r^2 h - \frac{1}{12}\pi r^2 h = \frac{7}{12}\pi r^2 h$$

8 **Calculate volume**

The sphere has radius r, so its volume is

$$\frac{4}{3}\pi r^3$$

9 **The two volumes are equal**

The two volumes are equal:

$$\frac{7}{12}\pi r^2 h = \frac{4}{3}\pi r^3$$

10 **Divide by**

Divide by πr^2:

$$\frac{7}{12}h = \frac{4}{3}r$$

$$7h = 16r$$

$$r = \frac{7h}{16}$$

✓ **FINAL ANSWER**

$$r = \frac{7h}{16}$$

4MA1/1HR JANUARY 2020

Question 21

6 marks | Sine & Cosine Rules

21 The diagram shows the prism $ABCDEF$ with cross section triangle ABC.

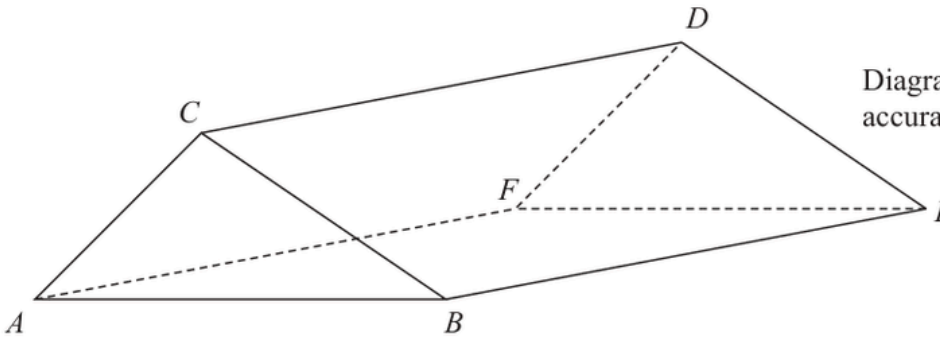


Diagram **NOT** accurately drawn

Angle $BEC = 40°$ and angle ACB is obtuse.
$AC = 6\,\text{cm}$ and $CE = 13\,\text{cm}$

The area of triangle ABC is $22\,\text{cm}^2$

Calculate the length of AB.
Give your answer correct to one decimal place.

..... cm

(Total for Question 21 is 6 marks)

WORKED SOLUTION

Question 21 · 4MA1/1HR January 2020

1 Use trigonometry

In triangle BEC, $CE = 13$ and $\angle BEC = 40°$.

2 Find the gradient

Since this is a prism, BE is perpendicular to the cross section, so triangle BEC is right-angled at B.

$$BC = 13 \sin 40°$$

$$BC = 8.356238\ldots$$

3 The area of triangle is

The area of triangle ABC is $22\ \text{cm}^2$.

4 **Use trigonometry**

Using

$$\text{Area} = \frac{1}{2}ab\sin C$$

$$22 = \frac{1}{2}(AC)(BC)\sin(\angle ACB)$$

$$22 = \frac{1}{2}(6)(8.356238\ldots)\sin(\angle ACB)$$

$$\sin(\angle ACB) = 0.877587\ldots$$

---

5 **Use trigonometry**

The angle ACB is obtuse, so

$$\angle ACB = 180° - \sin^{-1}(0.877587\ldots)$$

$$\angle ACB = 118.647\ldots°$$

---

6 **Use the cosine rule**

Use the cosine rule:

$$AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos(\angle ACB)$$

$$AB^2 = 6^2 + (8.356238\ldots)^2 - 2(6)(8.356238\ldots)\cos(118.647\ldots°)$$

$$AB = 12.4056\ldots$$

---

7 **Correct to one decimal place**

Correct to one decimal place:

$$12.4$$

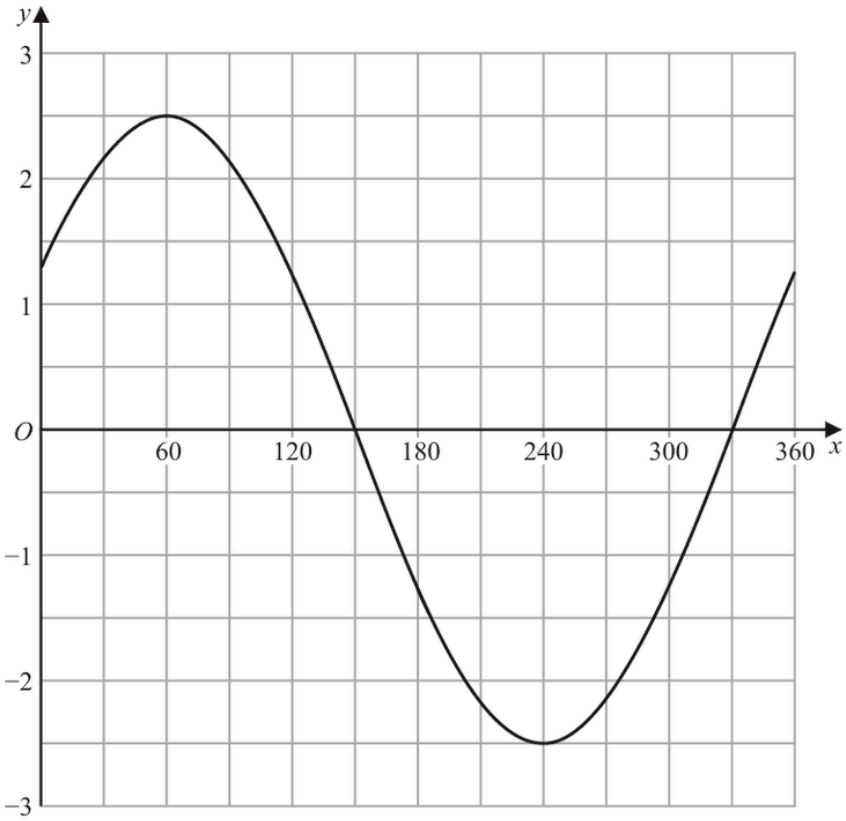
✓ **FINAL ANSWER**

$\boxed{12.4\text{ cm}}$

Question 22

4 marks | Transformations of Graphs

22 The graph of $y = a\cos(x + b)°$ for $0 \leqslant x \leqslant 360$ is drawn on the grid.



(a) Find the value of a and the value of b.

$a =$ .....

$b =$ .....

(2)

Another curve C has equation $y = f(x)$
The coordinates of the minimum point of C are (4, 5)

(b) Write down the coordinates of the minimum point of the curve with equation

(i) $y = f(2x)$

(..... , ...)

(ii) $y = f(x) - 7$

(..... , ...)

(2)

(Total for Question 22 is 4 marks)

1 **Read the graph**

For part (a), the graph has maximum 2.5 and minimum -2.5, so

$$a = 2.5$$

2 **Use trigonometry**

The maximum occurs at $x = 60°$. For $y = a\cos(x + b)°$, a maximum occurs when

$$x + b = 360$$

3 **Calculate value**

So

$$60 + b = 360$$

$$b = 300$$

4 **Calculate value**

For part (b), the minimum point of $y = f(x)$ is $(4, 5)$.

5 **Calculate value**

For

$$y = f(2x)$$

6 **the coordinate is divided by**

the x-coordinate is divided by 2:

$$(4, 5) \to (2, 5)$$

7 **Calculate value**

For

$$y = f(x) - 7$$

8 **the coordinate decreases by 7**

the y-coordinate decreases by 7:

$$(4, 5) \to (4, -2)$$

✓ **FINAL ANSWER**

$a = 2.5,\ b = 300;\ (2, 5)$ and $(4, -2)$

4MA1/1HR JANUARY 2020

Question 23

6 marks | Differentiation & Turning Points

23 A particle moves along a straight line.
The fixed point O lies on this line.
The displacement of the particle from O at time t seconds, $t \geqslant 0$, is s metres where

$$s = t^3 + 4t^2 - 5t + 7$$

At time T seconds the velocity of P is V m/s where $V \geqslant -5$

Find an expression for T in terms of V.

Give your expression in the form $\dfrac{-4 + \sqrt{k + mV}}{3}$ where k and m are integers to be found.

$T =$ .....

(Total for Question 23 is 6 marks)

WORKED SOLUTION

Question 23 · 4MA1/1HR January 2020

1 Calculate value

$$s = t^3 + 4t^2 - 5t + 7$$

2 Evaluate fraction

Velocity is the derivative of displacement:

$$v = \frac{ds}{dt} = 3t^2 + 8t - 5$$

3 Calculate value

At time T, the velocity is V, so

$$V = 3T^2 + 8T - 5$$

$$3T^2 + 8T - (V + 5) = 0$$

4 **Use the quadratic formula**

Use the quadratic formula:

$$T = \frac{-8 \pm \sqrt{8^2 - 4(3)(-(V+5))}}{2(3)}$$

$$T = \frac{-8 \pm \sqrt{64 + 12V + 60}}{6}$$

$$T = \frac{-8 \pm \sqrt{124 + 12V}}{6}$$

$$T = \frac{-4 \pm \sqrt{31 + 3V}}{3}$$

---

5 **Simplify surd**

Since $T \geq 0$, take the positive root:

$$T = \frac{-4 + \sqrt{31 + 3V}}{3}$$

✓ **FINAL ANSWER**

$$T = \frac{-4 + \sqrt{31 + 3V}}{3}$$