ELITE

IGCSE Mathematics A • Pearson Edexcel

12_May-Nov_2020 • Unit 1

1H + 2H source-order past paper

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L-to-M | CONVERTED MODULAR EDITION

Selected from Linear Higher papers; not an original Modular examination paper.

Contents

1H Q01 • Probability Diagrams - Venn & Tree Diagrams • 4 marks

1H Q02 • Probability Toolkit • 4 marks

1H Q03 • Linear Graphs (y = mx + c) • 3 marks

1H Q06 • Standard & Compound Units • 3 marks

1H Q07 • Factorising • 8 marks

1H Q11 • Area & Perimeter • 6 marks

1H Q15 • Probability Diagrams - Venn & Tree Diagrams • 5 marks

1H Q17 • Rounding, Estimation & Bounds • 4 marks

1H Q18 • Histograms • 5 marks

1H Q19 • Surds • 3 marks

1H Q21 • Algebraic Fractions • 5 marks

1H Q22 • Coordinate Geometry • 4 marks

2H Q03 • Angles in Polygons & Parallel Lines • 5 marks

2H Q06 • Linear Graphs (y = mx + c) • 2 marks

2H Q07 • Right-Angled Triangles - Pythagoras & Trigonometry • 3 marks

2H Q08 • Area & Perimeter • 4 marks

2H Q12 • Algebraic Roots & Indices • 3 marks

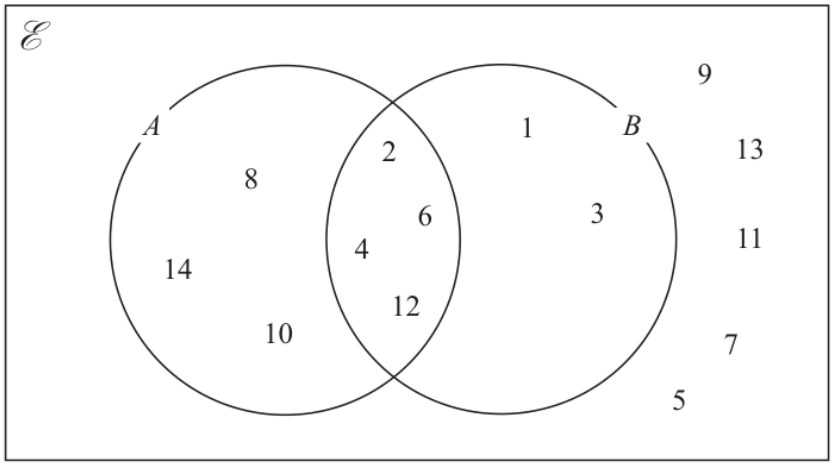
2H Q13 • Fractions, Decimals & Percentages • 2 marks

2H Q14 • Set Notation & Venn Diagrams • 6 marks

2H Q18 • 3D Pythagoras & Trigonometry • 5 marks

2H Q20 • Combined & Conditional Probability • 5 marks

1 The numbers from 1 to 14 are shown in the Venn diagram.



(a) List the members of the set $A \cap B$

.....

(1)

(b) List the members of the set B'

.....

(1)

A number is picked at random from the numbers in the Venn diagram.

(c) Find the probability that this number is in set A but is **not** in set B.

.....

(2)

(Total for Question 1 is 4 marks)

ANSWER / WORKED SOLUTION

1H • Family: Represent and read events with Venn diagrams • Variant: Read regions and probabilities from a Venn diagram • CHECKED LEGACY SOLUTION

Topic check: Probability Diagrams - Venn & Tree Diagrams. The tag is correct.

Solution

From the Venn diagram,

$A \cap B = \{2,4,6,12\}$

The values not in B are

$B' = \{5,7,8,9,10,11,13,14\}$

The values in A but not in B are

$\{8,10,14\}$

So

$P(A \cap B') = \frac{3}{14}$

Answers: $\{2,4,6,12\}$, $\{5,7,8,9,10,11,13,14\}$, $\frac{3}{14}$.

2 Toy cars are made in a factory.
The toy cars are made for 15 hours each day.
5 toy cars are made every 12 seconds.

For the toy cars made each day, the probability of a toy car being faulty is 0.002

Work out an estimate of the number of faulty toy cars that are made each day.

.....

(Total for Question 2 is 4 marks)

ANSWER / WORKED SOLUTION

1H • Family: Calculate theoretical single-event probability • Variant: Recover population composition from a single-event probability • CHECKED LEGACY SOLUTION

Topic check: Probability Toolkit. The tag is correct.

Solution

15 hours=15× 60× 60=54000 seconds

The machine makes 5 cars every 12 seconds, so it makes

(54000)/(12)× 5=22500

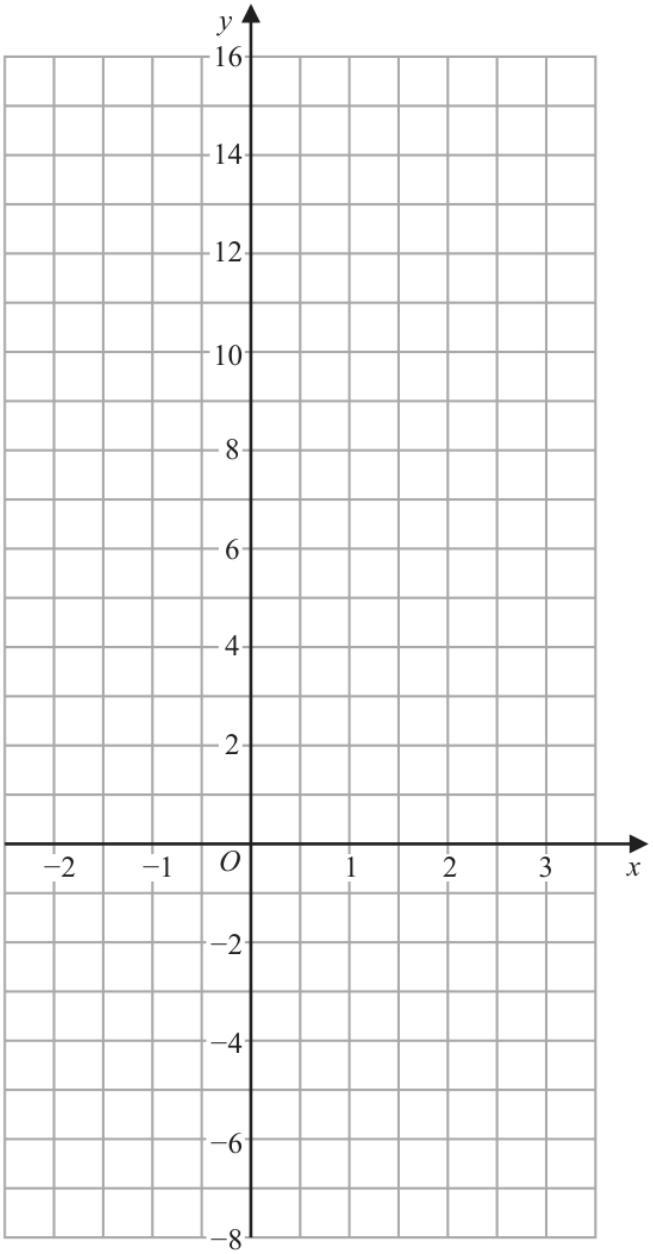
cars.

Expected faulty cars:

22500× 0.002=45

Answer: 45.

3 On the grid, draw the graph of $y = 7 - 4x$ for values of x from -2 to 3



(Total for Question 3 is 3 marks)

ANSWER / WORKED SOLUTION

1H • Family: Generate and plot a linear graph • Variant: Plot a line from generated values • CHECKED LEGACY SOLUTION

Topic check: Corrected to linear graphs. The function is a straight line.

Solution

$y=7-4x$

Useful points are:

$(-2,15),\quad (0,7),\quad (1,3),\quad (2,-1),\quad (3,-5)$

Plot these points and join them with a straight line.

Answer: The graph is the straight line $y=7-4x$.

6 A rocket travelled 100 km at an average speed of 28 440 km/h.

Work out how long it took the rocket to travel the 100 km.
Give your answer in seconds, correct to the nearest second.

..... seconds

(Total for Question 6 is 3 marks)

ANSWER / WORKED SOLUTION

1H • Family: Use speed, distance and time • Variant: Find average speed for one stage • CHECKED LEGACY SOLUTION

Topic check: Standard & Compound Units. The tag is correct.

Solution

Use

$time=\frac{distance}{speed}$

$time=(100)/(28440)$ hours

Convert hours to seconds:

$(100)/(28440)\times 3600=12.658\ldots$

To the nearest second,

$12.658\ldots \approx 13$

Answer: 13 seconds.

7 (a) Solve $5(4 - x) = 7 - 3x$
Show clear algebraic working.

$x =$ .....

(3)

(b) Factorise fully $16m^3g^3 + 24m^2g^5$

.....

(2)

(c) (i) Factorise $y^2 - 2y - 48$

.....

(2)

(ii) Hence, solve $y^2 - 2y - 48 = 0$

.....

(1)

(Total for Question 7 is 8 marks)

ANSWER / WORKED SOLUTION

1H • Family: Factorise a non-monic or special quadratic • Variant: Factorise as the first part of a factor-then-solve chain • CHECKED LEGACY SOLUTION

Topic check: Factorising. The tag is correct.

Solution

(a)

$5(4-x)=7-3x$

$20-5x=7-3x$

$13=2x$

$x=(13)/(2)$

(b)

$16m^3g^3+24m^2g^5=8m^2g^3(2m+3g^2)$

(c)(i)

$y^2-2y-48=(y-8)(y+6)$

(c)(ii) $y=8$ or $y=-6$.

Answer: $x=(13)/(2)$, $8m^2g^3(2m+3g^2)$, $(y-8)(y+6)$, $y=8$ or $y=-6$

11 The diagram shows trapezium $ABCD$ in which BC and AD are parallel.

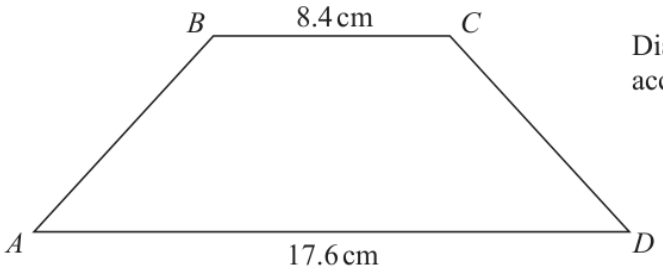


Diagram **NOT** accurately drawn

The trapezium has exactly one line of symmetry.

$BC = 8.4\,\text{cm}$
$AD = 17.6\,\text{cm}$

The trapezium has area $179.4\,\text{cm}^2$

Work out the size of angle ABC.
Give your answer correct to 1 decimal place.

.....°

(Total for Question 11 is 6 marks)

ANSWER / WORKED SOLUTION

1H • Family: Use rectangle and triangle area • Variant: Recover a missing dimension from area • CHECKED LEGACY SOLUTION

Topic check: Area & Perimeter. The tag is correct.

Solution

The average of the parallel sides is

$(8.4+17.6)/(2)=13$

So the perpendicular height is

$179.4\div 13=13.8$

The horizontal offset on each side is

$(17.6-8.4)/(2)=4.6$

The acute angle between AB and the horizontal is

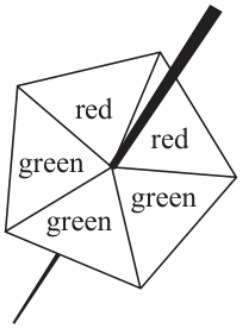
$\tan^{(-1)}((13.8)/(4.6))=71.565\ldots^{\circ}$

So the interior angle ABC is

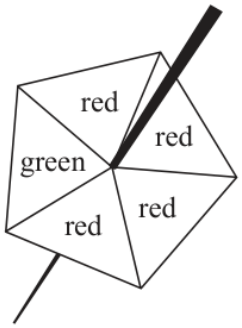
$180^{\circ}-71.565\ldots^{\circ}=108.434\ldots^{\circ}$

Answer: 108.4°.

15 Harry has two fair 5-sided spinners.



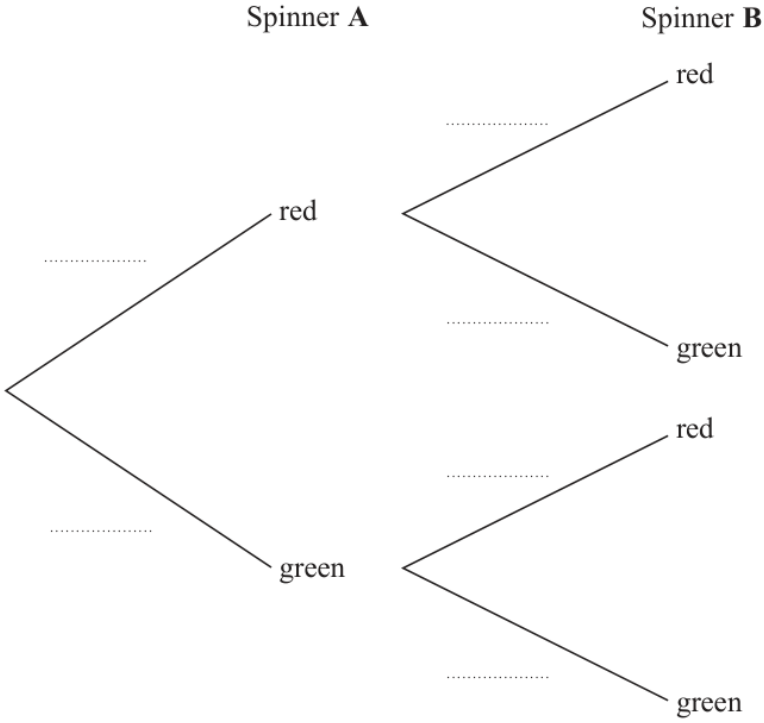
Spinner **A**



Spinner **B**

Harry is going to spin each spinner once.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that at least one of the spinners will land on green.

.....

(3)

(Total for Question 15 is 5 marks)

1H • Family: Represent and read events with Venn diagrams • Variant: Complete a two-set Venn diagram from totals • CHECKED LEGACY SOLUTION

Topic check: Probability Diagrams - Venn & Tree Diagrams. The tag is correct.

Solution

Spinner A has 2 red sectors and 3 green sectors:

$P(A_R)=\frac{2}{5},\qquad P(A_G)=\frac{3}{5}$

Spinner B has 4 red sectors and 1 green sector:

$P(B_R)=\frac{4}{5},\qquad P(B_G)=\frac{1}{5}$

At least one green is the complement of both red.

$P(\text{at least one green})=1-\frac{2}{5}\times\frac{4}{5}$

$=1-\frac{8}{25}=\frac{17}{25}$

Answer: $\frac{17}{25}$.

17 A metal block has a mass of 5 kg, correct to the nearest 50 grams.
The block has a volume of $(1.84 \times 10^{-3})\,m^3$, correct to 3 significant figures.

Work out the upper bound for the density of the block.
Give your answer in kg/m^3 correct to 1 decimal place.
Show your working clearly.

..... kg/m^3

(Total for Question 17 is 4 marks)

ANSWER / WORKED SOLUTION

1H • Family: Recover single-measurement bounds • Variant: Mixed stated accuracies • CHECKED LEGACY SOLUTION

Topic check: Corrected to Rounding, Estimation & Bounds. The question is mainly about using upper and lower bounds to find the upper bound for density.

Method

Density is

density=frac{mass}{volume}

To get the upper bound for density, use the upper bound for mass and the lower bound for volume.

The mass is 5 kg correct to the nearest 50 g.

50 g=0.05 kg

Half of this is 0.025 kg, so

upper bound for mass=5+0.025=5.025

The volume is 1.84× 10^(-3)=0.00184 m^3, correct to 3 significant figures.

The lower bound is

0.00184-0.000005=0.001835

So the upper bound for the density is

(5.025)/(0.001835)=2738.419ldots

Correct to 1 decimal place,

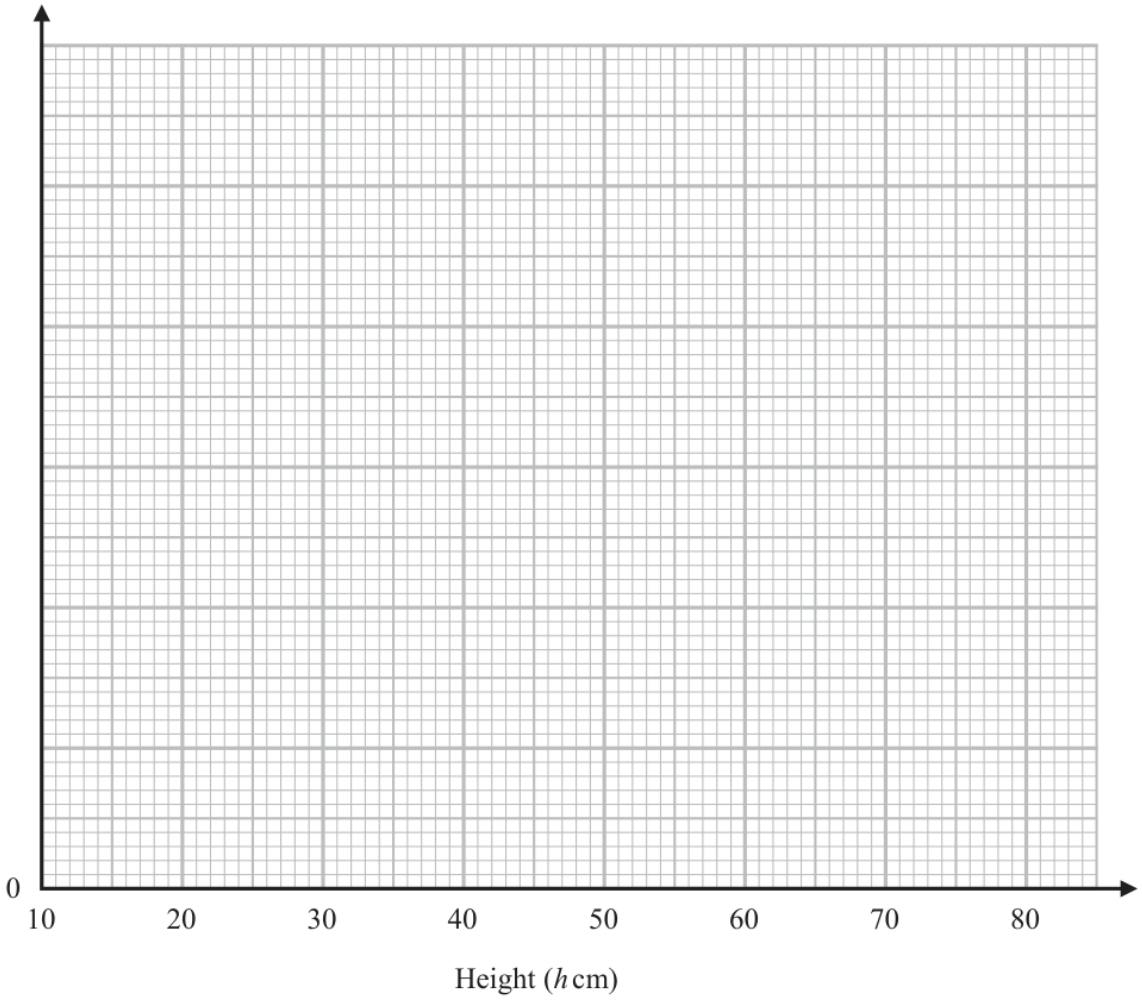
2738.4

Answer: 2738.4 kg/m^3

18 The table gives information about the heights, in centimetres, of some plants.

Height (h cm)	Frequency
$10 < h \leqslant 20$	35
$20 < h \leqslant 35$	45
$35 < h \leqslant 50$	75
$50 < h \leqslant 70$	40
$70 < h \leqslant 80$	8

(a) On the grid, draw a histogram for this information.



(3)

(b) Work out an estimate for the number of these plants with a height greater than 40 cm.

.....

(2)

(Total for Question 18 is 5 marks)

ANSWER / WORKED SOLUTION

1H • Family: Interpret histogram areas • Variant: Estimate a histogram proportion, fraction or percentage
• CHECKED LEGACY SOLUTION

Topic check: Histograms. The tag is correct.

Solution

The histogram densities are

$(35)/(10)=3.5,\quad (45)/(15)=3,\quad (75)/(15)=5,\quad (40)/(20)=2,\quad (8)/(10)=0.8$

For plants taller than 40 cm:

From 40<hle 50:

10(5)=50

From 50<hle 70:

40

From 70<hle 80:

8

50+40+8=98

Answer: draw bars with densities 3.5,3,5,2,0.8, and 98 plants.

19 Without using a calculator, rationalise the denominator of $\frac{6}{3-\sqrt{7}}$

Simplify your answer.
You must show each stage of your working.

.....

(Total for Question 19 is 3 marks)

ANSWER / WORKED SOLUTION

1H • Family: Rationalise a surd denominator • Variant: Single-term surd denominator • CHECKED LEGACY SOLUTION

Topic check: Surds. The tag is correct.

Method

(6)/(3-sqrt7)

Rationalise the denominator:

(6)/(3-sqrt7)× (3+sqrt7)/(3+sqrt7) =(6(3+sqrt7))/(9-7)

=(6(3+sqrt7))/(2)=3(3+sqrt7)

=9+3sqrt7

Answer: 9+3sqrt7

21 Express

$$\frac{1}{3x-2} \times \frac{9x^2-4}{3x^2-13x-10} - \frac{7}{x-1}$$

as a single fraction in its simplest form.

.....

(Total for Question 21 is 5 marks)

ANSWER / WORKED SOLUTION

1H • Family: Substitute or transform a rational expression • Variant: Substitute a variable relation into a fraction • CHECKED LEGACY SOLUTION

Topic check: Corrected from Sine/Cosine Rule to Algebraic Fractions.

Method

frac1{3x-2}×(9x^2-4)/(3x^2-13x-10) -frac7{x-1}

Factor:

9x^2-4=(3x-2)(3x+2)

3x^2-13x-10=(3x+2)(x-5)

So the first product becomes

frac1{3x-2}× ((3x-2)(3x+2))/((3x+2)(x-5))

Cancel common factors:

frac1{x-5}

Now subtract:

frac1{x-5}-frac7{x-1}

Use the common denominator (x-5)(x-1):

(x-1-7(x-5))/((x-5)(x-1))

=(x-1-7x+35)/((x-5)(x-1))

=(34-6x)/((x-5)(x-1))

Answer: displaystyle boxed{(34-6x)/((x-5)(x-1))}

22 $ABCD$ is a rhombus.

The diagonals, AC and BD, intersect at the point M.
The coordinates of M are $(6, -11)$

The points A and C both lie on the line with equation $2y + 7x = 20$

Find the exact coordinates of the point where the line through B and D intersects the y-axis.

(..... , ...)

(Total for Question 22 is 4 marks)

ANSWER / WORKED SOLUTION

1H • Family: Read and plot Cartesian coordinates • Variant: Read a plotted coordinate pair • CHECKED LEGACY SOLUTION

Topic check: Correct.

Method

In a rhombus, the diagonals are perpendicular and bisect each other.

The diagonal AC lies on

$2y+7x=20$

$2y=-7x+20$

$y=-\frac{7}{2}x+10$

So the gradient of AC is

$-\frac{7}{2}$

Therefore the gradient of BD is

$\frac{2}{7}$

The line BD passes through

$M=(6,-11)$

So

$y+11=\frac{2}{7}(x-6)$

To find where this line intersects the y-axis, put $x=0$:

$y+11=\frac{2}{7}(0-6)$

$y+11=-\frac{12}{7}$

$y=-11-\frac{12}{7}$

$y=-\frac{89}{7}$

Answer: $\displaystyle (0,-\frac{89}{7})$

3

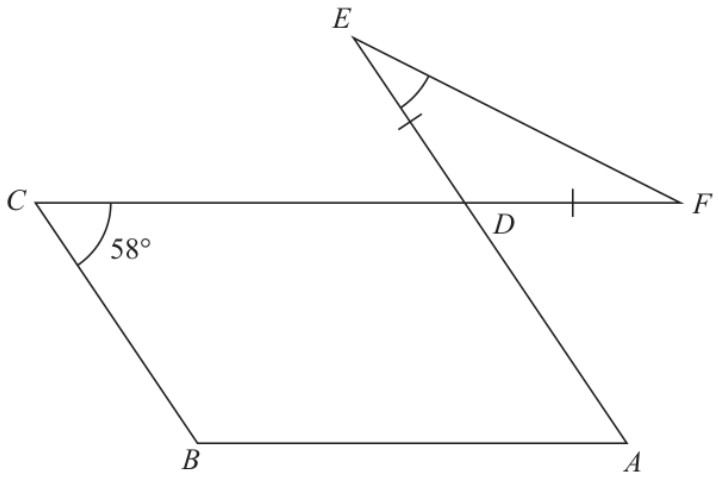


Diagram **NOT** accurately drawn

The diagram shows a parallelogram $ABCD$ and an isosceles triangle DEF in which $DE = DF$

CDF and ADE are straight lines.
Angle $BCD = 58°$

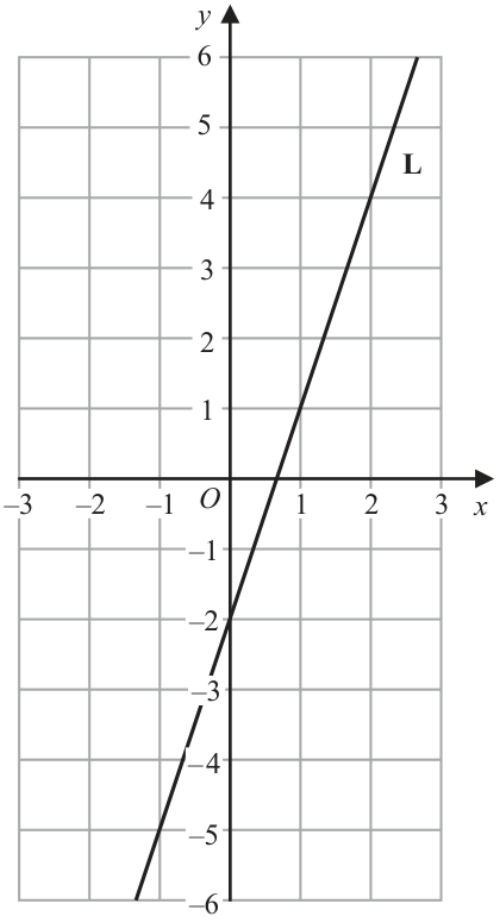
Work out the size of angle DEF.
Give a reason for each stage of your working.

.....°

(Total for Question 3 is 5 marks)

3	*ADC* = 180 – 58 (= 122) **or** *EDF* = 122 **or** *CDE* = 58 **or** *ADF* = 58			M1	may be seen marked on the diagram
	e.g. *DEF* = 58 ÷ 2 **or** *DEF* = (180 – 122) ÷ 2			M1	complete method to find angle *DEF*
		29		A1	
			5	B2	dep on M2 for fully correct reasons for their method (B1 dep on M1 for one correct reason stated and used) e.g. Allied angles, co-interior angles, Alternate angles, Corresponding angles, Vertically opposite angles are equal (or Vertically opposite angles are equal), Angles on a straight line add up to 180°(or angles on a straight line add to 180°), Sum of two angles in a triangle are equal to opposite exterior angle, Angles in a triangle add up to 180°(or Angles in a triangle add up to 180°), Base angles in an isosceles triangle Angles in a quadrilateral add up to 360. (accept "4-sided shape"or parallelogram) Opposite angles of a parallelogram are equal
					Total 5 marks

6 The line **L** is shown on the grid.



Find an equation for **L**.

.....

(Total for Question 6 is 2 marks)

6				M1	for $y = 3x + c$ oe **or** $y = mx - 2$ oe **or** $3x - 2$ **or** eg $L = 3x - 2$ **or** $y = 3(x \pm a)$
		$y = 3x - 2$	2	A1	oe eg $y - 4 = 3(x - 2)$ $y - 1 = 3(x - 1)$ $y - a = 3(x - b)$ where (a, b) is any coordinate on the line
					Total 2 marks

7 The diagram shows a right-angled triangle.

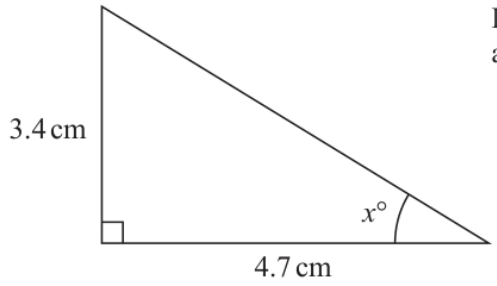


Diagram **NOT** accurately drawn

Calculate the value of x.
Give your answer correct to one decimal place.

$x =$ .....

(Total for Question 7 is 3 marks)

7	$\tan x = \frac{3.4}{4.7}$ oe eg $\cos x = \frac{4.7}{\sqrt{3.4^2 + 4.7^2}}$ oe			M1	or $\sin x = \frac{3.4 \sin 90}{\sqrt{3.4^2 + 4.7^2}}$ oe
	$(x =) \tan^{-1}\left(\frac{3.4}{4.7}\right)$ oe eg $(x =) \cos^{-1}\left(\frac{4.7}{\sqrt{3.4^2 + 4.7^2}}\right)$			M1	or $(x =) \sin^{-1}\left(\frac{3.4 \sin 90}{\sqrt{3.4^2 + 4.7^2}}\right)$ oe
		35.9	3	A1	accept 35.7 - 36.1
					Total 3 marks

8 The diagram shows an isosceles triangle.

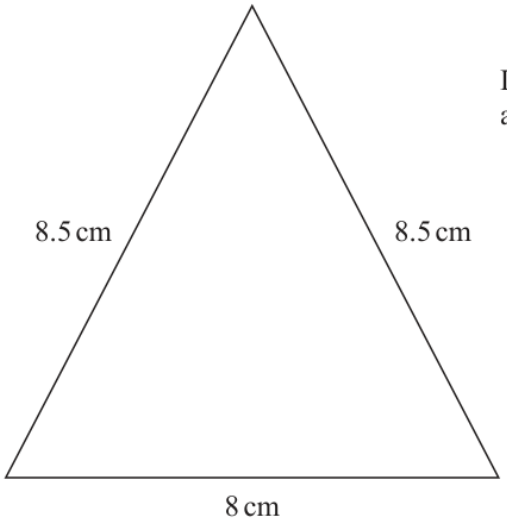


Diagram **NOT** accurately drawn

Work out the area of the triangle.

..... cm^2

(Total for Question 8 is 4 marks)

8	$8.5^2 - (8 \div 2)^2$ (= 56.25) **or** $\cos x = \frac{4}{8.5}$ oe			M1	**or** eg $\cos A = \frac{8^2 + 8.5^2 - 8.5^2}{2 \times 8 \times 8.5}$
	$\sqrt{"56.25"}$ (= 7.5) **or** $x = \cos^{-1}\left(\frac{4}{8.5}\right)$ (= 61.927...) oe			M1	**or** eg $(A =)\cos^{-1}\left(\frac{8^2 + 8.5^2 - 8.5^2}{2 \times 8 \times 8.5}\right)$ (61.927...) (other angle = 56.144...)
	8 × "7.5" ÷ 2 oe **or** 0.5 × 8 × 8.5 × sin "61.927..."			M1	**or** eg 0.5 × 8.5 × 8 × sin"61.927..." oe
		30	4	A1	
					Total 4 marks

12 Given that $\frac{3^x}{9^{3x}} = 81$

find the value of x.
Show clear algebraic working.

$x =$ .....

(Total for Question 12 is 3 marks)

12	$3^4 = \frac{3^x}{9^{3x}}$ **or** $81 = \frac{3^x}{(3^2)^{3x}}$	$9^2 = \frac{3^x}{9^{3x}}$ **or** $81 = \frac{(9^{0.5})^x}{9^{3x}}$			M1 replacing 81 with 3^4 **or** 9^{3x} with $(3^2)^{3x}$ (or 3^{6x}) **or** replacing 81 with 9^2 **or** 3^x with $(9^{0.5})^x$ (in an equation)
	eg $4 + 6x = x$ or $4 = x - 2(3x)$ oe	eg $2 = 0.5x - 3x$ oe			M1 a correct equation using powers
			-0.8	3	A1 oe, dep on at least M1
					Total 3 marks

13 Use algebra to show that $0.68\dot{1} = \frac{15}{22}$

(Total for Question 13 is 2 marks)

13	e.g. $x = 0.6\dot{8}\dot{1}$ and $100x = 68.\dot{1}\dot{8}$ **or** $10x = 6.\dot{8}\dot{1}$ and $1000x = 681.\dot{8}\dot{1}$			M1	e.g. two decimals that when subtracted give a finite decimal (must show understanding of recurring figures by 'dot' or at least 2 lots of 18 or 81 after the decimal point). Algebra required, use of any letter.
	$99x = 67.5$, $x = \frac{67.5}{99} = \frac{15}{22}$ **or** $990x = 675$, $x = \frac{675}{990} = \frac{15}{22}$ oe	show	2	A1	dep for completing the 'show that' arriving at given answer from correct working.
					Total 2 marks

14 $\mathscr{E} = \{\text{integers } x \text{ such that } 10 \leqslant x \leqslant 25\}$
$A = \{x : x < 18\}$
$B = \{x : 13 \leqslant x < 22\}$

(a) Write down n(A)

.....

(1)

(b) List the members of the set $(A \cup B)'$

.....

(2)

(c) List the members of the set $A' \cap B$

.....

(2)

$C \subset A$, $C \subset B$ and n(C) = 5

(d) List the members of the set C

.....

(1)

(Total for Question 14 is 6 marks)

14	a		8	1	B1	
	b	$A = \{10, 11, 12, 13, 14\ 15, 16, 17\}$ $B = \{13, 14, 15, 16, 17, 18, 19, 20, 21\}$ **or** $A \cup B = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21\}$			M1	may be seen in a Venn diagram (allow for example 10 – 17 for A and 13 – 21 for B or 10 – 21 for $A \cup B$) **or** for an answer with one missing element or one extra element
			22, 23, 24, 25	2	A1	
	c	$A' = \{18, 19, 20, 21, 22, 23, 24, 25\}$ $B = \{13, 14, 15, 16, 17, 18, 19, 20, 21\}$			M1	may be seen in a Venn diagram (allow 18 – 25 for A' and 13 – 21 for B) **or** for an answer with one missing element or one extra element
			18, 19, 20, 21	2	A1	
	d		13, 14, 15, 16, 17	1	B1	
						Total 6 marks

18 The diagram shows cuboid *ABCDEFGH*.

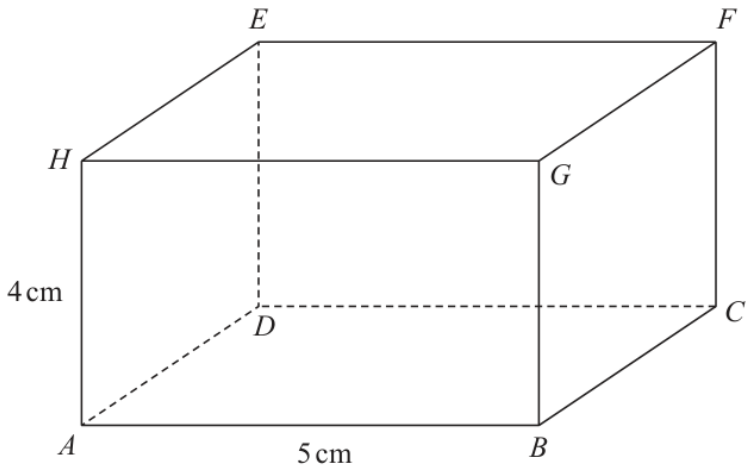


Diagram **NOT** accurately drawn

$AB = 5$ cm
$AH = 4$ cm
The size of the angle between CH and the plane $ABCD$ is 35°

Calculate the volume of the cuboid.
Give your answer correct to 3 significant figures.

..... cm^3

(Total for Question 18 is 5 marks)

18	eg $\frac{4}{AC} = \tan 35$ oe **or** $\frac{AC}{4} = \tan 55$ oe **or** $\frac{AC}{\sin 55} = \frac{4}{\sin 35}$ oe **or** $CH = \frac{4}{\sin 35}$ oe (= 6.97...) **and** $\frac{AC}{"6.97"} = \cos 35$ oe **or** $CH = \frac{4}{\sin 35}$ oe (=6.97...) **and** $AC^2 = 6.97^2 - 4^2$ oe			M1	A correct trig statement involving AC **or** trig and then Pythagoras involving AC
	$(AC =) \frac{4}{\tan 35}$ oe eg $(AC =) 4\tan 55$ (= 5.71...) **or** $(AC =) \frac{4\sin 55}{\sin 35}$ **or** "6.97" × cos35 oe **or** $(AC =) \sqrt{"6.97"^2 - 4^2}$			M1	complete method to find AC
	$(BC =) \sqrt{"5.71"^2 - 5^2}$ (= 2.76...)			M1	complete method to find BC
	4 × 5 × "2.76..."			M1	method to find volume
		55.3	5	A1	accept 55.1 – 55.5
					Total 5 marks

20 A bag contains X counters.

There are only red counters and blue counters in the bag.

There are 4 more blue counters than red counters in the bag.

Finty takes at random 2 counters from the bag.

The probability that Finty takes 2 blue counters from the bag is $\frac{3}{8}$

Work out the value of X.
Show clear algebraic working.

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(Total for Question 20 is 5 marks)

20	$\frac{\left(\frac{X+4}{2}\right)}{X}\left(=\frac{X+4}{2X}\right)$ **or** $\frac{\left(\frac{X+4}{2}\right)-1}{X-1}\left(=\frac{X+2}{2X-2}\right)$	eg, where b = number of blue counters $\frac{b}{2b-4}$ **or** $\frac{b-1}{2b-5}$	eg, where r = number of red counters $\frac{r+4}{2r+4}$ **or** $\frac{r+3}{2r+3}$			M1	for making a correct start by finding the probability of the first counter being blue for their method
	eg $\frac{X+4}{2X}\times\frac{X+2}{2X-2}$	eg $\frac{b}{2b-4}\times\frac{b-1}{2b-5}$	eg $\frac{r+4}{2r+4}\times\frac{r+3}{2r+3}$			M1	oe correct calculation for 2 blue (using one variable)
	eg $8(X^2+6X+8)=3(4X^2-4X)$	eg $8b(b-1)=3(2b-4)(2b-5)$	eg $8(r+4)(r+3)=3(2r+4)(2r+3)$			M1	dep for a correct equation with no algebraic fractions eg could have $X^2+6X+8=\frac{3}{8}(4X^2-4X)$
	Eg $4X^2-60X-64\ (=0)$ **or** $X^2-15X-16\ (=0)$ oe	eg $4b^2-46b+60\ (=0)$ **or** $2b^2-23b+30\ (=0)$ oe	eg $4r^2-14r-60\ (=0)$ **or** $2r^2-7r-30\ (=0)$ oe			M1	for rearranging their equation to a correct 3 term quadratic
				16	5	A1	cao dep on M4
							Total 5 marks