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WME01 Mechanics 1 Past Paper Solutions

The full WME01 bank rebuilt in original paper order, with each question followed by its worked solution.

138
paper-order questions

WME01
Mechanics 1

Past-paper solutions
standalone IAL route

Dr Eslam Ahmed

Prepared for Dr Eslam Ahmed - eliteigcse.com

M1

PAST PAPER

WME01/01 October 2019

October 2019 | 8 questions | 75 marks

8
questions

75
marks

Answers
worked solution after each question

WME01/01 OCTOBER 2019

6 marks

Question 1

Momentum, Impulse & Collisions

1. Two particles, P and Q, have masses $3m$ and $2m$ respectively. The particles are connected by a light inextensible string. Initially P and Q are at rest on a smooth horizontal plane with the string slack.

Particle P is then projected along the plane directly away from Q with speed $4u$. At the same instant, particle Q is projected along the plane in the opposite direction with speed $3u$.

Find

(a) the common speed of the particles immediately after the string becomes taut,
(3)

(b) the magnitude of the impulse exerted on Q at the instant when the string becomes taut.
(3)

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Worked Solution - Question 1

Momentum, Impulse & Collisions

Topic group

1. Choose a direction

Take P's initial direction as positive. Then P initially has velocity $4u$ and Q initially has velocity $-3u$.

2. Use conservation of momentum

When the string becomes taut, both particles move with common speed v. Hence $3m(4u) + 2m(-3u) = (3m + 2m)v$.

3. Find the common speed

$12mu - 6mu = 5mv$, so $v = \dfrac{6u}{5}$.

4. Use impulse on Q

Impulse on Q equals change in momentum: $I = 2m\left(\dfrac{6u}{5} - (-3u)\right)$.

5. Find the impulse magnitude

$$I = 2m\left(\frac{6u}{5} + \frac{15u}{5}\right) = 2m \cdot \frac{21u}{5} = \frac{42}{5}mu.$$

Final answer

$(a)\ v = \dfrac{6u}{5}. \quad (b)\ I = \dfrac{42}{5}mu.$

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9 marks

Question 2

Constant Acceleration in 1D

2. A small ball is released from rest from a point that is 40 m above horizontal ground. The ball bounces on the ground and rebounds vertically. Each time the ball bounces on the ground, the speed of the ball is instantaneously reduced by 50%. The ball is modelled as a particle moving freely under gravity, from the instant when it is released until it first hits the ground, and between each successive bounce.

(a) Find the time from the instant when the ball is released from rest to the instant when it hits the ground for the second time.

(5)

(b) Find the maximum height reached by the ball above the ground after the ball's third bounce.

(4)

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Worked Solution - Question 2

Constant Acceleration in 1D

Topic group

1. Find the time to the first hit

The ball is released from rest and falls 40 m. Use $s = ut + \frac{1}{2}gt^2$: $40 = \frac{1}{2}gt_1^2$.

Thus $t_1 = \sqrt{\frac{80}{g}} = \frac{20}{7}$ s.

2. Find the speed just before the first bounce

Using $v^2 = u^2 + 2gs$, $v_1^2 = 2g(40)$, so $v_1 = 28\,\mathrm{m\,s^{-1}}$.

3. Use the rebound speed

After the first bounce the speed is reduced by 50%, so the upward speed is $14\,\mathrm{m\,s^{-1}}$.

4. Find the time until the second hit

The time to go up and return to the ground is $\frac{2(14)}{g} = \frac{20}{7}$ s.

5. Find the total time

Total time to the second hit is $\frac{20}{7} + \frac{20}{7} = \frac{40}{7} = 5.71$ s approximately.

6. Find the speed after the third bounce

After each bounce, the speed is halved. The upward speeds after the first, second and third bounces are 14, 7 and $3.5\,\mathrm{m\,s^{-1}}$.

7. Find the height after the third bounce

At the top after the third bounce, $0 = (3.5)^2 - 2gh$. Therefore

$h = \frac{(3.5)^2}{2g} = 0.625$ m.

Final answer

(a) 5.71 s approximately. (b) 0.625 m.

Question 3

7 marks

Newton's Second Law

3. A car of mass 800 kg is towing a trailer of mass 400 kg up a straight road using a towbar. The towbar is parallel to the road and parallel to the direction of motion of the car. The road is inclined to the horizontal at an angle α, where $\sin \alpha = \frac{1}{7}$. The engine of the car produces a constant driving force of magnitude D newtons. The resistance to the motion of the car from non-gravitational forces is modelled as a single force of magnitude 420 N. The resistance to the motion of the trailer from non-gravitational forces is modelled as a single force of magnitude 300 N. The car and trailer are modelled as particles and the towbar is modelled as a light rod.

Given that the tension in the towbar is 2060 N, find the value of D.

(7)

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Worked Solution - Question 3

Newton's Second Law

Topic group

1. Use the trailer to find acceleration

For the trailer, take up the road as positive. The forces along the road are tension 2060 N up the road, resistance 300 N down the road, and weight component $400g\sin\alpha$ down the road.

2. Form the trailer equation

$2060 - 300 - 400g\sin\alpha = 400a$. Since $\sin\alpha = \frac{1}{7}$, this becomes $2060 - 300 - 400g \cdot \frac{1}{7} = 400a$.

3. Find the acceleration

Using $g = 9.8$, $400g \cdot \frac{1}{7} = 560$. Hence $1200 = 400a$ and $a = 3\ \mathrm{m\,s^{-2}}$.

4. Apply Newton's second law to the car

For the car: $D - 420 - 800g\sin\alpha - 2060 = 800a$.

5. Find the driving force

$D - 420 - 800g \cdot \frac{1}{7} - 2060 = 800(3)$. Since $800g \cdot \frac{1}{7} = 1120$, $D = 420 + 1120 + 2060 + 2400 = 6000$ N.

Final answer

$D = 6000\ \mathrm{N}$.

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12 marks

Question 4

Resolving Forces, Inclined Planes

4.

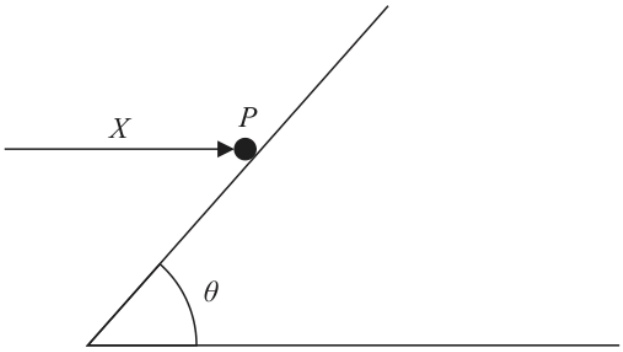

Figure 1

A particle, P, of mass km lies on a fixed rough plane. The plane is inclined to the horizontal at an acute angle θ. A horizontal force of magnitude X acts on P, as shown in Figure 1. The line of action of the force lies in the vertical plane which contains the line of greatest slope of the inclined plane that passes through P. The coefficient of friction between P and the inclined plane is μ.

When $X = mg$, the particle P is in equilibrium and on the point of sliding down the plane.

(a) Show that $\mu = \dfrac{k\tan\theta - 1}{k + \tan\theta}$

(10)

(b) Deduce that, when $k = 1$, θ must be greater than 45°

(2)

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Worked Solution - Question 4

Resolving Forces, Inclined Planes

Topic group

1. Set friction direction

The particle is on the point of sliding down the plane, so friction acts up the plane. Also $X = mg$.

2. Resolve perpendicular to the plane

Both the weight kmg and the horizontal force mg press the particle into the plane. Therefore $R = kmg\cos\theta + mg\sin\theta$.

3. Resolve parallel to the plane

Down the plane is the component $kmg\sin\theta$. Up the plane are friction F and the component $mg\cos\theta$. Thus $F + mg\cos\theta = kmg\sin\theta$.

4. Write friction in terms of theta

So $F = kmg\sin\theta - mg\cos\theta$.

5. Use limiting friction

$\mu = \dfrac{F}{R} = \dfrac{kmg\sin\theta - mg\cos\theta}{kmg\cos\theta + mg\sin\theta}$.

6. Simplify the expression

Cancel mg and divide numerator and denominator by $\cos\theta$: $\mu = \dfrac{k\tan\theta - 1}{k + \tan\theta}$, as required.

7. Deduce the condition when k is 1

When $k = 1$, $\mu = \dfrac{\tan\theta - 1}{1 + \tan\theta}$. Since $\mu > 0$ and the denominator is positive, $\tan\theta - 1 > 0$.

8. Finish the inequality

Therefore $\tan\theta > 1$, so because θ is acute, $\theta > 45°$.

Final answer

$(a)\ \mu = \dfrac{k\tan\theta - 1}{k + \tan\theta}$. $(b)\ \theta > 45°$.

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Question 5

8 marks

Moments

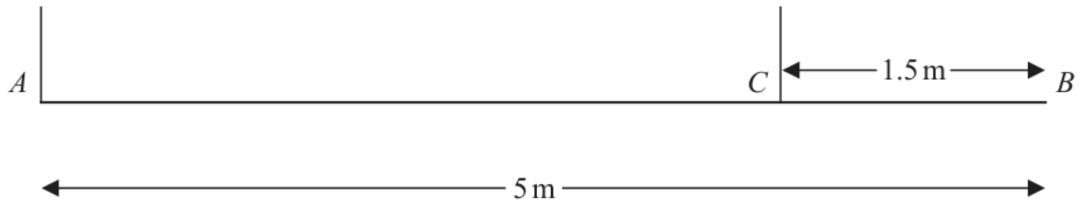

Figure 2

A non-uniform beam, AB, has length 5 m and mass 12 kg. The beam is suspended in a horizontal position by two vertical ropes. One rope is attached to the beam at A. The other rope is attached to the beam at C, where $CB = 1.5$ m, as shown in Figure 2. The distance of the centre of mass of the beam from A is 1.75 m. The beam is modelled as a non-uniform rod and the ropes are modelled as light inextensible strings.

A particle of mass M kg is now placed on the beam at B and the beam remains in equilibrium in a horizontal position.

(a) Find the largest possible value of M.

(3)

The particle at B is now removed and a particle of mass 15 kg is now placed on the beam at the point D, where $AD = x$ metres. The beam remains in equilibrium in a horizontal position.

Given that the tension in the rope attached to the beam at C is now twice the tension in the rope attached to the beam at A,

(b) find the value of x.

(5)

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Worked Solution - Question 5

Moments

Topic group

1. Locate the key distances

The beam is 5 m long. Since $CB = 1.5$ m, $AC = 3.5$ m. The centre of mass is 1.75 m from A, so it is also 1.75 m from C.

2. Use the largest M condition

For the largest possible M at B, the rope at A is just slack, so the beam is supported at C only.

3. Take moments about C

The particle at B is 1.5 m to the right of C, and the beam's weight acts 1.75 m to the left of C. Hence $Mg(1.5) = 12g(1.75)$.

4. Find M

$$M = \frac{12(1.75)}{1.5} = 14.$$

5. Set the tensions for part b

Let the tension at A be T. The tension at C is $2T$. Vertical equilibrium gives $T + 2T = 12g + 15g = 27g$, so $T = 9g$ and the tension at C is $18g$.

6. Take moments about A

For the new particle at $AD = x$, moments about A give $18g(3.5) = 12g(1.75) + 15gx$.

7. Solve for x

Cancel g: $63 = 21 + 15x$. Thus $15x = 42$ and $x = 2.8$ m.

Final answer

$(a)\ M = 14.\quad (b)\ x = 2.8\ \mathrm{m}.$

Question 6

12 marks

Kinematics Graphs

6. An athlete runs a 200 m race along a straight horizontal track.

In a model of the motion of the athlete, air resistance is ignored, the athlete starts from rest at time $t = 0$ seconds and moves with uniform acceleration $0.8\,\mathrm{m\,s^{-2}}$ for T seconds, reaching a speed of $V\,\mathrm{m\,s^{-1}}$. She then maintains this speed until she crosses the finishing line.

The total time from when the athlete starts to when she crosses the finishing line is 30 s.

(a) Sketch a speed-time graph for the model of the motion of the athlete from the instant when she starts to the instant when she crosses the finishing line.

(2)

(b) Write down an expression for V in terms of T.

(1)

(c) Show that $T^2 - kT + 500 = 0$, where k is a constant to be found.

(4)

(d) Hence find the value of T, justifying your answer carefully.

(3)

(e) Considering your speed-time graph or otherwise, state two ways, apart from including air resistance, in which the model could be made to be more realistic.

(2)

Worked Solution - Question 6

Kinematics Graphs

Topic group

1. Describe the speed-time graph

The graph starts at $(0, 0)$, rises in a straight line with gradient 0.8 until time T and speed V, then stays horizontal at speed V until $t = 30$.

2. Write V in terms of T

Uniform acceleration gives $V = 0.8T$.

3. Use area for the distance

The distance is the area under the speed-time graph: $\frac{1}{2}TV + V(30 - T) = 200$.

4. Substitute V

Using $V = 0.8T$: $\frac{1}{2}T(0.8T) + (0.8T)(30 - T) = 200$.

5. Form the quadratic

$0.4T^2 + 24T - 0.8T^2 = 200$, so $-0.4T^2 + 24T - 200 = 0$. Multiplying by -2.5 gives $T^2 - 60T + 500 = 0$, so $k = 60$.

6. Solve and choose the valid root

$T^2 - 60T + 500 = (T - 10)(T - 50) = 0$, so $T = 10$ or 50. Since the whole race lasts 30 s, $T = 10$.

7. Give two model improvements

A more realistic model could include reaction time at the start, and could replace the instantaneous change from acceleration to constant speed with a smoother transition.

Final answer

(a) Speed-time graph rises linearly to V at $t = T$, then is horizontal until $t = 30$.

(b) $V = 0.8T$. (c) $T^2 - 60T + 500 = 0$, so $k = 60$.

(d) $T = 10$. (e) For example, include reaction time at the start and avoid the instantaneous change to constant speed.

Question 7

7 marks

Working with Vectors

7. Two forces, **F** and **G**, act on a particle. The force **F** has magnitude 4 N and acts in a direction with a bearing of 120° and the force **G** has magnitude 6 N and acts due north.

Given that **P** = 2**F** + **G**, find

(i) the magnitude of **P**

(ii) the direction of **P**, giving your answer as a bearing to the nearest degree.

(7)

Worked Solution - Question 7

Working with Vectors

Topic group

1. Resolve 2F

Since $\mathbf{F}$ has magnitude 4 N on a bearing of $120°$, $\mathbf{2F}$ has magnitude 8 N on the same bearing.

2. Write the components

$\mathbf{2F} = 8\sin 120°\mathbf{i} + 8\cos 120°\mathbf{j} = 4\sqrt{3}\mathbf{i} - 4\mathbf{j}$. Also $\mathbf{G} = 6\mathbf{j}$.

3. Find P

$\mathbf{P} = \mathbf{2F} + \mathbf{G} = 4\sqrt{3}\mathbf{i} + 2\mathbf{j}$.

4. Find the magnitude

$|\mathbf{P}| = \sqrt{(4\sqrt{3})^2 + 2^2} = \sqrt{48+4} = \sqrt{52}$ N.

5. Find the bearing angle

The vector is in the north-east quadrant. The angle east of north satisfies

$\tan\theta = \dfrac{4\sqrt{3}}{2} = 2\sqrt{3}$.

6. State the bearing

$\theta = 73.9°$ approximately, so the bearing is $074°$ to the nearest degree.

Final answer

$(i)\ |\mathbf{P}| = \sqrt{52}\ \mathrm{N} \approx 7.21\ \mathrm{N}. \quad (ii)\ \text{bearing} = 074°.$

Question 8

8. [*In this question, the horizontal unit vectors* **i** *and* **j** *are directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two speedboats, A and B, are each moving with constant velocity. The velocity of A is $20\,\text{km}\,\text{h}^{-1}$ due west and the velocity of B is $40\,\text{km}\,\text{h}^{-1}$ on a bearing of 150°. The boats are modelled as particles.

At noon, the position vector of A is $60\mathbf{i}$ km and B is at the origin O. At time t hours after noon, the position vector of A is $\mathbf{r}$ km and the position vector of B is $\mathbf{s}$ km.

(a) Find the velocity of B in the form $(p\mathbf{i} + q\mathbf{j})\,\text{km}\,\text{h}^{-1}$

(3)

(b) Find expressions for $\mathbf{r}$ and $\mathbf{s}$ in terms of t, $\mathbf{i}$ and $\mathbf{j}$.

(3)

(c) Find the time, to the nearest minute, at which the distance between the boats is the same as it was at noon.

(8)

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Worked Solution - Question 8

Working with Vectors

Topic group

1. Resolve B's velocity

A bearing of $150°$ is $30°$ east of south. Therefore the east component is $40\sin 150° = 20$ and the north component is $40\cos 150° = -20\sqrt{3}$.

2. Write B's velocity

$\mathbf{v}_B = 20\mathbf{i} - 20\sqrt{3}\mathbf{j}\ \mathrm{km\,h^{-1}}$.

3. Write A's position

Boat A starts at $60\mathbf{i}$ and travels due west at $20\ \mathrm{km\,h^{-1}}$, so $\mathbf{r} = 60\mathbf{i} - 20t\mathbf{i} = (60 - 20t)\mathbf{i}$.

4. Write B's position

Boat B starts at O, so $\mathbf{s} = t(20\mathbf{i} - 20\sqrt{3}\mathbf{j}) = 20t\mathbf{i} - 20\sqrt{3}t\mathbf{j}$.

5. Find the separation vector

$\overrightarrow{AB} = \mathbf{s} - \mathbf{r} = (20t - (60 - 20t))\mathbf{i} - 20\sqrt{3}t\mathbf{j} = (40t - 60)\mathbf{i} - 20\sqrt{3}t\mathbf{j}$.

6. Use the noon distance

At noon the distance between the boats is 60 km. Set the later distance equal to 60: $(40t - 60)^2 + (-20\sqrt{3}t)^2 = 60^2$.

7. Solve for t

Expanding gives $2800t^2 - 4800t = 0$, so $t(2800t - 4800) = 0$. The later time is $t = \dfrac{4800}{2800} = \dfrac{12}{7}$ hours.

8. Convert to clock time

$\frac{12}{7}$ hours is 1 hour and $42.857\ldots$ minutes after noon, which rounds to $1{:}43$ pm, or $13{:}43$.

Final answer

$(a)\ \mathbf{v}_B = 20\mathbf{i} - 20\sqrt{3}\mathbf{j}\ \mathrm{km\,h^{-1}}.\quad (b)\ \mathbf{r} = (60-20t)\mathbf{i},\ \mathbf{s} = 20t\mathbf{i} - 20\sqrt{3}t\mathbf{j}.\quad (c)\ 13{:}43$, or $1{:}43\ \mathrm{pm}$.

PAST PAPER

WME01/01 January 2020

January 2020 | 7 questions | 75 marks

7
questions

75
marks

Answers
worked solution after each question

Question 1

8 marks

Momentum, Impulse & Collisions

1. Two particles, P and Q, of mass m_1 and m_2 respectively, are moving on a smooth horizontal plane. The particles are moving towards each other in opposite directions along the same straight line when they collide directly. Immediately before the collision, both particles are moving with speed u.

The direction of motion of each particle is reversed by the collision.

Immediately after the collision, the speed of Q is $\frac{1}{3}u$.

(a) Find, in terms of m_2 and u, the magnitude of the impulse exerted by P on Q in the collision.

(3)

(b) Find, in terms of m_1, m_2 and u, the speed of P immediately after the collision.

(3)

(c) Hence show that $m_2 > \frac{3}{4} m_1$

(2)

WME01/01 JANUARY 2020

Worked Solution - Question 1

Momentum, Impulse & Collisions

Topic group

1. Choose a positive direction

Take P's original direction as positive. Before collision, $v_P = u$ and $v_Q = -u$. After collision, Q has velocity $\frac{u}{3}$ because its direction is reversed.

2. Find the impulse on Q

Impulse on Q equals change in momentum:

$$I = m_2\left(\frac{u}{3} - (-u)\right) = m_2\left(\frac{4u}{3}\right) = \frac{4}{3}m_2u.$$

3. Use conservation of momentum

Let the speed of P after collision be v. Since P reverses direction, its velocity is $-v$.

Hence $m_1u - m_2u = -m_1v + \frac{1}{3}m_2u$.

4. Solve for P's speed

Rearrange: $m_1v = \frac{4}{3}m_2u - m_1u$, so $v = \frac{(4m_2 - 3m_1)u}{3m_1}$.

5. Use the fact that speed is positive

Since $u > 0$ and $m_1 > 0$, the numerator must be positive: $4m_2 - 3m_1 > 0$.

Therefore $m_2 > \frac{3}{4}m_1$.

Final answer

$(a)\ I = \frac{4}{3}m_2u. \quad (b)\ \text{speed of P} = \frac{(4m_2 - 3m_1)u}{3m_1}. \quad (c)\ m_2 > \frac{3}{4}m_1$

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Question 2

7 marks

Moments

2.

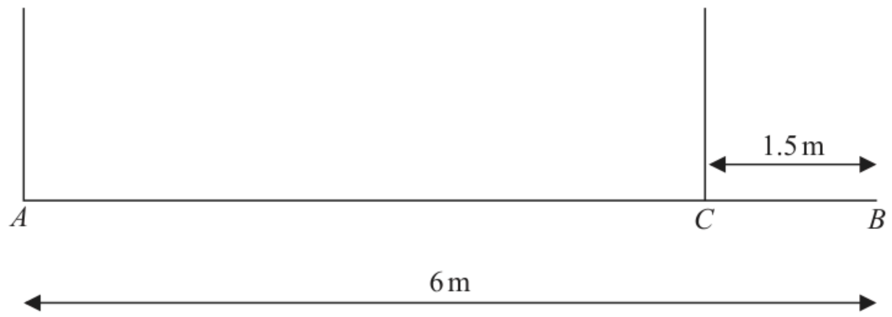

Figure 1

A non-uniform beam *AB* has length 6 m and weight *W* newtons. The beam is supported in equilibrium in a horizontal position by two vertical ropes, one attached to the beam at *A* and the other attached to the beam at *C*, where *CB* = 1.5 m, as shown in Figure 1.

The centre of mass of the beam is 2.625 m from *A*.

The ropes are modelled as light strings. The beam is modelled as a non-uniform rod.

Given that the tension in the rope attached at *C* is 20 N greater than the tension in the rope attached at *A*,

(a) find the value of *W*.

(6)

(b) State how you have used the fact that the beam is modelled as a rod.

(1)

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Worked Solution - Question 2

Moments

Topic group

1. Set the tensions

Let the tension at A be T. The tension at C is then $T + 20$. Since $CB = 1.5$ m and $AB = 6$ m, $AC = 4.5$ m.

2. Use vertical equilibrium

The upward tensions balance the weight: $T + (T + 20) = W$, so $2T + 20 = W$.

3. Take moments about A

The centre of mass is 2.625 m from A. Taking moments about A gives $4.5(T + 20) = 2.625W$.

4. Solve for W

From the moments equation, $T + 20 = \frac{2.625}{4.5}W = \frac{7}{12}W$. Also $T + 20 = \frac{W + 20}{2}$. Hence $\frac{W + 20}{2} = \frac{7W}{12}$, so $6W + 120 = 7W$ and $W = 120$ N.

5. State the modelling use

Modelling the beam as a rod means it stays straight and rigid, so the distances used in the moments equations do not change.

Final answer

(a) $W = 120\text{ N.}$ (b) The rod model keeps the beam straight and rigid, so the stated distances along the beam are fixed.

Question 3

11 marks

Constant Acceleration in 1D

3. A particle, P, is projected vertically upwards with speed U from a fixed point O. At the instant when P reaches its greatest height H above O, a second particle, Q, is projected with speed $\frac{1}{2}U$ vertically upwards from O.

(a) Find H in terms of U and g.

(2)

(b) Find, in terms of U and g, the time between the instant when Q is projected and the instant when the two particles collide.

(6)

(c) Find where the two particles collide.

(3)

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Worked Solution - Question 3

Constant Acceleration in 1D

Topic group

1. Find the greatest height

For P, at greatest height the speed is zero. Use $v^2 = u^2 + 2as$ with $u = U$, $v = 0$, $a = -g$ and $s = H$: $0 = U^2 - 2gH$. Hence $H = \frac{U^2}{2g}$.

2. Start timing when Q is projected

At this instant P is at height H and has zero velocity. After t seconds, P has fallen $\frac{1}{2}gt^2$, so its height above O is $H - \frac{1}{2}gt^2$.

3. Write Q's height

Q is projected upwards from O with speed $\frac{1}{2}U$, so its height after t seconds is $\frac{1}{2}Ut - \frac{1}{2}gt^2$.

4. Equate the heights

At collision, $H - \frac{1}{2}gt^2 = \frac{1}{2}Ut - \frac{1}{2}gt^2$. The gravity terms cancel, so $H = \frac{1}{2}Ut$.

5. Find the time

Substitute $H = \frac{U^2}{2g}$: $\frac{U^2}{2g} = \frac{1}{2}Ut$. Therefore $t = \frac{U}{g}$.

6. Find the collision position

Substitute $t = \frac{U}{g}$ into P's height: $H - \frac{1}{2}g\left(\frac{U}{g}\right)^2 = \frac{U^2}{2g} - \frac{U^2}{2g} = 0$. Thus they collide at O.

Final answer

$(a)\ H = \dfrac{U^2}{2g}$. $(b)\ t = \dfrac{U}{g}$. (c) The particles collide at O.

Question 4

10 marks

Forces

4.

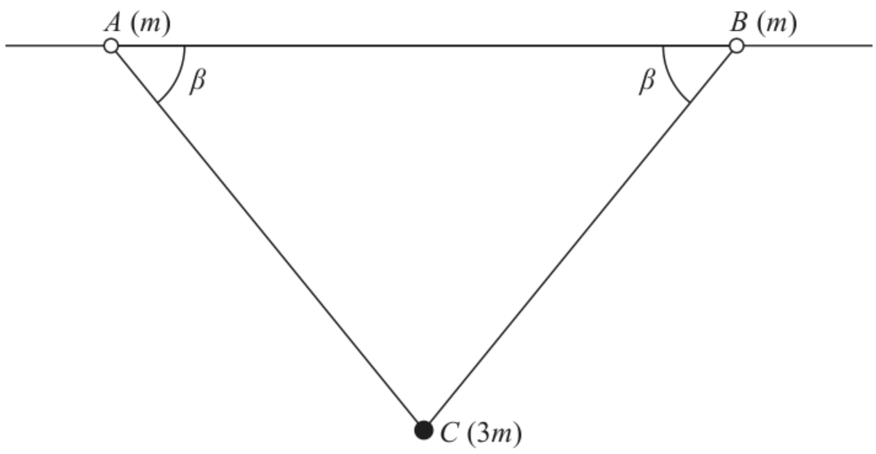

Figure 2

Two identical small rings, A and B, each of mass m, are threaded onto a rough horizontal wire. The rings are connected by a light inextensible string. A particle C of mass $3m$ is attached to the midpoint of the string. The particle C hangs in equilibrium below the wire with angle $BAC = \beta$, as shown in Figure 2.

The tension in each of the parts, AC and BC, of the string is T

(a) By considering particle C, find T in terms of m, g and β

(2)

(b) Find, in terms of m and g, the magnitude of the normal reaction between the wire and A.

(3)

The coefficient of friction between each ring and the wire is $\frac{4}{5}$

The two rings, A and B, are on the point of sliding along the wire towards each other.

(c) Find the value of $\tan\beta$

(5)

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Worked Solution - Question 4

Forces

Topic group

1. Resolve forces on C

The two tensions are equal and symmetric. Their vertical components support the weight of C, so $2T\sin\beta = 3mg$. Hence $T = \dfrac{3mg}{2\sin\beta}$.

2. Resolve vertically for ring A

For ring A, the normal reaction balances its own weight and the downward vertical component of the string tension: $R = mg + T\sin\beta$.

3. Find the normal reaction

From $2T\sin\beta = 3mg$, we have $T\sin\beta = \dfrac{3}{2}mg$. Therefore $R = mg + \dfrac{3}{2}mg = \dfrac{5}{2}mg$.

4. Use limiting friction

The rings are just about to slide towards each other, so friction is limiting. With $\mu = \dfrac{4}{5}$, $F = \mu R = \dfrac{4}{5} \cdot \dfrac{5}{2}mg = 2mg$.

5. Resolve horizontally for a ring

The horizontal component of the tension is balanced by friction: $T\cos\beta = F = 2mg$.

6. Find tan beta

Substitute $T = \dfrac{3mg}{2\sin\beta}$: $\dfrac{3mg}{2\sin\beta}\cos\beta = 2mg$. Hence $\dfrac{3}{2}\cot\beta = 2$, so $\cot\beta = \dfrac{4}{3}$ and $\tan\beta = \dfrac{3}{4}$.

Final answer

$(a)\ T = \dfrac{3mg}{2\sin\beta}$. $(b)\ R = \dfrac{5}{2}mg$. $(c)\ \tan\beta = \dfrac{3}{4}$.

10 marks

Question 5

Kinematics Graphs

5. A car travels at a constant speed of $40\,\text{m s}^{-1}$ in a straight line along a horizontal racetrack. At time $t = 0$, the car passes a motorcyclist who is at rest. The motorcyclist immediately sets off to catch up with the car.

The motorcyclist accelerates at $4\,\text{m s}^{-2}$ for 15 s and then accelerates at $1\,\text{m s}^{-2}$ for a further T seconds until he catches up with the car.

(a) Sketch, on the same axes, the speed-time graph for the motion of the car and the speed-time graph for the motion of the motorcyclist, from time $t = 0$ to the instant when the motorcyclist catches up with the car.

(2)

At the instant when $t = t_1$ seconds, the car and the motorcyclist are moving at the same speed.

(b) Find the value of t_1

(2)

(c) Show that $T^2 + kT - 300 = 0$, where k is a constant to be found.

(6)

WME01/01 JANUARY 2020

Worked Solution - Question 5

Kinematics Graphs

Topic group

1. Describe the graph

The car's graph is a horizontal line at $40\ \mathrm{m\,s^{-1}}$. The motorcyclist's graph starts at zero, rises with gradient 4 until $t = 15$, then rises with smaller gradient 1 for a further T seconds.

2. Find the equal-speed time

Before $t = 15$, the motorcyclist's speed is $4t$. Set this equal to the car's speed: $4t_1 = 40$, so $t_1 = 10$ s.

3. Find the motorcyclist's speed at 15 seconds

After the first 15 seconds, the motorcyclist's speed is $4(15) = 60\ \mathrm{m\,s^{-1}}$.

4. Find the final speed at catch-up

During the next T seconds the acceleration is $1\ \mathrm{m\,s^{-2}}$, so the speed at catch-up is $60 + T$.

5. Equate distances

Motorcyclist distance $= \frac{1}{2}(15)(60) + \frac{1}{2}[60 + (60 + T)]T$. Car distance $= 40(15 + T)$.

6. Simplify the equation

$450 + 60T + \frac{1}{2}T^2 = 600 + 40T$. Hence $\frac{1}{2}T^2 + 20T - 150 = 0$, so $T^2 + 40T - 300 = 0$ and $k = 40$.

Final answer

(a) Car: horizontal line at 40; motorcyclist: two rising straight-line sections ending at the catch-up time.

(b) $t_1 = 10\,\text{s.}$ (c) $T^2 + 40T - 300 = 0$, so $k = 40$.

Question 6

11 marks

Working with Vectors

6. A force $\mathbf{F}$ is given by $\mathbf{F} = (10\mathbf{i} + \mathbf{j})\,\text{N}$.

(a) Find the exact value of the magnitude of $\mathbf{F}$.

(2)

(b) Find, in degrees, the size of the angle between the direction of $\mathbf{F}$ and the direction of the vector $(\mathbf{i} + \mathbf{j})$.

(4)

The resultant of the force $\mathbf{F}$ and the force $(-15\mathbf{i} + a\mathbf{j})\,\text{N}$, where a is a constant, is parallel to, but in the opposite direction to, the vector $(2\mathbf{i} - 3\mathbf{j})$.

(c) Find the value of a.

(5)

WME01/01 JANUARY 2020

Worked Solution - Question 6

Working with Vectors

Topic group

1. Find the magnitude

For $\mathbf{F} = 10\mathbf{i} + \mathbf{j}$, $|\mathbf{F}| = \sqrt{10^2 + 1^2} = \sqrt{101}$ N.

2. Find the direction angle of F

The angle that $\mathbf{F}$ makes with the positive $\mathbf{i}$ direction is α, where $\tan\alpha = \dfrac{1}{10}$.

Thus $\alpha = 5.71^\circ$ approximately.

3. Compare with i plus j

The vector $\mathbf{i} + \mathbf{j}$ makes an angle of 45° with the positive $\mathbf{i}$ direction.

4. Find the angle between the vectors

So the required angle is $45^\circ - 5.71^\circ = 39.29^\circ$, approximately 39.3°.

5. Write the resultant

The resultant of $10\mathbf{i} + \mathbf{j}$ and $-15\mathbf{i} + a\mathbf{j}$ is $-5\mathbf{i} + (a+1)\mathbf{j}$.

6. Use the parallel opposite direction

Parallel but opposite to $2\mathbf{i} - 3\mathbf{j}$ means the resultant is a positive multiple of $-2\mathbf{i} + 3\mathbf{j}$.

7. Solve for a

Let $-5\mathbf{i} + (a+1)\mathbf{j} = \lambda(-2\mathbf{i} + 3\mathbf{j})$. From the $\mathbf{i}$ component, $-5 = -2\lambda$, so $\lambda = 2.5$. Then $a + 1 = 3(2.5) = 7.5$, hence $a = 6.5$.

Final answer

(a) $|\mathbf{F}| = \sqrt{101}$ N. (b) 39.3° approximately. (c) $a = 6.5$.

WME01/01 JANUARY 2020

Question 7

18 marks

Newton's Second Law

7.

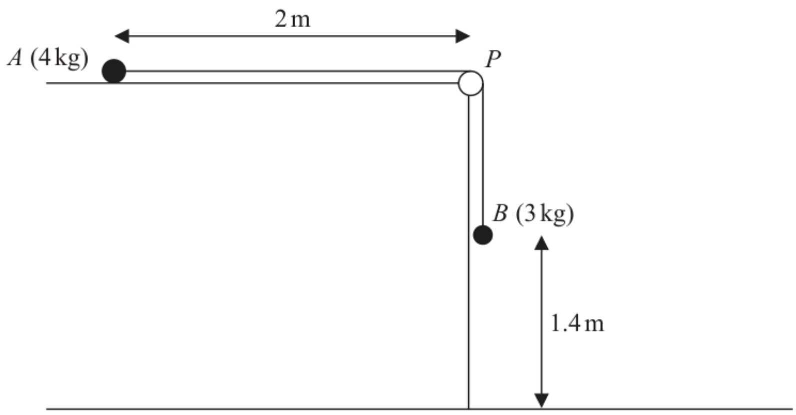

Figure 3

A particle A of mass 4 kg is held at rest on a rough horizontal table. Particle A is attached to one end of a string that passes over a pulley P. The pulley is fixed at the edge of the table. The other end of the string is attached to a particle B, of mass 3 kg, which hangs freely below P.

The part of the string from A to P is perpendicular to the edge of the table and A, P and B all lie in the same vertical plane.

The string is modelled as being light and inextensible and the pulley is modelled as being small, smooth and light.

The system is released from rest with the string taut. At the instant of release, A is 2 m from the edge of the table and B is 1.4 m above a horizontal floor, as shown in Figure 3.

After descending with constant acceleration for 2 seconds, B hits the floor and does not rebound.

(a) Show that the acceleration of A before B hits the floor is $0.7\,\text{m}\,\text{s}^{-2}$

(2)

(b) State which of the modelling assumptions you have used in order to answer part (a).

(1)

(c) Find the magnitude of the resultant force exerted on the pulley by the string.

(4)

The coefficient of friction between A and the table is μ.

(d) Find the value of μ.

(6)

(e) Determine, by calculation, whether or not A reaches the pulley.

(5)

(Total 18 marks)

WME01/01 JANUARY 2020

Worked Solution - Question 7

Newton's Second Law

Topic group

1. Use B's descent

Before B hits the floor, it descends 1.4 m from rest in 2 s. Use $s = ut + \frac{1}{2}at^2$: $1.4 = \frac{1}{2}a(2)^2$. Hence $a = 0.7\ \mathrm{m\,s^{-2}}$.

2. Name the modelling assumption

The string is inextensible, so A and B have the same magnitude of acceleration while the string is taut.

3. Find the tension

For B, taking downward as positive: $3g - T = 3(0.7)$. Thus $T = 3g - 2.1 = 27.3$ N.

4. Find the resultant force on the pulley

The pulley is pulled by two perpendicular tensions of magnitude T. Therefore the resultant is $\sqrt{T^2 + T^2} = T\sqrt{2} = 27.3\sqrt{2} = 38.6$ N approximately.

5. Find the friction on A

For A before B hits the floor, $T - F = 4(0.7)$. Hence $F = 27.3 - 2.8 = 24.5$ N.

6. Find mu

On the horizontal table, $R = 4g = 39.2$ N and $F = \mu R$. So $\mu = \frac{24.5}{39.2} = 0.625$.

7. Find A's speed when B hits the floor

At this instant, $v = at = 0.7(2) = 1.4\ \mathrm{m\,s^{-1}}$. A has also moved 1.4 m towards the pulley.

8. Find the extra stopping distance

After B hits the floor, A is slowed only by friction. Its deceleration is $\mu g = 0.625(9.8) = 6.125\ \mathrm{m\,s^{-2}}$. Use $0 = (1.4)^2 - 2(6.125)s$, giving $s = 0.16$ m.

9. Compare with the distance to the pulley

A's total distance is $1.4 + 0.16 = 1.56$ m, which is less than 2 m. Therefore A does not reach the pulley.

Final answer

(a) $a = 0.7\ \mathrm{m\,s^{-2}}$. (b) Inextensibility of the string.

(c) $38.6\ \mathrm{N}$. (d) $\mu = 0.625$. (e) A does not reach the pulley.

PAST PAPER

WME01/01 January 2021

January 2021 | 8 questions | 75 marks

8
questions

75
marks

Answers
worked solution after each question

WME01/01 JANUARY 2021

6 marks

Question 1

Constant Acceleration in 1D

1. A small stone is projected vertically upwards with speed 20 m s^{-1} from a point O which is 5 m above horizontal ground. The stone is modelled as a particle moving freely under gravity.

Find

(a) the speed of the stone at the instant when it is 2 m above the ground,

(2)

(b) the total time between the instant when the stone is projected from O and the instant when it first strikes the ground.

(4)

WME01/01 JANUARY 2021

Worked Solution - Question 1

Constant Acceleration in 1D

Topic group

1. Find displacement for part a

The stone starts 5 m above the ground. When it is 2 m above the ground, its displacement from O is -3 m.

2. Use SUVAT for speed

Taking upward as positive, $u = 20$, $a = -9.8$ and $s = -3$. Use $v^2 = u^2 + 2as$: $v^2 = 20^2 + 2(-9.8)(-3) = 458.8$.

3. State the speed

$v = 21.4\ \mathrm{m\,s^{-1}}$ to 3 significant figures.

4. Set up the ground equation

At the ground, displacement from O is -5 m. Use $s = ut + \frac{1}{2}at^2$: $-5 = 20t - 4.9t^2$.

5. Solve for time

$4.9t^2 - 20t - 5 = 0$. The positive root is $t = 4.3179\ldots$, so the total time is 4.32 s.

Final answer

$(a)\ 21.4\ \mathrm{m\,s^{-1}}. \quad (b)\ 4.32\ \mathrm{s}$ approximately.

WME01/01 JANUARY 2021

6 marks

Question 2

Momentum, Impulse & Collisions

2. Two particles, P and Q, have masses $2m$ and m respectively. The particles are moving towards each other in opposite directions along the same straight line on a smooth horizontal plane. The particles collide directly.

Immediately before the collision, the speed of P is $3u$ and the speed of Q is $2u$.

The magnitude of the impulse exerted on Q by P in the collision is $5mu$.

Find

(a) the speed of P immediately after the collision,

(3)

(b) the speed of Q immediately after the collision.

(3)

WME01/01 JANUARY 2021

Worked Solution - Question 2

Momentum, Impulse & Collisions

Topic group

1. Choose a positive direction

Take P's original direction as positive. Before collision, P has velocity $3u$ and Q has velocity $-2u$.

2. Use impulse on P

The impulse on P is opposite to P's original motion and has magnitude $5mu$. Let P's velocity after collision be v. Then $-5mu = 2m(v - 3u)$.

3. Find P speed

Divide by m: $-5u = 2v - 6u$, so $2v = u$ and $v = \dfrac{u}{2}$.

4. Use impulse on Q

The impulse on Q is in P's original direction. Let Q's velocity after collision be w. Then $5mu = m(w - (-2u))$.

5. Find Q speed

$5u = w + 2u$, so $w = 3u$. Therefore the speed of Q is $3u$.

Final answer

(a) speed of P $= \dfrac{u}{2}$. (b) speed of Q $= 3u$.

Question 3

9 marks

Forces

3.

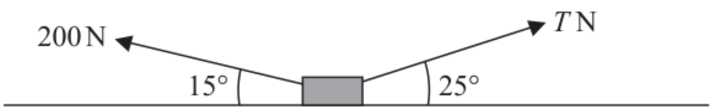

Figure 1

A parcel of mass 20 kg is at rest on a rough horizontal floor. The coefficient of friction between the parcel and the floor is 0.3

Two forces, both acting in the same vertical plane, of magnitudes 200 N and T N are applied to the parcel. The line of action of the 200 N force makes an angle of 15° with the horizontal and the line of action of the T N force makes an angle of 25° with the horizontal, as shown in Figure 1. The parcel is modelled as a particle P.

Find the smallest value of T for which P remains in equilibrium.

(9)

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Worked Solution - Question 3

Forces

Topic group

1. Resolve vertically

For vertical equilibrium, $R + 200\sin 15^\circ + T\sin 25^\circ = 20g$.

2. Resolve horizontally

For the smallest T, friction is limiting and balances the remaining horizontal tendency. Horizontally, $200\cos 15^\circ - T\cos 25^\circ - F = 0$.

3. Use limiting friction

$F = 0.3R$.

4. Eliminate R and F

From the vertical equation, $R = 20g - 200\sin 15^\circ - T\sin 25^\circ$. Substitute into $F = 0.3R$ and then into the horizontal equation.

5. Solve for T

$200\cos 15^\circ - T\cos 25^\circ - 0.3(20g - 200\sin 15^\circ - T\sin 25^\circ) = 0$. Solving gives $T = 192.3\ldots\ \mathrm{N}$, so the smallest value is about $192\ \mathrm{N}$.

Final answer

$T = 192\ \mathrm{N}$ approximately.

Question 4

4.

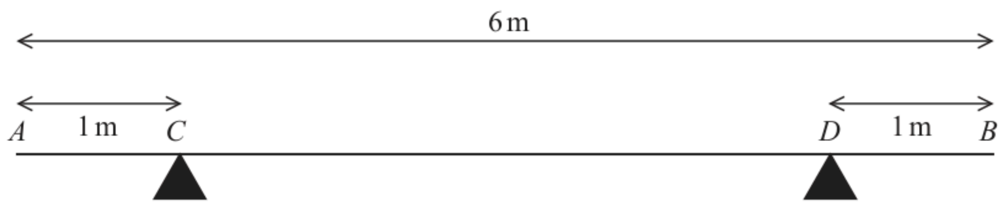

Figure 2

A metal girder AB has weight W newtons and length 6 m. The girder rests in a horizontal position on two supports C and D where $AC = DB = 1$ m, as shown in Figure 2.

When a force of magnitude 900 N is applied vertically upwards to the girder at A, the girder is about to tilt about D.

When a force of magnitude 1500 N is applied vertically upwards to the girder at B, the girder is about to tilt about C.

The girder is modelled as a non-uniform rod whose centre of mass is a distance x metres from A.

Find the value of x.

(6)

WME01/01 JANUARY 2021

Worked Solution - Question 4

Moments

Topic group

1. Set support positions

The girder has length 6 m, with C 1 m from A and D 1 m from B. So D is 5 m from A.

2. Use tilting about D

When the 900 N upward force is applied at A, the girder is about to tilt about D. Taking moments about D gives $900(5) = W(5 - x)$.

3. Use tilting about C

When the 1500 N upward force is applied at B, the girder is about to tilt about C. Taking moments about C gives $1500(5) = W(x - 1)$.

4. Solve the equations

The equations are $W(5 - x) = 4500$ and $W(x - 1) = 7500$. Add them: $W[(5 - x) + (x - 1)] = 12000$, so $4W = 12000$ and $W = 3000$.

5. Find x

Use $W(x - 1) = 7500$: $3000(x - 1) = 7500$, so $x - 1 = 2.5$ and $x = 3.5$ m.

Final answer

$x = 3.5\text{ m}$.

Question 5

7 marks

Working with Vectors

5. A particle is acted upon by two forces **F** and **G**. The force **F** has magnitude 8 N and acts in a direction with a bearing of 240°. The force **G** has magnitude 10 N and acts due South.

Given that **R** = **F** + **G**, find

(i) the magnitude of **R**,

(ii) the direction of **R**, giving your answer as a bearing to the nearest degree.

(7)

Worked Solution - Question 5

Working with Vectors

Topic group

1. Resolve F into components

A bearing of $240°$ is $60°$ west of south. Hence
$\mathbf{F} = (-8\sin 60°)\mathbf{i} + (-8\cos 60°)\mathbf{j} = -4\sqrt{3}\mathbf{i} - 4\mathbf{j}$.

2. Add G

The force $\mathbf{G}$ is due south with magnitude 10, so $\mathbf{G} = -10\mathbf{j}$. Therefore
$\mathbf{R} = \mathbf{F} + \mathbf{G} = -4\sqrt{3}\mathbf{i} - 14\mathbf{j}$.

3. Find the magnitude

$$|\mathbf{R}| = \sqrt{(-4\sqrt{3})^2 + (-14)^2} = \sqrt{48 + 196} = \sqrt{244}\,\mathrm{N}.$$

4. Find the bearing angle

The resultant points south-west. The angle west of south satisfies

$$\tan\theta = \frac{4\sqrt{3}}{14} = \frac{2\sqrt{3}}{7}.$$

5. State the bearing

$\theta = 26.3°$. Bearing $= 180° + 26.3° = 206.3°$, so the bearing is $206°$ to the nearest degree.

Final answer
$(i)\ |\mathbf{R}| = \sqrt{244}\,\mathrm{N} \approx 15.6\,\mathrm{N}. \quad (ii)\ \text{bearing} = 206°.$

Question 6

12 marks

Working with Vectors

6. Two girls, Agatha and Brionie, are roller skating inside a large empty building. The girls are modelled as particles.

At time $t = 0$, Agatha is at the point with position vector $(11\mathbf{i} + 11\mathbf{j})$ m and Brionie is at the point with position vector $(7\mathbf{i} + 16\mathbf{j})$ m. The position vectors are given relative to the door, O, and $\mathbf{i}$ and $\mathbf{j}$ are horizontal perpendicular unit vectors.

Agatha skates with constant velocity $(3\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$

Brionie skates with constant velocity $(4\mathbf{i} - 2\mathbf{j})\,\text{m s}^{-1}$

(a) Find the position vector of Agatha at time t seconds.

(2)

At time $t = 6$ seconds, Agatha passes through the point P.

(b) Show that Brionie also passes through P and find the value of t when this occurs.

(4)

At time t seconds, Agatha is at the point A and Brionie is at the point B.

(c) Show that $\overrightarrow{AB} = [(t - 4)\mathbf{i} + (5 - t)\mathbf{j}]$ m

(2)

(d) Find the distance between the two girls when they are closest together.

(4)

WME01/01 JANUARY 2021

Worked Solution - Question 6

Working with Vectors

Topic group

1. Write Agatha position

$\mathbf{r}_A = (11\mathbf{i} + 11\mathbf{j}) + t(3\mathbf{i} - \mathbf{j}) = (11 + 3t)\mathbf{i} + (11 - t)\mathbf{j}$.

2. Find point P

At $t = 6$, Agatha is at $(11 + 18)\mathbf{i} + (11 - 6)\mathbf{j} = 29\mathbf{i} + 5\mathbf{j}$.

3. Write Brionie position

$\mathbf{r}_B = (7\mathbf{i} + 16\mathbf{j}) + t(4\mathbf{i} - 2\mathbf{j}) = (7 + 4t)\mathbf{i} + (16 - 2t)\mathbf{j}$.

4. Show Brionie passes P

Set $7 + 4t = 29$ to get $t = 5.5$. Also $16 - 2t = 5$ gives $t = 5.5$. So Brionie also passes through P at $t = 5.5$ s.

5. Find AB vector

$\overrightarrow{AB} = \mathbf{r}_B - \mathbf{r}_A = [7 + 4t - (11 + 3t)]\mathbf{i} + [16 - 2t - (11 - t)]\mathbf{j}$.

6. Simplify AB

So $\overrightarrow{AB} = (t - 4)\mathbf{i} + (5 - t)\mathbf{j}$ m.

7. Minimise distance squared

$AB^2 = (t - 4)^2 + (5 - t)^2 = 2(t - 4.5)^2 + 0.5$.

8. State the minimum distance

The minimum occurs when $t = 4.5$, giving $AB^2 = 0.5$. Hence the minimum distance is $\sqrt{0.5} = 0.707\ldots\ \mathrm{m}$.

Final answer

(a) $\mathbf{r}_A = (11 + 3t)\mathbf{i} + (11 - t)\mathbf{j}$. (b) $t = 5.5$ s. (c) $\overrightarrow{AB} = (t - 4)\mathbf{i} + (5 - t)\mathbf{j}$. (d) minimum distance $= \sqrt{0.5}$ m ≈ 0.71 m

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Question 7

12 marks

Kinematics Graphs

7. A helicopter is hovering at rest above horizontal ground at the point H. A parachutist steps out of the helicopter and immediately falls vertically and freely under gravity from rest for 2.5 s. His parachute then opens and causes him to immediately decelerate at a constant rate of $3.9\,\text{m}\,\text{s}^{-2}$ for T seconds ($T < 6$), until his speed is reduced to $V\,\text{m}\,\text{s}^{-1}$. He then moves with this constant speed $V\,\text{m}\,\text{s}^{-1}$ until he hits the ground. While he is decelerating, he falls a distance of 73.75 m. The total time between the instant when he leaves H and the instant when he hits the ground is 20 s.

The parachutist is modelled as a particle.

(a) Find the speed of the parachutist at the instant when his parachute opens.

(1)

(b) Sketch a speed-time graph for the motion of the parachutist from the instant when he leaves H to the instant when he hits the ground.

(2)

(c) Find the value of T.

(5)

(d) Find, to the nearest metre, the height of the point H above the ground.

(4)

WME01/01 JANUARY 2021

Worked Solution - Question 7

Kinematics Graphs

Topic group

1. Find speed when parachute opens

The parachutist falls freely from rest for 2.5 s, so $v = gt = 9.8(2.5) = 24.5\ \mathrm{m\,s^{-1}}$.

2. Describe the graph

The speed-time graph starts at 0, rises linearly to 24.5 at $t = 2.5$, then decreases linearly with smaller gradient for T seconds, then stays horizontal at speed V until $t = 20$.

3. Use the deceleration distance

During the deceleration phase, initial speed is 24.5, acceleration is -3.9, time is T, and distance is 73.75.

4. Form the equation for T

$73.75 = 24.5T + \frac{1}{2}(-3.9)T^2 = 24.5T - 1.95T^2$.

5. Choose the valid T

Solving gives $T = 5$ or $T = 7.56\ldots$. Since $T < 6$, take $T = 5$.

6. Find V

$V = 24.5 - 3.9(5) = 5\ \mathrm{m\,s^{-1}}$.

7. Find the total height

Height is the total area under the speed-time graph: first triangle $\frac{1}{2}(2.5)(24.5) = 30.625$, deceleration distance 73.75, and final rectangle $(20 - 2.5 - 5)(5) = 62.5$.

8. Round the height

Total height $= 30.625 + 73.75 + 62.5 = 166.875$ m, which rounds to 167 m.

Final answer

(a) $24.5\ \mathrm{m\,s^{-1}}$. (b) Speed-time graph rises to 24.5 at $t = 2.5$, falls to V at $t = 2.5 + T$, then is horizontal to $t = 20$.

(c) $T = 5$. (d) $H = 167\ \mathrm{m}$ to the nearest metre.

WME01/01 JANUARY 2021

Question 8

17 marks

Resolving Forces, Inclined Planes

8.

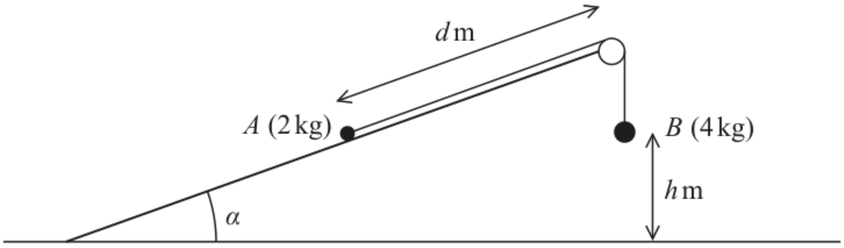

Figure 3

Two particles, A and B, have masses 2 kg and 4 kg respectively. The particles are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough plane. The plane is inclined to the horizontal ground at an angle α where $\tan\alpha = \frac{3}{4}$. The particle A is held at rest on the plane at a distance d metres from the pulley. The particle B hangs freely at rest, vertically below the pulley, at a distance h metres above the ground, as shown in Figure 3. The part of the string between A and the pulley is parallel to a line of greatest slope of the plane. The coefficient of friction between A and the plane is $\frac{1}{4}$

The system is released from rest with the string taut and B descends.

(a) Find the tension in the string as B descends.

(9)

On hitting the ground, B immediately comes to rest.

Given that A comes to rest before reaching the pulley,

(b) find, in terms of h, the range of possible values of d.

(7)

(c) State one physical factor, other than air resistance, that could be taken into account to make the model described above more realistic.

(1)

(Total 17 marks)

WME01/01 JANUARY 2021

Worked Solution - Question 8

Resolving Forces, Inclined Planes

Topic group

1. Use the trig ratios

Since $\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$.

2. Find friction on A

For A, $R = 2g\cos\alpha = \frac{8g}{5}$. With coefficient $\frac{1}{4}$, friction is $F = \frac{1}{4}R = \frac{2g}{5}$.

3. Equation for A

A moves up the plane. Taking up the plane as positive: $T - 2g\sin\alpha - F = 2a$.

4. Simplify A's equation

$T - 2g \cdot \frac{3}{5} - \frac{2g}{5} = 2a$, so $T - \frac{8g}{5} = 2a$.

5. Equation for B

B descends. Taking downward as positive: $4g - T = 4a$.

6. Solve for T

From A, $T = 2a + \frac{8g}{5}$. From B, $T = 4g - 4a$. Equating gives $6a = \frac{12g}{5}$, so $a = \frac{2g}{5}$. Then $T = 4g - 4\left(\frac{2g}{5}\right) = \frac{12g}{5}$ N.

7. Find speed when B reaches the ground

Before B hits the ground, both particles move distance h from rest with acceleration $\frac{2g}{5}$. Thus $v^2 = 2\left(\frac{2g}{5}\right)h = \frac{4gh}{5}$.

8. Find extra distance after B stops

After B hits the ground, A continues up the plane. The forces down the plane on A are $2g\sin\alpha + F = \frac{6g}{5} + \frac{2g}{5} = \frac{8g}{5}$, so its deceleration is $\frac{4g}{5}$.

9. Calculate extra distance

Let the extra distance be s. Then $0 = \frac{4gh}{5} - 2\left(\frac{4g}{5}\right)s$, so $s = \frac{h}{2}$.

10. Find the range for d

A travels total distance $h + \frac{h}{2} = \frac{3h}{2}$ before stopping. Since it comes to rest before reaching the pulley, $d > \frac{3h}{2}$.

11. Name a more realistic factor

A more realistic model could include friction at the pulley.

Final answer

$(a)\ T = \frac{12g}{5}\ \mathrm{N} = 23.5\ \mathrm{N}.\quad (b)\ d > \frac{3h}{2}.\quad (c)$ A more realistic model could include friction at the pulley.

PAST PAPER

WME01/01 May/June 2021

May/June 2021 | 8 questions | 75 marks

8 questions	**75** marks	**Answers** worked solution after each question

WME01/01 MAY/JUNE 2021

7 marks

Question 1

Momentum, Impulse & Collisions

1. A particle P has mass $3m$ and a particle Q has mass $5m$. The particles are moving towards each other in opposite directions along the same straight line on a smooth horizontal surface. The particles collide directly.

Immediately before the collision the speed of P is ku, where k is a constant, and the speed of Q is $2u$.

Immediately after the collision the speed of P is u and the speed of Q is $3u$.

The direction of motion of Q is reversed by the collision.

(a) Find, in terms of m and u, the magnitude of the impulse exerted on Q by P in the collision.

(2)

(b) Find the two possible values of k.

(5)

WME01/01 MAY/JUNE 2021

Worked Solution - Question 1

Momentum, Impulse & Collisions

Topic group

1. Choose a positive direction

Take P's original direction as positive. Q initially has velocity $-2u$. Since Q reverses direction and then has speed $3u$, its final velocity is $3u$.

2. Find the impulse on Q

Impulse equals change in momentum: $I = 5m(3u - (-2u)) = 25mu$.

3. Use conservation of momentum

Before collision, total momentum is $3m(ku) + 5m(-2u) = 3mku - 10mu$.

4. Case 1: P continues in its original direction

If P has final velocity u, final momentum is $3m(u) + 5m(3u) = 18mu$. So $3mku - 10mu = 18mu$, giving $3k = 28$ and $k = \frac{28}{3}$.

5. Case 2: P reverses direction

If P has final velocity $-u$, final momentum is $3m(-u) + 5m(3u) = 12mu$. So $3mku - 10mu = 12mu$, giving $3k = 22$ and $k = \frac{22}{3}$.

Final answer

$(a)\ I = 25mu. \quad (b)\ k = \frac{28}{3} \text{ or } \frac{22}{3}.$

8 marks

Question 2

Constant Acceleration in 1D

2. A car moves along a straight horizontal road with constant acceleration $a \, \mathrm{m\,s^{-2}}$ where $a > 0$

The car is modelled as a particle.

At time $t = 0$, the car passes point A and is moving with speed $u \, \mathrm{m\,s^{-1}}$

In the first three seconds after passing A the car travels 20 m.

In the fourth second after passing A the car travels 10 m.

The speed of the car as it passes point B is $20 \, \mathrm{m\,s^{-1}}$

Find the time taken for the car to travel from A to B.

(8)

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Worked Solution - Question 2

Constant Acceleration in 1D

Topic group

1. Use the first three seconds

Using $s = ut + \frac{1}{2}at^2$, the first 3 seconds give $20 = 3u + \frac{1}{2}a(3^2) = 3u + \frac{9}{2}a$.

2. Use the fourth second

The distance in the fourth second is $u + a(4 - \frac{1}{2}) = u + \frac{7}{2}a$. Hence $10 = u + \frac{7}{2}a$.

3. Solve for a and u

From $10 = u + \frac{7}{2}a$, $u = 10 - \frac{7}{2}a$. Substitute into $20 = 3u + \frac{9}{2}a$:
$20 = 30 - \frac{21}{2}a + \frac{9}{2}a = 30 - 6a$. Thus $a = \frac{5}{3}$.

4. Find u

$u = 10 - \frac{7}{2} \cdot \frac{5}{3} = 10 - \frac{35}{6} = \frac{25}{6}$.

5. Use the speed at B

At B, speed is 20. Use $v = u + at$: $20 = \frac{25}{6} + \frac{5}{3}t$.

6. Find the time

$\frac{5}{3}t = 20 - \frac{25}{6} = \frac{95}{6}$, so $t = \frac{95}{10} = 9.5 \text{ s}$.

Final answer

$t = 9.5 \text{ s}$.

Question 3

9 marks

Working with Vectors

3. [*In this question* **i** *and* **j** *are perpendicular horizontal unit vectors.*]

Three forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$, are given by

$$\mathbf{F}_1 = (5\mathbf{i} + 2\mathbf{j})\,\text{N} \qquad \mathbf{F}_2 = (-3\mathbf{i} + \mathbf{j})\,\text{N} \qquad \mathbf{F}_3 = (a\mathbf{i} + b\mathbf{j})\,\text{N}$$

where a and b are constants.

The forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a particle P of mass 4 kg.

Given that P rests in equilibrium on a smooth horizontal surface under the action of these three forces,

(a) find the size of the angle between the direction of $\mathbf{F}_3$ and the direction of $-\mathbf{j}$.

(4)

The force $\mathbf{F}_3$ is now removed and replaced by the force $\mathbf{F}_4$ given by $\mathbf{F}_4 = \lambda(\mathbf{i} + 3\mathbf{j})\,\text{N}$, where λ is a positive constant.

When the three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_4$ act on P, the acceleration of P has magnitude $3.25\ \text{m s}^{-2}$

(b) Find the value of λ.

(5)

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Worked Solution - Question 3

Working with Vectors

Topic group

1. Find F3 from equilibrium

Equilibrium gives $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0}$. Now $\mathbf{F}_1 + \mathbf{F}_2 = (5\mathbf{i} + 2\mathbf{j}) + (-3\mathbf{i} + \mathbf{j}) = 2\mathbf{i} + 3\mathbf{j}$.

2. Write F3

So $\mathbf{F}_3 = -2\mathbf{i} - 3\mathbf{j}$.

3. Find the angle with -j

The vector $-\mathbf{j}$ points vertically downward. For $\mathbf{F}_3 = -2\mathbf{i} - 3\mathbf{j}$, the angle from $-\mathbf{j}$ satisfies $\tan\theta = \dfrac{2}{3}$.

4. State the angle

$\theta = 33.7^\circ$, so the angle is about 34°.

5. Find the new resultant force

With $\mathbf{F}_4 = \lambda(\mathbf{i} + 3\mathbf{j})$, the resultant is $(2 + \lambda)\mathbf{i} + (3 + 3\lambda)\mathbf{j}$.

6. Use the acceleration magnitude

The mass is 4 kg and acceleration magnitude is 3.25, so the resultant force magnitude is $4(3.25) = 13$ N.

7. Form the equation

$(2 + \lambda)^2 + (3 + 3\lambda)^2 = 13^2$.

8. Solve for lambda

Expanding gives $10\lambda^2 + 22\lambda - 156 = 0$. The roots are 3 and -5.2, and since λ is positive, $\lambda = 3$.

Final answer

(a) 34° approximately. (b) $\lambda = 3$.

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6 marks

Question 4

Newton's Second Law

4.

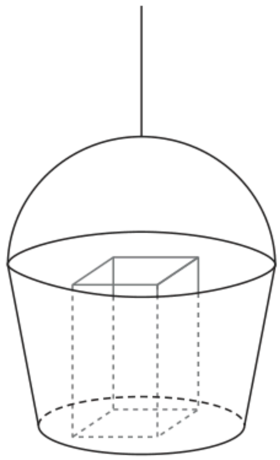

Figure 1

Figure 1 shows a large bucket used by a crane on a building site to move materials between the ground and the top of the building. The mass of the bucket is 15 kg.

The bucket is attached to a vertical cable with the bottom of the bucket horizontal. The cable is modelled as light and inextensible.

When the bucket is on the ground, a bag of cement of mass 25 kg is placed in the bucket.

The bucket with the bag of cement moves vertically upwards with constant acceleration $0.2\,\text{m s}^{-2}$. Air resistance is modelled as being negligible.

(a) Find the tension in the cable.

(3)

At the top of the building, the bag of cement is removed. A box of tools of mass 12 kg is now placed in the bucket.

Later on the bucket with the box of tools is moving vertically downwards with constant deceleration $0.1\,\text{m s}^{-2}$. Air resistance is again modelled as being negligible.

(b) Find the magnitude of the normal reaction between the bucket and the box of tools.

(3)

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Worked Solution - Question 4

Newton's Second Law

Topic group

1. Find total mass going up

The bucket and cement have total mass $15 + 25 = 40$ kg and acceleration $0.2\ \mathrm{m\,s^{-2}}$ upward.

2. Apply Newtons second law

Taking upward as positive: $T - 40g = 40(0.2)$.

3. Find the tension

$T - 392 = 8$, so $T = 400\ \mathrm{N}$.

4. Interpret the downward deceleration

The bucket and tools are moving downward but decelerating, so their acceleration is upward with magnitude $0.1\ \mathrm{m\,s^{-2}}$.

5. Consider the box of tools

For the 12 kg box, upward force is the normal reaction R, and downward force is weight $12g$.

6. Find R

$R - 12g = 12(0.1)$. Thus $R = 117.6 + 1.2 = 118.8\ \mathrm{N}$, about $119\ \mathrm{N}$.

Final answer

$(a)\ T = 400\ \mathrm{N}. \quad (b)\ R = 119\ \mathrm{N}$ approximately.

Question 5

9 marks

Working with Vectors

5. [*In this question* **i** *and* **j** *are perpendicular horizontal unit vectors.*]

A particle P is moving with constant acceleration. At 2 pm, the velocity of P is $(3\mathbf{i} + 5\mathbf{j})$ km h^{-1} and at 2.30 pm the velocity of P is $(\mathbf{i} + 7\mathbf{j})$ km h^{-1}

At time T hours after 2 pm, P is moving in the direction of the vector $(-\mathbf{i} + 2\mathbf{j})$

(a) Find the value of T.

(6)

Another particle, Q, has velocity $\mathbf{v}_Q$ km h^{-1} at time t hours after 2 pm, where

$$\mathbf{v}_Q = (-4 - 2t)\mathbf{i} + (\mu + 3t)\mathbf{j}$$

and μ is a constant.

Given that there is an instant when the velocity of P is equal to the velocity of Q,

(b) find the value of μ.

(3)

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Worked Solution - Question 5

Working with Vectors

Topic group

1. Find P acceleration

From 2:00 to 2:30 is 0.5 hours. The velocity changes from $3\mathbf{i} + 5\mathbf{j}$ to $\mathbf{i} + 7\mathbf{j}$.

2. Calculate acceleration

$\mathbf{a} = \dfrac{(\mathbf{i} + 7\mathbf{j}) - (3\mathbf{i} + 5\mathbf{j})}{0.5} = -4\mathbf{i} + 4\mathbf{j}$.

3. Write P velocity

At time t hours after 2:00, $\mathbf{v}_P = (3 - 4t)\mathbf{i} + (5 + 4t)\mathbf{j}$.

4. Use the direction vector

At time T, the velocity is parallel to $-\mathbf{i} + 2\mathbf{j}$, so $\dfrac{5 + 4T}{3 - 4T} = \dfrac{2}{-1} = -2$.

5. Find T

$5 + 4T = -2(3 - 4T) = -6 + 8T$. Hence $11 = 4T$ and $T = \dfrac{11}{4}$ hours.

6. Equate i-components for P and Q

For equal velocities, $3 - 4t = -4 - 2t$. This gives $7 = 2t$, so $t = 3.5$.

7. Equate j-components

At $t = 3.5$, P's j-component is $5 + 4(3.5) = 19$. Q's j-component is $\mu + 3(3.5) = \mu + 10.5$.

8. Find mu

$\mu + 10.5 = 19$, so $\mu = 8.5$.

Final answer

(a) $T = \dfrac{11}{4}$ h. (b) $\mu = 8.5$.

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Question 6

13 marks

Resolving Forces, Inclined Planes

6. A fixed rough plane is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$

A particle of mass 6 kg is projected with speed 5 $\mathrm{m\,s^{-1}}$ from a point A on the plane, up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is $\frac{1}{4}$

(a) Find the magnitude of the frictional force acting on the particle as it moves up the plane.

(3)

The particle comes to instantaneous rest at the point B.

(b) Find the distance AB.

(5)

The particle now slides down the plane from B. At the instant when the particle passes through the point C on the plane, the speed of the particle is again 5 $\mathrm{m\,s^{-1}}$

(c) Find the distance BC.

(5)

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Worked Solution - Question 6

Resolving Forces, Inclined Planes

Topic group

1. Use the trig ratios

Since $\tan\theta = \frac{5}{12}$, $\sin\theta = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$.

2. Find the normal reaction

$R = 6g\cos\theta = 6g \cdot \frac{12}{13} = \frac{72g}{13}$.

3. Find friction

With $\mu = \frac{1}{4}$, $F = \frac{1}{4}R = \frac{18g}{13} = 13.6\ \mathrm{N}$ approximately.

4. Find acceleration while moving up

While moving up, both friction and the component of weight act down the plane. Taking up the plane as positive: $6a = -6g\sin\theta - F$.

5. Simplify the upward acceleration

$6a = -6g \cdot \frac{5}{13} - \frac{18g}{13} = -\frac{48g}{13}$, so $a = -\frac{8g}{13}$.

6. Find AB

Use $v^2 = u^2 + 2as$ with $u = 5$ and $v = 0$: $0 = 25 + 2\left(-\frac{8g}{13}\right)AB$. Hence $AB = 2.07\ldots\ \mathrm{m}$.

7. Find acceleration while sliding down

When sliding down, friction acts up the plane. Taking down the plane as positive: $6a = 6g\sin\theta - F = \frac{30g}{13} - \frac{18g}{13} = \frac{12g}{13}$. Thus $a = \frac{2g}{13}$.

8. Find BC

Starting from rest at B and reaching speed 5 at C: $25 = 2\left(\frac{2g}{13}\right)BC$. Hence $BC = 8.29\ldots\ \text{m}$.

Final answer

$(a)\ F = 13.6\ \text{N}. \quad (b)\ AB = 2.07\ \text{m} \approx 2.1\ \text{m}. \quad (c)\ BC = 8.29\ \text{m} \approx 8.3\ \text{m}$

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10 marks

Question 7

Moments

7.

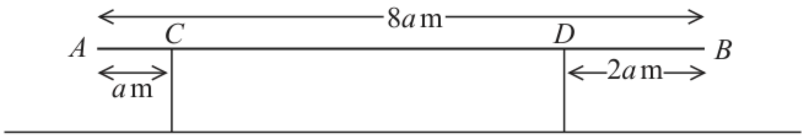

Figure 2

A non-uniform beam AB, of mass 60 kg and length $8a$ metres, rests in equilibrium in a horizontal position on two vertical supports. One support is at C, where $AC = a$ metres and the other support is at D, where $DB = 2a$ metres, as shown in Figure 2.

The magnitude of the normal reaction between the beam and the support at D is three times the magnitude of the normal reaction between the beam and the support at C.

By modelling the beam as a non-uniform rod whose centre of mass is at a distance x metres from A,

(a) find an expression for x in terms of a.

(5)

A box of mass M kg is placed on the beam at E, where $AE = 2a$ metres.

The beam remains in equilibrium in a horizontal position.

The magnitude of the normal reaction between the beam and the support at C is now equal to the magnitude of the normal reaction between the beam and the support at D.

By modelling the box as a particle,

(b) find the value of M.

(5)

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Worked Solution - Question 7

Moments

Topic group

1. Set the support positions

The beam length is $8a$. C is at distance a from A, and D is at distance $6a$ from A because $DB = 2a$.

2. Name the reactions

Let the reaction at C be R. The reaction at D is $3R$.

3. Use vertical equilibrium

$R + 3R = 60g$, so $4R = 60g$ and $R = 15g$.

4. Take moments about A

The beam's weight acts at distance x from A. Hence $60gx = R(a) + 3R(6a)$.

5. Find x

Substitute $R = 15g$: $60gx = 15ga + 45g(6a) = 285ga$. Thus $x = \dfrac{285a}{60} = \dfrac{19a}{4}$.

6. Use equal reactions with the box

When the box is added, let each support reaction be S. Vertical equilibrium gives $2S = (60 + M)g$.

7. Take moments about A again

The box is at $AE = 2a$. Moments about A give

$$60g\left(\frac{19a}{4}\right) + Mg(2a) = S(a) + S(6a) = 7aS.$$

8. Solve for M

Substitute $S = \dfrac{(60+M)g}{2}$ and cancel ag:

$285 + 2M = \dfrac{7}{2}(60+M) = 210 + \dfrac{7}{2}M$. Hence $75 = \dfrac{3}{2}M$ and $M = 50$.

Final answer

$(a)\ x = \dfrac{19a}{4}$. $\quad (b)\ M = 50$.

Question 8

8. Two trams, tram A and tram B, run on parallel straight horizontal tracks. Initially the two trams are at rest in the depot and level with each other.

At time $t = 0$, tram A starts to move. Tram A moves with constant acceleration $2\,\text{m}\,\text{s}^{-2}$ for 5 seconds and then continues to move along the track at constant speed.

At time $t = 20$ seconds, tram B starts from rest and moves in the same direction as tram A. Tram B moves with constant acceleration $3\,\text{m}\,\text{s}^{-2}$ for 4 seconds and then continues to move along the track at constant speed.

The trams are modelled as particles.

(a) Sketch, on the same axes, a speed-time graph for the motion of tram A and a speed-time graph for the motion of tram B, from $t = 0$ to the instant when tram B overtakes tram A.

(3)

At the instant when the two trams are moving with the same speed, tram A is d metres in front of tram B.

(b) Find the value of d.

(5)

(c) Find the distance of the trams from the depot at the instant when tram B overtakes tram A.

(5)

(Total 13 marks)

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Worked Solution - Question 8

Kinematics Graphs

Topic group

1. Describe tram A graph

Tram A starts at $t = 0$, accelerates at $2\ \mathrm{m\,s^{-2}}$ for 5 s, reaching speed $10\ \mathrm{m\,s^{-1}}$, then continues at this constant speed.

2. Describe tram B graph

Tram B starts at $t = 20$, accelerates at $3\ \mathrm{m\,s^{-2}}$ for 4 s, reaching speed $12\ \mathrm{m\,s^{-1}}$, then continues at this constant speed. Its graph crosses A's distance later.

3. Find when speeds are equal

Tram A's constant speed is 10. Tram B reaches speed 10 after $\frac{10}{3}$ s of acceleration, so the time from $t = 0$ is $20 + \frac{10}{3} = \frac{70}{3}$ s.

4. Distance of A then

A travels $\frac{1}{2}(5)(10) = 25$ m in its acceleration phase, then $10\left(\frac{70}{3} - 5\right) = \frac{550}{3}$ m. Total $= \frac{625}{3}$ m.

5. Distance of B then

B is still accelerating from rest, so its distance is $\frac{1}{2}\left(\frac{10}{3}\right)(10) = \frac{50}{3}$ m.

6. Find d

$d = \frac{625}{3} - \frac{50}{3} = \frac{575}{3}$ m.

7. Set overtake distances equal

For $t > 24$, A's distance is $25 + 10(t - 5) = 10t - 25$. B's distance is $\frac{1}{2}(4)(12) + 12(t - 24) = 12t - 264$.

8. Find the overtake time

Set $10t - 25 = 12t - 264$. Then $2t = 239$ and $t = 119.5$ s.

9. Find the distance from the depot

Using A's distance, $10(119.5) - 25 = 1170$ m.

Final answer

(a) Tram A reaches speed 10 at $t = 5$; tram B starts at $t = 20$, reaches speed 12 at $t = 24$, then later overtakes A.

$(b)\ d = \dfrac{575}{3}\ \mathrm{m} \approx 192\ \mathrm{m}.\quad (c)\ 1170\ \mathrm{m}.$

PAST PAPER

WME01/01 October 2021

October 2021 | 8 questions | 75 marks

8
questions

75
marks

Answers
worked solution after each question

WME01/01 OCTOBER 2021

Question 1

6 marks

Moments

1.

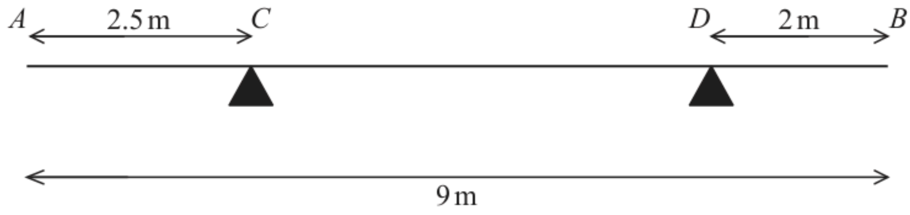

Figure 1

A non-uniform rod AB has length 9 m and mass M kg.

The rod rests in equilibrium in a horizontal position on two supports, one at C where $AC = 2.5$ m and the other at D where $DB = 2$ m, as shown in Figure 1.

The magnitude of the force acting on the rod at D is twice the magnitude of the force acting on the rod at C.

The centre of mass of the rod is d metres from A.

Find the value of d.

(6)

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Worked Solution - Question 1

Moments

Topic group

1. Set the support positions

C is 2.5 m from A. Since $DB = 2$ m and $AB = 9$ m, D is 7 m from A.

2. Name the reactions

Let the reaction at C be R. The reaction at D is twice this, so it is $2R$.

3. Use vertical equilibrium

$R + 2R = Mg$, so $3R = Mg$.

4. Take moments about A

$R(2.5) + 2R(7) = Mgd$.

5. Substitute Mg = 3R

$2.5R + 14R = 3Rd$. Hence $16.5R = 3Rd$.

6. Find d

Cancel R: $d = \dfrac{16.5}{3} = 5.5\text{ m}$.

Final answer

$d = 5.5\text{ m}$.

Question 2

10 marks

Momentum, Impulse & Collisions

2. A particle P of mass $2m$ is moving on a rough horizontal plane when it collides directly with a particle Q of mass $4m$ which is at rest on the plane. The speed of P immediately before the collision is $3u$. The speed of Q immediately after the collision is $2u$.

(a) Find, in terms of u, the speed of P immediately after the collision.

(3)

(b) State clearly the direction of motion of P immediately after the collision.

(1)

Following the collision, Q comes to rest after travelling a distance $\frac{6u^2}{g}$ along the plane.

The coefficient of friction between Q and the plane is μ.

(c) Find the value of μ.

(6)

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Worked Solution - Question 2

Momentum, Impulse & Collisions

Topic group

1. Use conservation of momentum

Take P's original direction as positive. Before collision, P has velocity $3u$ and Q is at rest. After collision, Q has velocity $2u$. Let P's velocity be v.

2. Find P velocity

$2m(3u) = 2mv + 4m(2u)$. Thus $6mu = 2mv + 8mu$, so $2mv = -2mu$ and $v = -u$.

3. State speed and direction

The speed of P is u. The negative sign shows that P moves in the opposite direction after the collision.

4. Find the deceleration of Q

After collision, Q starts with speed $2u$ and stops after distance $\frac{6u^2}{g}$. Use $v^2 = u^2 + 2as$: $0 = (2u)^2 + 2a\left(\frac{6u^2}{g}\right)$.

5. Simplify acceleration

$0 = 4u^2 + \frac{12au^2}{g}$, so $a = -\frac{g}{3}$. The deceleration magnitude is $\frac{g}{3}$.

6. Use friction on Q

For Q, $R = 4mg$, so friction is $\mu R = 4\mu mg$. This friction produces deceleration: $4\mu mg = 4m \cdot \frac{g}{3}$.

7. Find mu

Cancel $4mg$: $\mu = \frac{1}{3}$.

Final answer

(a) speed of $\mathrm{P} = u$. (b) P reverses direction. (c) $\mu = \dfrac{1}{3}$.

Question 3

10 marks

Constant Acceleration in 1D

3. A car is moving at a constant speed of $25\,\text{m}\,\text{s}^{-1}$ along a straight horizontal road.

The car is modelled as a particle.

At time $t = 0$, the car is at the point A and the driver sees a road sign 48 m ahead.

Let t seconds be the time that elapses after the car passes A.

In a **first** model, the car is assumed to decelerate uniformly at $6\,\text{m}\,\text{s}^{-2}$ from A until the car reaches the road sign.

(a) Use this first model to find the speed of the car as it reaches the sign.

(2)

The road sign indicates that the speed limit immediately after the sign is $13\,\text{m}\,\text{s}^{-1}$.

In a **second** model, the car is assumed to decelerate uniformly at $6\,\text{m}\,\text{s}^{-2}$ from A until it reaches a speed of $13\,\text{m}\,\text{s}^{-1}$. The car then maintains this speed until it reaches the road sign.

(b) Use this second model to find the value of t at which the car reaches the sign.

(4)

In a **third** model, the car is assumed to move with constant speed $25\,\text{m}\,\text{s}^{-1}$ from A until time $t = 0.2$, the car then decelerates uniformly at $6\,\text{m}\,\text{s}^{-2}$ until it reaches a speed of $13\,\text{m}\,\text{s}^{-1}$. The car then maintains this speed until it reaches the road sign.

(c) Use this third model to find the value of t at which the car reaches the sign.

(4)

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Worked Solution - Question 3

Constant Acceleration in 1D

Topic group

1. Use the first model

From A to the sign, $u = 25$, $a = -6$ and $s = 48$. Use $v^2 = u^2 + 2as$.

2. Find the speed at the sign

$v^2 = 25^2 + 2(-6)(48) = 625 - 576 = 49$, so $v = 7\ \mathrm{m\,s^{-1}}$.

3. Second model: time to reach 13

Use $v = u + at$: $13 = 25 - 6t$, so $t = 2$ s to slow to $13\ \mathrm{m\,s^{-1}}$.

4. Second model: distance while slowing

Distance while slowing is $\dfrac{25+13}{2} \cdot 2 = 38$ m.

5. Second model: finish time

The remaining distance is $48 - 38 = 10$ m at speed 13. Total time $= 2 + \dfrac{10}{13} = \dfrac{36}{13}$ s.

6. Third model: first 0.2 seconds

The car first travels at $25\ \mathrm{m\,s^{-1}}$ for 0.2 s, covering 5 m.

7. Third model: slowing distance

It then takes another 2 s to slow from 25 to 13, covering 38 m as before. Total covered before constant-speed part is $5 + 38 = 43$ m.

8. Third model: finish time

The remaining distance is $48 - 43 = 5$ m at speed 13. Total time $= 0.2 + 2 + \dfrac{5}{13} = \dfrac{168}{65}$ s.

Final answer

(a) $7\,\mathrm{m\,s^{-1}}$. (b) $t = \dfrac{36}{13}\,\mathrm{s} \approx 2.77\,\mathrm{s}$. (c) $t = \dfrac{168}{65}\,\mathrm{s} \approx 2.58\,\mathrm{s}$.

Question 4

8 marks

Working with Vectors

4. The position vector, **r** metres, of a particle P at time t seconds, relative to a fixed origin O, is given by

$$\mathbf{r} = (t - 3)\mathbf{i} + (1 - 2t)\mathbf{j}$$

(a) Find, to the nearest degree, the size of the angle between **r** and the vector **j**, when $t = 2$ **(3)**

(b) Find the values of t for which the distance of P from O is 2.5 m. **(5)**

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Worked Solution - Question 4

Working with Vectors

Topic group

1. Find r when t = 2

$\mathbf{r} = (2-3)\mathbf{i} + (1-2(2))\mathbf{j} = -\mathbf{i} - 3\mathbf{j}$.

2. Find the angle with j

The vector is in the third quadrant. The angle between $\mathbf{r}$ and the positive $\mathbf{j}$ direction satisfies $\tan\alpha = \frac{1}{3}$ from the vertical comparison.

3. State the angle

$\alpha = 18.4^\circ$ from the negative $\mathbf{j}$ direction, so the size of the angle with positive $\mathbf{j}$ is $180^\circ - 18.4^\circ = 161.6^\circ$, which is 162°.

4. Use the distance condition

Distance from O is 2.5, so $(t-3)^2 + (1-2t)^2 = 2.5^2$.

5. Form the quadratic

$t^2 - 6t + 9 + 1 - 4t + 4t^2 = 6.25$, so $5t^2 - 10t + 3.75 = 0$.

6. Solve

Multiply by 4: $20t^2 - 40t + 15 = 0$, or $4t^2 - 8t + 3 = 0$. Hence $(2t-1)(2t-3) = 0$, so $t = \frac{1}{2}$ or $t = \frac{3}{2}$.

Final answer

$(a)\ 162^\circ. \quad (b)\ t = \frac{1}{2} \text{ or } t = \frac{3}{2}.$

Question 5

10 marks

Forces

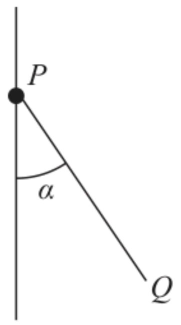

Figure 2

A small bead of mass 0.2 kg is attached to the end P of a light rod PQ. The bead is threaded onto a fixed vertical rough wire.

The bead is held in equilibrium with the rod PQ inclined to the wire at an angle α, where $\tan\alpha = \frac{4}{3}$, as shown in Figure 2.

The thrust in the rod is T newtons.

The bead is modelled as a particle.

(a) Find the magnitude and direction of the friction force acting on the bead when $T = 2.5$

(3)

The coefficient of friction between the bead and the wire is μ.

Given that the greatest possible value of T is 6.125

(b) find the value of μ.

(7)

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Worked Solution - Question 5

Forces

Topic group

1. Use the trig ratios

Since $\tan\alpha = \frac{4}{3}$, $\sin\alpha = \frac{4}{5}$ and $\cos\alpha = \frac{3}{5}$.

2. Resolve vertically when T = 2.5

The vertical component of the thrust is $2.5\cos\alpha = 2.5 \cdot \frac{3}{5} = 1.5$ N upward. The weight is $0.2g = 1.96$ N downward.

3. Find friction and direction

For equilibrium, friction must supply the missing upward force: $F = 1.96 - 1.5 = 0.46\ \mathrm{N}$. So friction acts upwards.

4. Use greatest T

When $T = 6.125$, the vertical component of the thrust is larger than the weight, so friction acts downward at the limiting value.

5. Find normal reaction

The normal reaction is the horizontal component of the thrust: $R = 6.125\sin\alpha = 6.125 \cdot \frac{4}{5} = 4.9\ \mathrm{N}$.

6. Find limiting friction

Vertical equilibrium gives $0.2g + F = 6.125\cos\alpha$. Thus $1.96 + F = 6.125 \cdot \frac{3}{5} = 3.675$, so $F = 1.715$ N.

7. Find mu

$F = \mu R$, so $\mu = \frac{1.715}{4.9} = 0.35$.

Final answer

(a) $F = 0.46$ N upwards. (b) $\mu = 0.35$.

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Question 6

5 marks

Constant Acceleration in 1D

6.

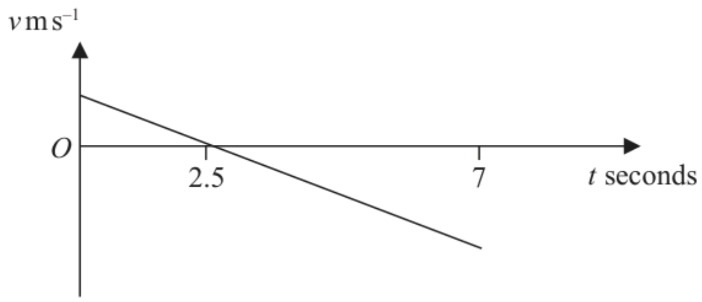

Figure 3

A small ball is thrown vertically upwards at time $t = 0$ from a point A which is above horizontal ground. The ball hits the ground 7 s later.

The ball is modelled as a particle moving freely under gravity.

The velocity-time graph shown in Figure 3 represents the motion of the ball for $0 \leqslant t \leqslant 7$

(a) Find the speed with which the ball is thrown.

(2)

(b) Find the height of A above the ground.

(3)

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Worked Solution - Question 6

Constant Acceleration in 1D

Topic group

1. Use the velocity-time graph

The velocity becomes zero at $t = 2.5$ s. Since acceleration is $-g = -9.8\ \mathrm{m\,s^{-2}}$, the initial velocity is $u = 9.8(2.5)$.

2. Find the initial speed

$u = 24.5\ \mathrm{m\,s^{-1}}$.

3. Use displacement to the ground

From launch to $t = 7$, the displacement is the area under the velocity-time graph. Algebraically, $s = ut - \frac{1}{2}gt^2$ with $t = 7$.

4. Calculate displacement

$s = 24.5(7) - \frac{1}{2}(9.8)(7^2) = 171.5 - 240.1 = -68.6$ m.

5. State the height

The negative sign shows the ground is below A, so the height of A above the ground is $68.6\ \mathrm{m}$.

Final answer

$(a)\ 24.5\ \mathrm{m\,s^{-1}}. \quad (b)\ 68.6\ \mathrm{m\ approximately}.$

Question 7

14 marks

Resolving Forces, Inclined Planes

7.

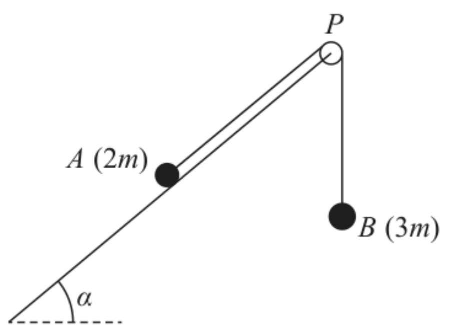

Figure 4

One end of a light inextensible string is attached to a particle A of mass $2m$. The other end of the string is attached to a particle B of mass $3m$. The string passes over a small, smooth, light pulley P which is fixed at the top of a rough inclined plane. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

Particle A is held at rest on the plane with the string taut and B hanging freely below P, as shown in Figure 4. The section of the string AP is parallel to a line of greatest slope of the plane.

The coefficient of friction between A and the plane is $\frac{1}{2}$

Particle A is released and begins to move up the plane.

For the motion before A reaches the pulley,

(a) (i) write down an equation of motion for A,

(ii) write down an equation of motion for B,

(4)

(b) find, in terms of g, the acceleration of A,

(5)

(c) find the magnitude of the force exerted on the pulley by the string.

(4)

(d) State how you have used the information that P is a smooth pulley.

(1)

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Worked Solution - Question 7

Resolving Forces, Inclined Planes

Topic group

1. Equation for A

A moves up the plane, so friction and the component of weight act down the plane. Taking up the plane as positive: $T - 2mg\sin\alpha - F = 2ma$.

2. Equation for B

B moves downward. Taking downward as positive: $3mg - T = 3ma$.

3. Find friction on A

Since $\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$. The normal reaction on A is $R = 2mg\cos\alpha = \frac{8mg}{5}$, so $F = \frac{1}{2}R = \frac{4mg}{5}$.

4. Simplify A's equation

$T - 2mg \cdot \frac{3}{5} - \frac{4mg}{5} = 2ma$, so $T - 2mg = 2ma$.

5. Solve with B's equation

From B, $T = 3mg - 3ma$. Substitute into $T - 2mg = 2ma$: $3mg - 3ma - 2mg = 2ma$. Thus $mg = 5ma$ and $a = \frac{g}{5}$.

6. Find the tension

Using $3mg - T = 3m\left(\frac{g}{5}\right)$ gives $T = 3mg - \frac{3mg}{5} = \frac{12mg}{5}$.

7. Find the pulley resultant

The two tensions are one vertical and one along the plane. Components of the resultant are $T\cos\alpha$ horizontally and $T + T\sin\alpha$ vertically.

8. Calculate the magnitude

Resultant $= T\sqrt{\cos^2\alpha + (1+\sin\alpha)^2} = T\sqrt{(\frac{4}{5})^2 + (\frac{8}{5})^2} = \frac{4\sqrt{5}}{5}T$. With $T = \frac{12mg}{5}$, this is $\frac{48\sqrt{5}}{25}mg$.

9. Use the smooth pulley information

The pulley is smooth, so the tension is the same on both sides of the pulley.

Final answer

$(a)(i)\ T - 2mg\sin\alpha - F = 2ma. \quad (a)(ii)\ 3mg - T = 3ma. \quad (b)\ a = \frac{g}{5}. \quad (c)\ \text{resultant} = \frac{48\sqrt{5}}{25}mg. \quad (d)$
The tension is the same on both sides of the pulley.

Question 8

12 marks

Working with Vectors

8. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin.*]

At 7 am a ship leaves a port and moves with constant velocity. The position vector of the port is $(-2\mathbf{i} + 9\mathbf{j})$ km.

At 7.36 am the ship is at the point with position vector $(4\mathbf{i} + 6\mathbf{j})$ km.

(a) Show that the velocity of the ship is $(10\mathbf{i} - 5\mathbf{j})\,\text{km h}^{-1}$

(2)

(b) Find the position vector of the ship t hours after leaving port.

(2)

At 8.48 am a passenger on the ship notices that a lighthouse is due east of the ship.

At 9 am the same passenger notices that the lighthouse is now north east of the ship.

(c) Find the position vector of the lighthouse.

(4)

(d) Find the position vector of the ship when it is due south of the lighthouse.

(4)

(Total 12 marks)

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Worked Solution - Question 8

Working with Vectors

Topic group

1. Convert the time

From 7:00 to 7:36 is 36 minutes, which is 0.6 hours.

2. Find velocity

Displacement $= (4\mathbf{i} + 6\mathbf{j}) - (-2\mathbf{i} + 9\mathbf{j}) = 6\mathbf{i} - 3\mathbf{j}$. Divide by 0.6: $\mathbf{v} = 10\mathbf{i} - 5\mathbf{j}$.

3. Write the ship position

$\mathbf{r} = (-2\mathbf{i} + 9\mathbf{j}) + t(10\mathbf{i} - 5\mathbf{j}) = (-2 + 10t)\mathbf{i} + (9 - 5t)\mathbf{j}$.

4. Use 8:48

At 8:48, $t = 1.8$. The ship position is $(-2 + 18)\mathbf{i} + (9 - 9)\mathbf{j} = 16\mathbf{i}$. The lighthouse is due east, so it has the same j-component, 0.

5. Use 9:00

At 9:00, $t = 2$. The ship position is $18\mathbf{i} - \mathbf{j}$. The lighthouse is north-east of the ship, so its east and north displacements from the ship are equal.

6. Find the lighthouse position

Since the lighthouse has j-component 0, from $18\mathbf{i} - \mathbf{j}$ it is 1 km north, so it is also 1 km east. Hence its position is $19\mathbf{i}$.

7. Find when the ship is due south

Due south of the lighthouse means the ship has i-component 19. Set $-2 + 10t = 19$, so $t = 2.1$.

8. Find the ship position

At $t = 2.1$, $\mathbf{r} = (-2 + 21)\mathbf{i} + (9 - 10.5)\mathbf{j} = 19\mathbf{i} - 1.5\mathbf{j}$.

Final answer

(a) $\mathbf{v} = 10\mathbf{i} - 5\mathbf{j}\ \mathrm{km\,h^{-1}}$. (b) $\mathbf{r} = (-2 + 10t)\mathbf{i} + (9 - 5t)\mathbf{j}$. (c) lighthouse $= 19\mathbf{i}$. (d) ship position $= 19\mathbf{i} - 1.5\mathbf{j}$

.

PAST PAPER

WME01/01 January 2022

January 2022 | 8 questions | 75 marks

8 questions	**75** marks	**Answers** worked solution after each question

WME01/01 JANUARY 2022

6 marks

Question 1

Forces

1.

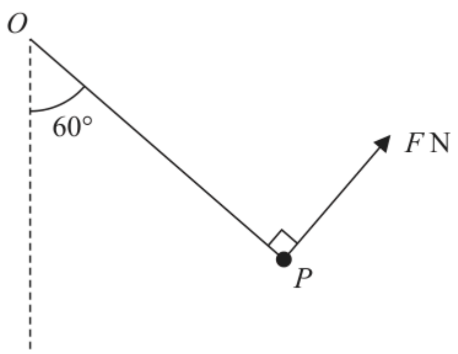

Figure 1

A particle P of weight 5 N is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. The particle P is held in equilibrium by a force of magnitude F newtons. The direction of this force is perpendicular to the string and OP makes an angle of 60° with the vertical, as shown in Figure 1.

Find

(a) the value of F

(3)

(b) the tension in the string.

(3)

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Worked Solution - Question 1

Forces

Topic group

1. Resolve perpendicular to the string

The force F is perpendicular to the string. The weight is 5 N vertically downward, and the string makes 60° with the vertical.

2. Find F

The component of the weight perpendicular to the string is $5\cos 30^\circ$, so

$$F = 5\cos 30^\circ = \frac{5\sqrt{3}}{2}\ \mathrm{N}.$$

3. Resolve along the string

The tension balances the component of the weight along the string. This component is $5\sin 30^\circ$.

4. Find T

$$T = 5\sin 30^\circ = \frac{5}{2}\ \mathrm{N}.$$

Final answer

$$(a)\ F = \frac{5\sqrt{3}}{2}\ \mathrm{N} \approx 4.3\ \mathrm{N}. \quad (b)\ T = \frac{5}{2}\ \mathrm{N}.$$

Question 2

2. A particle P has mass km and a particle Q has mass m. The particles are moving towards each other in opposite directions along the same straight line when they collide directly.

Immediately before the collision, P has speed $3u$ and Q has speed u.

As a result of the collision, the direction of motion of each particle is reversed and the speed of each particle is halved.

(a) Find the value of k.

(4)

(b) Find, in terms of m and u, the magnitude of the impulse exerted on Q in the collision.

(3)

Worked Solution - Question 2

Momentum, Impulse & Collisions

Topic group

1. Choose a positive direction

Take the original direction of P as positive. Before the collision, P has velocity $3u$ and Q has velocity $-u$.

2. Write the velocities after collision

After collision, each direction is reversed and each speed is halved. So P has velocity $-\frac{3}{2}u$ and Q has velocity $\frac{1}{2}u$.

3. Use conservation of momentum

$$km(3u) + m(-u) = km\left(-\frac{3}{2}u\right) + m\left(\frac{1}{2}u\right).$$

4. Find k

Divide by mu: $3k - 1 = -\frac{3}{2}k + \frac{1}{2}$. Hence $\frac{9}{2}k = \frac{3}{2}$, so $k = \frac{1}{3}$.

5. Find impulse on Q

Impulse on Q equals change in momentum of Q:
$I = m\left(\frac{1}{2}u - (-u)\right) = \frac{3}{2}mu.$

6. State the magnitude

The magnitude of the impulse is $\frac{3}{2}mu$.

Final answer
$(a)\ k = \frac{1}{3}. \quad (b)\ I = \frac{3}{2}mu.$

Question 3

7 marks

Moments

3.

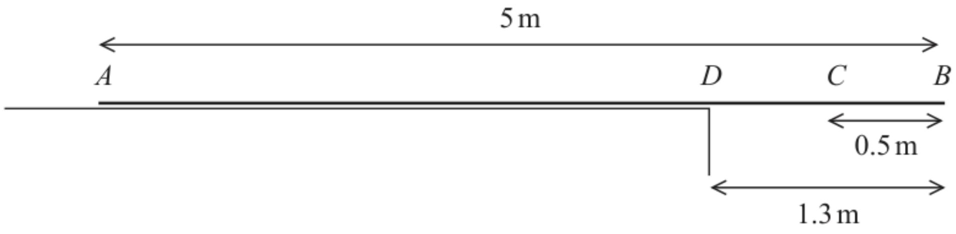

Figure 2

A beam *ADCB* has length 5 m. The beam lies on a horizontal step with the end *A* on the step and the end *B* projecting over the edge of the step. The edge of the step is at the point *D* where $DB = 1.3$ m, as shown in Figure 2.

When a small boy of mass 30 kg stands on the beam at *C*, where $CB = 0.5$ m, the beam is on the point of tilting.

The boy is modelled as a particle and the beam is modelled as a uniform rod.

(a) Find the mass of the beam.

(3)

A block of mass X kg is now placed on the beam at *A*.

The block is modelled as a particle.

(b) Find the smallest value of X that will enable the boy to stand on the beam at *B* without the beam tilting.

(3)

(c) State how you have used the modelling assumption that the block is a particle in your calculations.

(1)

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Worked Solution - Question 3

Moments

Topic group

1. Set the distances from D

The beam length is 5 m and $DB = 1.3$ m, so D is 3.7 m from A. The centre of mass of the uniform beam is at its midpoint, so it is 1.2 m to the left of D. Point C is 0.8 m to the right of D.

2. Use the first tilting condition

When the boy stands at C, the beam is about to tilt about D. Let the mass of the beam be M kg.

3. Take moments about D

$Mg(1.2) = 30g(0.8)$. Hence $1.2M = 24$ and $M = 20\text{ kg}$.

4. Set up the second tilting condition

With the boy at B, his moment about D is $30g(1.3)$. The block at A is 3.7 m to the left of D, and the beam's weight acts 1.2 m to the left of D.

5. Take moments about D again

$Xg(3.7) + 20g(1.2) = 30g(1.3)$.

6. Find X

$3.7X + 24 = 39$, so $3.7X = 15$ and $X = \dfrac{150}{37} = 4.05\ldots\text{ kg}$.

7. Use the particle model

The block is modelled as a particle, so its mass is treated as acting at one point, A.

Final answer

(a) beam mass $= 20$ kg. (b) $X = \dfrac{150}{37}$ kg ≈ 4.05 kg. (c) The block's mass is treated as concentrated at A.

Question 4

8 marks

Constant Acceleration in 1D

4. At time $t = 0$, a small ball is projected vertically upwards from a point A which is 24.5 m above the ground. The ball first comes to instantaneous rest at the point B, where $AB = 19.6$ m and first hits the ground at time $t = T$ seconds.

The ball is modelled as a particle moving freely under gravity.

(a) Find the value of T.

(6)

(b) Sketch a speed-time graph for the motion of the ball from $t = 0$ to $t = T$ seconds.

(No further calculations are needed in order to draw this sketch.)

(2)

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Worked Solution - Question 4

Constant Acceleration in 1D

Topic group

1. Find the initial speed

At the highest point B, the velocity is 0 and $AB = 19.6$ m. Use $v^2 = u^2 + 2as$ upward: $0 = u^2 - 2g(19.6)$.

2. Calculate u

$u^2 = 2(9.8)(19.6) = 384.16$, so $u = 19.6\,\mathrm{m\,s^{-1}}$.

3. Use displacement to the ground

From A to the ground, taking upward as positive, displacement is -24.5 m. Use $s = ut + \frac{1}{2}at^2$: $-24.5 = 19.6T - 4.9T^2$.

4. Solve for T

$4.9T^2 - 19.6T - 24.5 = 0$. Divide by 4.9: $T^2 - 4T - 5 = 0$, so $(T-5)(T+1) = 0$. The positive time is $T = 5$ s.

5. Describe the speed-time graph

The speed starts at 19.6, decreases linearly to 0 at $t = 2$, then increases linearly until the ball hits the ground at $t = 5$. The final speed is 29.4, so the second line is longer than the first.

Final answer

(a) $T = 5$ s. (b) Speed-time graph is V-shaped, from speed 19.6 at $t = 0$ to 0 at $t = 2$, then to speed 29.4 at $t = 5$.

Question 5

8 marks

Resolving Forces, Inclined Planes

5.

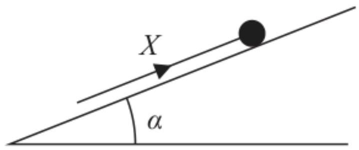

Figure 3

A particle of mass m rests in equilibrium on a fixed rough plane under the action of a force of magnitude X. The force acts up a line of greatest slope of the plane, as shown in Figure 3.

The plane is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$

The coefficient of friction between the particle and the plane is μ.

- When $X = 2P$, the particle is on the point of sliding up the plane.
- When $X = P$, the particle is on the point of sliding down the plane.

Find the value of μ.

(8)

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Worked Solution - Question 5

Resolving Forces, Inclined Planes

Topic group

1. Use the trig ratios

Since $\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$.

2. Resolve perpendicular to the plane

The force X acts along the plane, so the normal reaction is $R = mg\cos\alpha = \frac{4}{5}mg$.

3. Use friction

The limiting friction is $F = \mu R = \frac{4}{5}\mu mg$.

4. Use the sliding-up condition

When $X = 2P$, the particle is about to slide up, so friction acts down the plane. Hence $2P = mg\sin\alpha + F = \frac{3}{5}mg + \frac{4}{5}\mu mg$.

5. Use the sliding-down condition

When $X = P$, the particle is about to slide down, so friction acts up the plane. Hence $P + F = mg\sin\alpha$, so $P = \frac{3}{5}mg - \frac{4}{5}\mu mg$.

6. Eliminate P

Double the second equation: $2P = \frac{6}{5}mg - \frac{8}{5}\mu mg$. This equals the first expression for $2P$.

7. Solve for mu

$\frac{6}{5}mg - \frac{8}{5}\mu mg = \frac{3}{5}mg + \frac{4}{5}\mu mg$. Cancel mg and multiply by 5: $6 - 8\mu = 3 + 4\mu$. Hence $3 = 12\mu$ and $\mu = \frac{1}{4}$.

Final answer

$\mu = \frac{1}{4}.$

Question 6

12 marks

Working with Vectors

6. [*In this question* **i** *and* **j** *are horizontal unit vectors.*]

A particle P of mass 2 kg moves under the action of two forces, $(p\mathbf{i} + q\mathbf{j})$ N and $(2q\mathbf{i} + p\mathbf{j})$ N, where p and q are constants.

Given that the acceleration of P is $(\mathbf{i} - \mathbf{j})\,\text{m}\,\text{s}^{-2}$

(a) find the value of p and the value of q.

(5)

(b) Find the size of the angle between the direction of the acceleration and the vector **j**.

(2)

At time $t = 0$, the velocity of P is $(3\mathbf{i} - 4\mathbf{j})\,\text{m}\,\text{s}^{-1}$

At $t = T$ seconds, P is moving in the direction of the vector $(11\mathbf{i} - 13\mathbf{j})$.

(c) Find the value of T.

(5)

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Worked Solution - Question 6

Working with Vectors

Topic group

1. Use F = ma

The total force is $(p\mathbf{i} + q\mathbf{j}) + (2q\mathbf{i} + p\mathbf{j}) = (p + 2q)\mathbf{i} + (p + q)\mathbf{j}$. Since the mass is 2 kg and acceleration is $\mathbf{i} - \mathbf{j}$, the total force is $2\mathbf{i} - 2\mathbf{j}$.

2. Equate components

$p + 2q = 2$ and $p + q = -2$.

3. Find p and q

Subtract the equations to get $q = 4$. Then $p + 4 = -2$, so $p = -6$.

4. Find the angle with j

The acceleration is $\mathbf{i} - \mathbf{j}$. Its dot product with $\mathbf{j}$ is -1, so the angle with the positive $\mathbf{j}$ direction is obtuse. Since the components are equal in magnitude, the angle is $135°$.

5. Write the velocity at time T

$\mathbf{v} = (3\mathbf{i} - 4\mathbf{j}) + T(\mathbf{i} - \mathbf{j}) = (3 + T)\mathbf{i} + (-4 - T)\mathbf{j}$.

6. Use the direction vector

The velocity is parallel to $11\mathbf{i} - 13\mathbf{j}$, so $\dfrac{3 + T}{11} = \dfrac{-4 - T}{-13}$.

7. Solve for T

$13(3 + T) = 11(4 + T)$, so $39 + 13T = 44 + 11T$. Hence $2T = 5$ and $T = 2.5$.

Final answer

$(a)\ p = -6,\ q = 4.\quad (b)\ 135^\circ.\quad (c)\ T = 2.5.$

Question 7

Resolving Forces, Inclined Planes

7.

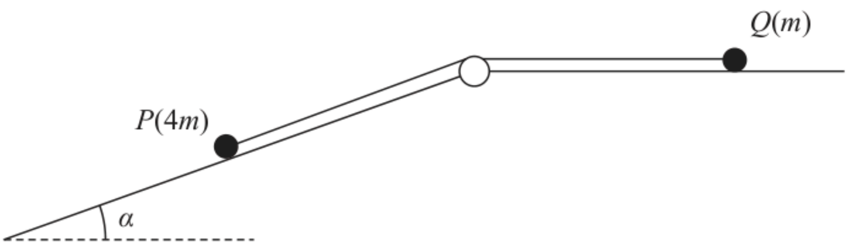

Figure 4

A particle P of mass $4m$ lies on the surface of a fixed rough inclined plane.

The plane is inclined to the horizontal at an angle α where $\tan\alpha = \frac{3}{4}$

The particle P is attached to one end of a light inextensible string.

The string passes over a small smooth pulley that is fixed at the top of the plane. The other end of the string is attached to a particle Q of mass m which lies on a smooth horizontal plane.

The string lies along the horizontal plane and in the vertical plane that contains the pulley and a line of greatest slope of the inclined plane.

The system is released from rest with the string taut, as shown in Figure 4, and P moves down the plane.

The coefficient of friction between P and the plane is $\frac{1}{4}$

For the motion before Q reaches the pulley

(a) write down an equation of motion for Q,

(1)

(b) find, in terms of m and g, the tension in the string,

(7)

(c) find the magnitude of the force exerted on the pulley by the string.

(4)

(d) State where in your working you have used the information that the string is light.

(1)

WME01/01 JANUARY 2022

Worked Solution - Question 7

Resolving Forces, Inclined Planes

Topic group

1. Write the equation for Q

Q lies on a smooth horizontal plane. The only horizontal force on Q is the tension, so $T = ma$.

2. Use the trig ratios

Since $\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$.

3. Find friction on P

For P, $R = 4mg\cos\alpha = 4mg \cdot \frac{4}{5} = \frac{16mg}{5}$. With coefficient $\frac{1}{4}$, friction is $F = \frac{1}{4}R = \frac{4mg}{5}$.

4. Apply Newtons second law to P

P moves down the plane. Taking down the plane as positive:
$4mg\sin\alpha - T - F = 4ma$.

5. Substitute values

$4mg \cdot \frac{3}{5} - T - \frac{4mg}{5} = 4ma$, so $\frac{8mg}{5} - T = 4ma$.

6. Use T = ma

From Q, $a = \frac{T}{m}$. Substitute into the equation for P: $\frac{8mg}{5} - T = 4T$.

7. Find T

$\frac{8mg}{5} = 5T$, so $T = \frac{8}{25}mg$.

8. Find the pulley resultant

The pulley is pulled by two tensions of magnitude T: one horizontal and one along the inclined string. Their resultant has components $T - T\cos\alpha$ horizontally and $T\sin\alpha$ vertically.

9. Calculate the resultant

Magnitude $= \sqrt{(T - T\cos\alpha)^2 + (T\sin\alpha)^2} = T\sqrt{(1-\frac{4}{5})^2 + (\frac{3}{5})^2} = \frac{T}{5}\sqrt{10}$. With $T = \frac{8}{25}mg$, the resultant is $\frac{8\sqrt{10}}{125}mg$.

10. Use the light-string model

The string is light, so the tension can be treated as the same on the P side and the Q side of the pulley.

Final answer

$(a)\ T = ma. \quad (b)\ T = \frac{8}{25}mg. \quad (c)\ \text{resultant} = \frac{8\sqrt{10}}{125}mg. \quad (d)$ Same tension was used on both sides of the pulley.

Question 8

14 marks

Working with Vectors

8. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin.*]

A ship A moves with constant velocity $(3\mathbf{i} - 10\mathbf{j})\,\text{km h}^{-1}$

At time t hours, the position vector of A is $\mathbf{r}$ km.

At time $t = 0$, A is at the point with position vector $(13\mathbf{i} + 5\mathbf{j})$ km.

(a) Find $\mathbf{r}$ in terms of t.

(2)

Another ship B moves with constant velocity $(15\mathbf{i} + 14\mathbf{j})\,\text{km h}^{-1}$

At time $t = 0$, B is at the point with position vector $(3\mathbf{i} - 5\mathbf{j})$ km.

(b) Show that, at time t hours,

$$\overrightarrow{AB} = [(12t - 10)\mathbf{i} + (24t - 10)\mathbf{j}]\,\text{km}$$

(4)

Given that the two ships do not change course,

(c) find the shortest distance between the two ships,

(6)

(d) find the bearing of ship B from ship A when the ships are closest.

(2)

(Total 14 marks)

Worked Solution - Question 8

Working with Vectors

Topic group

1. Write the position of A

$\mathbf{r} = (13\mathbf{i} + 5\mathbf{j}) + t(3\mathbf{i} - 10\mathbf{j}) = (13 + 3t)\mathbf{i} + (5 - 10t)\mathbf{j}$.

2. Write the position of B

$\mathbf{s} = (3\mathbf{i} - 5\mathbf{j}) + t(15\mathbf{i} + 14\mathbf{j}) = (3 + 15t)\mathbf{i} + (-5 + 14t)\mathbf{j}$.

3. Find AB

$\overrightarrow{AB} = \mathbf{s} - \mathbf{r} = [3 + 15t - (13 + 3t)]\mathbf{i} + [-5 + 14t - (5 - 10t)]\mathbf{j}$.

4. Show the expression

This simplifies to $\overrightarrow{AB} = (12t - 10)\mathbf{i} + (24t - 10)\mathbf{j}$ km.

5. Use distance squared

The square of the distance is $d^2 = (12t - 10)^2 + (24t - 10)^2$.

6. Expand and minimise

$d^2 = 720t^2 - 720t + 200$. This quadratic is minimum at $t = \dfrac{720}{2 \cdot 720} = \dfrac{1}{2}$.

7. Find the shortest distance

At $t = \frac{1}{2}$, $\overrightarrow{AB} = (-4)\mathbf{i} + 2\mathbf{j}$, so the shortest distance is $\sqrt{(-4)^2 + 2^2} = \sqrt{20} = 2\sqrt{5}\ \mathrm{km}$.

8. Find the bearing

At closest approach, B is west and north of A. The angle west of north satisfies $\tan\theta = \frac{4}{2} = 2$, so $\theta = 63.4°$. Bearing $= 360° - 63.4° = 296.6°$, which is $297°$.

Final answer

(a) $\mathbf{r} = (13 + 3t)\mathbf{i} + (5 - 10t)\mathbf{j}$. (b) $\overrightarrow{AB} = (12t - 10)\mathbf{i} + (24t - 10)\mathbf{j}$. (c) $2\sqrt{5}\ \text{km} \approx 4.5\ \text{km}$. (d) 297°

.

PAST PAPER

WME01/01 May/June 2022

May/June 2022 | 8 questions | 75 marks

8 questions	**75** marks	**Answers** worked solution after each question

WME01/01 MAY/JUNE 2022

4 marks

Question 1

Momentum, Impulse & Collisions

1. Two particles, P and Q, are moving towards each other in opposite directions along the same straight line when they collide directly. Immediately before the collision the speed of Q is $2u$. The mass of Q is $3m$ and the magnitude of the impulse exerted by P on Q in the collision is $4mu$.

Find

(a) the speed of Q immediately after the collision,

(3)

(b) the direction of motion of Q immediately after the collision.

(1)

WME01/01 MAY/JUNE 2022

Worked Solution - Question 1

Momentum, Impulse & Collisions

Topic group

1. Use impulse as change in momentum

Take the original direction of Q as positive. Q has mass $3m$ and initial velocity $2u$. Let its velocity after collision be v.

2. Set up the impulse equation

The impulse has magnitude $4mu$ and acts opposite to Q's original motion, so $4mu = 3m(2u - v)$.

3. Find the speed

Cancel m: $4u = 6u - 3v$. Thus $3v = 2u$ and $v = \dfrac{2u}{3}$.

4. State the direction

Since v is positive in Q's original direction, Q continues in the same direction after the collision.

Final answer

$(a)\ \text{speed} = \dfrac{2u}{3}. \quad (b)\ \text{same direction as before the collision.}$

WME01/01 MAY/JUNE 2022

Question 2

7 marks

Constant Acceleration in 1D

2. A motorbike is moving with constant acceleration along a straight horizontal road.

The motorbike passes a point P and 10 seconds later passes a point Q.

The speed of the motorbike as it passes Q is $28\,\text{m s}^{-1}$

Given that $PQ = 220\,\text{m}$,

(a) find the acceleration of the motorbike,

(3)

(b) find the distance travelled by the motorbike during the fifth second after passing P

(4)

WME01/01 MAY/JUNE 2022

Worked Solution - Question 2

Constant Acceleration in 1D

Topic group

1. Find the speed at P

Let the speed at P be u. Over the 10 seconds from P to Q, the speed changes uniformly from u to 28, and the distance is 220 m.

2. Use average speed

$220 = \frac{u+28}{2} \cdot 10 = 5(u+28)$. Hence $u + 28 = 44$ and $u = 16\ \mathrm{m\,s^{-1}}$.

3. Find the acceleration

Use $v = u + at$: $28 = 16 + 10a$, so $a = 1.2\ \mathrm{m\,s^{-2}}$.

4. Write the distance from P after t seconds

$s = 16t + \frac{1}{2}(1.2)t^2 = 16t + 0.6t^2$.

5. Find the distance in the fifth second

The fifth second is from $t = 4$ to $t = 5$. $s(5) = 16(5) + 0.6(25) = 95$ and $s(4) = 16(4) + 0.6(16) = 73.6$.

6. Subtract

Distance in the fifth second $= 95 - 73.6 = 21.4\ \mathrm{m}$.

Final answer

$(a)\ a = 1.2\ \mathrm{m\,s^{-2}}$. $(b)\ 21.4\ \mathrm{m}$.

Question 3

7 marks

Newton's Second Law

3. A tractor of mass 6 tonnes is dragging a large block of mass 2 tonnes along rough horizontal ground. The cable connecting the tractor to the block is horizontal and parallel to the direction of motion.

The cable is modelled as being light and inextensible.

The driving force of the tractor is 7400 N and the resistance to the motion of the tractor is 200 N. The resistance to the motion of the block is R newtons, where R is a constant.

Given that the tension in the cable is 6000 N and the tractor is accelerating,

(a) find the value of R.

(6)

(b) State how you have used the fact that the cable is modelled as being inextensible.

(1)

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Worked Solution - Question 3

Newton's Second Law

Topic group

1. Apply Newtons second law to the tractor

The tractor has mass 6000 kg. Its forward driving force is 7400 N, and the backward forces are 200 N resistance and 6000 N tension.

2. Find the acceleration

$7400 - 200 - 6000 = 6000a$, so $1200 = 6000a$ and $a = 0.2\,\mathrm{m\,s^{-2}}$.

3. Apply Newtons second law to the block

The block has mass 2000 kg. The forward force is the tension 6000 N and the resistance is R N.

4. Find R

$6000 - R = 2000(0.2) = 400$. Therefore $R = 5600\ \mathrm{N}$.

5. Use the inextensible cable model

The cable is inextensible, so the tractor and block move together with the same acceleration.

Final answer

(a) $R = 5600\ \mathrm{N}$. (b) The tractor and block have the same acceleration.

Question 4

12 marks

Forces

4.

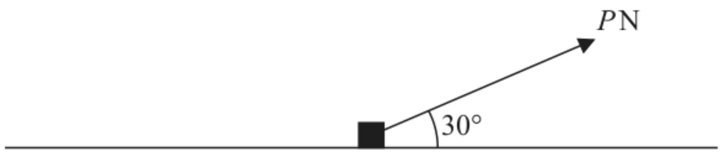

Figure 1

A small block of mass 5 kg lies at rest on a rough horizontal plane.

The coefficient of friction between the block and the plane is $\frac{3}{7}$

A force of magnitude P newtons is applied to the block in a direction which makes an angle of 30° with the plane, as shown in Figure 1.

The block is modelled as a particle.

Given that $P = 14$

(a) find the magnitude of the frictional force exerted on the block by the plane and describe what happens to the block, justifying your answer.

(6)

The value of P is now changed so that the block is on the point of slipping along the plane.

(b) Find the value of P

(6)

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Worked Solution - Question 4

Forces

Topic group

1. Find the normal reaction when P = 14

Resolve vertically. The upward component of the applied force is $14 \sin 30°$, so $R + 14 \sin 30° = 5g$.

2. Calculate R

$R + 7 = 49$, so $R = 42$ N.

3. Find the maximum friction

$F_{\text{max}} = \mu R = \frac{3}{7}(42) = 18$ N.

4. Compare with the horizontal pull

The horizontal component of the applied force is $14 \cos 30° = 12.1\ldots$ N. Since this is less than the maximum possible friction, the block does not move.

5. State the actual friction

For equilibrium, the actual friction equals the horizontal pull, so $F = 12.1$ N approximately.

6. Set up the limiting case

When the block is on the point of slipping, friction is limiting. Resolve vertically: $R + P \sin 30° = 5g$, so $R = 5g - P \sin 30°$.

7. Resolve horizontally

At the limiting case, $P \cos 30° = \frac{3}{7}R = \frac{3}{7}(5g - P \sin 30°)$.

8. Find P

Substitute $g = 9.8$, $\sin 30° = \frac{1}{2}$: $P \cos 30° = \frac{3}{7}(49 - \frac{1}{2}P)$. Solving gives $P = 19.4\ldots$ N.

Final answer

(a) $F = 12.1$ N approximately, and the block does not move. (b) $P = 19.4$ N approximately

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Question 5

9 marks

Moments

5.

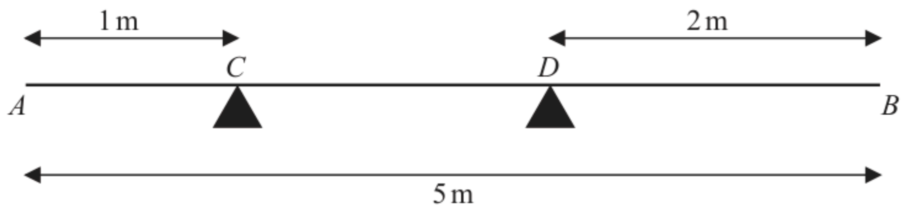

Figure 2

A uniform rod AB has length 5 m and mass 5 kg. The rod rests in equilibrium in a horizontal position on two supports C and D, where $AC = 1\text{ m}$ and $DB = 2\text{ m}$, as shown in Figure 2.

A particle of mass 10 kg is placed on the rod at A and a particle of mass M kg is placed on the rod at B. The rod remains horizontal and in equilibrium.

(a) Find, in terms of M, the magnitude of the reaction on the rod at C.

(3)

(b) Find, in terms of M, the magnitude of the reaction on the rod at D.

(3)

(c) Hence, or otherwise, find the range of possible values of M.

(3)

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Worked Solution - Question 5

Moments

Topic group

1. Set the positions

Take distances from A. Then C is at 1 m, D is at 3 m, the rod's centre of mass is at 2.5 m, and B is at 5 m.

2. Find RC by moments about D

Taking moments about D: $R_C(2) + Mg(2) = 10g(3) + 5g(0.5)$.

3. Simplify RC

$2R_C + 2Mg = 30g + 2.5g = 32.5g$, so $R_C = 16.25g - Mg = (16.25 - M)g$.

4. Find RD by moments about C

Taking moments about C: $10g(1) + R_D(2) = 5g(1.5) + Mg(4)$.

5. Simplify RD

$10g + 2R_D = 7.5g + 4Mg$, so $2R_D = (4M - 2.5)g$ and $R_D = (2M - 1.25)g$.

6. Use non-negative reactions

For the rod to rest on both supports, both reactions must be non-negative.

7. Find the range

$R_C \geq 0$ gives $16.25 - M \geq 0$, so $M \leq 16.25$. Also $R_D \geq 0$ gives $2M - 1.25 \geq 0$, so $M \geq 0.625$. Therefore $0.625 \leq M \leq 16.25$.

Final answer

$(a)\ R_C = (16.25 - M)g. \quad (b)\ R_D = (2M - 1.25)g. \quad (c)\ 0.625 \leq M \leq 16.25$

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WME01/01 MAY/JUNE 2022

Question 6

6 marks

Working with Vectors

6. A particle P is moving with constant acceleration.

At time $t = 1$ second, P has velocity $(-\mathbf{i} + 4\mathbf{j})\,\text{m}\,\text{s}^{-1}$

At time $t = 4$ seconds, P has velocity $(5\mathbf{i} - 8\mathbf{j})\,\text{m}\,\text{s}^{-1}$

Find the speed of P at time $t = 3.5$ seconds.

(6)

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Worked Solution - Question 6

Working with Vectors

Topic group

1. Find the acceleration

The velocity changes from $-\mathbf{i} + 4\mathbf{j}$ at $t = 1$ to $5\mathbf{i} - 8\mathbf{j}$ at $t = 4$. The time difference is 3 s.

2. Calculate acceleration

$$\mathbf{a} = \frac{(5\mathbf{i} - 8\mathbf{j}) - (-\mathbf{i} + 4\mathbf{j})}{3} = \frac{6\mathbf{i} - 12\mathbf{j}}{3} = 2\mathbf{i} - 4\mathbf{j}.$$

3. Find velocity at t = 3.5

From $t = 1$ to $t = 3.5$ is 2.5 s, so $\mathbf{v} = (-\mathbf{i} + 4\mathbf{j}) + 2.5(2\mathbf{i} - 4\mathbf{j})$.

4. Simplify the velocity

$$\mathbf{v} = -\mathbf{i} + 4\mathbf{j} + 5\mathbf{i} - 10\mathbf{j} = 4\mathbf{i} - 6\mathbf{j}.$$

5. Find the speed

Speed $= \sqrt{4^2 + (-6)^2} = \sqrt{52} = 2\sqrt{13}\,\mathrm{m\,s^{-1}}$, about $7.2\,\mathrm{m\,s^{-1}}$.

Final answer

$$\text{speed} = \sqrt{52} = 2\sqrt{13}\,\mathrm{m\,s^{-1}} \approx 7.2\,\mathrm{m\,s^{-1}}.$$

Question 7

13 marks

Kinematics Graphs

7. Two small children, Ajaz and Beth, are running a 100 m race along a straight horizontal track.

They both start from rest, leaving the start line at the same time.

Ajaz accelerates at $0.8\,\text{m}\,\text{s}^{-2}$ up to a speed of $4\,\text{m}\,\text{s}^{-1}$ and then maintains this speed until he crosses the finish line.

Beth accelerates at $1\,\text{m}\,\text{s}^{-2}$ for T seconds and then maintains a constant speed until she crosses the finish line.

Ajaz and Beth cross the finish line at the same time.

(a) Sketch, on the same axes, a speed-time graph for each child, from the instant when they leave the start line to the instant when they cross the finish line.

(3)

(b) Find the time taken by Ajaz to complete the race.

(4)

(c) Find the value of T

(4)

(d) Find the difference in the speeds of the two children as they cross the finish line.

(2)

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Worked Solution - Question 7

Kinematics Graphs

Topic group

1. Describe Ajaz graph

Ajaz starts from rest, accelerates linearly to speed $4\,\mathrm{m\,s^{-1}}$, then continues with a horizontal speed-time line at speed 4.

2. Describe Beth graph

Beth starts from rest, has a steeper straight acceleration line for T seconds, then a horizontal line. Both graphs finish at the same final time.

3. Find Ajaz acceleration time

Ajaz reaches speed 4 with acceleration 0.8, so the time is $\frac{4}{0.8} = 5$ s.

4. Find Ajaz total time

Distance during acceleration is $\frac{1}{2}(5)(4) = 10$ m. The remaining 90 m at speed 4 takes 22.5 s, so Ajaz's total time is 27.5 s.

5. Set Beth distance equation

Beth accelerates for T seconds to speed T, then runs at speed T until time 27.5. Her distance is $\frac{1}{2}T^2 + T(27.5 - T)$.

6. Solve for T

$100 = \frac{1}{2}T^2 + 27.5T - T^2 = 27.5T - \frac{1}{2}T^2$. Thus $T^2 - 55T + 200 = 0$. The valid root is $T = 3.915\ldots$ s.

7. Find the finish speeds

Ajaz finishes at $4\,\mathrm{m\,s^{-1}}$. Beth's constant speed is $T = 3.915\ldots\ \mathrm{m\,s^{-1}}$.

8. Find the difference

Difference $= 4 - 3.915\ldots = 0.085\ \mathrm{m\,s^{-1}}$ approximately.

Final answer

(a) Ajaz reaches speed 4 then runs horizontally; Beth has a steeper initial line to speed T, then horizontal, both ending at the same time.

$(b)\ 27.5\text{ s.}\quad (c)\ T = 3.92\text{ s approximately.}\quad (d)\ 0.085\ \mathrm{m\,s^{-1}}\text{ approximately}$

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WME01/01 MAY/JUNE 2022

Question 8

17 marks

Working with Vectors

8. [*In this question,* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two boats, P and Q, are moving with constant velocities.

The velocity of P is $15\mathbf{i}\,\text{m s}^{-1}$ and the velocity of Q is $(20\mathbf{i} - 20\mathbf{j})\,\text{m s}^{-1}$

(a) Find the direction in which Q is travelling, giving your answer as a bearing.

(2)

The boats are modelled as particles.

At time $t = 0$, P is at the origin O and Q is at the point with position vector $200\mathbf{j}$ m.

At time t seconds, the position vector of P is $\mathbf{p}$ m and the position vector of Q is $\mathbf{q}$ m.

(b) Show that

$$\overrightarrow{PQ} = [5t\mathbf{i} + (200 - 20t)\mathbf{j}]\,\text{m}$$

(5)

(c) Find the bearing of P from Q when $t = 10$

(2)

(d) Find the distance between P and Q when Q is north east of P

(5)

(e) Find the times when P and Q are 200 m apart.

(3)

(Total 17 marks)

TOTAL FOR PAPER: 75 MARKS

WME01/01 MAY/JUNE 2022

Worked Solution - Question 8

Working with Vectors

Topic group

1. Find the bearing of Q

Q has velocity $20\mathbf{i} - 20\mathbf{j}$, so it travels south-east with equal east and south components. Its bearing is $135°$.

2. Write P position

P starts at the origin and has velocity $15\mathbf{i}$, so $\mathbf{p} = 15t\mathbf{i}$.

3. Write Q position

Q starts at $200\mathbf{j}$ and has velocity $20\mathbf{i} - 20\mathbf{j}$, so $\mathbf{q} = 20t\mathbf{i} + (200 - 20t)\mathbf{j}$.

4. Show PQ

$\overrightarrow{PQ} = \mathbf{q} - \mathbf{p} = (20t - 15t)\mathbf{i} + (200 - 20t)\mathbf{j} = 5t\mathbf{i} + (200 - 20t)\mathbf{j}$.

5. Find the bearing at t = 10

At $t = 10$, $\overrightarrow{PQ} = 50\mathbf{i}$. This means Q is east of P, so P is west of Q. The bearing of P from Q is $270°$.

6. Use the north-east condition

Q is north-east of P when the i and j components of $\overrightarrow{PQ}$ are equal and positive. So $5t = 200 - 20t$.

7. Find the distance then

The equation gives $t = 8$. Then $\overrightarrow{PQ} = 40\mathbf{i} + 40\mathbf{j}$, so the distance is $\sqrt{40^2 + 40^2} = 40\sqrt{2}\,\text{m}$.

8. Set the 200 m distance equation

For distance 200 m, $(5t)^2 + (200 - 20t)^2 = 200^2$.

9. Solve for t

This simplifies to $425t^2 - 8000t = 0$, so $t(425t - 8000) = 0$. Hence $t = 0$ or $t = \dfrac{8000}{425} = \dfrac{320}{17}$ s.

Final answer

$(a)\ 135^\circ. \quad (b)\ \overrightarrow{PQ} = 5t\mathbf{i} + (200 - 20t)\mathbf{j}. \quad (c)\ 270^\circ. \quad (d)\ 40\sqrt{2}\ \mathrm{m}. \quad (e)\ t = 0\ \mathrm{s}\ \text{or}\ \dfrac{320}{17}\ \mathrm{s}$

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PAST PAPER

WME01/01 October 2022

October 2022 | 8 questions | 75 marks

8
questions

75
marks

Answers
worked solution after each question

Question 1

5 marks

Momentum, Impulse & Collisions

1. A railway truck S of mass 20 tonnes is moving along a straight horizontal track when it collides with another railway truck T of mass 30 tonnes which is at rest. Immediately before the collision the speed of S is $4\,\mathrm{m\,s^{-1}}$
As a result of the collision, the two railway trucks join together.

Find
(a) the common speed of the railway trucks immediately after the collision,

(2)

(b) the magnitude of the impulse exerted on S in the collision, stating the units of your answer.

(3)

WME01/01 OCTOBER 2022

Worked Solution - Question 1

Momentum, Impulse & Collisions

Topic group

1. Use conservation of momentum

The trucks join together, so they move with a common speed v. Using tonnes consistently, $20(4) + 30(0) = (20 + 30)v$.

2. Find the common speed

$80 = 50v$, so $v = 1.6\,\mathrm{m\,s^{-1}}$.

3. Use change in momentum of S

Truck S has mass $20\,000$ kg. Its speed changes from 4 to $1.6\,\mathrm{m\,s^{-1}}$.

4. Find the impulse magnitude

$I = 20000|1.6 - 4| = 20000(2.4) = 48000\,\mathrm{N\,s}$.

Final answer

$(a)\ v = 1.6\,\mathrm{m\,s^{-1}}$. $(b)\ I = 48000\,\mathrm{N\,s}$.

WME01/01 OCTOBER 2022

Question 2

6 marks

Moments

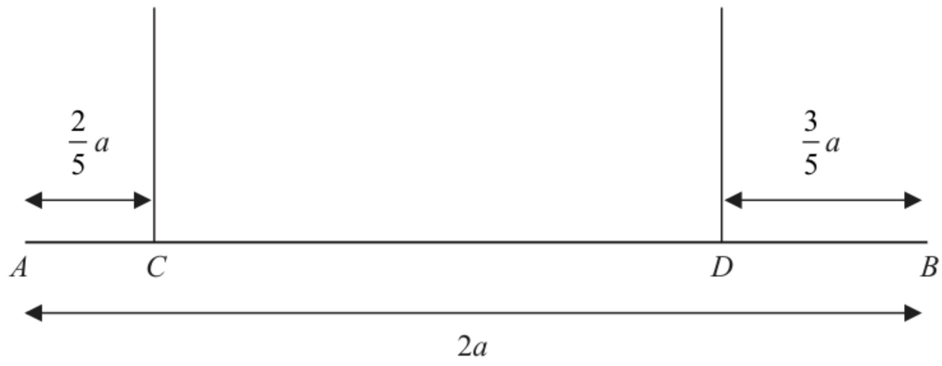

Figure 1

A uniform rod AB has length $2a$ and mass M. The rod is held in equilibrium in a horizontal position by two vertical light strings which are attached to the rod at C and D, where $AC = \frac{2}{5}a$ and $DB = \frac{3}{5}a$, as shown in Figure 1.

A particle P is placed on the rod at B.

The rod remains horizontal and in equilibrium.

(a) Find, in terms of M, the largest possible mass of the particle P

(3)

Given that the mass of P is $\frac{1}{2}M$

(b) find, in terms of M and g, the tension in the string that is attached to the rod at C.

(3)

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Worked Solution - Question 2

Moments

Topic group

1. Set the distances

The rod length is $2a$. The centre of mass is at the midpoint, distance a from A. Also $AC = \dfrac{2a}{5}$ and $AD = 2a - \dfrac{3a}{5} = \dfrac{7a}{5}$.

2. Use the largest mass condition

For the largest possible mass at B, the string at C is just slack and the rod is about to tilt about D.

3. Take moments about D

Let the particle mass be X. The distance from D to B is $\dfrac{3a}{5}$ and the distance from D to the rod's centre of mass is $\dfrac{2a}{5}$. Hence $Xg\left(\dfrac{3a}{5}\right) = Mg\left(\dfrac{2a}{5}\right)$.

4. Find the largest mass

Cancel $ga/5$: $3X = 2M$, so $X = \dfrac{2M}{3}$.

5. Use the given mass

Now the particle has mass $\dfrac{1}{2}M$. Let the tensions at C and D be T_C and T_D.

6. Take moments about D

About D, $T_C(a) + \dfrac{1}{2}Mg\left(\dfrac{3a}{5}\right) = Mg\left(\dfrac{2a}{5}\right)$.

7. Find the tension at C

$T_C a + \dfrac{3}{10}Mga = \dfrac{2}{5}Mga$, so $T_C = \dfrac{1}{10}Mg$.

Final answer

(a) largest mass $= \dfrac{2M}{3}$. (b) $T_C = \dfrac{1}{10} Mg$.

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11 marks

Question 3

Resolving Forces, Inclined Planes

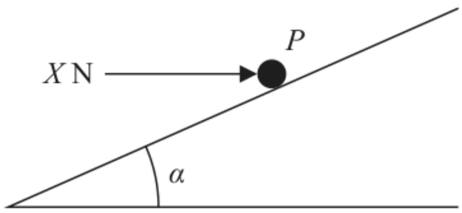

Figure 2

A rough plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$

A particle P of mass 2 kg is held in equilibrium on the plane by a horizontal force of magnitude X newtons, as shown in Figure 2. The force acts in a vertical plane which contains a line of greatest slope of the inclined plane.

(a) Show that when $X = 14.7$ there is no frictional force acting on P

(3)

The coefficient of friction between P and the plane is 0.5

(b) Find the smallest possible value of X.

(8)

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Worked Solution - Question 3

Resolving Forces, Inclined Planes

Topic group

1. Use the trig ratios

Since $\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$.

2. Check the no-friction case

If there is no friction, the component of the horizontal force up the plane balances the component of weight down the plane: $X\cos\alpha = 2g\sin\alpha$.

3. Substitute X = 14.7

$14.7 \cdot \frac{4}{5} = 11.76$, and $2(9.8) \cdot \frac{3}{5} = 11.76$. The components balance exactly, so the frictional force is zero.

4. Set the direction for smallest X

For the smallest possible X, the particle is about to slide down the plane, so friction acts up the plane.

5. Resolve perpendicular to the plane

The horizontal force increases the normal reaction, so $R = 2g\cos\alpha + X\sin\alpha$.

6. Use limiting friction

With $\mu = 0.5$, the limiting friction is $F = 0.5R = 0.5(2g\cos\alpha + X\sin\alpha)$.

7. Resolve along the plane

At the limiting case, $X\cos\alpha + F = 2g\sin\alpha$.

8. Solve for X

Substitute the exact trig values: $\frac{4}{5}X + 0.5\left(2g \cdot \frac{4}{5} + X \cdot \frac{3}{5}\right) = 2g \cdot \frac{3}{5}$. This gives $\frac{11}{10}X + \frac{4g}{5} = \frac{6g}{5}$, so $X = \frac{4g}{11} = 3.56\ldots$ N.

Final answer

(a) When $X = 14.7$, $F = 0$.

(b) $X_{\text{min}} = \dfrac{4g}{11} = 3.56\ldots\ \text{N}$.

WME01/01 OCTOBER 2022

6 marks

Question 4

Newton's Second Law

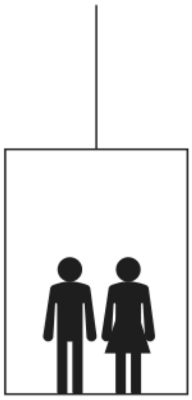

Figure 3

Two children, Alan and Bhavana, are standing on the horizontal floor of a lift, as shown in Figure 3.

The lift has mass 250 kg. The lift is raised vertically upwards with constant acceleration by a vertical cable which is attached to the top of the lift. The cable is modelled as being light and inextensible. While the lift is accelerating upwards, the tension in the cable is 3616 N.

As the lift accelerates upwards, the floor of the lift exerts a force of magnitude 565 N on Alan and a force of magnitude 226 N on Bhavana.

Air resistance is modelled as being negligible and Alan and Bhavana are modelled as particles.

(a) By considering the forces acting on the lift only, find the acceleration of the lift.

(3)

(b) Find the mass of Alan.

(3)

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Worked Solution - Question 4

Newton's Second Law

Topic group

1. Consider the lift only

For the lift, the upward force is the cable tension 3616 N. Downward forces on the lift are its own weight $250g$ and the contact forces from Alan and Bhavana, 565 N and 226 N.

2. Apply Newtons second law

Taking upward as positive: $3616 - 250g - 565 - 226 = 250a$.

3. Find the acceleration

$3616 - 2450 - 565 - 226 = 250a$, so $375 = 250a$ and $a = 1.5\ \mathrm{m\,s^{-2}}$.

4. Consider Alan

Let Alan's mass be m_A. The upward force on Alan is 565 N and his weight is $m_A g$.

5. Find Alan's mass

$565 - m_A g = m_A(1.5)$, so $565 = m_A(9.8 + 1.5) = 11.3m_A$. Hence $m_A = 50\ \mathrm{kg}$.

Final answer

$(a)\ a = 1.5\ \mathrm{m\,s^{-2}}.\quad (b)$ mass of Alan $= 50\ \mathrm{kg}$.

Question 5

9 marks

Constant Acceleration in 1D

5. A small ball is projected vertically upwards with speed $29.4\,\text{m s}^{-1}$ from a point A which is $19.6\,\text{m}$ above horizontal ground.

The ball is modelled as a particle moving freely under gravity until it hits the ground. It is assumed that the ball does not rebound.

(a) Find the distance travelled by the ball while its speed is less than $14.7\,\text{m s}^{-1}$

(3)

(b) Find the time for which the ball is moving with a speed of more than $29.4\,\text{m s}^{-1}$

(3)

(c) Sketch a speed-time graph for the motion of the ball from the instant when it is projected from A to the instant when it hits the ground. Show clearly where your graph meets the axes.

(3)

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Worked Solution - Question 5

Constant Acceleration in 1D

Topic group

1. Find one side of the slow-speed interval

The speed is less than 14.7 m s^{-1} near the highest point. From speed 14.7 to speed 0, use $v^2 = u^2 + 2as$: $0 = 14.7^2 - 2gs$.

2. Find the total distance

This gives $s = 11.025$ m on the way up. The same distance occurs on the way down before the speed reaches 14.7 again, so the total distance is $2s = 22.05 \text{ m}$, about 22.1 m.

3. Interpret speed greater than 29.4

The ball has speed 29.4 again when it passes A on the way down. It is faster than 29.4 only after passing A downward until it hits the ground.

4. Find that time

From A to the ground, taking downward as positive, $19.6 = 29.4t + \frac{1}{2}gt^2$.

5. Solve the time equation

$19.6 = 29.4t + 4.9t^2$, giving the positive root $t = 0.606\ldots \text{ s}$.

6. Describe the speed-time graph

The graph starts at speed 29.4 when $t = 0$, decreases linearly to speed 0 at $t = 3$, then increases linearly until the ball hits the ground. The final speed is greater than 29.4, so the right-hand arm ends above the starting height on the speed axis.

Final answer

(a) 22.1 m approximately. (b) 0.606 s approximately. (c) V-shaped speed-time graph through $(0, 29.4)$ and $(3, 0)$, ending above 29.4.

Question 6

9 marks

Working with Vectors

6. [*In this question,* **i** *and* **j** *are horizontal unit vectors.*]

A particle A of mass 0.5 kg is at rest on a smooth horizontal plane.

At time $t = 0$, two forces, $\mathbf{F}_1 = (-3\mathbf{i} + 2\mathbf{j})$ N and $\mathbf{F}_2 = (p\mathbf{i} + q\mathbf{j})$ N, where p and q are constants, are applied to A.

Given that A moves in the direction of the vector $(\mathbf{i} - 2\mathbf{j})$,

(a) show that $2p + q - 4 = 0$

(4)

Given that $p = 5$

(b) find the speed of A at time $t = 4$ seconds.

(5)

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Worked Solution - Question 6

Working with Vectors

Topic group

1. Find the resultant force

$\mathbf{F} = (-3\mathbf{i} + 2\mathbf{j}) + (p\mathbf{i} + q\mathbf{j}) = (p-3)\mathbf{i} + (q+2)\mathbf{j}$.

2. Use the direction of motion

Since the particle starts from rest, its motion is in the direction of the resultant force. This direction is $\mathbf{i} - 2\mathbf{j}$, so $\dfrac{q+2}{p-3} = -2$.

3. Show the relation

$q + 2 = -2(p-3) = -2p + 6$, hence $2p + q - 4 = 0$.

4. Find q when p = 5

Substitute $p = 5$ into $2p + q - 4 = 0$: $10 + q - 4 = 0$, so $q = -6$.

5. Find the resultant force

The resultant force is $(5-3)\mathbf{i} + (-6+2)\mathbf{j} = 2\mathbf{i} - 4\mathbf{j}$ N.

6. Find acceleration

The mass is 0.5 kg, so $\mathbf{a} = \dfrac{2\mathbf{i} - 4\mathbf{j}}{0.5} = 4\mathbf{i} - 8\mathbf{j}$.

7. Find velocity at t = 4

Starting from rest, $\mathbf{v} = \mathbf{a}t = (4\mathbf{i} - 8\mathbf{j})4 = 16\mathbf{i} - 32\mathbf{j}$.

8. Find the speed

Speed $= \sqrt{16^2 + (-32)^2} = \sqrt{1280} = 16\sqrt{5}\ \mathrm{m\,s^{-1}}$, about $36\ \mathrm{m\,s^{-1}}$.

Final answer

(a) $2p + q - 4 = 0$. (b) speed $= 16\sqrt{5}\ \mathrm{m\,s^{-1}} \approx 36\ \mathrm{m\,s^{-1}}$.

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13 marks

Question 7

Newton's Second Law

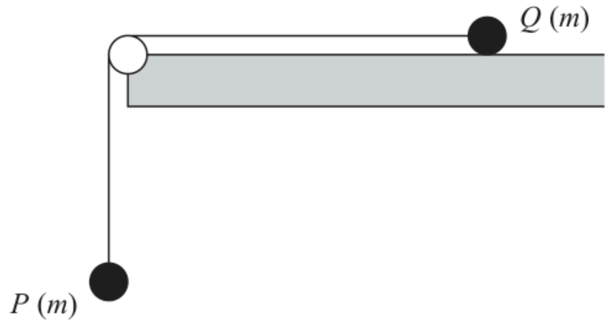

Figure 4

A particle P of mass m is attached to one end of a light inextensible string. Another particle Q, also of mass m, is attached to the other end of the string. The string passes over a small smooth pulley which is fixed at the edge of a rough horizontal table. Particle Q is held at rest on the table and particle P hangs vertically below the pulley with the string taut, as shown in Figure 4.

The pulley, P and Q all lie in the same vertical plane.

The coefficient of friction between Q and the table is μ, where $\mu < 1$

Particle Q is released from rest.

The tension in the string before Q hits the pulley is kmg, where k is a constant.

(a) Find k in terms of μ.

(7)

Given that Q is initially a distance d from the pulley,

(b) find, in terms of d, g and μ, the time taken by Q, after release, to reach the pulley.

(4)

(c) Describe what would happen if $\mu \geqslant 1$, giving a reason for your answer.

(2)

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Worked Solution - Question 7

Newton's Second Law

Topic group

1. Write the friction on Q

For Q on the horizontal table, $R = mg$, so the friction is $F = \mu mg$.

2. Apply Newtons second law to P

P moves downward and Q moves towards the pulley. For P, taking downward as positive: $mg - T = ma$.

3. Apply Newtons second law to Q

For Q, taking motion towards the pulley as positive: $T - \mu mg = ma$.

4. Use T = kmg

From P, $a = g(1-k)$. From Q, $a = g(k-\mu)$.

5. Find k

Equate the two accelerations: $1-k = k-\mu$. Hence $2k = 1+\mu$ and $k = \dfrac{1+\mu}{2}$.

6. Find the acceleration

$$a = g(1-k) = g\left(1 - \frac{1+\mu}{2}\right) = \frac{g(1-\mu)}{2}.$$

7. Use motion from rest

Q starts from rest and travels distance d, so $d = \dfrac{1}{2}at^2 = \dfrac{1}{2} \cdot \dfrac{g(1-\mu)}{2}t^2$.

8. Find the time

$d = \dfrac{g(1-\mu)t^2}{4}$, so $t = \sqrt{\dfrac{4d}{g(1-\mu)}}$.

9. Consider mu at least 1

If $\mu \geq 1$, the maximum available friction on Q is at least mg, so it can balance the pull caused by the hanging particle. The system would not start moving.

Final answer

$(a)\ k = \dfrac{1+\mu}{2}$. $(b)\ t = \sqrt{\dfrac{4d}{g(1-\mu)}}$. (c) If $\mu \geq 1$, the system would not move because friction can balance the pull from P.

Question 8

16 marks

Working with Vectors

8. [*In this question,* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two ships, A and B, are moving with constant velocities.
The velocity of A is $(3\mathbf{i} + 12\mathbf{j})\,\text{km h}^{-1}$ and the velocity of B is $(p\mathbf{i} + q\mathbf{j})\,\text{km h}^{-1}$

(a) Find the speed of A.

(2)

The ships are modelled as particles.
At 12 noon, A is at the point with position vector $(-9\mathbf{i} + 6\mathbf{j})\,\text{km}$ and B is at the point with position vector $(16\mathbf{i} + 6\mathbf{j})\,\text{km}$.

At time t hours after 12 noon,

$$\overrightarrow{AB} = [(25 - 12t)\mathbf{i} - 9t\mathbf{j}]\,\text{km}$$

(b) Find the value of p and the value of q.

(7)

(c) Find the bearing of A from B when the ships are 15 km apart, giving your answer to the nearest degree.

(7)

(Total 16 marks)

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Worked Solution - Question 8

Working with Vectors

Topic group

1. Find the speed of A

$|\mathbf{v}_A| = \sqrt{3^2 + 12^2} = \sqrt{153} = 3\sqrt{17}\,\mathrm{km\,h^{-1}}$.

2. Write A position

At time t, $\mathbf{a} = (-9\mathbf{i} + 6\mathbf{j}) + t(3\mathbf{i} + 12\mathbf{j}) = (-9 + 3t)\mathbf{i} + (6 + 12t)\mathbf{j}$.

3. Write B position

$\mathbf{b} = (16\mathbf{i} + 6\mathbf{j}) + t(p\mathbf{i} + q\mathbf{j}) = (16 + pt)\mathbf{i} + (6 + qt)\mathbf{j}$.

4. Use AB vector

$\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = [25 + (p-3)t]\mathbf{i} + [(q-12)t]\mathbf{j}$.

5. Compare components

Given $\overrightarrow{AB} = (25 - 12t)\mathbf{i} - 9t\mathbf{j}$, compare coefficients: $p - 3 = -12$ and $q - 12 = -9$.

6. Find p and q

Therefore $p = -9$ and $q = 3$.

7. Find the time when the distance is 15

Use $|\overrightarrow{AB}| = 15$: $(25 - 12t)^2 + (-9t)^2 = 15^2$.

8. Solve the distance equation

This gives $225t^2 - 600t + 400 = 0$, so $(3t-4)^2 = 0$ and $t = \dfrac{4}{3}$.

9. Find BA at this time

At $t = \dfrac{4}{3}$, $\overrightarrow{AB} = 9\mathbf{i} - 12\mathbf{j}$. Therefore $\overrightarrow{BA} = -9\mathbf{i} + 12\mathbf{j}$.

10. Convert to a bearing

A is north-west of B. The angle west of north satisfies $\tan\theta = \dfrac{9}{12}$, so $\theta = 36.9^\circ$.

Bearing $= 360^\circ - 36.9^\circ = 323^\circ$ to the nearest degree.

Final answer

$(a)\ 3\sqrt{17}\ \mathrm{km\,h^{-1}}. \quad (b)\ p = -9,\ q = 3. \quad (c)\ \text{bearing} = 323^\circ.$

PAST PAPER

WME01/01 January 2023

January 2023 | 8 questions | 75 marks

8
questions

75
marks

Answers
worked solution after each question

Question 1

1. A train travels along a straight horizontal track between two stations A and B.

The train starts from rest at station A and accelerates uniformly for T seconds until it reaches a speed of $20\,\mathrm{m\,s^{-1}}$

The train then travels at a constant speed of $20\,\mathrm{m\,s^{-1}}$ for 3 minutes before decelerating uniformly until it comes to rest at station B.

The magnitude of the acceleration of the train is twice the magnitude of the deceleration.

(a) On the axes below, sketch a speed–time graph to illustrate the motion of the train as it moves from station A to station B.

If you need to redraw your graph, use the axes on page 3

(3)

Stations A and B are 4.8 km apart.

(b) Find the value of T

(5)

(c) Find the acceleration of the train during the first T seconds of its motion.

(2)

Only use these axes if you need to redraw your graph.

(Total for Question 1 is 10 marks)

WME01/01 JANUARY 2023

Worked Solution - Question 1

Kinematics Graphs

Topic group

1. Relate the acceleration and deceleration times

The train reaches $20\ \mathrm{m\,s^{-1}}$ in T seconds. Since the acceleration magnitude is twice the deceleration magnitude, the time needed to decelerate from 20 to 0 is $2T$.

2. Describe the graph

The speed-time graph rises linearly from $(0,0)$ to $(T,20)$, stays horizontal at speed 20 from T to $T+180$, then falls linearly to zero at $3T+180$.

3. Use area under the graph

The distance from A to B is $4.8\ \mathrm{km} = 4800\ \mathrm{m}$. The area is $\frac{1}{2}(T)(20) + 20(180) + \frac{1}{2}(2T)(20)$.

4. Solve for T

$4800 = 10T + 3600 + 20T$, so $1200 = 30T$ and $T = 40\ \mathrm{s}$.

5. Find the acceleration

During the first T seconds, $20 = aT$. With $T = 40$, $a = \frac{20}{40} = 0.5\ \mathrm{m\,s^{-2}}$.

Final answer

(a) Speed-time graph with times T, $T+180$, $3T+180$ and top speed 20 .

(b) $T = 40\ \mathrm{s}$. (c) $a = 0.5\ \mathrm{m\,s^{-2}}$.

Question 2

8 marks

Momentum, Impulse & Collisions

2. Two particles, A and B, are moving in a straight line in opposite directions towards each other on a smooth horizontal surface when they collide directly.

Particle A has mass $3m$ kg and particle B has mass m kg.

Immediately before the collision, both particles have a speed of $1.5\,\text{m s}^{-1}$

Immediately after the collision, the direction of motion of A is unchanged and the difference between the speed of A and speed of B is $1\,\text{m s}^{-1}$

(a) Find (i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision.

(5)

(b) Find, in terms of m, the magnitude of the impulse exerted on B in the collision.

(3)

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Worked Solution - Question 2

Momentum, Impulse & Collisions

Topic group

1. Choose a positive direction

Take the original direction of A as positive. Before the collision, A has velocity 1.5 and B has velocity -1.5.

2. Use the speed difference

After the collision, A keeps the same direction. Let A's speed be v. The valid motion has B moving in A's direction with speed $v + 1$.

3. Use conservation of momentum

$3m(1.5) + m(-1.5) = 3mv + m(v + 1)$.

4. Find the two speeds

$4.5m - 1.5m = 3mv + mv + m$, so $3 = 4v + 1$. Hence $v = 0.5$, and B's speed is $v + 1 = 1.5$.

5. Find the impulse on B

For B, impulse equals change in momentum: $I = m(1.5 - (-1.5)) = 3m$.

6. State the magnitude

The magnitude of the impulse is $3m\ \mathrm{N\,s}$.

Final answer

$(a)(i)$ speed of A $= 0.5\ \mathrm{m\,s^{-1}}$. $(a)(ii)$ speed of B $= 1.5\ \mathrm{m\,s^{-1}}$. (b) $I = 3m\ \mathrm{N\,s}$
.

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10 marks

Question 3

Working with Vectors

3. A particle P is moving with constant acceleration $(-4\mathbf{i} + \mathbf{j})\,\text{m}\,\text{s}^{-2}$

At time $t = 0$, P has velocity $(14\mathbf{i} - 5\mathbf{j})\,\text{m}\,\text{s}^{-1}$

(a) Find the speed of P at time $t = 2$ seconds.

(3)

(b) Find the size of the angle between the direction of **i** and the direction of motion of P at time $t = 2$ seconds.

(3)

At time $t = T$ seconds, P is moving in the direction of vector $(2\mathbf{i} - 3\mathbf{j})$

(c) Find the value of T

(4)

Worked Solution - Question 3

Working with Vectors

Topic group

1. Find the velocity at t = 2

$\mathbf{v} = (14\mathbf{i} - 5\mathbf{j}) + 2(-4\mathbf{i} + \mathbf{j}) = 6\mathbf{i} - 3\mathbf{j}$.

2. Find the speed

The speed is $\sqrt{6^2 + (-3)^2} = \sqrt{45} = 3\sqrt{5}\,\mathrm{m\,s^{-1}}$.

3. Find the angle with i

The velocity $6\mathbf{i} - 3\mathbf{j}$ is below the positive $\mathbf{i}$ direction. The acute angle satisfies $\tan\theta = \dfrac{3}{6}$.

4. State the angle

$\theta = 26.6^\circ$, so the size of the angle is 27° to the nearest degree.

5. Write the velocity at time T

$\mathbf{v}(T) = (14 - 4T)\mathbf{i} + (-5 + T)\mathbf{j}$.

6. Use the given direction

The direction is parallel to $2\mathbf{i} - 3\mathbf{j}$, so the component ratio must match: $\dfrac{14 - 4T}{2} = \dfrac{-5 + T}{-3}$.

7. Solve for T

$-3(14 - 4T) = 2(-5 + T)$, so $-42 + 12T = -10 + 2T$. Hence $10T = 32$ and $T = 3.2$.

Final answer

(a) $3\sqrt{5}\,\mathrm{m\,s^{-1}} \approx 6.7\,\mathrm{m\,s^{-1}}$. (b) 27°. (c) $T = 3.2$.

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Question 4

8 marks

Moments

4.

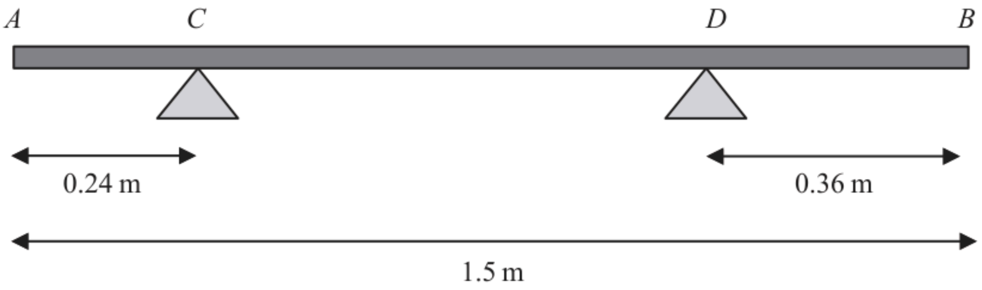

Figure 1

A branch AB, of length 1.5 m, rests horizontally in equilibrium on two supports.

The two supports are at the points C and D, where $AC = 0.24\text{ m}$ and $DB = 0.36\text{ m}$, as shown in Figure 1.

When a force of 150 N is applied vertically upwards at B, the branch is on the point of tilting about C.

When a force of 225 N is applied vertically downwards at B, the branch is on the point of tilting about D.

The branch is modelled as a non-uniform rod AB of weight W newtons.

The distance from the point C to the centre of mass of the rod is x metres.

Use the model to find

(i) the value of W

(ii) the value of x

(8)

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Worked Solution - Question 4

Moments

Topic group

1. Set the distances

The branch has length 1.5 m. Since $AC = 0.24$ m and $DB = 0.36$ m, $CB = 1.26$ m and $CD = 0.90$ m. The centre of mass is x m from C.

2. Use the first tilting condition

With the 150 N upward force at B, the branch is on the point of tilting about C, so the support at D has zero reaction.

3. Take moments about C

The upward force at B balances the weight moment: $150(1.26) = Wx$. Thus $Wx = 189$.

4. Use the second tilting condition

With the 225 N downward force at B, the branch is on the point of tilting about D, so the support at C has zero reaction.

5. Take moments about D

The distance from D to the centre of mass is $0.90 - x$. Hence $225(0.36) = W(0.90 - x)$.

6. Solve the simultaneous equations

From the second equation, $81 = 0.90W - Wx$. Since $Wx = 189$, $81 = 0.90W - 189$, so $0.90W = 270$ and $W = 300\ \mathrm{N}$.

7. Find x

Using $Wx = 189$, $300x = 189$, so $x = 0.63\ \mathrm{m}$.

Final answer

$(i)\ W = 300\ \text{N}.\quad (ii)\ x = 0.63\ \text{m}.$

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9 marks

Question 5

Constant Acceleration in 1D

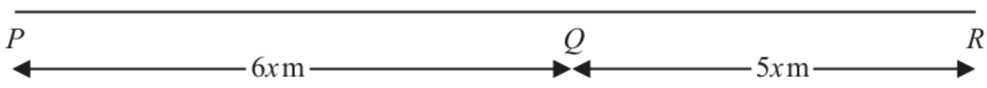

Figure 2

Three points P, Q and R are on a horizontal road where PQR is a straight line.

The point Q is between P and R, with $PQ = 6x$ metres and $QR = 5x$ metres, as shown in Figure 2.

A vehicle moves along the road from P to Q with constant acceleration.

The vehicle is modelled as a particle.

At time $t = 0$, the vehicle passes P with speed $u\,\text{m s}^{-1}$

At time $t = 12$ s, the vehicle passes Q with speed $2u\,\text{m s}^{-1}$

Using the model,

(a) show that $x = 3u$

(2)

As the vehicle passes Q, the acceleration of the vehicle changes instantaneously to $1.5\,\text{m s}^{-2}$

The vehicle continues to move with a constant acceleration of $1.5\,\text{m s}^{-2}$ and passes R with speed $3u\,\text{m s}^{-1}$

Using the model,

(b) find the value of u,

(3)

(c) find the distance travelled by the vehicle during the first 14 seconds after passing P

(4)

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Worked Solution - Question 5

Constant Acceleration in 1D

Topic group

1. Use average speed from P to Q

From P to Q, the speed changes uniformly from u to $2u$ over 12 s. The average speed is $\dfrac{u+2u}{2} = \dfrac{3u}{2}$.

2. Show x = 3u

Distance $PQ = 6x$, so $6x = \dfrac{3u}{2} \cdot 12 = 18u$. Hence $x = 3u$.

3. Use Q to R

The distance $QR = 5x = 15u$. The speed changes from $2u$ to $3u$ with acceleration $1.5\ \mathrm{m\,s^{-2}}$.

4. Find u

Use $v^2 = u^2 + 2as$: $(3u)^2 = (2u)^2 + 2(1.5)(15u)$. Thus $9u^2 = 4u^2 + 45u$, so $5u^2 = 45u$ and $u = 9$.

5. Find the first 12 seconds distance

In the first 12 seconds, the distance is $PQ = 6x = 18u = 18(9) = 162$ m.

6. Find the next 2 seconds distance

From $t = 12$ to $t = 14$, the vehicle starts this part at speed $2u = 18$ and accelerates at 1.5. Distance $= 18(2) + \dfrac{1}{2}(1.5)(2^2) = 36 + 3 = 39$ m.

7. Add the distances

Total distance in the first 14 seconds is $162 + 39 = 201\ \mathrm{m}$.

Final answer

$(a)\ x = 3u.$ $(b)\ u = 9.$ (c) 201 m.

Question 6

8 marks

Forces

6.

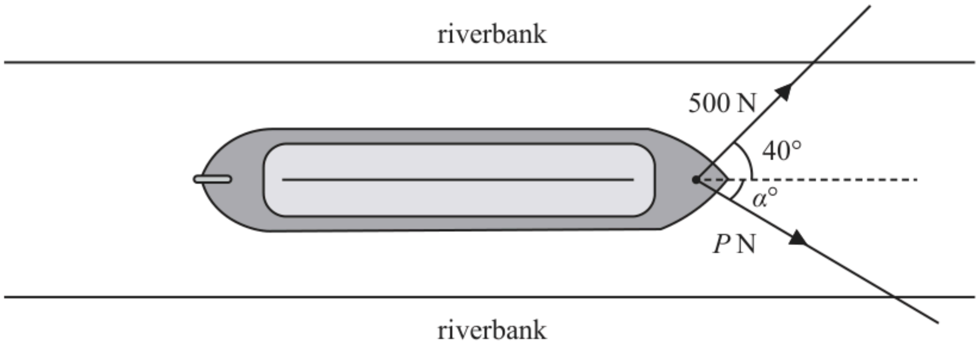

Figure 3

A boat is pulled along a river at a constant speed by two ropes.

The banks of the river are parallel and the boat travels horizontally in a straight line, parallel to the riverbanks.

- The tension in the first rope is 500 N acting at an angle of 40° to the direction of motion, as shown in Figure 3.
- The tension in the second rope is P newtons, acting at an angle of $\alpha°$ to the direction of motion, also shown in Figure 3.
- The resistance to motion of the boat as it moves through the water is a constant force of magnitude 900 N

The boat is modelled as a particle. The ropes are modelled as being light and lying in a horizontal plane.

Use the model to find

(i) the value of α

(ii) the value of P

(8)

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Worked Solution - Question 6

Forces

Topic group

1. Resolve perpendicular to the motion

The boat travels straight, so the sideways components balance: $500 \sin 40^\circ = P \sin \alpha$.

2. Resolve along the motion

The speed is constant, so the forward components balance the resistance: $500 \cos 40^\circ + P \cos \alpha = 900$.

3. Eliminate P

From the two equations, $P \sin \alpha = 500 \sin 40^\circ$ and $P \cos \alpha = 900 - 500 \cos 40^\circ$.

4. Find alpha

Divide the sine equation by the cosine equation: $\tan \alpha = \dfrac{500 \sin 40^\circ}{900 - 500 \cos 40^\circ}$.

This gives $\alpha = 31.9^\circ$, so $\alpha = 32^\circ$ approximately.

5. Find P

Use $P \sin \alpha = 500 \sin 40^\circ$: $P = \dfrac{500 \sin 40^\circ}{\sin 31.9^\circ} = 608.7\ldots$ N. Hence $P \approx 610$ N.

Final answer

$(i)\ \alpha = 32^\circ$ approximately. $(ii)\ P = 610$ N approximately.

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7 marks

Question 7

Newton's Second Law

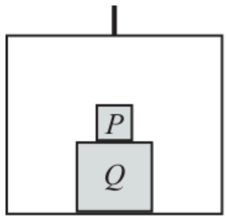

Figure 4

A simple lift operates by means of a vertical cable which is attached to the top of the lift.
The lift has mass m
A box Q is placed on the floor of the lift.
A box P is placed directly on top of box Q, as shown in Figure 4.

The cable is modelled as being light and inextensible and air resistance is modelled as being negligible.

The tension in the cable is $\frac{42mg}{5}$

The lift and its contents move vertically upwards with acceleration $\frac{2g}{5}$

Using the model,

(a) find, in terms of m, the combined mass of boxes P and Q

(4)

During the motion of the lift, the force exerted on box P by box Q is $\frac{14mg}{5}$

Using the model,

(b) find, in terms of m, the mass of box P

(3)

WME01/01 JANUARY 2023

Worked Solution - Question 7

Newton's Second Law

Topic group

1. Define the box mass

Let the combined mass of boxes P and Q be M. The lift itself has mass m, so the total moving mass is $m + M$.

2. Apply Newtons second law to the whole system

Taking upward as positive, $\frac{42}{5}mg - (m + M)g = (m + M)\frac{2g}{5}$.

3. Find M

Move the weight term to the right:
$\frac{42}{5}mg = (m + M)\left(g + \frac{2g}{5}\right) = \frac{7g}{5}(m + M)$. Hence $42m = 7(m + M)$ and $M = 5m$.

4. Let the mass of P be p

For box P alone, the upward force from Q is $\frac{14}{5}mg$, while its weight is pg. The acceleration is still $\frac{2g}{5}$ upward.

5. Apply Newtons second law to P

$\frac{14}{5}mg - pg = p\frac{2g}{5}$.

6. Find p

Divide by g: $\frac{14}{5}m = p + \frac{2p}{5} = \frac{7p}{5}$. Therefore $p = 2m$.

Final answer

(a) combined mass of P and Q $= 5m$. (b) mass of P $= 2m$.

WME01/01 JANUARY 2023

15 marks

Question 8

Resolving Forces, Inclined Planes

8.

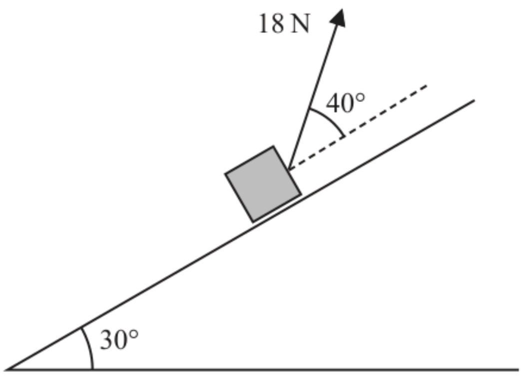

Figure 5

A parcel of mass 2 kg is pulled up a rough inclined plane by the action of a constant force.

The force has magnitude 18 N and acts at an angle of 40° to the plane.

The line of action of the force lies in a vertical plane containing a line of greatest slope of the inclined plane.

The plane is inclined at an angle of 30° to the horizontal, as shown in Figure 5.

The coefficient of friction between the plane and the parcel is 0.3

The parcel is modelled as a particle P

(a) Find the acceleration of P

(8)

The points A and B lie on a line of greatest slope of the plane, where $AB = 5\,\text{m}$ and B is above A. Particle P passes through A with speed $2\,\text{m}\,\text{s}^{-1}$ in the direction AB.

(b) Find the speed of P as it passes through B.

(3)

The force of 18 N is removed at the instant P passes through B. As a result, P comes to rest at the point C.

(c) Determine whether P will remain at rest at C. You must show all stages of your working clearly.

(4)

WME01/01 JANUARY 2023

Worked Solution - Question 8

Resolving Forces, Inclined Planes

Topic group

1. Resolve perpendicular to the plane

The force of 18 N is angled away from the plane, so $R + 18\sin 40^\circ = 2g\cos 30^\circ$. Hence $R = 2g\cos 30^\circ - 18\sin 40^\circ$.

2. Use friction

The parcel moves up the plane, so friction acts down the plane. With $\mu = 0.3$, $F = 0.3R$.

3. Resolve parallel to the plane

Taking up the plane as positive: $18\cos 40^\circ - F - 2g\sin 30^\circ = 2a$.

4. Substitute for R and F

$18\cos 40^\circ - 0.3(2g\cos 30^\circ - 18\sin 40^\circ) - 2g\sin 30^\circ = 2a$.

5. Find the acceleration

Using $g = 9.8$, this gives $a = 1.18\ldots\ \mathrm{m\,s^{-2}}$ up the plane.

6. Find the speed at B

From A to B, $u = 2$, $s = 5$ and $a = 1.18\ldots$. Use $v^2 = u^2 + 2as$: $v^2 = 2^2 + 2(1.18\ldots)(5)$.

7. State the speed

$v = 3.98\ldots\ \mathrm{m\,s^{-1}}$, about $4.0\ \mathrm{m\,s^{-1}}$.

8. Check rest after the force is removed

At C, with the pulling force removed, $R = 2g\cos 30^\circ$. The maximum friction is $0.3R = 0.3(2g\cos 30^\circ) = 5.09\ldots\ \mathrm{N}$.

9. Compare with the downslope weight component

The component of weight down the plane is $2g\sin 30^\circ = 9.8\ \mathrm{N}$. Since $9.8 > 5.09\ldots$, friction is not large enough to hold P at rest, so P will not remain at rest at C.

Final answer

$(a)\ a = 1.18\ \mathrm{m\,s^{-2}}$ up the plane. $\quad(b)\ v = 3.98\ \mathrm{m\,s^{-1}} \approx 4.0\ \mathrm{m\,s^{-1}}$. $\quad(c)$ P will not remain at rest at C.

PAST PAPER

WME01/01 May/June 2023

May/June 2023 | 8 questions | 75 marks

8
questions

75
marks

Answers
worked solution after each question

WME01/01 MAY/JUNE 2023

7 marks

Question 1

Momentum, Impulse & Collisions

1. A particle A has mass 4 kg and a particle B has mass 2 kg.

The particles move towards each other in opposite directions along the same straight line on a smooth horizontal table and collide directly.

Immediately before the collision, the speed of A is $2u\,\text{m s}^{-1}$ and the speed of B is $3u\,\text{m s}^{-1}$

Immediately after the collision, the speed of B is $2u\,\text{m s}^{-1}$

The direction of motion of B is reversed by the collision.

(a) Find, in terms of u, the speed of A immediately after the collision.

(3)

(b) State the direction of motion of A immediately after the collision.

(1)

(c) Find, in terms of u, the magnitude of the impulse received by B in the collision. State the units of your answer.

(3)

WMEO1/01 MAY/JUNE 2023

Worked Solution - Question 1

Momentum, Impulse & Collisions

Topic group

1. Choose a positive direction

Take the original direction of A as positive. Before the collision, A has velocity $2u$ and B has velocity $-3u$. After the collision, B has velocity $2u$ because its direction is reversed.

2. Use conservation of momentum

Let the velocity of A after the collision be v. Then $4(2u) + 2(-3u) = 4v + 2(2u)$.

3. Solve for the velocity of A

$8u - 6u = 4v + 4u$, so $2u = 4v + 4u$. Hence $4v = -2u$ and $v = -\frac{u}{2}$.

4. State the speed and direction

The speed of A is the magnitude of its velocity, so it is $\frac{u}{2}$. The negative sign shows that A has reversed direction.

5. Find the impulse on B

Impulse equals change in momentum. For B,
$I = 2\left(2u - (-3u)\right) = 2(5u) = 10u$.

6. Attach the units

The impulse is $10u\ \mathrm{N\,s}$, equivalently $10u\ \mathrm{kg\,m\,s^{-1}}$.

Final answer

(a) speed of A $= \frac{u}{2}$. (b) A reverses direction. (c) $I = 10u\ \mathrm{N\,s}$.

Question 2

10 marks

Working with Vectors

[*In this question* **i** *and* **j** *are horizontal perpendicular unit vectors.*]

2. A particle P rests in equilibrium on a smooth horizontal plane.

A system of **three** forces, $\mathbf{F}_1$ N, $\mathbf{F}_2$ N and $\mathbf{F}_3$ N where

$$\mathbf{F}_1 = (3c\mathbf{i} + 4c\mathbf{j})$$
$$\mathbf{F}_2 = (-14\mathbf{i} + 7\mathbf{j})$$

is applied to P.

Given that P remains in equilibrium,

(a) find $\mathbf{F}_3$ in terms of c, **i** and **j**.

(2)

The force $\mathbf{F}_3$ is **removed** from the system.

Given that $c = 2$

(b) find the size of the angle between the direction of **i** and the direction of the resultant force acting on P.

(4)

The mass of P is m kg.

Given that the magnitude of the acceleration of P is $8.5\,\mathrm{m\,s^{-2}}$

(c) find the value of m.

(4)

WME01/01 MAY/JUNE 2023

Worked Solution - Question 2

Working with Vectors

Topic group

1. Use equilibrium

For equilibrium, $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0}$. Hence $\mathbf{F}_3 = -(\mathbf{F}_1 + \mathbf{F}_2)$.

2. Find F3

$\mathbf{F}_1 + \mathbf{F}_2 = (3c - 14)\mathbf{i} + (4c + 7)\mathbf{j}$, so $\mathbf{F}_3 = (14 - 3c)\mathbf{i} + (-7 - 4c)\mathbf{j}$.

3. Find the resultant when c = 2

When $c = 2$, $\mathbf{F}_1 = 6\mathbf{i} + 8\mathbf{j}$ and $\mathbf{F}_2 = -14\mathbf{i} + 7\mathbf{j}$. Therefore the resultant is $-8\mathbf{i} + 15\mathbf{j}$.

4. Find the angle

The resultant is in the second quadrant. Its angle from the positive $\mathbf{i}$ direction is $180^\circ - \tan^{-1}\left(\frac{15}{8}\right) = 118.1^\circ$, so the angle is about 118°.

5. Find the magnitude of the resultant

$|\mathbf{R}| = \sqrt{(-8)^2 + 15^2} = 17\ \mathrm{N}$.

6. Use F = ma

The acceleration magnitude is $8.5\ \mathrm{m\,s^{-2}}$, so $17 = 8.5m$. Hence $m = 2$.

Final answer

$(a)\ \mathbf{F}_3 = (14 - 3c)\mathbf{i} + (-7 - 4c)\mathbf{j}. \quad (b)\ 118^\circ \text{ approximately}. \quad (c)\ m = 2$

.

Question 3

8 marks

Constant Acceleration in 1D

3. Two students observe a book of mass 0.2 kg fall vertically from rest from a shelf that is 1.5 m above the floor.

Student A suggests that the book is modelled as a particle falling freely under gravity.

(a) Use student A's model to find the time taken for the book to reach the floor.

(3)

Student B suggests an improved model where the book is modelled as a particle experiencing a constant resistance to motion of magnitude R newtons.

Given that the time taken for the book to reach the floor is 0.6 seconds,

(b) use student B's model to find the value of R

(5)

WME01/01 MAY/JUNE 2023

Worked Solution - Question 3

Constant Acceleration in 1D

Topic group

1. Use free fall for student A

From rest, with downward displacement 1.5 m and acceleration g, use $s = ut + \frac{1}{2}at^2$.

2. Find the time

$1.5 = 0 + \frac{1}{2}(9.8)t^2$, so $t^2 = \frac{3}{9.8}$ and $t = 0.553\ldots$ s.

3. Find the acceleration in student B's model

Now the time is 0.6 s. Again starting from rest, $1.5 = \frac{1}{2}a(0.6)^2 = 0.18a$, so $a = 8.333\ldots\ \mathrm{m\,s^{-2}}$ downward.

4. Apply Newtons second law

Taking downward as positive, weight is $0.2g$ and resistance R acts upward. Therefore $0.2g - R = 0.2a$.

5. Find R

$1.96 - R = 0.2(8.333\ldots) = 1.666\ldots$, so $R = 0.293\ldots$ N.

Final answer

$(a)\ t = 0.553$ s approximately. $(b)\ R = 0.293$ N.

WME01/01 MAY/JUNE 2023

Question 4

12 marks

Moments

4.

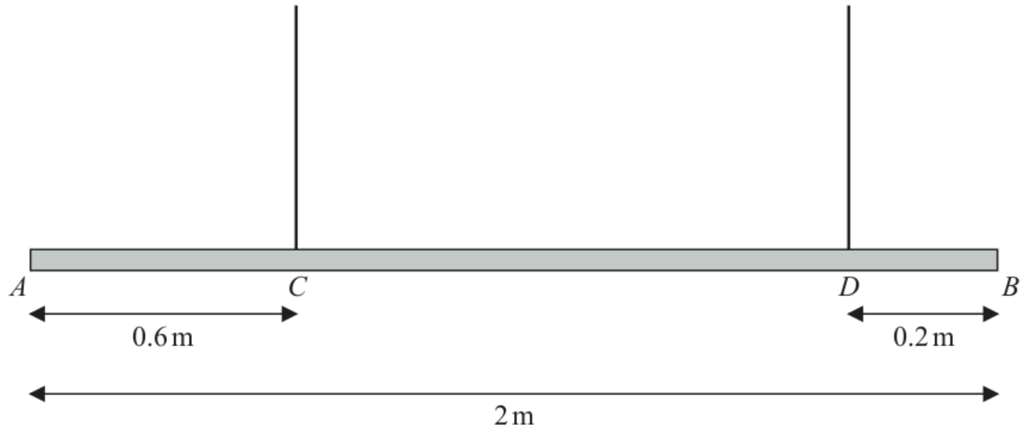

Figure 1

Figure 1 shows a beam AB, of mass m kg and length 2 m, suspended by two light vertical ropes.
The ropes are attached to the points C and D on the beam, where $AC = 0.6$ m and $DB = 0.2$ m
The beam is in equilibrium in a horizontal position.

A particle of mass pm kg is attached to the beam at A and the beam remains in equilibrium in a horizontal position.

The beam is modelled as a uniform rod.

(a) Given that the tension in the rope attached at C is four times the tension in the rope attached at D, use the model to find the exact value of p.

(7)

The particle of mass pm kg at A is removed and replaced by a particle of mass qm kg at A.
The beam remains in equilibrium in a horizontal position but is now on the point of tilting.

(b) Using the model, find the exact value of q

(4)

(c) State how you have used the modelling assumption that the beam is uniform.

(1)

WME01/01 MAY/JUNE 2023

Worked Solution - Question 4

Moments

Topic group

1. Set the geometry

The beam is 2 m long. Since $AC = 0.6$ m and $DB = 0.2$ m, point D is 1.8 m from A. The uniform beam has weight mg acting 1.0 m from A.

2. Name the tensions

Let the tension at D be T. The tension at C is four times this, so it is $4T$.

3. Use vertical equilibrium

$4T + T = pmg + mg$, so $5T = (p+1)mg$.

4. Take moments about A

The particle at A has no moment about A. Hence $4T(0.6) + T(1.8) = mg(1.0)$.

5. Find T

$2.4T + 1.8T = mg$, so $4.2T = mg$ and $T = \dfrac{5mg}{21}$.

6. Find p

Substitute into $5T = (p+1)mg$: $5 \cdot \dfrac{5mg}{21} = (p+1)mg$. Thus $p + 1 = \dfrac{25}{21}$ and $p = \dfrac{4}{21}$.

7. Use the tilting condition

When the particle of mass qm is at A and the beam is on the point of tilting, the rope at D is just slack, so its tension is zero.

8. Take moments about C

About C, the particle at A has moment $qmg(0.6)$ and the beam weight has moment $mg(0.4)$ in the opposite direction. Thus $qmg(0.6) = mg(0.4)$, giving $q = \frac{2}{3}$.

9. Use the modelling assumption

The beam is uniform, so its centre of mass is at the midpoint of the beam and its weight acts there.

Final answer

$(a)\ p = \frac{4}{21}$. $(b)\ q = \frac{2}{3}$. (c) The beam weight acts at its midpoint.

WME01/01 MAY/JUNE 2023

11 marks

Question 5

Kinematics Graphs

5.

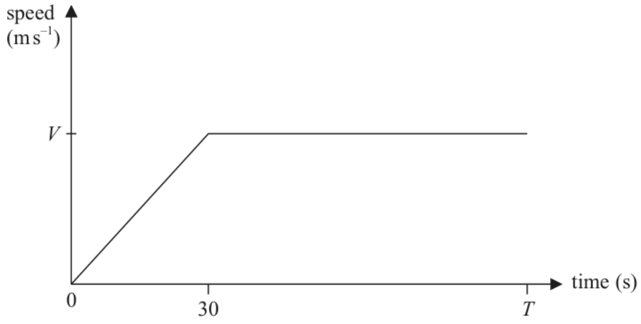

Figure 2

The speed-time graph in Figure 2 illustrates the motion of a car travelling along a straight horizontal road.
At time $t = 0$, the car starts from rest and accelerates uniformly for 30 s until it reaches a speed of V m s^{-1}
The car then travels at a constant speed of V m s^{-1} until time $t = T$ seconds.

(a) Show that the distance travelled by the car between $t = 0$ and $t = T$ seconds is $V(T - 15)$ metres.

(2)

A motorbike also travels along the same road.

- The motorbike starts from rest at time $\boldsymbol{t} = \mathbf{10}$ **s** and accelerates uniformly for 40 s
- The acceleration of the motorbike is the **same** as the acceleration of the car
- The motorbike then travels at a constant speed for a further 10 s before decelerating uniformly until it reaches a speed of V m s^{-1} at time T seconds

(b) On Figure 2, sketch a speed-time graph for the motion of the motorbike.

[If you need to redraw your sketch, there is a copy of Figure 2 on page 15.]

(2)

(c) Show that the constant speed of the motorbike is $\frac{4V}{3}$ m s^{-1}

(2)

At time $t = T$ seconds, the distance travelled by each vehicle is the same.

(d) Find the value of T

(5)

Only use this copy of Figure 2 if you need to redraw your sketch.

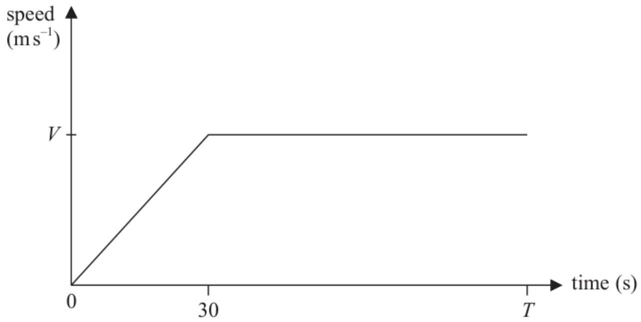

Figure 2

(Total for Question 5 is 11 marks)

WME01/01 MAY/JUNE 2023

Worked Solution - Question 5

Kinematics Graphs

Topic group

1. Find the car distance

The car graph is a triangle from 0 to 30 plus a rectangle from 30 to T. Distance $= \frac{1}{2}(30)V + V(T-30)$.

2. Simplify the car distance

$\frac{1}{2}(30)V + V(T-30) = 15V + VT - 30V = V(T-15)$, as required.

3. Describe the motorbike graph

The motorbike starts at $(10, 0)$, accelerates with the same gradient as the car until $t = 50$, then travels at constant speed until $t = 60$, then decelerates linearly to (T, V).

4. Find the motorbike constant speed

The car acceleration is $\frac{V}{30}$. The motorbike accelerates for 40 s, so its speed is $40 \cdot \frac{V}{30} = \frac{4V}{3}$.

5. Find the motorbike distance

Its distance is $\frac{1}{2}(40)\left(\frac{4V}{3}\right) + 10\left(\frac{4V}{3}\right) + \frac{1}{2}\left(\frac{4V}{3} + V\right)(T-60)$.

6. Simplify the motorbike distance

This becomes $\frac{80V}{3} + \frac{40V}{3} + \frac{7V}{6}(T-60) = 40V + \frac{7V}{6}(T-60) = \frac{7VT}{6} - 30V$.

7. Equate the two distances

At time T, the distances are equal: $V(T-15) = \frac{7VT}{6} - 30V$. Divide by V to get $T - 15 = \frac{7T}{6} - 30$.

8. Find T

Multiplying by 6 gives $6T - 90 = 7T - 180$, so $T = 90$.

Final answer

(a) car distance $= V(T - 15)$.

(b) Motorbike graph starts at $t = 10$, reaches $\dfrac{4V}{3}$ at $t = 50$, stays constant to $t = 60$, then falls to V at $t = T$.

(c) constant speed $= \dfrac{4V}{3}$. (d) $T = 90$.

Question 6

7 marks

Forces

6.

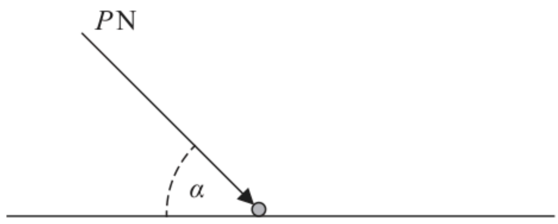

Figure 3

A particle of weight W newtons lies at rest on a rough horizontal surface, as shown in Figure 3.
A force of magnitude P newtons is applied to the particle.

The force acts at an angle α to the horizontal, where $\tan\alpha = \frac{4}{3}$

The coefficient of friction between the particle and the surface is $\frac{1}{4}$

Given that the particle does not move, show that

$$P \leqslant \frac{5W}{8}$$

(7)

WME01/01 MAY/JUNE 2023

Worked Solution - Question 6

Forces

Topic group

1. Use the trig ratios

Given $\tan\alpha = \frac{4}{3}$, use $\sin\alpha = \frac{4}{5}$ and $\cos\alpha = \frac{3}{5}$.

2. Resolve vertically

The applied force increases the normal reaction in this model, so $R = W + P\sin\alpha$.

3. Resolve horizontally

For the particle not to move, friction must be able to balance the horizontal component of the applied force, so $P\cos\alpha \le F_{\max}$.

4. Use limiting friction

Since $\mu = \frac{1}{4}$, $F_{\max} = \frac{1}{4}R = \frac{1}{4}(W + P\sin\alpha)$.

5. Substitute the exact trig values

Thus $P \cdot \frac{3}{5} \le \frac{1}{4}\left(W + P \cdot \frac{4}{5}\right)$.

6. Rearrange the inequality

Multiplying by 20 gives $12P \le 5W + 4P$. Hence $8P \le 5W$, so $P \le \frac{5}{8}W$.

Final answer

$P \le \frac{5}{8}W$.

7.

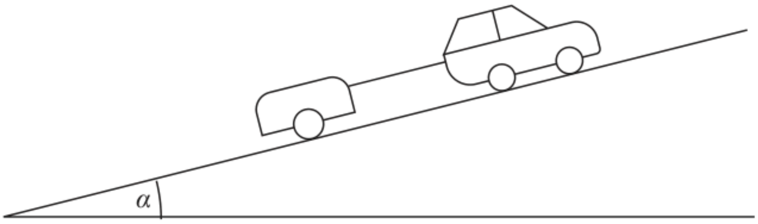

Figure 4

A car of mass 1200 kg is towing a trailer of mass 600 kg up a straight road, as shown in Figure 4.

The road is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{12}$

The driving force produced by the engine of the car is 3000 N.

The car moves with acceleration $0.75\,\text{m}\,\text{s}^{-2}$

The non-gravitational resistance to motion of

- the **car** is modelled as a constant force of magnitude $2R$ newtons
- the **trailer** is modelled as a constant force of magnitude R newtons

The car and the trailer are modelled as particles.
The tow bar between the car and trailer is modelled as a light rod that is parallel to the direction of motion.

Using the model,

(a) show that the value of R is 60

(4)

(b) find the tension in the tow bar.

(3)

When the car and trailer are moving at a speed of $12\,\text{m}\,\text{s}^{-1}$, the tow bar breaks.

Given that the non-gravitational resistance to motion of the trailer remains unchanged,

(c) use the model to find the further distance moved by the trailer before it first comes to rest.

(4)

WME01/01 MAY/JUNE 2023

Worked Solution - Question 7

Newton's Second Law

Topic group

1. Use the whole system

For car plus trailer, the total mass is 1800 kg and the total resistance is $2R + R = 3R$. Taking up the slope as positive: $3000 - 1800g\sin\alpha - 3R = 1800(0.75)$.

2. Show R is 60

Since $\sin\alpha = \frac{1}{12}$, $1800g\sin\alpha = 1800 \cdot 9.8 \cdot \frac{1}{12} = 1470$. So $3000 - 1470 - 3R = 1350$, giving $180 = 3R$ and $R = 60\text{ N}$.

3. Apply Newtons second law to the trailer

For the trailer while it is being towed, $T - 600g\sin\alpha - R = 600(0.75)$.

4. Find the tow-bar tension

$T - 600 \cdot 9.8 \cdot \frac{1}{12} - 60 = 450$, so $T - 490 - 60 = 450$ and $T = 1000\text{ N}$.

5. Find the trailer acceleration after the break

After the tow bar breaks, the trailer is still moving up the slope, but the component of weight and resistance act down the slope. Taking up the slope as positive: $-600g\sin\alpha - 60 = 600a$.

6. Simplify the acceleration

$-490 - 60 = 600a$, so $a = -\frac{550}{600} = -\frac{11}{12}\text{ m s}^{-2}$.

7. Find the stopping distance

With initial speed 12 m s^{-1} and final speed 0, use $v^2 = u^2 + 2as$: $0 = 12^2 + 2\left(-\frac{11}{12}\right)d$. Hence $d = \frac{864}{11} = 78.5\ldots\text{ m}$.

Final answer

(a) $R = 60$ N. (b) $T = 1000$ N. (c) $d = 78.5$ m approximately.

WME01/01 MAY/JUNE 2023

Question 8

9 marks

Working with Vectors

8.

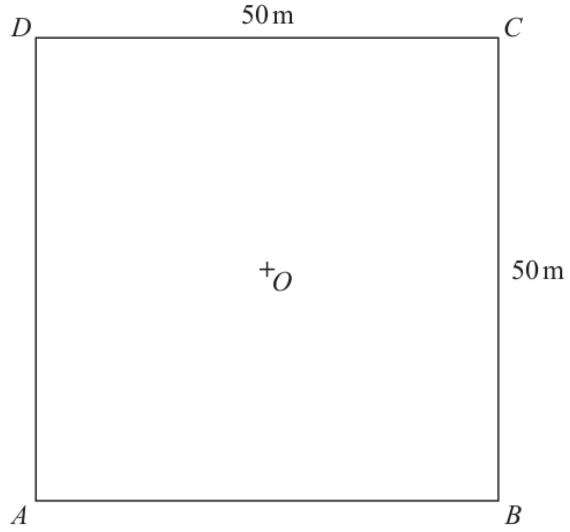

Figure 5

A square floor space $ABCD$, with centre O, is modelled as a flat horizontal surface measuring 50 m by 50 m, as shown in Figure 5.
The horizontal unit vectors **i** and **j** are in the direction of $\overrightarrow{AB}$ and $\overrightarrow{AD}$ respectively.

All position vectors are given relative to O.

A small robot R is programmed to travel across the floor at a constant velocity.

- At time $t = 0$, R is at the point with position vector $(-2\mathbf{i} + \mathbf{j})$ m
- At time $t = 11$ s, R is at the point with position vector $(9\mathbf{i} + 23\mathbf{j})$ m
- At time t seconds, the position vector of R is **r** metres

(a) Find, in terms of t, **i** and **j**, an expression for **r**

(3)

A second robot S is at the point C.

- At time $t = 0$, S leaves C and moves with constant velocity $(-\mathbf{i} - \mathbf{j})\,\mathrm{m\,s^{-1}}$
- At time t seconds, the position vector of S is **s** metres

(b) Write down, in terms of t, **i** and **j**, an expression for **s**

(1)

(c) Show that

$$\overrightarrow{SR} = [(2t - 27)\mathbf{i} + (3t - 24)\mathbf{j}]\ \mathrm{m}$$

(2)

(d) Find the time when the distance between R and S is a minimum.

(3)

Worked Solution - Question 8

Working with Vectors

Topic group

1. Find the velocity of R

R moves from $-2\mathbf{i}+\mathbf{j}$ to $9\mathbf{i}+23\mathbf{j}$ in 11 s. Its displacement is $11\mathbf{i}+22\mathbf{j}$, so its velocity is $\mathbf{i}+2\mathbf{j}$.

2. Write r

$\mathbf{r}=(-2\mathbf{i}+\mathbf{j})+t(\mathbf{i}+2\mathbf{j})=(t-2)\mathbf{i}+(2t+1)\mathbf{j}$.

3. Write s

Point C has position vector $25\mathbf{i}+25\mathbf{j}$. Since S moves with velocity $-\mathbf{i}-\mathbf{j}$, $\mathbf{s}=(25-t)\mathbf{i}+(25-t)\mathbf{j}$.

4. Find SR vector

$\overrightarrow{SR}=\mathbf{r}-\mathbf{s}=[(t-2)-(25-t)]\mathbf{i}+[(2t+1)-(25-t)]\mathbf{j}$.

5. Simplify SR

Therefore $\overrightarrow{SR}=(2t-27)\mathbf{i}+(3t-24)\mathbf{j}$ m.

6. Minimise the distance squared

The distance squared is $d^2=(2t-27)^2+(3t-24)^2$.

7. Expand the quadratic

$d^2=4t^2-108t+729+9t^2-144t+576=13t^2-252t+1305$.

8. Find the minimum time

The quadratic is minimum at $t=\dfrac{252}{2\cdot 13}=\dfrac{126}{13}=9.69\ldots$ s.

Final answer

(a) $\mathbf{r} = (t-2)\mathbf{i} + (2t+1)\mathbf{j}$. (b) $\mathbf{s} = (25-t)\mathbf{i} + (25-t)\mathbf{j}$. (c) $\overrightarrow{SR} = (2t-27)\mathbf{i} + (3t-24)\mathbf{j}$. (d) $t = \frac{126}{13}\ \mathrm{s} \approx 9.7\ \mathrm{s}$

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PAST PAPER

WME01/01 October 2023

October 2023 | 7 questions | 75 marks

7
questions

75
marks

Answers
worked solution after each question

WME01/01 OCTOBER 2023

5 marks

Question 1

Moments

1.

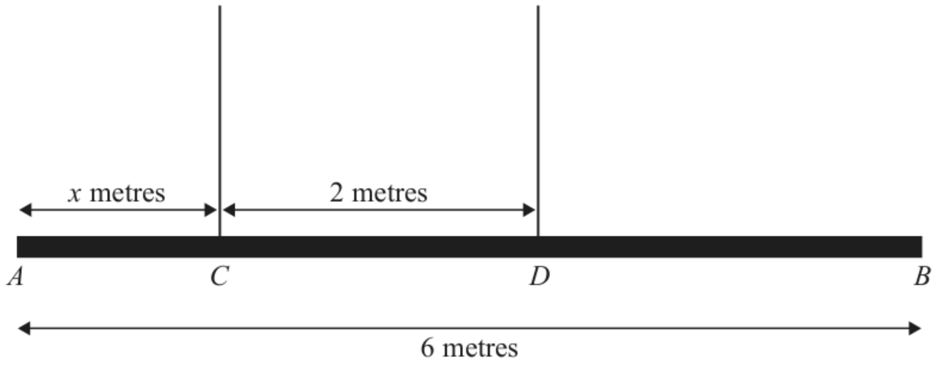

Figure 1

Figure 1 shows a beam AB with weight 24 N and length 6 m.

The beam is suspended by two light vertical ropes. The ropes are attached to the points C and D on the beam where $AC = x$ metres and $CD = 2$ m.

The tension in the rope attached to the beam at C is double the tension in the rope attached to the beam at D.

The beam is modelled as a uniform rod, resting horizontally in equilibrium.

Find

(i) the tension in the rope attached to the beam at D.

(ii) the value of x.

(5)

WME01/01 OCTOBER 2023

Worked Solution - Question 1

Moments

Topic group

1. Name the tensions

Let the tension at D be T. The tension at C is double this, so it is $2T$.

2. Use vertical equilibrium

The beam is in equilibrium, so the upward forces equal the weight: $2T + T = 24$. Hence $3T = 24$ and $T = 8\text{ N}$.

3. Set the moment distances

The beam is uniform, so its weight acts at the midpoint, 3 m from A. Also $AC = x$ and $AD = x + 2$.

4. Take moments about A

Clockwise and anticlockwise moments balance: $(2T)x + T(x+2) = 24 \cdot 3$.

5. Solve for x

Substitute $T = 8$: $16x + 8(x+2) = 72$. Thus $24x + 16 = 72$, so $24x = 56$ and $x = \dfrac{7}{3}$ m.

Final answer

$(i)\ T_D = 8\text{ N}. \quad (ii)\ x = \dfrac{7}{3}\text{ m} \approx 2.33\text{ m}.$

Question 2

2.

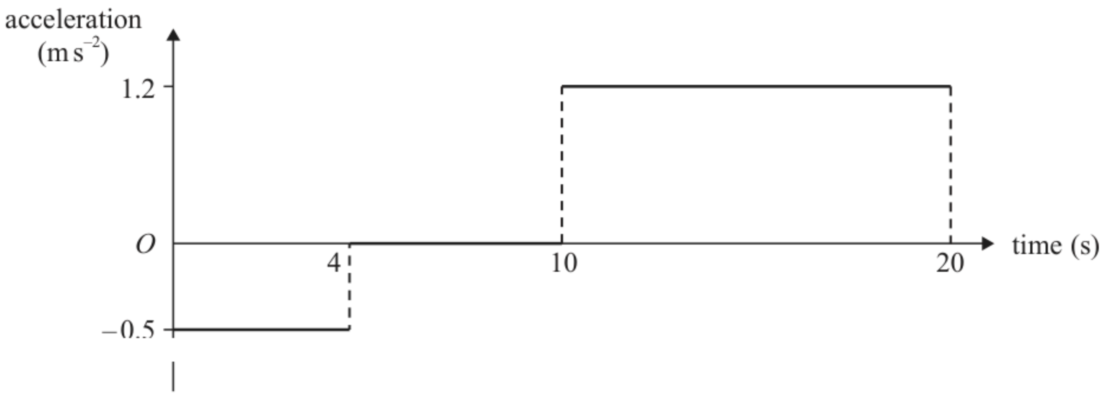

Figure 2

Two fixed points, A and B, are on a straight horizontal road.

The **acceleration-time** graph in Figure 2 represents the motion of a car travelling along the road as it moves from A to B.

At time $t = 0$, the car passes through A with speed $8\,\text{m}\,\text{s}^{-1}$

At time $t = 20\,\text{s}$, the car passes through B with speed $v\,\text{m}\,\text{s}^{-1}$

(a) Show that $v = 18$

(3)

(b) Sketch a speed-time graph for the motion of the car from A to B.

(3)

(c) Find the distance AB.

(4)

WME01/01 OCTOBER 2023

Worked Solution - Question 2

Kinematics Graphs

Topic group

1. Find the speed at t = 4

From 0 to 4 s the acceleration is $-0.5\ \mathrm{m\,s^{-2}}$. Therefore $w = 8 + (-0.5)(4) = 6\ \mathrm{m\,s^{-1}}$.

2. Find the final speed

From 10 to 20 s the acceleration is $1.2\ \mathrm{m\,s^{-2}}$, while the speed at $t = 10$ is still 6. Hence $v = 6 + 1.2(10) = 18\ \mathrm{m\,s^{-1}}$.

3. Describe the speed-time graph

The graph starts at $(0, 8)$, falls linearly to $(4, 6)$, stays horizontal to $(10, 6)$, then rises linearly to $(20, 18)$.

4. Use area under the speed-time graph

The distance is the area under the graph. From 0 to 4: $\dfrac{8+6}{2} \cdot 4 = 28$. From 4 to 10: $6 \cdot 6 = 36$. From 10 to 20: $\dfrac{6+18}{2} \cdot 10 = 120$.

5. Add the distances

$AB = 28 + 36 + 120 = 184\ \mathrm{m}$.

Final answer

(a) $v = 18\ \mathrm{m\,s^{-1}}$.

(b) Speed-time graph through $(0, 8), (4, 6), (10, 6), (20, 18)$.

(c) $AB = 184\ \mathrm{m}$.

Question 3

3. A hammer is used to hit a tent peg into soft ground.

The hammer has mass 1.8 kg and the tent peg has mass 0.2 kg.

The hammer and tent peg are both modelled as particles and the impact is modelled as a direct collision.

Immediately before the impact, the tent peg is stationary and the hammer is moving vertically downwards with speed $10\,\text{m s}^{-1}$

Immediately after the impact, the hammer and tent peg move together, vertically downwards, with the **same** speed $v\,\text{m s}^{-1}$

(a) Find the value of v

(2)

(b) Find the magnitude of the impulse exerted on the tent peg by the hammer, stating the units of your answer.

(3)

The ground exerts a constant vertical resistive force of magnitude R newtons, bringing the hammer and tent peg to rest after they travel a distance of 12 cm.

(c) Find the value of R.

(5)

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Worked Solution - Question 3

Momentum, Impulse & Collisions

Topic group

1. Use conservation of momentum

Take downward as positive during the impact. Before impact, the hammer has momentum $1.8(10)$ and the peg has momentum 0. After impact, the combined mass is $1.8 + 0.2 = 2.0$ kg.

2. Find the common speed

$1.8(10) = 2.0v$, so $18 = 2v$ and $v = 9\ \mathrm{m\,s^{-1}}$.

3. Use impulse on the peg

The peg changes from rest to speed $9\ \mathrm{m\,s^{-1}}$ downward. The impulse magnitude is $0.2(9 - 0) = 1.8\ \mathrm{N\,s}$.

4. Find the deceleration in the ground

The hammer and peg move together from speed 9 to rest over 0.12 m. With downward positive, $0 = 9^2 + 2a(0.12)$, so $a = -337.5\ \mathrm{m\,s^{-2}}$.

5. Apply Newtons second law

For the combined mass 2.0 kg, forces are weight $2g$ downward and resistance R upward. With downward positive, $2g - R = 2a$.

6. Find R

$19.6 - R = 2(-337.5) = -675$, so $R = 694.6\ \mathrm{N}$, which is about $695\ \mathrm{N}$.

Final answer

(a) $v = 9\,\mathrm{m\,s^{-1}}$.

(b) $I = 1.8\,\mathrm{N\,s}$.

(c) $R \approx 695\,\mathrm{N}$.

Question 4

10 marks

Working with Vectors

4. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively.*]

A particle P moves with constant acceleration $(-\lambda\mathbf{i} + 2\lambda\mathbf{j})\,\mathrm{m\,s^{-2}}$, where λ is a positive constant.

At time $t = 0$, the velocity of P is $(5\mathbf{i} - 8\mathbf{j})\,\mathrm{m\,s^{-1}}$

(a) Find the velocity of P when $t = 5\,\mathrm{s}$, giving your answer in terms of **i**, **j** and λ.

(2)

The speed of P when $t = 5\,\mathrm{s}$ is $13\,\mathrm{m\,s^{-1}}$

(b) Show that

$$25\lambda^2 - 42\lambda - 16 = 0$$

(3)

(c) Find the direction of motion of P when $t = 4\,\mathrm{s}$, giving your answer as a bearing to the nearest degree.

(5)

Worked Solution - Question 4

Working with Vectors

Topic group

1. Use v = u + at

The initial velocity is $(5\mathbf{i} - 8\mathbf{j})$ and the acceleration is $(-\lambda\mathbf{i} + 2\lambda\mathbf{j})$. At $t = 5$, $\mathbf{v} = (5\mathbf{i} - 8\mathbf{j}) + 5(-\lambda\mathbf{i} + 2\lambda\mathbf{j})$.

2. Write the velocity at t = 5

So $\mathbf{v} = (5 - 5\lambda)\mathbf{i} + (-8 + 10\lambda)\mathbf{j}$.

3. Use the speed at t = 5

The speed is 13, so $(5 - 5\lambda)^2 + (-8 + 10\lambda)^2 = 13^2$.

4. Simplify to the printed equation

Expanding gives $25 - 50\lambda + 25\lambda^2 + 64 - 160\lambda + 100\lambda^2 = 169$. Therefore $125\lambda^2 - 210\lambda - 80 = 0$, and dividing by 5 gives $25\lambda^2 - 42\lambda - 16 = 0$.

5. Find lambda

Solving $25\lambda^2 - 42\lambda - 16 = 0$ gives $\lambda = 2$ or $\lambda = -\dfrac{8}{25}$. Since λ is positive, $\lambda = 2$.

6. Find the velocity at t = 4

$\mathbf{v}(4) = (5\mathbf{i} - 8\mathbf{j}) + 4(-2\mathbf{i} + 4\mathbf{j}) = -3\mathbf{i} + 8\mathbf{j}$.

7. Convert to a bearing

The motion is north-west. The angle west of north satisfies $\tan\alpha = \dfrac{3}{8}$, so $\alpha = 20.6°$. The bearing is $360° - 20.6° = 339.4°$, so $339°$ to the nearest degree.

Final answer

(a) $\mathbf{v} = (5 - 5\lambda)\mathbf{i} + (-8 + 10\lambda)\mathbf{j}$.

(b) $25\lambda^2 - 42\lambda - 16 = 0$.

(c) bearing $= 339°$.

Question 5

12 marks

Forces

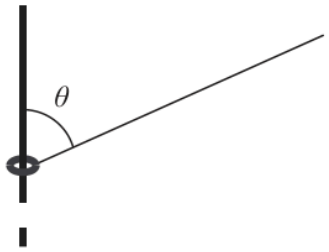

Figure 3

A small ring of mass 0.2 kg is attached to one end of a light inextensible string.

The ring is **threaded** onto a fixed rough vertical rod.

The string is taut and makes an angle θ with the rod, as shown in Figure 3, where $\tan\theta = \frac{12}{5}$

Given that the ring is in equilibrium and that the tension in the string is 10 N,

(a) find the magnitude of the frictional force acting on the ring,

(3)

(b) state the direction of the frictional force acting on the ring.

(1)

The coefficient of friction between the ring and the rod is $\frac{1}{4}$

Given that the ring is in equilibrium, and that the tension in the string, T newtons, can now vary,

(c) (i) find the minimum value of T

(ii) find the maximum value of T

(8)

Worked Solution - Question 5

Forces

Topic group

1. Use the trig ratios

The string makes angle θ with the vertical rod and $\tan\theta = \frac{12}{5}$. Hence $\sin\theta = \frac{12}{13}$ and $\cos\theta = \frac{5}{13}$.

2. Find the friction for T = 10

The upward component of the tension is $10\cos\theta = 10\cdot\frac{5}{13} = \frac{50}{13}$ N. The weight is $0.2g = 1.96$ N.

3. Balance vertical forces

Since the upward component of tension is larger than the weight, friction must act downwards. Its magnitude is $\frac{50}{13} - 1.96 = 1.89\ldots$ N, so $F = 1.9$ N.

4. Write the normal reaction

For variable tension T, the normal reaction from the rod is produced by the horizontal component of tension: $R = T\sin\theta = \frac{12T}{13}$.

5. Use limiting friction

Since $\mu = \frac{1}{4}$, limiting friction is $F = \mu R = \frac{1}{4}\cdot\frac{12T}{13} = \frac{3T}{13}$.

6. Find the minimum tension

For the minimum tension, the ring would tend to slide down, so friction acts upwards. Vertical equilibrium gives $T\cos\theta + F = 0.2g$, hence $\frac{5T}{13} + \frac{3T}{13} = 1.96$. Thus $T = 3.185\ldots$ N, so $T_{\text{min}} = 3.2$ N.

7. Find the maximum tension

For the maximum tension, the ring would tend to move up, so friction acts downwards. Vertical equilibrium gives $T\cos\theta = 0.2g + F$, hence $\dfrac{5T}{13} = 1.96 + \dfrac{3T}{13}$. Thus $T = 12.74\ldots\ \mathrm{N}$, so $T_{\max} = 13\ \mathrm{N}$.

Final answer

(a) $F = 1.9\ \mathrm{N}$.

(b) Friction acts downwards.

$(c)(i)$ $T_{\min} = 3.2\ \mathrm{N}$. $(c)(ii)$ $T_{\max} = 13\ \mathrm{N}$.

Question 6

15 marks

Working with Vectors

6. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

At 12:00, a ship P sets sail from a harbour with position vector $(15\mathbf{i} + 36\mathbf{j})$ km.
At 12:30, P is at the point with position vector $(20\mathbf{i} + 34\mathbf{j})$ km.

Given that P moves with constant velocity,

(a) show that the velocity of P is $(10\mathbf{i} - 4\mathbf{j})\,\text{km h}^{-1}$

(2)

At time t hours after 12:00, the position vector of P is $\mathbf{p}$ km.

(b) Find an expression for $\mathbf{p}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and t.

(2)

A second ship Q is also travelling at a constant velocity.

At time t hours after 12:00, the position vector of Q is given by $\mathbf{q}$ km, where

$$\mathbf{q} = (42 - 8t)\mathbf{i} + (9 + 14t)\mathbf{j}$$

Ships P and Q are modelled as particles.

If both ships maintained their course,

(c) (i) verify that they would collide at 13:30

(ii) find the position vector of the point at which the collision would occur.

(4)

At 12:30 Q changes speed and direction to avoid the collision.

Ship Q now travels due north with a constant speed of $15\,\text{km h}^{-1}$

Ship P maintains the same constant velocity throughout.

(d) Find the exact distance between P and Q at 14:30

(7)

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Worked Solution - Question 6

Working with Vectors

Topic group

1. Find the velocity of P

From 12:00 to 12:30, the displacement of P is $(20\mathbf{i} + 34\mathbf{j}) - (15\mathbf{i} + 36\mathbf{j}) = 5\mathbf{i} - 2\mathbf{j}$ km. The time is 0.5 hours, so $\mathbf{v}_P = \dfrac{5\mathbf{i} - 2\mathbf{j}}{0.5} = 10\mathbf{i} - 4\mathbf{j}$.

2. Write the position of P

At t hours after 12:00, $\mathbf{p} = (15\mathbf{i} + 36\mathbf{j}) + t(10\mathbf{i} - 4\mathbf{j}) = (15 + 10t)\mathbf{i} + (36 - 4t)\mathbf{j}$.

3. Verify the collision time

At 13:30, $t = 1.5$. Then $\mathbf{p} = (15 + 15)\mathbf{i} + (36 - 6)\mathbf{j} = 30\mathbf{i} + 30\mathbf{j}$. Also $\mathbf{q} = (42 - 8(1.5))\mathbf{i} + (9 + 14(1.5))\mathbf{j} = 30\mathbf{i} + 30\mathbf{j}$.

4. State the collision position

Since both position vectors are the same, the collision point has position vector $30\mathbf{i} + 30\mathbf{j}$ km.

5. Find P at 14:30

At 14:30, $t = 2.5$, so $\mathbf{p} = (15 + 25)\mathbf{i} + (36 - 10)\mathbf{j} = 40\mathbf{i} + 26\mathbf{j}$.

6. Find Q after changing course

At 12:30, $t = 0.5$, so $\mathbf{q} = 38\mathbf{i} + 16\mathbf{j}$. From 12:30 to 14:30 is 2 hours due north at $15\ \mathrm{km\,h^{-1}}$, so the extra displacement is $30\mathbf{j}$. Thus Q is at $38\mathbf{i} + 46\mathbf{j}$.

7. Find the distance

The separation vector is $(40 - 38)\mathbf{i} + (26 - 46)\mathbf{j} = 2\mathbf{i} - 20\mathbf{j}$. The distance is $\sqrt{2^2 + 20^2} = \sqrt{404} = 2\sqrt{101}\ \mathrm{km}$.

Final answer

(a) $\mathbf{v}_P = 10\mathbf{i} - 4\mathbf{j}\ \mathrm{km\,h^{-1}}$. (b) $\mathbf{p} = (15 + 10t)\mathbf{i} + (36 - 4t)\mathbf{j}$. $(c)(ii)$ $30\mathbf{i} + 30\mathbf{j}$. (d) $2\sqrt{101}\ \mathrm{km}$

.

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13 marks

Question 7

Resolving Forces, Inclined Planes

7.

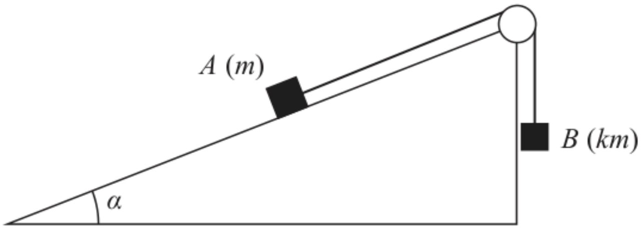

Figure 4

Figure 4 shows a block A of mass m held at rest on a rough plane.
The plane is inclined at an angle α to the horizontal and the coefficient of friction between the block and the plane is μ.

One end of a light inextensible string is now attached to A. The string passes over a small smooth pulley which is fixed at the top of the plane.
The other end of the string is attached to a block B of mass km.
Block B hangs vertically below the pulley, with the string taut.

The string from A to the pulley lies along a line of greatest slope of the plane.

Both A and B are modelled as particles.

When the system is released from rest, A moves up the plane and the tension in the string is $\frac{4mg}{3}$

Given that $\mu = \frac{1}{3}$ and $\tan\alpha = \frac{7}{24}$

(a) (i) find the magnitude of the acceleration of A, giving your answer in terms of g,

(ii) find the value of k.

(9)

(b) Find the magnitude of the resultant force exerted on the pulley by the string, giving your answer in terms of m and g.

(4)

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Worked Solution - Question 7

Resolving Forces, Inclined Planes

Topic group

1. Use the trig ratios

Given $\tan\alpha = \frac{7}{24}$, use $\sin\alpha = \frac{7}{25}$ and $\cos\alpha = \frac{24}{25}$.

2. Find friction on A

For block A, $R = mg\cos\alpha = \frac{24mg}{25}$. Since $\mu = \frac{1}{3}$, friction is $F = \frac{1}{3}R = \frac{8mg}{25}$.

3. Apply Newtons second law to A

A moves up the plane, so friction and the component of weight act down the plane. Taking up the plane as positive: $T - mg\sin\alpha - F = ma$.

4. Find the acceleration

Substitute $T = \frac{4}{3}mg$, $mg\sin\alpha = \frac{7mg}{25}$ and $F = \frac{8mg}{25}$:

$ma = \frac{4}{3}mg - \frac{15mg}{25} = \left(\frac{4}{3} - \frac{3}{5}\right)mg = \frac{11mg}{15}$. Hence $a = \frac{11g}{15}$.

5. Apply Newtons second law to B

Block B moves downwards with the same acceleration. Taking downward as positive: $kmg - T = kma$.

6. Find k

Substitute $T = \frac{4}{3}mg$ and $a = \frac{11g}{15}$: $kmg - \frac{4}{3}mg = km\frac{11g}{15}$. Divide by mg to get $k - \frac{4}{3} = \frac{11k}{15}$, so $15k - 20 = 11k$ and $k = 5$.

7. Resolve the force on the pulley

The pulley is pulled by two tensions of magnitude T. One is vertical and the other is along the plane. The horizontal component of the resultant is $T\cos\alpha$ and the vertical component is $T + T\sin\alpha$.

8. Find the resultant

The magnitude is $T\sqrt{\cos^2\alpha + (1+\sin\alpha)^2}$. With $\sin\alpha = \dfrac{7}{25}$ and $\cos\alpha = \dfrac{24}{25}$, this is $T\sqrt{\dfrac{576}{625} + \dfrac{1024}{625}} = \dfrac{8T}{5}$. Since $T = \dfrac{4}{3}mg$, the resultant is $\dfrac{32}{15}mg$.

Final answer

$(a)(i)\ a = \dfrac{11g}{15}. \quad (a)(ii)\ k = 5. \quad (b)\ \text{resultant} = \dfrac{32}{15}mg.$

PAST PAPER

WME01/01 January 2024

January 2024 | 8 questions | 75 marks

8
questions

75
marks

Answers
worked solution after each question

Question 1

1.

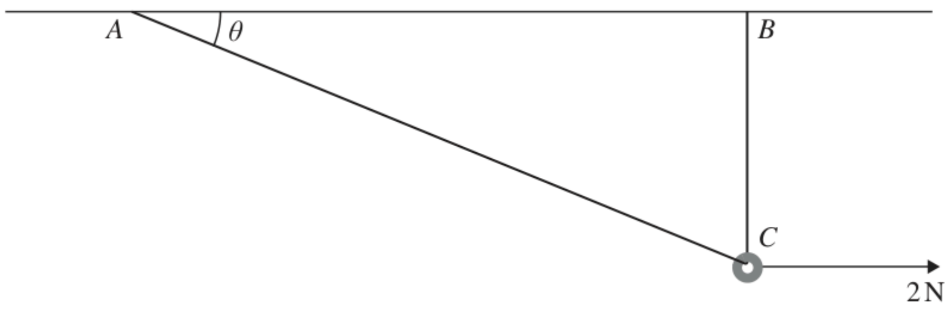

Figure 1

Figure 1 shows a small smooth ring **threaded** onto a light inextensible string.

One end of the string is attached to a fixed point A on a horizontal ceiling and the other end of the string is attached to a fixed point B on the ceiling.

A horizontal force of magnitude 2 N acts on the ring so that the ring rests in equilibrium at a point C, vertically below B, with the string taut.

The line of action of the horizontal force and the string both lie in the same vertical plane.

The angle that the string makes with the ceiling at A is θ, where $\tan\theta = \frac{3}{4}$

The tension in the string is T newtons. The mass of the ring is M kg.

(a) Find the value of T

(3)

(b) Find the value of M

(3)

WME01/01 JANUARY 2024

Worked Solution - Question 1

Forces

Topic group

1. Use the trig ratios

Given $\tan\theta = \frac{3}{4}$, use $\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$.

2. Resolve horizontally

The horizontal force of 2 N is balanced by the horizontal component of the tension in the sloping part of the string: $T\cos\theta = 2$.

3. Find the tension

$T \cdot \frac{4}{5} = 2$, so $T = 2.5\text{ N}$.

4. Resolve vertically

The ring is smooth, so the tension is the same in both parts of the string. Upward components are T from the vertical part and $T\sin\theta$ from the sloping part. Therefore $T + T\sin\theta = Mg$.

5. Find the mass

$Mg = 2.5 + 2.5 \cdot \frac{3}{5} = 4.0$, so $M = \frac{4.0}{9.8} = 0.408\text{ kg}$.

Final answer

$(a)\ T = 2.5\text{ N}. \quad (b)\ M = 0.408\text{ kg approximately}.$

WME01/01 JANUARY 2024

6 marks

Question 2

Momentum, Impulse & Collisions

2.

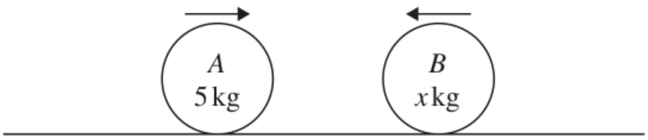

Figure 2

Figure 2 shows two particles, A and B, moving in opposite directions on a smooth horizontal surface. Particle A has mass 5 kg and particle B has mass x kg.

The particles collide directly.

Immediately before the collision, the speed of A is $3\,\text{m}\,\text{s}^{-1}$ and the speed of B is $x\,\text{m}\,\text{s}^{-1}$

Immediately after the collision, the speed of A is $1\,\text{m}\,\text{s}^{-1}$ and its direction of motion is unchanged.

Immediately after the collision, the speed of B is $1.5\,\text{m}\,\text{s}^{-1}$

(a) Find the value of x.

(3)

(b) Find the magnitude of the impulse exerted on A by B in the collision.

(3)

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Worked Solution - Question 2

Momentum, Impulse & Collisions

Topic group

1. Choose a sign convention

Take the initial direction of A as positive. Before collision, A has velocity 3 and B has velocity $-x$.

2. Set up conservation of momentum

After collision, A still moves in the same direction with velocity 1. B has speed 1.5 in the positive direction for the valid solution. Hence $5(3) + x(-x) = 5(1) + 1.5x$.

3. Solve for x

$15 - x^2 = 5 + 1.5x$, so $x^2 + 1.5x - 10 = 0$. This gives $x = 2.5$ or $x = -4$. Since mass is positive, $x = 2.5$.

4. Use change in momentum of A

The impulse exerted on A equals the change in momentum of A: $I = 5(1-3) = -10\ \mathrm{N\,s}$ using the chosen positive direction.

5. Give the magnitude

The magnitude of the impulse is $10\ \mathrm{N\,s}$.

Final answer

$(a)\ x = 2.5. \quad (b)\ I = 10\ \mathrm{N\,s}.$

Question 3

10 marks

Constant Acceleration in 1D

3. A van travels with constant acceleration along a straight horizontal road.

The van passes a point *A* with speed $u\,\mathrm{m\,s^{-1}}$ and 20 seconds later passes a point *B* with speed $28\,\mathrm{m\,s^{-1}}$

The distance *AB* is 400 m.

(a) Show that $u = 12$

(2)

(b) Find the time taken for the van to travel from *A* to the midpoint of *AB*.

(5)

The van has mass 1200 kg.

During its motion the van experiences a constant resistive force of magnitude 260 N

(c) Find the magnitude of the driving force exerted by the engine of the van as it travels from *A* to *B*.

(3)

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Worked Solution - Question 3

Constant Acceleration in 1D

Topic group

1. Use average speed from A to B

With constant acceleration, distance equals average speed times time. So $400 = \dfrac{u+28}{2} \cdot 20$.

2. Show u is 12

$400 = 10(u+28)$, so $u + 28 = 40$ and $u = 12$.

3. Find the acceleration

Use $v = u + at$: $28 = 12 + 20a$, hence $a = 0.8\ \mathrm{m\,s^{-2}}$.

4. Use the midpoint distance

The midpoint of AB is 200 m from A. Use $s = ut + \frac{1}{2}at^2$: $200 = 12t + \frac{1}{2}(0.8)t^2$.

5. Solve for the time to the midpoint

$200 = 12t + 0.4t^2$. The positive solution is $t = 5\sqrt{29} - 15 = 11.9258\ldots$ s.

6. Use Newtons second law for the van

Let the driving force be D. The resultant forward force is $D - 260$. Since $ma = 1200(0.8) = 960$, $D - 260 = 960$.

7. Find the driving force

$D = 1220$ N.

Final answer

$(a)\ u = 12.$ $\quad (b)\ t = 5\sqrt{29} - 15 \approx 11.9\ \text{s}.$ $\quad (c)\ \text{driving force} = 1220\ \text{N}$

.

Question 4

8 marks

Forces

4.

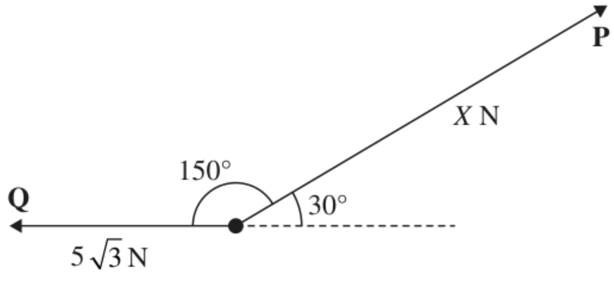

Figure 3

Figure 3 shows two horizontal forces **P** and **Q** acting on a particle.

The angle between the direction of **P** and the direction of **Q** is 150°

Force **P** has magnitude X newtons.

Force **Q** has magnitude $5\sqrt{3}$ N.

The resultant of **P** and **Q** has magnitude $\sqrt{129}$ N.

Find

(i) the value of X.

(ii) the angle between **Q** and the resultant, giving your answer to the nearest degree.

(8)

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Worked Solution - Question 4

Forces

Topic group

1. Use the included angle in the force triangle

The two given forces have angle 150° between them, so the triangle used for adding them has the supplementary angle 30° between the side of length X and the side of length $5\sqrt{3}$.

2. Use the cosine rule to find X

The resultant has magnitude $\sqrt{129}$. Therefore $129 = X^2 + (5\sqrt{3})^2 - 2X(5\sqrt{3})\cos 30^\circ$.

3. Solve for X

Since $(5\sqrt{3})^2 = 75$ and $2(5\sqrt{3})\cos 30^\circ = 15$, the equation becomes $129 = X^2 + 75 - 15X$. Thus $X^2 - 15X - 54 = 0$, giving $X = 18$.

4. Find the angle with Q

Let β be the angle between Q and the resultant. Using the cosine rule in the same triangle, $\dfrac{18}{\sin\beta} = \dfrac{\sqrt{129}}{\sin 30^\circ}$.

5. Choose the correct angle

This gives an obtuse angle for β. Equivalently, $\cos\beta = \dfrac{(5\sqrt{3})^2 + 129 - 18^2}{2(5\sqrt{3})\sqrt{129}}$, so $\beta = 127.6\ldots^\circ$.

6. Round the angle

To the nearest degree, the angle between Q and the resultant is 128°.

Final answer

(i) $X = 18$ N. (ii) angle $= 128^\circ$ to the nearest degree.

Question 5

10 marks

Moments

5.

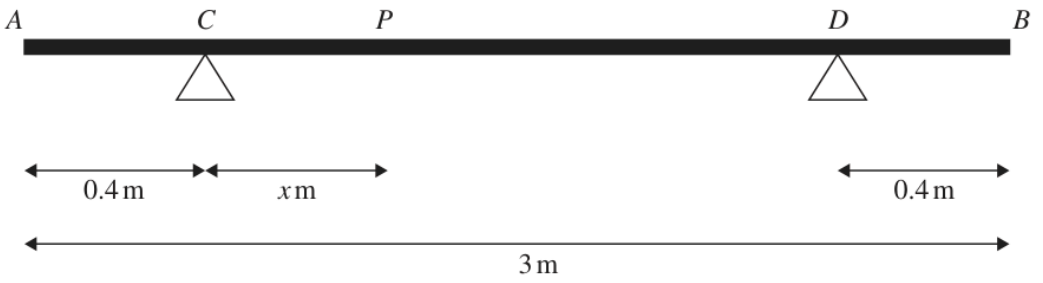

Figure 4

A beam AB has mass 30 kg and length 3 m.

The beam rests on supports at C and D where $AC = 0.4$ m and $DB = 0.4$ m, as shown in Figure 4.

A person of mass 55 kg stands on the beam between C and D.

The person is modelled as a particle at the point P, where $CP = x$ metres and $0 < x < 2.2$

The beam is modelled as a uniform rod resting in equilibrium in a horizontal position.

Using the model,

(a) show that the magnitude of the reaction at C is $(686 - 245x)$ N.

(3)

The magnitude of the reaction at C is **four** times the magnitude of the reaction at D.

Using the model,

(b) find the value of x

(4)

The person steps off the beam and places a package of mass M kg at A.

The package is modelled as a particle at the point A.

The beam is now on the point of tilting about C.

Using the model,

(c) find the value of M

(3)

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Worked Solution - Question 5

Moments

Topic group

1. Set the positions on the beam

$AC = 0.4$ m and $DB = 0.4$ m, so $CD = 2.2$ m. The beam is uniform, so its weight $30g$ acts at the midpoint, which is 1.1 m from D. The person is $2.2 - x$ m from D.

2. Take moments about D

Taking moments about D removes R_D: $R_C(2.2) = 55g(2.2 - x) + 30g(1.1)$.

3. Show the expression for RC

Using $g = 9.8$, $R_C = \dfrac{539(2.2 - x) + 294(1.1)}{2.2} = 686 - 245x$.

4. Use the reaction ratio

Given $R_C = 4R_D$ and vertical equilibrium gives $R_C + R_D = (55 + 30)g = 833$. Hence $5R_D = 833$ and $R_C = 666.4$.

5. Find x

$686 - 245x = 666.4$, so $245x = 19.6$ and $x = 0.08$.

6. Use the tilting condition with the package

When the package is at A and the beam is on the point of tilting about C, the reaction at D is zero.

7. Find M

Take moments about C: $Mg(0.4) = 30g(1.1)$. Thus $M = \dfrac{33}{0.4} = 82.5$.

Final answer

$(a)\ R_C = 686 - 245x.$ $(b)\ x = 0.08.$ $(c)\ M = 82.5.$

Question 6

12 marks

Constant Acceleration in 1D

6. A particle is projected vertically upwards from a point A with speed $24\,\text{m}\,\text{s}^{-1}$

The point A is 2.5 m vertically above the point B.

Point B lies on horizontal ground.

The particle moves freely under gravity until it hits the ground at B with speed $V\,\text{m}\,\text{s}^{-1}$

After hitting the ground the particle does not rebound.

(a) Find the value of V.

(3)

(b) Find the time taken for the particle to reach B.

(3)

The point C is 10 m vertically above A.

(c) Find the length of time for which the particle is above C.

(4)

(d) Sketch a speed-time graph for the motion of the particle from projection to the instant that it reaches B. (No further calculations are required.)

(2)

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Worked Solution - Question 6

Constant Acceleration in 1D

Topic group

1. Find the impact speed

Take upward as positive from A. At B, the displacement is -2.5 m, with $u = 24$ and $a = -9.8$. Use $v^2 = u^2 + 2as$: $V^2 = 24^2 + 2(-9.8)(-2.5) = 625$.

2. State V

$V = 25\ \mathrm{m\,s^{-1}}$.

3. Find the time to reach B

The velocity at B is $-25\ \mathrm{m\,s^{-1}}$. Using $v = u + at$, $-25 = 24 - 9.8t$, so $t = 5$ s.

4. Set up the height C equation

C is 10 m above A. The particle is above C between the two times when $s = 10$: $10 = 24t - 4.9t^2$.

5. Solve for the two crossing times

$4.9t^2 - 24t + 10 = 0$, giving $t = 0.459\ldots$ and $t = 4.438\ldots$.

6. Find the time above C

The length of time above C is $4.438\ldots - 0.459\ldots = 3.98\ldots$ s, about 4.0 s.

7. Describe the speed-time graph

The graph starts at speed 24, decreases linearly to 0 at the highest point, then increases linearly to speed 25 at $t = 5$. It is a V-shaped speed-time graph.

Final answer

(a) $V = 25\ \mathrm{m\,s^{-1}}$. (b) $t = 5$ s. (c) 4.0 s approximately. (d) Speed-time graph is V-shaped, from speed 24 to 0, then to speed 25 at t=5.

Question 7

11 marks

Working with Vectors

7. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

At midnight, a ship S is at the point with position vector $(19\mathbf{i} + 22\mathbf{j})$ km

The ship travels with constant velocity $(12\mathbf{i} - 16\mathbf{j})\,\text{km h}^{-1}$

(a) Find the speed of S.

(2)

At time t hours after midnight, the position vector of S is $\mathbf{s}$ km.

(b) Find an expression for $\mathbf{s}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and t.

(2)

A lighthouse stands on a small rocky island. The lighthouse is modelled as being at the point with position vector $(26\mathbf{i} + 15\mathbf{j})$ km.

It is not safe for ships to be within 1.3 km of the lighthouse.

(c) (i) Find the value of t when S is closest to the lighthouse.

(ii) Hence determine whether it is safe for S to continue its course.

(7)

Worked Solution - Question 7

Working with Vectors

Topic group

1. Find the speed

The velocity is $12\mathbf{i} - 16\mathbf{j}$, so the speed is $\sqrt{12^2 + (-16)^2} = 20\ \mathrm{km\,h^{-1}}$.

2. Write the ship position vector

At time t hours after midnight,
$\mathbf{s} = (19\mathbf{i} + 22\mathbf{j}) + t(12\mathbf{i} - 16\mathbf{j}) = (19 + 12t)\mathbf{i} + (22 - 16t)\mathbf{j}$.

3. Form the vector from lighthouse to ship

The lighthouse is at $26\mathbf{i} + 15\mathbf{j}$. Therefore $\overrightarrow{LS} = \mathbf{s} - \mathbf{l} = (12t - 7)\mathbf{i} + (7 - 16t)\mathbf{j}$.

4. Use distance squared

The square of the distance is
$d^2 = (12t - 7)^2 + (7 - 16t)^2 = 400t^2 - 392t + 98$.

5. Find the time of closest approach

This quadratic is minimum when $t = \dfrac{392}{2 \cdot 400} = 0.49$.

6. Find the minimum distance

At $t = 0.49$, $d^2 = 1.96$, so $d = 1.4\ \mathrm{km}$.

7. Decide if the course is safe

The unsafe distance is 1.3 km. Since $1.4 > 1.3$, it is safe for the ship to continue its course.

Final answer

(a) $20\ \mathrm{km\,h^{-1}}$. (b) $\mathbf{s} = (19 + 12t)\mathbf{i} + (22 - 16t)\mathbf{j}$. $(c)(i)$ $t = 0.49$. $(c)(ii)$ minimum distance $= 1.4$ km, so it is safe

.

Question 8

8.

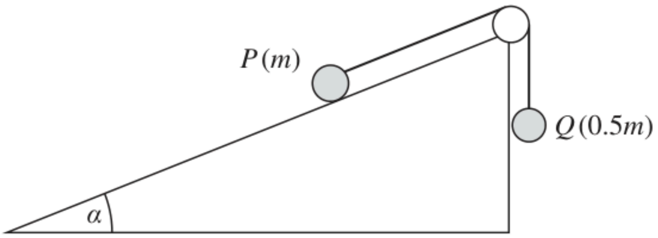

Figure 5

A fixed rough plane is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{5}{12}$

A small smooth pulley is fixed at the top of the plane.

One end of a light inextensible string is attached to a particle P which is at rest on the plane. The string passes over the pulley and the other end of the string is attached to a particle Q which hangs vertically below the pulley, as shown in Figure 5.

Particle P has mass m and particle Q has mass $0.5m$

The string from P to the pulley lies along a line of greatest slope of the plane.

The coefficient of friction between P and the plane is μ.

The system is in **limiting equilibrium** with the string taut and P is on the point of slipping **up** the plane.

(a) Find the value of μ.

(8)

The string breaks and P begins to move down the plane.

When particle P has travelled a distance of 0.8 m down the plane, the speed of P is $V\,\text{m}\,\text{s}^{-1}$

(b) Find the value of V.

(4)

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Worked Solution - Question 8

Resolving Forces, Inclined Planes

Topic group

1. Use the trig ratios

Given $\tan\alpha = \frac{5}{12}$, use $\sin\alpha = \frac{5}{13}$ and $\cos\alpha = \frac{12}{13}$.

2. Find the tension

Q has mass $0.5m$ and the system is in equilibrium, so the tension is $T = 0.5mg$.

3. Resolve P parallel to the plane

P is on the point of slipping up the plane, so friction acts down the plane. Along the plane, $T = mg\sin\alpha + F$.

4. Find the friction force

$F = T - mg\sin\alpha = \frac{1}{2}mg - \frac{5}{13}mg = \frac{3}{26}mg$.

5. Find the normal reaction and mu

$R = mg\cos\alpha = \frac{12}{13}mg$. Since $F = \mu R$, $\mu = \frac{3mg/26}{12mg/13} = \frac{1}{8}$.

6. Find acceleration after the string breaks

P moves down the plane, so friction acts up the plane. The resultant down the plane is $mg\sin\alpha - F = \frac{5}{13}mg - \frac{1}{8}\cdot\frac{12}{13}mg = \frac{7}{26}mg$.

7. Apply Newtons second law

As P has mass m, $a = \frac{7g}{26}$ down the plane.

8. Find V

From rest over 0.8 m, $V^2 = 2\left(\frac{7g}{26}\right)(0.8)$. With $g = 9.8$, $V = 2.05\ \mathrm{m\,s^{-1}}$ to 3 significant figures.

Final answer

(a) $\mu = \dfrac{1}{8}$. (b) $V = 2.05\ \mathrm{m\,s^{-1}}$ approximately.

PAST PAPER

WME01/01 May/June 2024

May/June 2024 | 8 questions | 75 marks

8 questions	**75** marks	**Answers** worked solution after each question

Question 1

1. Two particles, A and B, have masses m and $3m$ respectively. The particles are connected by a light inextensible string. Initially A and B are at rest on a smooth horizontal plane with the string slack.

Particle A is then projected along the plane away from B with speed U.

Given that the common speed of the particles immediately after the string becomes taut is S

(a) find S in terms of U.

(2)

(b) Find, in terms of m and U, the magnitude of the impulse exerted on A immediately after the string becomes taut.

(3)

Worked Solution - Question 1

Momentum, Impulse & Collisions

Topic group

1. Use conservation of momentum

When the string becomes taut, A and B move with the same speed S. Before this, only A is moving. Hence $mU = (m + 3m)S = 4mS$.

2. Find the common speed

From $mU = 4mS$, cancel m to get $S = \dfrac{U}{4}$.

3. Use impulse on A

The impulse on A equals the magnitude of its change in momentum:
$I = m\left|\dfrac{U}{4} - U\right|$.

4. Simplify the impulse

$I = m\left(\dfrac{3U}{4}\right) = \dfrac{3mU}{4}$.

Final answer

$(a)\ S = \dfrac{U}{4}. \quad (b)\ I = \dfrac{3mU}{4}.$

Question 2

4 marks

Forces

2. Two forces, **P** and **Q**, act on a particle.

- **P** has magnitude 10 N and acts due west
- **Q** has magnitude 8 N and acts on a bearing of 330°

Given that **F** = **P** + **Q**, find the magnitude of **F**.

(4)

Worked Solution - Question 2

Forces

Topic group

1. Resolve the two forces

Take east as $\mathbf{i}$ and north as $\mathbf{j}$. The force due west is $-10\mathbf{i}$. A bearing of $330°$ is $30°$ west of north, so the 8 N force has components $-8\sin 30°\mathbf{i} + 8\cos 30°\mathbf{j}$.

2. Add the components

$\mathbf{F} = (-10 - 4)\mathbf{i} + 4\sqrt{3}\mathbf{j} = -14\mathbf{i} + 4\sqrt{3}\mathbf{j}$.

3. Find the magnitude

$|\mathbf{F}| = \sqrt{(-14)^2 + (4\sqrt{3})^2} = \sqrt{196 + 48} = \sqrt{244} = 2\sqrt{61}\ \mathrm{N}$.

4. Give a decimal value

$2\sqrt{61} = 15.620\ldots$, so the magnitude is about $16\ \mathrm{N}$ to 2 significant figures.

Final answer

$|\mathbf{F}| = 2\sqrt{61}\ \mathrm{N} \approx 15.6\ \mathrm{N}$.

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6 marks

Question 3

Newton's Second Law

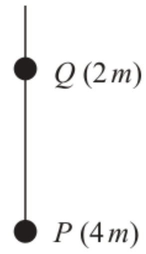

Figure 1

Two particles, P and Q, have masses $4m$ and $2m$ respectively. The particles are connected by a light inextensible string. A second light inextensible string has one end attached to Q. Both strings are taut and vertical, as shown in Figure 1.

The particles are **accelerating** vertically **downwards**.

Given that the tension in the string connecting the two particles is $3mg$, find, in terms of m and g, the tension in the upper string.

(6)

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Worked Solution - Question 3

Newton's Second Law

Topic group

1. Use the lower particle to find acceleration

For P, take downward as positive. Its weight is $4mg$ and the tension in the string between P and Q is $3mg$ upward, so $4mg - 3mg = 4ma$.

2. Find the acceleration

The equation gives $mg = 4ma$, hence $a = \frac{g}{4}$ downward.

3. Write the equation for Q

For Q, downward forces are its weight $2mg$ and the lower string tension $3mg$. The upper string tension T acts upward. Taking downward as positive:
$2mg + 3mg - T = 2ma$.

4. Substitute acceleration

$5mg - T = 2m\left(\frac{g}{4}\right) = \frac{mg}{2}$.

5. Find the upper tension

$T = 5mg - \frac{mg}{2} = \frac{9mg}{2}$.

Final answer

$T = \frac{9mg}{2} = 4.5mg$.

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Question 4

6 marks

Moments

4.

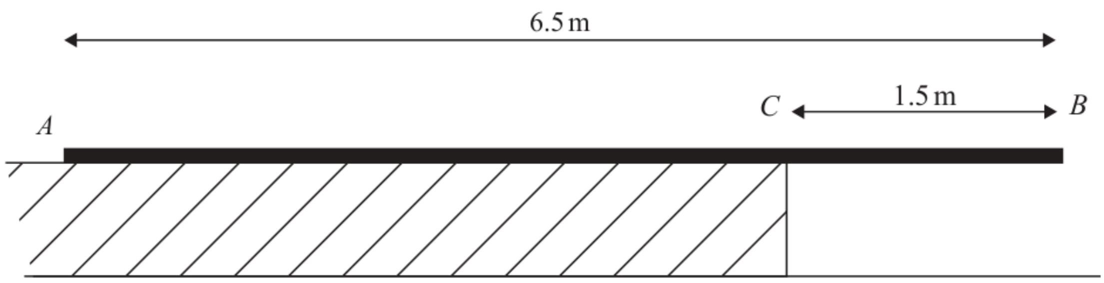

Figure 2

A non-uniform rod *AB* has length 6.5 m and mass 1.2 kg. The centre of mass of the rod is 3 m from *A*. The rod rests on a horizontal step and overhangs the end of the step *C* by 1.5 m, as shown in Figure 2.

The rod is perpendicular to the edge of the step.

A particle of mass 4 kg is placed on the rod at *B* and another particle, whose mass is *M* kg, is placed on the rod at *D*, where $AD = 0.5$ m.

The rod remains in equilibrium in a horizontal position.

(a) Find the smallest possible value of *M*.

(3)

The particle at *B* **and** the particle at *D* are now **removed**.

A new particle is placed on the rod at the point *E*, where $EB = 0.9$ m.

The rod remains in equilibrium in a horizontal position but is on the point of tilting about *C*.

(b) Find the magnitude of the force acting on the rod at *C*.

(3)

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Worked Solution - Question 4

Moments

Topic group

1. Set the positions relative to C

The rod is 6.5 m long and overhangs C by 1.5 m, so C is 5.0 m from A. The rod centre of mass is 3.0 m from A, so it is 2.0 m to the left of C. Point D is 0.5 m from A, so it is 4.5 m to the left of C.

2. Use the limiting balance for smallest M

For the smallest M, the rod is just prevented from tilting clockwise about C. Taking moments about C: $Mg(4.5) + 1.2g(2) = 4g(1.5)$.

3. Find M

$4.5M + 2.4 = 6$, so $4.5M = 3.6$ and $M = 0.8$.

4. Place the new particle E

Now the old particles are removed. Since $EB = 0.9$ m and $AB = 6.5$ m, E is 5.6 m from A, so E is 0.6 m to the right of C.

5. Find the new particle mass from moments

At tilting about C, moments about C give $Xg(0.6) = 1.2g(2)$, so $X = 4$.

6. Find the force at C

The upward force at C balances the rod and the new particle:
$R_C = 1.2g + 4g = 5.2g\,\mathrm{N}$, which is about $51\ \mathrm{N}$.

Final answer

$(a)\ M = 0.8. \quad (b)\ R_C = 5.2g\,\mathrm{N} \approx 51\ \mathrm{N}.$

Question 5

5. A parachute is used to deliver a box of supplies. The parachute is attached to the box.

- the parachute and box are dropped from rest from a helicopter that is hovering at a height of 520 m above the ground
- the parachute and box fall vertically and freely under gravity for 5 seconds, then the parachute opens
- from the instant the parachute opens, it provides a resistance to motion of magnitude 3200 N
- the parachute and box continue to fall vertically downwards after the parachute opens
- the parachute and box are modelled throughout the motion as a particle *P* of mass 250 kg

(a) Find the distance fallen by *P* in the first 5 seconds.

(2)

(b) Find the speed with which *P* lands on the ground.

(7)

(c) Find the total time from the instant when *P* is dropped from the helicopter to the instant when *P* lands on the ground.

(3)

(d) Sketch a speed-time graph for the motion of *P* from the instant when *P* is dropped from the helicopter to the instant when *P* lands on the ground.

(2)

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Worked Solution - Question 5

Constant Acceleration in 1D

Topic group

1. Find the first distance

For the first 5 seconds, P falls freely from rest. Using $s = ut + \frac{1}{2}at^2$, $s = 0 + \frac{1}{2}(9.8)(5^2) = 122.5\ \mathrm{m}$.

2. Find the speed when the parachute opens

After 5 seconds of free fall, $v = 0 + 9.8(5) = 49\ \mathrm{m\,s^{-1}}$ downward.

3. Find the new acceleration

After the parachute opens, weight is $250g = 2450$ N downward and resistance is 3200 N upward. Taking downward as positive, the resultant is $2450 - 3200 = -750$ N, so $a = \frac{-750}{250} = -3\ \mathrm{m\,s^{-2}}$.

4. Use the remaining distance

The remaining distance is $520 - 122.5 = 397.5$ m. Using $v^2 = u^2 + 2as$: $v^2 = 49^2 + 2(-3)(397.5) = 16$.

5. Find the landing speed

$v = 4\ \mathrm{m\,s^{-1}}$, so P lands with speed $4\ \mathrm{m\,s^{-1}}$.

6. Find the time after the parachute opens

Using $v = u + at$, $4 = 49 - 3t$, so $t = 15$ s after the parachute opens.

7. Find the total time and describe the graph

The total time is $5 + 15 = 20$ s. The speed-time graph rises linearly from 0 to 49 over the first 5 s, then falls linearly to 4 at $t = 20$; it does not meet the time-axis.

Final answer

(a) $122.5\ \mathrm{m}$ or $123\ \mathrm{m}$. (b) $4\ \mathrm{m\,s^{-1}}$. (c) 20 s. (d) Speed-time graph: straight line from $(0, 0)$ to $(5, 49)$ then straight line down to $(20, 4)$.

Question 6

6.

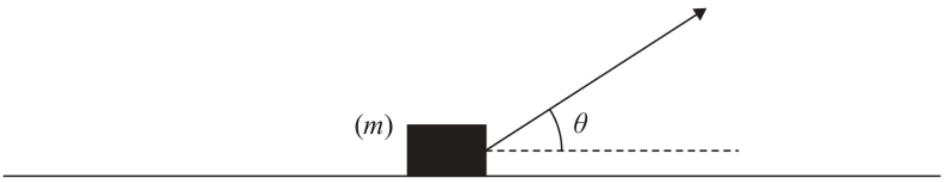

Figure 3

A box of mass m lies on a rough horizontal plane. The box is pulled along the plane in a straight line at **constant speed** by a light rope. The rope is inclined at an angle θ to the plane, as shown in Figure 3.

The coefficient of friction between the box and the plane is $\frac{1}{3}$

The box is modelled as a particle.

Given that $\tan\theta = \frac{3}{4}$

(a) find, in terms of m and g, the tension in the rope.

(7)

The rope is now removed and the box is placed at rest on the plane.
The box is then projected horizontally along the plane with speed u.

The box is again modelled as a particle.

When the box has moved a distance d along the plane, the speed of the box is $\frac{1}{2}u$.

(b) Find d in terms of u and g.

(5)

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Worked Solution - Question 6

Resolving Forces, Inclined Planes

Topic group

1. Use the trig ratios

Given $\tan\theta = \frac{3}{4}$, use $\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$.

2. Resolve vertically

The box moves at constant speed, so it is in equilibrium. Vertically, $R + T\sin\theta = mg$, hence $R = mg - T\sin\theta$.

3. Resolve horizontally

Horizontally, the tension component balances friction: $T\cos\theta = F$.

4. Use friction

With $\mu = \frac{1}{3}$, $F = \frac{1}{3}R$. Therefore $T\cos\theta = \frac{1}{3}(mg - T\sin\theta)$.

5. Find T

Substitute $\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$: $\frac{4}{5}T = \frac{1}{3}\left(mg - \frac{3}{5}T\right)$. This gives $\frac{4}{5}T = \frac{mg}{3} - \frac{1}{5}T$, so $T = \frac{mg}{3}$.

6. Find the deceleration when the rope is removed

On the horizontal plane without the rope, $R = mg$ and friction is $\frac{1}{3}mg$. Hence the acceleration is $-\frac{g}{3}$.

7. Use SUVAT to find d

When the speed has fallen from u to $\frac{1}{2}u$, use $v^2 = u^2 + 2as$: $\left(\frac{u}{2}\right)^2 = u^2 + 2\left(-\frac{g}{3}\right)d$.

8. Solve for d

$\frac{u^2}{4} = u^2 - \frac{2gd}{3}$, so $\frac{2gd}{3} = \frac{3u^2}{4}$ and $d = \frac{9u^2}{8g}$.

Final answer

$(a)\ T = \frac{mg}{3}$. $\quad (b)\ d = \frac{9u^2}{8g}$.

Question 7

13 marks

Working with Vectors

7. [*In this question, the horizontal unit vectors* **i** *and* **j** *are directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two speedboats, A and B, are each moving with constant velocity.

- the velocity of A is $40\,\text{km}\,\text{h}^{-1}$ due east
- the velocity of B is $20\,\text{km}\,\text{h}^{-1}$ on a bearing of angle α ($0° < \alpha < 90°$), where $\tan\alpha = \frac{4}{3}$

The boats are modelled as particles.

(a) Find, in terms of **i** and **j**, the velocity of B in $\text{km}\,\text{h}^{-1}$

(2)

At noon

- the position vector of A is $20\mathbf{j}$ km
- the position vector of B is $(10\mathbf{i} + 5\mathbf{j})$ km

At time t hours after noon

- the position vector of A is $\mathbf{r}$ km, where $\mathbf{r} = 20\mathbf{j} + 40t\mathbf{i}$
- the position vector of B is $\mathbf{s}$ km

(b) Find an expression for $\mathbf{s}$ in terms of t, **i** and **j**.

(2)

(c) Show that at time t hours after noon,

$$\overrightarrow{AB} = [(10 - 24t)\mathbf{i} + (12t - 15)\mathbf{j}]\,\text{km}$$

(2)

(d) Show that the boats will never collide.

(3)

(e) Find the distance between the boats when the bearing of B from A is 225°

(4)

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Worked Solution - Question 7

Working with Vectors

Topic group

1. Resolve the velocity of B

For a bearing angle α with $\tan\alpha = \frac{4}{3}$, use $\sin\alpha = \frac{4}{5}$ and $\cos\alpha = \frac{3}{5}$. Therefore $\mathbf{v}_B = 20\sin\alpha\mathbf{i} + 20\cos\alpha\mathbf{j} = 16\mathbf{i} + 12\mathbf{j}$.

2. Write the position of B

At noon, B is at $10\mathbf{i} + 5\mathbf{j}$. With constant velocity $16\mathbf{i} + 12\mathbf{j}$, $\mathbf{s} = (10 + 16t)\mathbf{i} + (5 + 12t)\mathbf{j}$.

3. Find AB vector

A has position vector $\mathbf{r} = 40t\mathbf{i} + 20\mathbf{j}$. Hence $\overrightarrow{AB} = \mathbf{s} - \mathbf{r} = (10 + 16t - 40t)\mathbf{i} + (5 + 12t - 20)\mathbf{j}$.

4. Show the printed expression

This simplifies to $\overrightarrow{AB} = (10 - 24t)\mathbf{i} + (12t - 15)\mathbf{j}$ km.

5. Show the boats do not collide

For collision, both components must be zero. From $10 - 24t = 0$, $t = \frac{5}{12}$. From $12t - 15 = 0$, $t = \frac{5}{4}$. These times are different, so the boats never collide.

6. Use the bearing 225 degrees condition

A bearing of $225°$ means B is south-west of A, so the i and j components of $\overrightarrow{AB}$ are equal and negative. Set $10 - 24t = 12t - 15$.

7. Find the distance

The equation gives $t = \frac{25}{36}$. Then each component is $-\frac{20}{3}$, so the distance is $\sqrt{\left(\frac{20}{3}\right)^2 + \left(\frac{20}{3}\right)^2} = \frac{20\sqrt{2}}{3}$ km.

Final answer

(*a*) $\mathbf{v}_B = 16\mathbf{i} + 12\mathbf{j}\ \mathrm{km\,h^{-1}}$. (*b*) $\mathbf{s} = (10 + 16t)\mathbf{i} + (5 + 12t)\mathbf{j}$. (*c*) $\overrightarrow{AB} = (10 - 24t)\mathbf{i} + (12t - 15)\mathbf{j}$. (*d*) no collision. (*e*) $\frac{20\sqrt{2}}{3}$ km ≈ 9.4 km

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15 marks

Question 8

Resolving Forces, Inclined Planes

8.

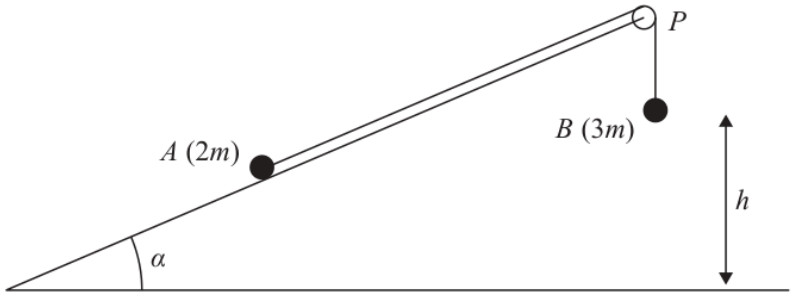

Figure 4

One end of a light inextensible string is attached to a particle A of mass $2m$. The other end of the string is attached to a particle B of mass $3m$. Particle A is held at rest on a rough plane which is inclined to horizontal ground at an angle α, where $\tan\alpha = \frac{5}{12}$

The string passes over a small smooth pulley P which is fixed at the top of the plane. Particle B hangs vertically below P with the string taut, at a height h above the ground, as shown in Figure 4.

The part of the string between A and P lies along a line of greatest slope of the plane. The two particles, the string and the pulley all lie in the same vertical plane.

The coefficient of friction between A and the plane is $\frac{11}{36}$

The particle A is released from rest and begins to move up the plane.

(a) Show that the frictional force acting on A as it moves up the plane is $\frac{22mg}{39}$

(3)

(b) Write down an equation of motion for B.

(2)

(c) Show that the acceleration of A immediately after its release is $\frac{1}{3}g$

(4)

In the subsequent motion, A comes to rest before it reaches the pulley.

(d) Find, in terms of h, the total distance travelled by A from when it was released from rest to when it first comes to rest again.

(6)

WME01/01 MAY/JUNE 2024

Worked Solution - Question 8

Resolving Forces, Inclined Planes

Topic group

1. Use the trig ratios

Here $\tan\alpha = \frac{5}{12}$, so $\sin\alpha = \frac{5}{13}$ and $\cos\alpha = \frac{12}{13}$.

2. Show the frictional force on A

For A, $R = 2mg\cos\alpha = 2mg \cdot \frac{12}{13} = \frac{24mg}{13}$. Since $\mu = \frac{11}{36}$, $F = \mu R = \frac{11}{36} \cdot \frac{24mg}{13} = \frac{22mg}{39}$.

3. Write the equation for B

B moves down as A moves up. Taking downward as positive for B: $3mg - T = 3ma$.

4. Write the equation for A

Taking up the plane as positive for A: $T - 2mg\sin\alpha - F = 2ma$. Substituting $\sin\alpha = \frac{5}{13}$ and $F = \frac{22mg}{39}$ gives $T - \frac{10mg}{13} - \frac{22mg}{39} = 2ma$.

5. Show the acceleration

From B, $T = 3mg - 3ma$. Substitute into A equation: $3mg - 3ma - \frac{10mg}{13} - \frac{22mg}{39} = 2ma$. The constant part is $\frac{5mg}{3}$, so $\frac{5mg}{3} = 5ma$ and $a = \frac{g}{3}$.

6. Find the speed when B reaches the ground

While B descends distance h, A moves distance h up the plane from rest with acceleration $g/3$. Thus $v^2 = 2\left(\frac{g}{3}\right)h = \frac{2gh}{3}$.

7. Find the later deceleration of A

After B reaches the ground, the string no longer pulls A. The forces down the plane on A are $2mg\sin\alpha + F = \frac{10mg}{13} + \frac{22mg}{39} = \frac{4mg}{3}$. Since A has mass $2m$, its deceleration is $\frac{2g}{3}$.

8. Find the extra distance and total distance

Let the extra distance be d. Using $0 = v^2 - 2\left(\frac{2g}{3}\right)d$ with $v^2 = \frac{2gh}{3}$ gives $d = \frac{h}{2}$. Total distance travelled by A is $h + \frac{h}{2} = \frac{3h}{2}$.

Final answer

$(a)\ F = \frac{22}{39}mg. \quad (b)\ 3mg - T = 3ma. \quad (c)\ a = \frac{g}{3}. \quad (d)\ \text{total distance} = \frac{3h}{2}$

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PAST PAPER

WME01/01 October 2024

October 2024 | 7 questions | 75 marks

7
questions

75
marks

Answers
worked solution after each question

WME01/01 OCTOBER 2024

6 marks

Question 1

Momentum, Impulse & Collisions

1. Particle A has mass $4m$ and particle B has mass $3m$.

The particles are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.

Immediately **before** the collision, the speed of A is $2x$ and the speed of B is x.

Immediately **after** the collision, the speed of A is y and the speed of B is $5y$.

The direction of motion of each particle is reversed as a result of the collision.

(a) Show that $y = \frac{5}{11}x$.

(3)

(b) Find, in terms of m and x, the magnitude of the impulse received by A in the collision.

(3)

WME01/01 OCTOBER 2024

Worked Solution - Question 1

Momentum, Impulse & Collisions

Topic group

1. Choose a sign convention

Take the initial direction of A as positive. Before collision, A has velocity $2x$ and B has velocity $-x$. After collision, both reverse direction, so A has velocity $-y$ and B has velocity $5y$.

2. Apply conservation of momentum

$4m(2x) + 3m(-x) = 4m(-y) + 3m(5y)$. The left side is $5mx$ and the right side is $11my$.

3. Show the ratio

$5mx = 11my$, so $\dfrac{y}{x} = \dfrac{5}{11}$.

4. Use change in momentum for A

The impulse received by A is the change in momentum of A:
$I = 4m|-y-2x| = 4m(y+2x)$.

5. Substitute y

Using $y = \dfrac{5x}{11}$, $I = 4m\left(\dfrac{5x}{11} + 2x\right) = 4m\left(\dfrac{27x}{11}\right) = \dfrac{108}{11}mx$.

Final answer

$(a)\ \dfrac{y}{x} = \dfrac{5}{11}$. $\quad (b)\ I = \dfrac{108}{11}mx$.

Question 2

9 marks

Moments

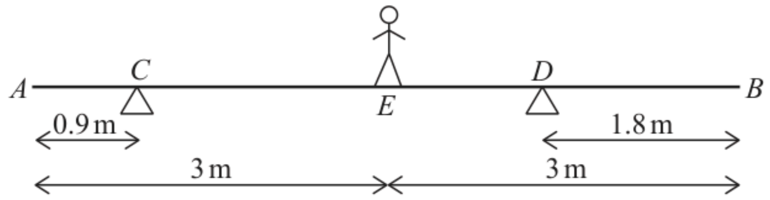


Figure 1

A non-uniform beam AB has length 6 m and mass 50 kg. The beam rests horizontally on two supports at C and D, where $AC = 0.9$ m and $DB = 1.8$ m.

A child of mass 25 kg stands on the beam at E, where $AE = EB = 3$ m, as shown in Figure 1.

The beam is in equilibrium.

The magnitude of the normal reaction between the beam and the support at C is R_C newtons.

The magnitude of the normal reaction between the beam and the support at D is R_D newtons.

The beam is modelled as a rod and the child is modelled as a particle.

The centre of mass of the beam is between C and D and is a distance x metres from D.

Given that $2R_D = 3R_C$

(a) show that $x = 1.38$

(6)

The child remains at E and a block of mass M kg is placed on the beam at B.

The block is modelled as a particle.

Given that the beam is on the point of tilting,

(b) find the value of M.

(3)

WME01/01 OCTOBER 2024

Worked Solution - Question 2

Moments

Topic group

1. Find the reactions from vertical equilibrium

Vertically, $R_C + R_D = 50g + 25g = 75g$. Given $2R_D = 3R_C$, we have $R_D = 1.5R_C$. Therefore $2.5R_C = 75g$, so $R_C = 30g$ and $R_D = 45g$.

2. Set the distances from D

D is $6 - 1.8 = 4.2$ m from A. The support C is $4.2 - 0.9 = 3.3$ m to the left of D, the child at E is $4.2 - 3 = 1.2$ m to the left of D, and the centre of mass is x m to the left of D.

3. Take moments about D

Balancing moments about D gives $50gx + 25g(1.2) = R_C(3.3)$.

4. Show x is 1.38

Substitute $R_C = 30g$: $50x + 30 = 99$, so $50x = 69$ and $x = \dfrac{69}{50} = 1.38$.

5. Use the tilting condition with the block

When the block at B is on the point of making the beam tilt, the beam is about to pivot about D and the reaction at C is zero.

6. Take moments about D for the block

The child and the beam act to the left of D, and the block at B acts 1.8 m to the right of D. Hence $25g(1.2) + 50g(1.38) = Mg(1.8)$.

7. Find M

$30 + 69 = 1.8M$, so $M = \dfrac{99}{1.8} = 55$.

Final answer

$(a)\ x = 1.38.$ $(b)\ M = 55.$

Question 3

3. [*In this question* **i** *and* **j** *are horizontal unit vectors and position vectors are given relative to a fixed origin.*]

A ship A is moving with constant velocity.

At 1 pm, the position vector of A is $(25\mathbf{i} + 10\mathbf{j})$ km.

At 3 pm, the position vector of A is $(55\mathbf{i} + 34\mathbf{j})$ km.

At time t hours after 1 pm, the position vector of A is $\mathbf{r}_A$ km.

(a) Show that $\mathbf{r}_A = (25 + 15t)\mathbf{i} + (10 + 12t)\mathbf{j}$

(4)

The speed of A is V m s^{-1}

(b) Find the value of V.

(2)

A ship B is moving with constant velocity $(20\mathbf{i} - 6\mathbf{j})$ km h^{-1}

At 1 pm, the position vector of B is $(35\mathbf{i} + 51\mathbf{j})$ km.

At 2:30 pm, B passes through the point P.

(c) Show that A also passes through P.

(5)

WME01/01 OCTOBER 2024

Worked Solution - Question 3

Working with Vectors

Topic group

1. Find the velocity of A

From 1 pm to 3 pm is 2 hours. The displacement of A is $(55\mathbf{i}+34\mathbf{j})-(25\mathbf{i}+10\mathbf{j})=30\mathbf{i}+24\mathbf{j}$ km, so $\mathbf{v}_A=15\mathbf{i}+12\mathbf{j}\ \mathrm{km\,h^{-1}}$.

2. Write the position vector of A

At time t hours after 1 pm,
$\mathbf{r}_A=(25\mathbf{i}+10\mathbf{j})+t(15\mathbf{i}+12\mathbf{j})=(25+15t)\mathbf{i}+(10+12t)\mathbf{j}$.

3. Convert the speed to metres per second

The speed of A is $\sqrt{15^2+12^2}=\sqrt{369}\ \mathrm{km\,h^{-1}}$. Converting to $\mathrm{m\,s^{-1}}$,
$V=\sqrt{369}\times\dfrac{1000}{3600}=\dfrac{5\sqrt{41}}{6}\ \mathrm{m\,s^{-1}}$.

4. Find the point P reached by B

At 2:30 pm, $t=1.5$ hours after 1 pm. For B,
$\mathbf{r}_B=(35\mathbf{i}+51\mathbf{j})+1.5(20\mathbf{i}-6\mathbf{j})=65\mathbf{i}+42\mathbf{j}$.

5. Check whether A reaches P

Set $\mathbf{r}_A=65\mathbf{i}+42\mathbf{j}$. From the i-component, $25+15t=65$, so $t=\dfrac{8}{3}$.

6. Confirm with the j-component

When $t=\dfrac{8}{3}$, $10+12t=10+32=42$. Both components match, so A also passes through P.

Final answer

(a) $\mathbf{r}_A = (25 + 15t)\mathbf{i} + (10 + 12t)\mathbf{j}$. (b) $V = \dfrac{5\sqrt{41}}{6}\ \mathrm{m\,s^{-1}} \approx 5.3\ \mathrm{m\,s^{-1}}$. (c) A also passes through P

.

WME01/01 OCTOBER 2024

Question 4

13 marks

Kinematics Graphs

4. The points A and B lie on the same straight horizontal road.

Figure 2, on page 11, shows the speed-time graph of a cyclist P, for his journey from A to B.
At time $t = 0$, P starts from rest at A and accelerates uniformly for 9 seconds until his speed is $V \text{ m s}^{-1}$

He then travels at constant speed $V \text{ m s}^{-1}$

When $t = 42$, cyclist P passes B.

Given that the distance AB is 120 m,

(a) show that $V = 3.2$

(3)

(b) Find the acceleration of cyclist P between $t = 0$ and $t = 9$

(2)

Cyclist P continues to cycle along the road in the same direction at the same constant speed, $V \text{ m s}^{-1}$

When $t = 6$, a second cyclist Q sets off from A and travels in the same direction as P along the same road. She accelerates for T seconds until her speed is 3.6 m s^{-1}

She then travels at constant speed 3.6 m s^{-1}

Cyclist Q catches up with P when $t = 54$

(c) On Figure 2, on page 11, sketch a speed-time graph showing the journeys of **both** cyclists, for the interval $0 \leqslant t \leqslant 54$

(3)

(d) Find the value of T

(5)

Question 4 continued

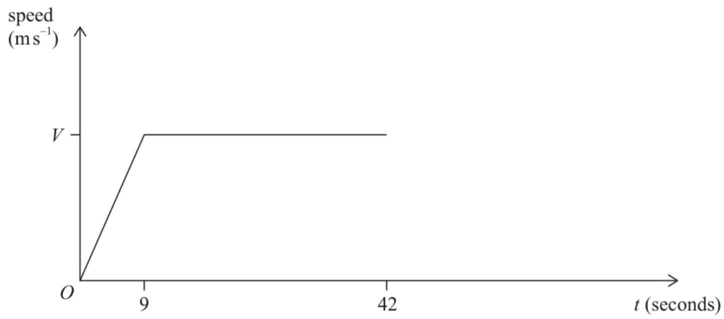
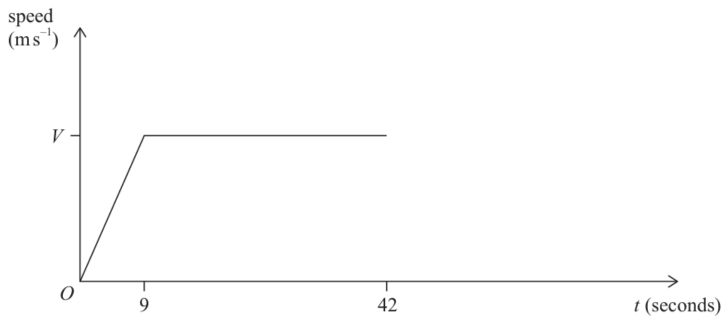


Figure 2

A copy of Figure 2 is on page 13 if you need to redraw your answer to part (c).

Question 4 continued

Only use this copy of Figure 2 if you need to redraw your answer to part (c).



Copy of Figure 2

Worked Solution - Question 4

Kinematics Graphs

Topic group

1. Use the area under P graph

For cyclist P, the distance from A to B is the area under the speed-time graph from 0 to 42. This is $\frac{1}{2}(9)V + 33V = 120$.

2. Show V is 3.2

$4.5V + 33V = 37.5V = 120$, so $V = 3.2\ \mathrm{m\,s^{-1}}$.

3. Find P acceleration

Between $t = 0$ and $t = 9$, P accelerates from 0 to 3.2. Hence $a = \frac{3.2}{9} = \frac{16}{45} = 0.356\ldots\ \mathrm{m\,s^{-2}}$.

4. Describe Q graph

Cyclist Q starts at $t = 6$ from rest, accelerates in a straight line for T seconds to speed $3.6\ \mathrm{m\,s^{-1}}$, then continues horizontally at $3.6\ \mathrm{m\,s^{-1}}$ until $t = 54$.

5. Find P distance by t equals 54

By $t = 54$, P has travelled $\frac{1}{2}(9)(3.2) + 45(3.2) = 14.4 + 144 = 158.4\ \mathrm{m}$.

6. Form Q distance in terms of T

Q travels for 48 seconds after starting at $t = 6$. Her distance is $\frac{1}{2}T(3.6) + (48 - T)(3.6) = 172.8 - 1.8T$.

7. Solve for T

Q catches P at $t = 54$, so $172.8 - 1.8T = 158.4$. Thus $1.8T = 14.4$ and $T = 8$ s.

Final answer

$(a)\ V = 3.2.\quad (b)\ a = \dfrac{16}{45}\ \mathrm{m\,s^{-2}} \approx 0.36\ \mathrm{m\,s^{-2}}.\quad (d)\ T = 8\ \mathrm{s}.$

Question 5

13 marks

Newton's Second Law

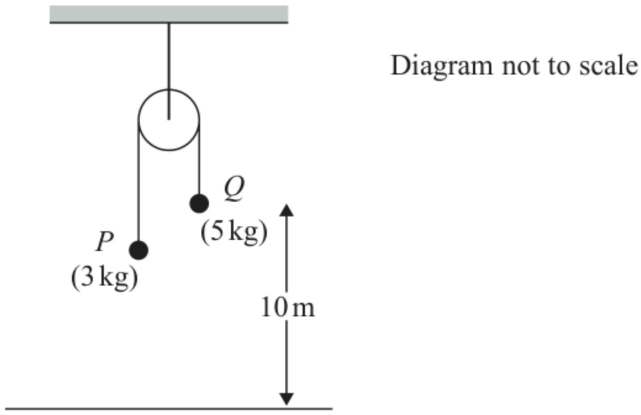


Figure 3

Two particles, P and Q, have masses 3 kg and 5 kg respectively. The particles are connected by a light inextensible string which passes over a small smooth fixed pulley.

The particles are released from rest with the string taut and the hanging parts of the string vertical, as shown in Figure 3.

Immediately after the particles are released from rest, P moves upwards with acceleration $a\,\text{m s}^{-2}$ and the tension in the string is T newtons.

(a) Write down an equation of motion for P.

(2)

(b) Find the value of T.

(4)

The total force acting on the pulley due to the string has magnitude F newtons.

(c) Find the value of F.

(2)

Initially, Q is 10 m above horizontal ground and P is more than 2 m below the pulley.

At the instant when Q has descended a distance of 2 m, the string breaks and Q falls to the ground.

(d) Find the speed of Q at the instant it hits the ground.

(5)

WME01/01 OCTOBER 2024

Worked Solution - Question 5

Newton's Second Law

Topic group

1. Write the equation for P

P has mass 3 kg and moves upwards, so taking upwards as positive gives $T - 3g = 3a$.

2. Write the equation for Q

Q has mass 5 kg and moves downwards, so taking downwards as positive for Q gives $5g - T = 5a$.

3. Find acceleration and tension

Adding the two equations gives $2g = 8a$, so $a = \frac{g}{4}$. Substitute into $T - 3g = 3a$: $T = 3g + 3\left(\frac{g}{4}\right) = \frac{15g}{4} = 36.8\ \mathrm{N}$.

4. Find the force on the pulley

The pulley is pulled by two vertical tensions, each of magnitude T. Hence $F = 2T = \frac{15g}{2} = 73.5\ \mathrm{N}$.

5. Find Q speed when the string breaks

Before the string breaks, Q descends 2 m from rest with acceleration $g/4$. Thus $v^2 = 2\left(\frac{g}{4}\right)(2) = g$, so $v = \sqrt{g}$.

6. Find the remaining fall

After the string breaks, Q is still 8 m above the ground and accelerates freely under gravity. Let the impact speed be w. Then $w^2 = v^2 + 2g(8) = g + 16g = 17g$.

7. Calculate the impact speed

$w = \sqrt{17g} = \sqrt{166.6} = 12.9\ \mathrm{m\,s^{-1}}$ to 3 significant figures.

Final answer

$(a)\ T - 3g = 3a.$ $(b)\ T = 36.8\ \text{N}.$ $(c)\ F = 73.5\ \text{N}.$ (d) speed $= 12.9\ \text{m}\,\text{s}^{-1}$

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WME01/01 OCTOBER 2024

12 marks

Question 6

Resolving Forces, Inclined Planes

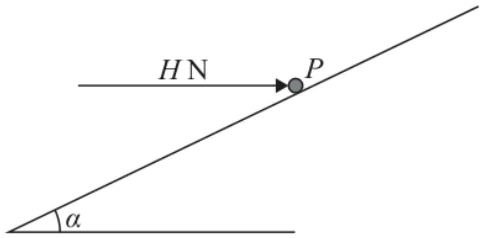

Figure 4

A particle P of mass 5 kg lies on the surface of a rough plane.

The plane is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$

The particle is held in equilibrium by a horizontal force of magnitude H newtons, as shown in Figure 4.

The horizontal force acts in a vertical plane containing a line of greatest slope of the inclined plane.

The coefficient of friction between the particle and the plane is $\frac{1}{4}$

(a) Find the smallest possible value of H.

(6)

The horizontal force is now removed, and P starts to slide down the slope.

In the first T seconds after P is released from rest, P slides 1.5 m down the slope.

(b) Find the value of T.

(6)

WME01/01 OCTOBER 2024

Worked Solution - Question 6

Resolving Forces, Inclined Planes

Topic group

1. Use the triangle for alpha

From $\tan\alpha = \frac{3}{4}$, use $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$.

2. Resolve perpendicular to the plane

The horizontal force has a component into the plane, so

$R = 5g\cos\alpha + H\sin\alpha = 4g + \frac{3}{5}H$.

3. Resolve parallel for the smallest H

For the smallest H, the particle is on the point of sliding down, so friction acts up the plane. Parallel to the plane, $H\cos\alpha + F = 5g\sin\alpha$.

4. Use friction to find H

With $F = \frac{1}{4}R$, the parallel equation becomes $\frac{4}{5}H + \frac{1}{4}\left(4g + \frac{3}{5}H\right) = 3g$.

Hence $0.95H + g = 3g$, so $H = \frac{40g}{19} = 20.6\ \mathrm{N}$.

5. Find the acceleration after H is removed

When H is removed, $R = 5g\cos\alpha = 4g$ and friction is $\frac{1}{4}R = g$. The component of weight down the plane is $5g\sin\alpha = 3g$, so the resultant down the plane is $2g$.

6. Apply Newtons second law

$2g = 5a$, so $a = \frac{2g}{5} = 3.92\ \mathrm{m\,s^{-2}}$ down the plane.

7. Find T from the displacement

From rest, the particle moves 1.5 m with acceleration $2g/5$. Using $s = \frac{1}{2}at^2$:

$1.5 = \frac{1}{2}\left(\frac{2g}{5}\right)T^2 = \frac{g}{5}T^2$. Thus $T = \sqrt{\frac{7.5}{9.8}} = 0.875\ \mathrm{s}$.

Final answer

$(a)\ H = \dfrac{40g}{19}\ \text{N} \approx 20.6\ \text{N}. \quad (b)\ T = 0.875\ \text{s}.$

Question 7

11 marks

Constant Acceleration in 1D

7 At time $t = 0$, a small ball A is projected vertically upwards with speed $8\,\mathrm{m\,s^{-1}}$ from a fixed point on horizontal ground.
The ball hits the ground again for the first time at time $t = T_1$ seconds.

Ball A is modelled as a particle moving freely under gravity.

(a) Show that $T_1 = 1.63$ to 3 significant figures.

(2)

After the first impact with the ground, A rebounds to a height of 2 m above the ground.

Given that the mass of A is 0.1 kg,

(b) find the magnitude of the impulse received by A as a result of its first impact with the ground.

(5)

At time $t = 1$ second, another small ball B is projected vertically upwards from another point on the ground with speed $5\,\mathrm{m\,s^{-1}}$

Ball B is modelled as a particle moving freely under gravity.

At time $t = T_2$ seconds ($T_2 > 1$), A and B are at the same height above the ground for the first time.

(c) Find the value of T_2

(4)

WME01/01 OCTOBER 2024

Worked Solution - Question 7

Constant Acceleration in 1D

Topic group

1. Find the first flight time

For ball A, take upward as positive. Returning to the ground gives $0 = 8T_1 - 4.9T_1^2$. The non-zero solution is $T_1 = \frac{8}{4.9} = 1.63$ s to 3 significant figures.

2. Find the rebound speed

After impact, A rebounds to a maximum height of 2 m. At the top, $v = 0$, so $0 = u^2 - 2g(2)$. Hence $u = \sqrt{4g} = 6.261\ \mathrm{m\,s^{-1}}$.

3. Use impulse as change in momentum

Just before the first impact, A has velocity $-8\ \mathrm{m\,s^{-1}}$. Just after impact, it has velocity $+6.261\ \mathrm{m\,s^{-1}}$. With mass 0.1 kg, $I = 0.1[6.261 - (-8)]$.

4. Calculate the impulse

$I = 1.426\ldots\ \mathrm{N\,s}$, so the impulse is $1.43\ \mathrm{N\,s}$ to 3 significant figures.

5. Write the height of A before first impact

For the first meeting with B, A is still in its first flight. Its height at time T_2 is $h_A = 8T_2 - 4.9T_2^2$.

6. Write the height of B

B starts at time $t = 1$, so at time T_2 it has been moving for $T_2 - 1$ seconds. Its height is $h_B = 5(T_2 - 1) - 4.9(T_2 - 1)^2$.

7. Set the heights equal

$8T_2 - 4.9T_2^2 = 5(T_2 - 1) - 4.9(T_2 - 1)^2$. Expanding and simplifying gives $6.8T_2 = 9.9$.

8. Find T2

$T_2 = \frac{9.9}{6.8} = 1.456\ldots$, so $T_2 = 1.46$ s to 3 significant figures.

Final answer

$(a)\ T_1 = 1.63\text{ s.} \quad (b)\ I = 1.43\text{ N s.} \quad (c)\ T_2 = 1.46\text{ s.}$

PAST PAPER

WME01/01 January 2025

January 2025 | 7 questions | 75 marks

7 questions	**75** marks	**Answers** worked solution after each question

WME01/01 JANUARY 2025

8 marks

Question 1

Forces

1. A particle of mass 2.5 kg moves on a smooth horizontal plane under the action of three horizontal forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$, where

$$\mathbf{F}_1 = (6\mathbf{i} + 8\mathbf{j})\,\text{N}$$

$$\mathbf{F}_2 = (-16\mathbf{i} + 2\mathbf{j})\,\text{N}$$

$$\mathbf{F}_3 = (-2\mathbf{i} + 8\mathbf{j})\,\text{N}$$

(a) Find the magnitude of the acceleration of the particle.

(4)

A fourth force, $\mathbf{F}_4 = (p\mathbf{i} + p\mathbf{j})\,\text{N}$, where p is a constant, is added.

The resultant of the four forces acts in the direction of the vector $(7\mathbf{i} + 2\mathbf{j})$.

(b) Find the value of p.

(4)

Worked Solution - Question 1

Forces

Topic group

1. Find the resultant of the first three forces

$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = (6\mathbf{i} + 8\mathbf{j}) + (-16\mathbf{i} + 2\mathbf{j}) + (-2\mathbf{i} + 8\mathbf{j}) = -12\mathbf{i} + 18\mathbf{j}$.

2. Apply Newtons second law

The mass is 2.5 kg, so $-12\mathbf{i} + 18\mathbf{j} = 2.5\mathbf{a}$. Hence $\mathbf{a} = -4.8\mathbf{i} + 7.2\mathbf{j}\ \mathrm{m\,s^{-2}}$.

3. Find the magnitude of the acceleration

$|\mathbf{a}| = \sqrt{(-4.8)^2 + 7.2^2} = 8.653\ldots$, so the magnitude is $8.7\ \mathrm{m\,s^{-2}}$ to 2 significant figures. Exactly, $|\mathbf{a}| = \dfrac{12\sqrt{13}}{5}$.

4. Add the fourth force

With $\mathbf{F}_4 = p\mathbf{i} + p\mathbf{j}$, the new resultant is $(p - 12)\mathbf{i} + (p + 18)\mathbf{j}$.

5. Use the given direction

The resultant acts in the direction $7\mathbf{i} + 2\mathbf{j}$, so $\dfrac{p - 12}{p + 18} = \dfrac{7}{2}$.

6. Solve for p

$2(p - 12) = 7(p + 18)$, so $2p - 24 = 7p + 126$. Therefore $-150 = 5p$ and $p = -30$.

Final answer

$(a)\ |\mathbf{a}| = \dfrac{12\sqrt{13}}{5}\ \mathrm{m\,s^{-2}} \approx 8.7\ \mathrm{m\,s^{-2}}$. $\quad (b)\ p = -30$.

Question 2

11 marks

Kinematics Graphs

2. The fixed points A, B and C lie in a straight line on a horizontal road.

- At time $t = 0$, a motorbike passes through A with speed $5\,\text{m s}^{-1}$
- From A, the motorbike accelerates uniformly until it reaches B with a speed of $V\,\text{m s}^{-1}$
- The motorbike takes T_1 seconds to travel from A to B
- From B, the motorbike decelerates uniformly until it comes to rest at C
- The motorbike takes T_2 seconds to travel from B to C

(a) Sketch a speed-time graph for the motion of the motorbike as it moves from A to C.

(3)

The distance AB is 132 m and the distance BC is 136 m.

(b) Find, in terms of V, an expression for

(i) T_1

(ii) T_2

(4)

Given that the motorbike takes 28 s to travel from A to C,

(c) find the value of V,

(2)

(d) find the deceleration of the motorbike.

(2)

Worked Solution - Question 2

Kinematics Graphs

Topic group

1. Sketch the speed-time graph

The graph starts at speed 5 when $t = 0$, rises linearly to speed V at time T_1, then falls linearly to speed 0 at time $T_1 + T_2$.

2. Use area from A to B

The area under the first section is the distance $AB = 132$. Thus $\frac{1}{2}(5 + V)T_1 = 132$, so $T_1 = \frac{264}{V + 5}$.

3. Use area from B to C

The second section is a triangle of height V and base T_2. Since $BC = 136$, $\frac{1}{2}VT_2 = 136$, so $T_2 = \frac{272}{V}$.

4. Use the total time

The whole journey takes 28 s, so $\frac{264}{V + 5} + \frac{272}{V} = 28$.

5. Solve for V

Multiplying by $V(V + 5)$ gives $264V + 272(V + 5) = 28V(V + 5)$. This simplifies to $7V^2 - 99V - 340 = 0$, so $(V - 17)(7V + 20) = 0$. Since speed is positive, $V = 17$.

6. Find the deceleration

For the final section, $T_2 = \frac{272}{17} = 16$ s. The speed falls from 17 to 0, so the deceleration is $\frac{17}{16} = 1.0625\ \mathrm{m\,s^{-2}}$.

Final answer

$(b)(i)\ T_1 = \dfrac{264}{V+5}$. $(b)(ii)\ T_2 = \dfrac{272}{V}$. $(c)\ V = 17$. (d) deceleration $= \dfrac{17}{16} = 1.0625\ \mathrm{m\,s^{-2}}$

.

Question 3

3.

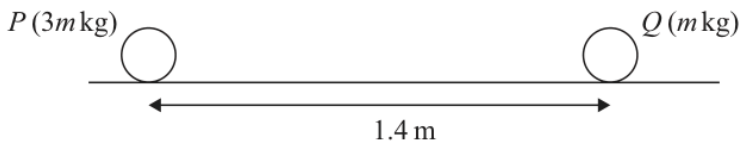

Figure 1

A particle P of mass $3m$ kg and a particle Q of mass m kg are at rest on a rough horizontal surface. The distance between P and Q is 1.4 m, as shown in Figure 1.

An impulse of magnitude λ N s is now applied to P in the direction PQ. Immediately after the impulse is applied, the speed of P is $5\,\text{m s}^{-1}$

(a) Find λ in terms of m.

(2)

Immediately before P collides with Q, the speed of P is $2.5\,\text{m s}^{-1}$
The coefficient of friction between P and the surface is μ

(b) Find the value of μ

(7)

Immediately after P collides with Q, the speed of Q is $2.1\,\text{m s}^{-1}$

(c) Find the speed of P immediately after the collision.

(3)

Worked Solution - Question 3

Momentum, Impulse & Collisions

Topic group

1. Use impulse to find lambda

P starts from rest and immediately after the impulse has speed $5\ \mathrm{m\,s^{-1}}$. Since the mass of P is $3m$, $\lambda = 3m(5-0) = 15m$.

2. Find the acceleration before collision

Between the impulse and the collision, P slows from 5 to $2.5\ \mathrm{m\,s^{-1}}$ over 1.4 m. Using $v^2 = u^2 + 2as$: $(2.5)^2 = 5^2 + 2a(1.4)$.

3. Calculate the acceleration

$6.25 = 25 + 2.8a$, so $a = \dfrac{-18.75}{2.8} = -6.696\ldots\ \mathrm{m\,s^{-2}}$.

4. Connect friction to acceleration

On the horizontal surface, $R = 3mg$ and friction is $F = \mu R = 3\mu mg$. This friction is opposite the motion, so $-3\mu mg = 3ma$ and therefore $a = -\mu g$.

5. Find mu

$\mu = \dfrac{-a}{g} = \dfrac{6.696\ldots}{9.8} = 0.683\ldots$, so $\mu = 0.68$ to 2 significant figures.

6. Use momentum in the collision

Immediately before collision, P has speed 2.5 and Q is at rest. Immediately after, Q has speed 2.1 and P has speed v. Conservation of momentum gives $3m(2.5) + m(0) = 3mv + m(2.1)$.

7. Find the speed after collision

$7.5 = 3v + 2.1$, so $3v = 5.4$ and $v = 1.8\ \mathrm{m\,s^{-1}}$.

Final answer

$(a)\ \lambda = 15m.$ $\quad (b)\ \mu = 0.68$ or $0.683.$ $\quad (c)$ speed of $\mathrm{P} = 1.8\ \mathrm{m\,s^{-1}}.$

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10 marks

Question 4

Moments

4.

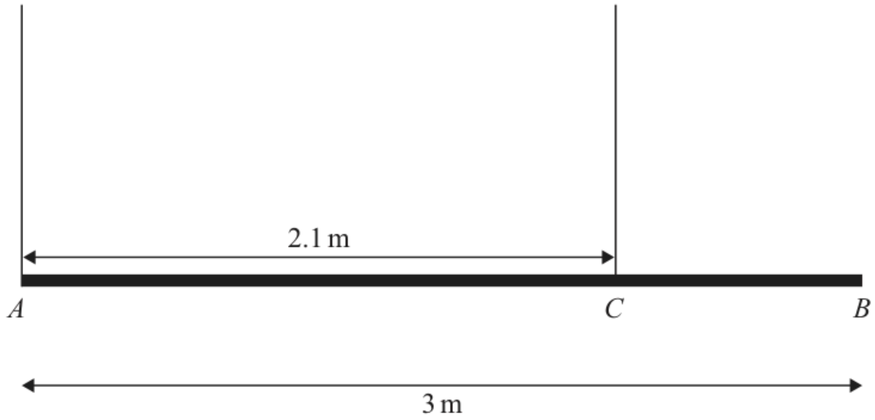

Figure 2

A uniform rod AB has length 3 m and weight W newtons.
The rod is suspended by two light vertical ropes.
One rope is attached to the rod at A and the other rope is attached to the rod at C, where $AC = 2.1$ m.

The rod is in equilibrium in a horizontal position, as shown in Figure 2.

The tension in the rope at C is 350 N.

(a) Show that $W = 490$

(3)

A particle P of weight 210 N is attached to the rod at a distance d metres from A.
The tension in the rope at C is now 600 N.
The rod remains in equilibrium in a horizontal position.

(b) Find the value of d.

(3)

Particle P is removed from the rod.
A particle Q of weight X newtons is now attached at B.
The rod remains in equilibrium in a horizontal position and is now on the point of tilting.

(c) Find the value of X.

(4)

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Worked Solution - Question 4

Moments

Topic group

1. Use moments to find W

Take moments about A. The rope at C is 2.1 m from A and the weight of the uniform rod acts at its midpoint, 1.5 m from A. Hence $350(2.1) = W(1.5)$.

2. Show W is 490

$735 = 1.5W$, so $W = 490$ N.

3. Set up moments with particle P

When P is attached, the tension at C is 600 N. Taking moments about A: $600(2.1) = 490(1.5) + 210d$.

4. Find d

$1260 = 735 + 210d$, so $525 = 210d$ and $d = 2.5$.

5. Use the tilting condition

When Q is attached at B and the rod is on the point of tilting, the rope at A is just slack. The rod is about to tilt about C.

6. Find X

Take moments about C. The rod weight acts $2.1 - 1.5 = 0.6$ m to the left of C, and Q acts $3 - 2.1 = 0.9$ m to the right. Thus $490(0.6) = X(0.9)$, so $X = \dfrac{980}{3}$ N.

Final answer

$(a)\ W = 490. \quad (b)\ d = 2.5. \quad (c)\ X = \dfrac{980}{3}\ \text{N} \approx 327\ \text{N}.$

Question 5

7 marks

Working with Vectors

5. [*In this question* **i** *and* **j** *are horizontal perpendicular unit vectors and position vectors are given relative to a fixed origin.*]

In a game, a ball B is rolled across a horizontal surface towards a fixed target. The ball is modelled as a particle moving with constant velocity.

At time $t = 1$ s, the position vector of B is $(-2\mathbf{i} + 5\mathbf{j})$ m.
At time $t = 9$ s, the position vector of B is $(10\mathbf{i} - 3\mathbf{j})$ m.

(a) Find the velocity of the ball.

(3)

The position vector of the target is $(13\mathbf{i} - 5\mathbf{j})$ m.

(b) Use the model to find the distance of B from the target at time $t = 7$ s.

(4)

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Worked Solution - Question 5

Working with Vectors

Topic group

1. Find the displacement over 8 seconds

From $t = 1$ to $t = 9$, the displacement is $(10\mathbf{i} - 3\mathbf{j}) - (-2\mathbf{i} + 5\mathbf{j}) = 12\mathbf{i} - 8\mathbf{j}$.

2. Find the velocity

The time interval is 8 seconds, so $\mathbf{v} = \dfrac{12\mathbf{i} - 8\mathbf{j}}{8} = 1.5\mathbf{i} - \mathbf{j}\ \mathrm{m\,s^{-1}}$.

3. Find the position at t=7

Time $t = 7$ is 6 seconds after $t = 1$. Therefore $\mathbf{r}_B(7) = (-2\mathbf{i} + 5\mathbf{j}) + 6(1.5\mathbf{i} - \mathbf{j}) = 7\mathbf{i} - \mathbf{j}$.

4. Find the vector from B to the target

The target has position vector $13\mathbf{i} - 5\mathbf{j}$. The vector from the ball to the target is $(13\mathbf{i} - 5\mathbf{j}) - (7\mathbf{i} - \mathbf{j}) = 6\mathbf{i} - 4\mathbf{j}$.

5. Find the distance

The distance is $\sqrt{6^2 + (-4)^2} = \sqrt{52} = 2\sqrt{13}\ \mathrm{m}$, which is about $7.2\ \mathrm{m}$.

Final answer

$(a)\ \mathbf{v} = 1.5\mathbf{i} - \mathbf{j}\ \mathrm{m\,s^{-1}}. \quad (b)\ \text{distance} = 2\sqrt{13}\ \mathrm{m} \approx 7.2\ \mathrm{m}.$

Question 6

6.

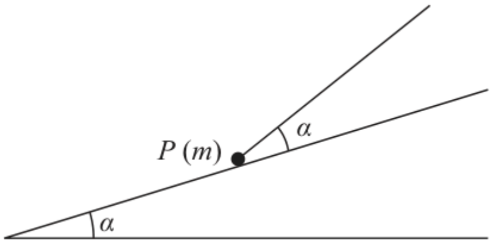

Figure 3

A particle P of mass m is held in equilibrium on a fixed rough inclined plane by a light inextensible string.

The plane is inclined at an angle α to the horizontal, where $\alpha < 45°$
The string is inclined to the plane at angle α, as shown in Figure 3.

The string lies in a vertical plane that contains a line of greatest slope of the inclined plane.

When the tension in the string is $0.75\,mg$, P is on the point of moving up the plane.

(a) Find an expression for the magnitude of the frictional force acting on P, giving your answer in terms of m, g and α.

(3)

The coefficient of friction between P and the plane is $\dfrac{1}{2}$

(b) Show that

$$\tan\alpha = \frac{2}{5}$$

(6)

The string breaks.

(c) Determine whether P remains at rest. You must justify your reasoning.

(3)

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Worked Solution - Question 6

Resolving Forces, Inclined Planes

Topic group

1. Resolve parallel to the plane

P is on the point of moving up the plane, so friction acts down the plane. The tension has component $0.75mg\cos\alpha$ up the plane, and the weight component is $mg\sin\alpha$ down the plane.

2. Find the frictional force

For equilibrium parallel to the plane, $0.75mg\cos\alpha = mg\sin\alpha + F$. Hence $F = 0.75mg\cos\alpha - mg\sin\alpha$.

3. Resolve perpendicular to the plane

The string has a component away from the plane, so $R = mg\cos\alpha - 0.75mg\sin\alpha$.

4. Use the coefficient of friction

Given $\mu = \frac{1}{2}$, limiting friction gives $F = \frac{1}{2}R$. Therefore $0.75mg\cos\alpha - mg\sin\alpha = \frac{1}{2}(mg\cos\alpha - 0.75mg\sin\alpha)$.

5. Show the tangent result

Divide by mg and collect terms: $0.75\cos\alpha - \sin\alpha = 0.5\cos\alpha - 0.375\sin\alpha$. Thus $0.25\cos\alpha = 0.625\sin\alpha$, so $\tan\alpha = \frac{0.25}{0.625} = \frac{2}{5}$.

6. Check what happens when the string breaks

Without the string, $R = mg\cos\alpha$ and the maximum friction is $\frac{1}{2}mg\cos\alpha$. The component of weight down the plane is $mg\sin\alpha$.

7. Compare friction with the downslope component

Since $\tan\alpha = \frac{2}{5} < \frac{1}{2}$, we have $mg\sin\alpha < \frac{1}{2}mg\cos\alpha$. The available friction is large enough to hold P, so P remains at rest.

Final answer

$(a)\ F = 0.75mg\cos\alpha - mg\sin\alpha. \quad (b)\ \tan\alpha = \frac{2}{5}. \quad (c)\ P \text{ remains at rest}$

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15 marks

Question 7

Newton's Second Law

7.

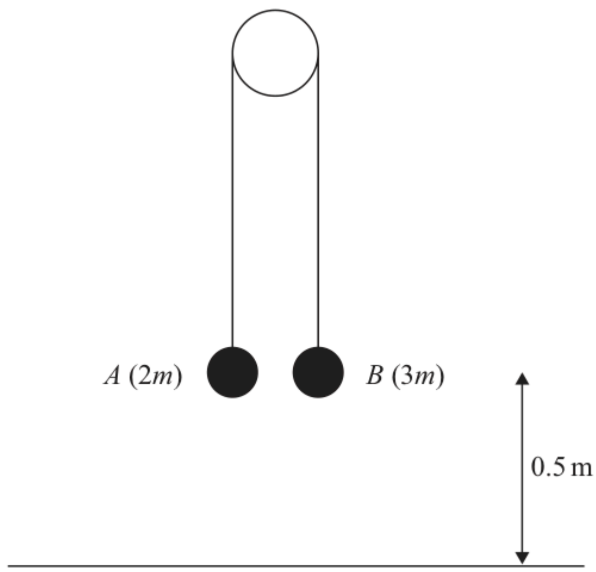

Figure 4

One end of a light inextensible string is attached to a particle A of mass $2m$.
The other end is attached to a particle B of mass $3m$.
The string passes over a small smooth fixed pulley.
The string is taut and both straight parts of the string are vertical.
Both particles are held at a distance 0.5 m above a horizontal surface, as shown in Figure 4.

The system is released from rest and B moves downwards.

(a) Find the tension in the string in terms of m and g.

(5)

(b) Find the speed of B at the instant it strikes the surface.

(4)

In the subsequent motion, A does not reach the pulley and B does not rebound after it strikes the surface.

(c) Find the time from the instant when the system is released from rest to the instant when A first reaches a height of 1.06 m above the surface.

(6)

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Worked Solution - Question 7

Newton's Second Law

Topic group

1. Write the equations of motion

B moves down and A moves up with the same acceleration a. For A: $T - 2mg = 2ma$. For B: $3mg - T = 3ma$.

2. Find the acceleration

Adding the two equations gives $mg = 5ma$, so $a = \frac{g}{5}$.

3. Find the tension

Use $T - 2mg = 2ma$: $T = 2mg + 2m\left(\frac{g}{5}\right) = \frac{12mg}{5}$.

4. Find the speed when B hits the surface

B starts from rest and moves 0.5 m with acceleration $g/5$. Using $v^2 = u^2 + 2as$, $v^2 = 2\left(\frac{g}{5}\right)(0.5) = \frac{g}{5} = 1.96$.

5. Calculate the impact speed

$v = \sqrt{1.96} = 1.4\,\mathrm{m\,s^{-1}}$.

6. Find the first time interval

The time until B hits the surface is found from $0.5 = \frac{1}{2}\left(\frac{g}{5}\right)t_1^2$. With $g = 9.8$, $t_1 = \frac{5}{7} = 0.714285\ldots$ s.

7. Find the extra time after B stops

After B hits the surface, A continues upwards freely under gravity. It is then at height 1.00 m and must rise another 0.06 m with initial speed $1.4\,\mathrm{m\,s^{-1}}$. So $0.06 = 1.4t_2 - 4.9t_2^2$.

8. Find the total time

Solving $0.06 = 1.4t_2 - 4.9t_2^2$ gives the first positive time $t_2 = 0.0525\ldots$ s. Therefore the total time is $0.714285\ldots + 0.0525\ldots = 0.767$ s.

Final answer

$(a)\ T = \dfrac{12mg}{5}$. $\quad (b)\ v = 1.4\,\mathrm{m\,s^{-1}}$. $\quad (c)\ t = 0.767$ s approximately

.

PAST PAPER

WME01/01 May/June 2025

May/June 2025 | 8 questions | 75 marks

8
questions

75
marks

Answers
worked solution after each question

WME01/01 MAY/JUNE 2025

6 marks

Question 1

Momentum, Impulse & Collisions

1.

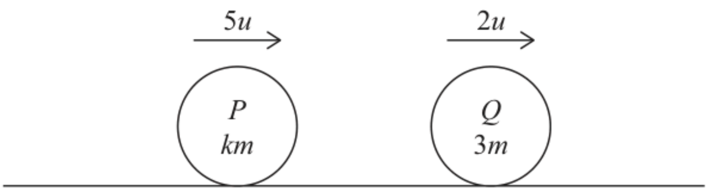

Figure 1

Figure 1 shows two particles, P and Q, moving in the same direction along the same straight line on a smooth horizontal surface.
Particle P has mass km and particle Q has mass $3m$
The particles collide directly.

Immediately before the collision, the speed of P is $5u$ and the speed of Q is $2u$

Immediately after the collision, the speed of P is $2u$ and its direction of motion is unchanged.

Immediately after the collision, the speed of Q is v

The impulse exerted on Q by P in the collision has magnitude $4.5mu$

(a) Find v in terms of u only.

(3)

(b) Find the value of k

(3)

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Worked Solution - Question 1

Momentum, Impulse & Collisions

Topic group

1. Part (a): Find v in terms of u - Apply impulse-momentum theorem to Q

$\text{Impulse} = \text{Final momentum} - \text{Initial momentum}$ For particle Q: ; Mass = $3m$; Initial velocity = $2u$; Final velocity = v ; Impulse received = $4.5mu$

2. Part (a): Find v in terms of u - Set up equation

$4.5mu = 3m \times v - 3m \times 2u$ $4.5mu = 3mv - 6mu$ $4.5u = 3v - 6u$ $4.5u + 6u = 3v$ $10.5u = 3v$ $v = \frac{10.5u}{3}$ $v = 3.5u$

3. Part (b): Find the value of k - Apply conservation of momentum

$\text{Total momentum before} = \text{Total momentum after}$ Before collision: $\text{P momentum} = km \times 5u = 5kmu$ $\text{Q momentum} = 3m \times 2u = 6mu$ $\text{Total} = 5kmu + 6mu$ After collision: $\text{P momentum} = km \times 2u = 2kmu$ $\text{Q momentum} = 3m \times 3.5u = 10.5mu$ $\text{Total} = 2kmu + 10.5mu$

4. Part (b): Find the value of k - Set up equation

$5kmu + 6mu = 2kmu + 10.5mu$ $5km + 6m = 2km + 10.5m$ $5k + 6 = 2k + 10.5$ $5k - 2k = 10.5 - 6$ $3k = 4.5$ $k = \frac{4.5}{3}$ $k = 1.5$

Final answer

$(a)\ v = 3.5u = \frac{7u}{2}. \quad (b)\ k = 1.5 = \frac{3}{2}.$

Question 2

7 marks

Working with Vectors

2. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively. Position vectors are given relative to a fixed origin.*]

Two particles, A and B, are moving on a smooth horizontal surface.
Each particle is moving with constant velocity.

At time t seconds, the position vector of A is given by $\mathbf{r}$ metres.

- When $t = 2$, $\mathbf{r} = (-5\mathbf{i} + 16\mathbf{j})$
- When $t = 5$, $\mathbf{r} = (10\mathbf{i} + 4\mathbf{j})$

(a) Find, in terms of **i** and **j**, the velocity of A.

(2)

(b) Find an expression for $\mathbf{r}$ at time t seconds.
Give your answer in the form $p\mathbf{i} + q\mathbf{j}$, where p and q are functions of t

(2)

At time t seconds, the position vector of B is given by $\mathbf{s}$ metres where

$$\mathbf{s} = -5\mathbf{i} + 7\mathbf{j} + t(2\mathbf{i} - 3\mathbf{j})$$

(c) Find, to the nearest degree, the bearing of B from A when $t = 5$

(3)

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Worked Solution - Question 2

Working with Vectors

Topic group

1. Part (a): Find velocity of A - Find change in position

Given positions for particle A: ; At $t = 2$: $\mathbf{r} = -5\mathbf{i} + 16\mathbf{j}$; At $t = 5$: $\mathbf{r} = 10\mathbf{i} + 4\mathbf{j}$
$\mathbf{v} = \frac{\Delta \mathbf{r}}{\Delta t}$ $\mathbf{v}_A = \frac{(10\mathbf{i}+4\mathbf{j})-(-5\mathbf{i}+16\mathbf{j})}{5-2} = \frac{(10+5)\mathbf{i}+(4-16)\mathbf{j}}{3} = \frac{15\mathbf{i}-12\mathbf{j}}{3} = 5\mathbf{i} - 4\mathbf{j} \text{ m s}^{-1}$

2. Part (b): Find expression for r at time t - Use position-velocity relationship

$\mathbf{r}(t) = \mathbf{r}(t_0) + \mathbf{v}(t - t_0)$ Using $t_0 = 2$ with $\mathbf{r}(2) = -5\mathbf{i} + 16\mathbf{j}$:
$\mathbf{r}(t) = (-5\mathbf{i} + 16\mathbf{j}) + (5\mathbf{i} - 4\mathbf{j})(t - 2) = -5\mathbf{i} + 16\mathbf{j} + 5(t-2)\mathbf{i} - 4(t-2)\mathbf{j}$
$= -5\mathbf{i} + 16\mathbf{j} + (5t - 10)\mathbf{i} + (-4t + 8)\mathbf{j} = (-5 + 5t - 10)\mathbf{i} + (16 - 4t + 8)\mathbf{j}$
$= (5t - 15)\mathbf{i} + (24 - 4t)\mathbf{j}$

3. Part (c): Find bearing of B from A when t = 5 - Find positions at t = 5

For A at $t = 5$: $\mathbf{r}_A = 10\mathbf{i} + 4\mathbf{j}$ (given) For B at $t = 5$:
$\mathbf{s}_B = -5\mathbf{i} + 7\mathbf{j} + 5(2\mathbf{i} - 3\mathbf{j}) = -5\mathbf{i} + 7\mathbf{j} + 10\mathbf{i} - 15\mathbf{j} = 5\mathbf{i} - 8\mathbf{j}$

4. Part (c): Find bearing of B from A when t = 5 - Find vector from A to B

$\overrightarrow{AB} = \mathbf{s}_B - \mathbf{r}_A = (5\mathbf{i} - 8\mathbf{j}) - (10\mathbf{i} + 4\mathbf{j}) = -5\mathbf{i} - 12\mathbf{j}$

5. Part (c): Find bearing of B from A when t = 5 - Find angle and bearing

Angle from South (negative j direction): $\tan\alpha = \frac{5}{12}$ $\alpha = \tan^{-1}\left(\frac{5}{12}\right)$
$\alpha = 22.619\ldots^\circ$ Bearing = $180^\circ + \alpha = 180^\circ + 22.619\ldots^\circ = 202.619\ldots^\circ$

Final answer

$(a)\ \mathbf{v}_A = 5\mathbf{i} - 4\mathbf{j}\ \text{m s}^{-1}. \quad (b)\ \mathbf{r} = (5t - 15)\mathbf{i} + (24 - 4t)\mathbf{j}. \quad (c)\ \text{bearing} = 203^\circ$
.

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Question 3

8 marks

Constant Acceleration in 1D

3. A particle is projected vertically upwards with speed $U\,\text{m s}^{-1}$ from a point A.

The point A is 12 m vertically above the point B.

Point B is on horizontal ground.

The particle moves freely under gravity until it hits the ground at B.

The time taken for the particle to travel from A to B is 4 seconds.

(a) Find the value of U.

(3)

(b) Find the speed of the particle as it hits the ground at B.

(3)

(c) Sketch a speed-time graph for the motion of the particle from the instant it leaves A to the instant it reaches B. (No further calculations are required.)

(2)

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Worked Solution - Question 3

Constant Acceleration in 1D

Topic group

1. Part (a): Find the value of U - Identify SUVAT variables

Taking upward as positive: ; $u = U$ m s^{-1} (unknown, upward) ; $s = -12$ m (displacement from A to B, downward) ; $t = 4$ seconds ; $a = -9.8$ m s^{-2} (gravity, downward)

2. Part (a): Find the value of U - Apply $s = ut + \frac{1}{2}at^2$

$s = ut + \frac{1}{2}at^2$ $-12 = U(4) + \frac{1}{2}(-9.8)(4)^2$ $-12 = 4U + \frac{1}{2}(-9.8)(16)$ $-12 = 4U - 78.4$ $4U = -12 + 78.4$ $4U = 66.4$ $U = 16.6 \text{ m s}^{-1}$

3. Part (b): Find speed as particle hits ground - Use SUVAT to find velocity at B

$v^2 = u^2 + 2as$ $v^2 = (16.6)^2 + 2(-9.8)(-12)$ $v^2 = 275.56 + 235.2$ $v^2 = 510.76$ $v = \pm 22.6$ Since particle is moving downward at B, $v = -22.6$ m s $^{-1}$ (negative)

4. Part (b): Find speed as particle hits ground - Find speed

Speed $= |v| = 22.6 \text{ m s}^{-1}$, or about 23 m s^{-1} to 2 significant figures.

5. Part (c): Sketch speed-time graph - Analyze the motion

The particle: ; Starts at A with speed $U = 16.6$ m s^{-1} upward ; Decelerates (speed decreases) as it goes up ; Reaches maximum height when speed = 0 ; Accelerates (speed increases) as it falls down ; Hits ground at B with speed 22.6 m s^{-1} Time to reach maximum height: $v = u + at$ $0 = 16.6 - 9.8t$ $t = \frac{16.6}{9.8} = 1.69 \text{ seconds}$

6. Part (c): Sketch speed-time graph - Sketch the graph

Key Features: ; V-shaped graph (straight lines due to constant acceleration) ; First line: negative gradient (speed decreasing) ; Second line: positive gradient (speed increasing) ; Vertex on horizontal axis (speed = 0 at max height) ; Second line is longer and steeper (falls from greater height)

Final answer

$(a)\ U = 16.6\ \mathrm{m\,s^{-1}}$ (about $17\ \mathrm{m\,s^{-1}}$ to 2 s.f.).

(b) speed $= 22.6\ \mathrm{m\,s^{-1}}$. (c) Correct V-shaped velocity-time graph with the labelled values from the working.

Question 4

7 marks

Forces

4.

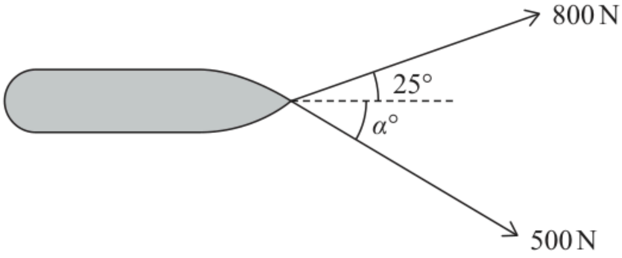

Figure 2

Two ropes are attached to a point on the front of a barge.
The barge is being pulled horizontally in a straight line along the centre of a long straight canal.

One rope makes an angle of 25° with the direction of motion of the barge and has a tension of 800 N.
The other rope makes an angle of α° with the direction of motion of the barge and has a tension of 500 N, as shown in Figure 2.

Both ropes are horizontal.

(a) Find the value of α

(3)

The mass of the barge is 15 tonnes and the resistance to the motion of the barge is a constant force of magnitude 750 N.

(b) Find the acceleration of the barge.

(4)

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Worked Solution - Question 4

Forces

Topic group

1. Part (a): Find the value of α - Resolve perpendicular to direction of motion

$\sum F_{\perp} = 0 \text{ (no sideways motion)}$ Perpendicular component of 800 N rope: $800 \sin 25^{\circ} = 800 \times 0.4226 = 338.1 \text{ N}$ Perpendicular component of 500 N rope: $500 \sin \alpha$

2. Part (a): Find the value of α - Set components equal

$800 \sin 25^{\circ} = 500 \sin \alpha$ $338.1 = 500 \sin \alpha$ $\sin \alpha = \frac{338.1}{500}$ $\sin \alpha = 0.6762$ $\alpha = \sin^{-1}(0.6762)$ $\alpha = 42.5^{\circ}$

3. Part (b): Find acceleration of barge - Resolve parallel to direction of motion

$\sum F_{\parallel} = ma$ Parallel component of 800 N rope: $800 \cos 25^{\circ} = 800 \times 0.9063 = 725.0 \text{ N}$ Parallel component of 500 N rope: $500 \cos 42.5^{\circ} = 500 \times 0.7373 = 368.7 \text{ N}$ Resistance force: 750 N (opposes motion) Mass: 15 tonnes $= 15000$ kg

4. Part (b): Find acceleration of barge - Apply Newton's Second Law

$725.0 + 368.7 - 750 = 15000 \times a$ $343.7 = 15000a$ $a = \frac{343.7}{15000}$ $a = 0.02291\ldots$ $a = 0.023 \text{ m s}^{-2} \text{ (2 sf)}$

Final answer

(a) $\alpha = 43^{\circ}$ to the nearest degree.

(b) $a = 0.023 \text{ m s}^{-2}$.

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Question 5

6 marks

Moments

5.

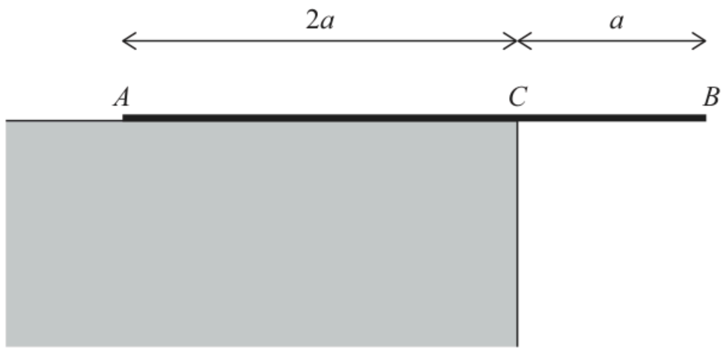

Figure 3

A non-uniform rod AB of length $3a$ rests in equilibrium on a horizontal ledge and overhangs the edge of the ledge at C.
The point C is such that $AC = 2a$ and $CB = a$, as shown in Figure 3.

The rod has weight W.

The distance of the centre of mass of the rod from A is d.

The rod is perpendicular to the edge of the ledge.

When a force of magnitude P, **acting vertically upwards**, is applied to the rod at B, the rod is on the point of tilting about A.

When the force applied at B is replaced by a force of magnitude $1.25P$, **acting vertically downwards** at B, the rod is on the point of tilting about C.

Find d in terms of a.

(6)

Worked Solution - Question 5

Moments

Topic group

1. Setup Diagrams - Scenario 1 - Tilting about A

Taking moments about A: $\sum M_A = 0$ Clockwise moments = Anticlockwise moments $P \times 3a = W \times d$ $3aP = Wd \quad ...(1)$

2. Setup Diagrams - Scenario 2 - Tilting about C

Taking moments about C: $\sum M_C = 0$ Distance from C to COM = $(2a - d)$ Distance from C to B = a $1.25P \times a = W \times (2a - d)$ $1.25Pa = W(2a - d)$ $1.25Pa = 2aW - Wd \quad ...(2)$

3. Setup Diagrams - Substitute equation (1) into equation (2)

From (1): $Wd = 3aP$ $1.25Pa = 2aW - 3aP$ $1.25Pa + 3aP = 2aW$ $4.25Pa = 2aW$ $P = \frac{2aW}{4.25a}$ $P = \frac{2W}{4.25}$ Substitute back into (1): $3a \times \frac{2W}{4.25} = Wd$ $\frac{6aW}{4.25} = Wd$ $d = \frac{6a}{4.25}$ $d = \frac{6a}{17/4}$ $d = \frac{24a}{17}$ $d = 1.41a$ (3 sf) Or in exact form: $d = \frac{24a}{17}$

Final answer

$$d = \frac{24a}{17} = 1.41a.$$

Question 6

6.

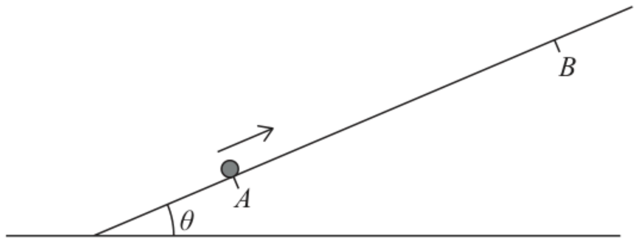

Figure 4

Figure 4 shows a rough plane inclined at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$

The points A and B lie on a line of greatest slope of the plane, with B above A.

A package P of mass m is projected up the plane from A towards B.

The coefficient of friction between the plane and the package is $\frac{1}{4}$

The package P is modelled as a particle.

(a) Show that the **deceleration** of P, as it moves from A to B, is $\frac{4}{5}g$

(6)

The package P comes to rest at B.

Given that P is projected from A with speed $U\,\text{m}\,\text{s}^{-1}$ and that $AB = 1.5\,\text{m}$,

(b) find the value of U.

(2)

On reaching B, P is held at rest there by a force of magnitude X newtons acting up the plane in the direction AB.

Given that the mass of P is 2 kg,

(c) find the smallest possible value of X.

(3)

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Worked Solution - Question 6

Resolving Forces, Inclined Planes

Topic group

1. Part (a): Show deceleration = $\frac{4g}{5}$ - Find exact trig values

Given: $\tan\theta = \frac{3}{4}$ Using 3-4-5 right triangle: $\sin\theta = \frac{3}{5}, \quad \cos\theta = \frac{4}{5}$

2. Part (a): Show deceleration = $\frac{4g}{5}$ - Resolve perpendicular to plane

$\perp$: $\sum F = 0$ $R = mg\cos\theta$ $R = \frac{4mg}{5}$

3. Part (a): Show deceleration = $\frac{4g}{5}$ - Calculate friction force

Package moving UP, so friction acts DOWN the plane: $F = \mu R = \frac{1}{4} \times \frac{4mg}{5} = \frac{mg}{5}$

4. Part (a): Show deceleration = $\frac{4g}{5}$ - Resolve parallel to plane

Taking up the plane as positive: $\|$: $\sum F = ma$ $-mg\sin\theta - F = ma$ $-mg \times \frac{3}{5} - \frac{mg}{5} = ma$ $-\frac{3mg}{5} - \frac{mg}{5} = ma$ $-\frac{4mg}{5} = ma$ $a = -\frac{4g}{5}$
Deceleration = $\left|-\frac{4g}{5}\right| = \frac{4g}{5}$

5. Part (b): Find initial speed U - Identify SUVAT variables

Package projected up plane, comes to rest at B: ; $u = U$ m s^{-1} (unknown) ; $v = 0$ m s^{-1} (comes to rest) ; $a = -\frac{4g}{5}$ m s^{-2} (from part a) ; $s = 1.5$ m

6. Part (b): Find initial speed U - Apply $v^2 = u^2 + 2as$

$v^2 = u^2 + 2as$ $0^2 = U^2 + 2\left(-\frac{4g}{5}\right)(1.5)$ $0 = U^2 - 2 \times \frac{4\times9.8}{5} \times 1.5$ $0 = U^2 - \frac{8\times9.8\times1.5}{5}$ $0 = U^2 - 23.52$ $U^2 = 23.52$ $U = 4.85\ \mathrm{m\,s^{-1}}$ Or using exact form: $U = \sqrt{2 \times \frac{4g}{5} \times 1.5} = \sqrt{\frac{12g}{5}}$

7. Part (c): Find smallest value of X - Analyze forces when package at rest at B

Package held at rest on slope by force X up the plane. Without X, package would slide down (component down > friction up). For MINIMUM X, friction acts UP the plane (at limiting friction).

8. Part (c): Find smallest value of X - Resolve perpendicular to plane

Mass = 2 kg, so weight = $2g$ N $R = 2g\cos\theta = 2g \times \frac{4}{5} = \frac{8g}{5}$ N

9. Part (c): Find smallest value of X - Calculate friction

Friction acts UP the plane (limiting): $F = \mu R = \frac{1}{4} \times \frac{8g}{5} = \frac{2g}{5}$ N

10. Part (c): Find smallest value of X - Resolve parallel to plane

For equilibrium (package at rest): $\|$: $\sum F = 0$ Taking up plane as positive: $X + F - 2g\sin\theta = 0$ $X + \frac{2g}{5} - 2g \times \frac{3}{5} = 0$ $X + \frac{2g}{5} - \frac{6g}{5} = 0$ $X = \frac{6g}{5} - \frac{2g}{5}$ $X = \frac{4g}{5}$ $X = \frac{4 \times 9.8}{5}$ $X = 7.84$ N

Final answer

(a) deceleration $= \dfrac{4g}{5}$, as required.

(b) $U = 4.85\ \mathrm{m\,s^{-1}}$ (about $4.8\ \mathrm{m\,s^{-1}}$).

(c) $X = 7.84$ N (about 7.8 N).

Question 7

13 marks

Working with Vectors

7. [*In this question* **i** *and* **j** *are horizontal unit vectors.*]

Two boats, A and B, are each moving with constant acceleration.

At time t hours after noon, boat A has velocity $\mathbf{v}_A$ km h^{-1}, where

$$\mathbf{v}_A = 2\mathbf{i} + 3\mathbf{j} + (\mathbf{i} - 4\mathbf{j})t$$

(a) Find the magnitude of the acceleration of A.

(2)

When $t = 2$, the velocity of B is $(4\mathbf{i} + \mathbf{j})$ km h^{-1}

When $t = 5$, the velocity of B is $(\mathbf{i} - 5\mathbf{j})$ km h^{-1}

(b) Find the acceleration of B, giving your answer in terms of **i** and **j**.

(2)

(c) Find the velocity of B at time $t = 0$, giving your answer in terms of **i** and **j**.

(2)

At time T_1 hours after noon, both boats are moving with the same speed.

(d) Find the exact value of T_1

(4)

At time T_2 hours after noon, both boats are moving in the same direction.

(e) Show that $3T_2^2 + pT_2 + q = 0$, where p and q are integers to be found.

(3)

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Worked Solution - Question 7

Working with Vectors

Topic group

1. Part (a): Find magnitude of acceleration of A - Identify acceleration from velocity function

Given: $\mathbf{v}_A = 2\mathbf{i} + 3\mathbf{j} + (\mathbf{i} - 4\mathbf{j})t$ This is in form: $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ Therefore: ; Initial velocity: $\mathbf{u}_A = 2\mathbf{i} + 3\mathbf{j}$; Acceleration: $\mathbf{a}_A = \mathbf{i} - 4\mathbf{j}$ km h^{-2}

2. Part (a): Find magnitude of acceleration of A - Find magnitude

$|\mathbf{a}| = \sqrt{a_x^2 + a_y^2}$ $|\mathbf{a}_A| = \sqrt{1^2 + (-4)^2} = \sqrt{1+16} = \sqrt{17} = 4.123\ldots$
$= 4.1 \text{ km h}^{-2} \text{ (2 sf)}$

3. Part (b): Find acceleration of B - Use two velocity points to find acceleration

Given: ; At $t = 2$: $\mathbf{v}_B = 4\mathbf{i} + \mathbf{j}$; At $t = 5$: $\mathbf{v}_B = \mathbf{i} - 5\mathbf{j}$ $\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t}$
$\mathbf{a}_B = \frac{(\mathbf{i}-5\mathbf{j})-(4\mathbf{i}+\mathbf{j})}{5-2} = \frac{(1-4)\mathbf{i}+(-5-1)\mathbf{j}}{3} = \frac{-3\mathbf{i}-6\mathbf{j}}{3} = -\mathbf{i} - 2\mathbf{j} \text{ km h}^{-2}$

4. Part (c): Find velocity of B at t = 0 - Use velocity equation backward

$\mathbf{v}(t) = \mathbf{v}(t_0) + \mathbf{a}(t - t_0)$ From $t = 2$ back to $t = 0$:
$\mathbf{v}_B(0) = \mathbf{v}_B(2) + \mathbf{a}_B(0-2) = (4\mathbf{i} + \mathbf{j}) + (-\mathbf{i} - 2\mathbf{j})(-2) = 4\mathbf{i} + \mathbf{j} + 2\mathbf{i} + 4\mathbf{j}$
$= 6\mathbf{i} + 5\mathbf{j} \text{ km h}^{-1}$

5. Part (d): Find exact value of T_1 when both have same speed - Express velocities at time t

For boat A: $\mathbf{v}_A(t) = 2\mathbf{i} + 3\mathbf{j} + (\mathbf{i} - 4\mathbf{j})t = (2+t)\mathbf{i} + (3-4t)\mathbf{j}$ For boat B:
$\mathbf{v}_B(t) = (6\mathbf{i} + 5\mathbf{j}) + (-\mathbf{i} - 2\mathbf{j})t = (6-t)\mathbf{i} + (5-2t)\mathbf{j}$

6. Part (d): Find exact value of T_1 when both have same speed - Find magnitudes

$|\mathbf{v}_A|^2 = (2+t)^2 + (3-4t)^2 = 4 + 4t + t^2 + 9 - 24t + 16t^2$
$= 17t^2 - 20t + 13$ $|\mathbf{v}_B|^2 = (6-t)^2 + (5-2t)^2$
$= 36 - 12t + t^2 + 25 - 20t + 4t^2 = 5t^2 - 32t + 61$

7. Part (d): Find exact value of T_1 when both have same speed - Set magnitudes equal

$17t^2 - 20t + 13 = 5t^2 - 32t + 61$ $12t^2 + 12t - 48 = 0$ $t^2 + t - 4 = 0$

8. Part (d): Find exact value of T_1 when both have same speed - Solve using quadratic formula

$t = \frac{-1\pm\sqrt{1+16}}{2} = \frac{-1\pm\sqrt{17}}{2}$ Taking positive value: $T_1 = \frac{-1+\sqrt{17}}{2}$

9. Part (e): Show $3T_2^2 + pT_2 + q = 0$ - Condition for same direction

Velocities parallel when: $\frac{2+T_2}{6-T_2} = \frac{3-4T_2}{5-2T_2}$

10. Part (e): Show $3T_2^2 + pT_2 + q = 0$ - Cross-multiply

$(2 + T_2)(5 - 2T_2) = (3 - 4T_2)(6 - T_2)$
$10 - 4T_2 + 5T_2 - 2T_2^2 = 18 - 3T_2 - 24T_2 + 4T_2^2$
$10 + T_2 - 2T_2^2 = 18 - 27T_2 + 4T_2^2$ $0 = 4T_2^2 + 2T_2^2 - 27T_2 - T_2 + 18 - 10$
$0 = 6T_2^2 - 28T_2 + 8$

11. Part (e): Show $3T_2^2 + pT_2 + q = 0$ - Divide by 2 to get required form

$0 = 3T_2^2 - 14T_2 + 4$ Therefore: $p = -14$ and $q = 4$

Final answer

(a) $|\mathbf{a}_A| = \sqrt{17}\,\mathrm{km\,h^{-2}} \approx 4.1\,\mathrm{km\,h^{-2}}$. (b) $\mathbf{a}_B = -\mathbf{i} - 2\mathbf{j}\,\mathrm{km\,h^{-2}}$. (c) $\mathbf{v}_B(0) = 6\mathbf{i} + 5\mathbf{j}\,\mathrm{km\,h^{-1}}$. (d) $T_1 = \frac{-1+\sqrt{17}}{2}$ hours. (e) $3T_2^2 - 14T_2 + 4 = 0,\ p = -14,\ q = 4$

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Question 8

17 marks

Newton's Second Law

8.

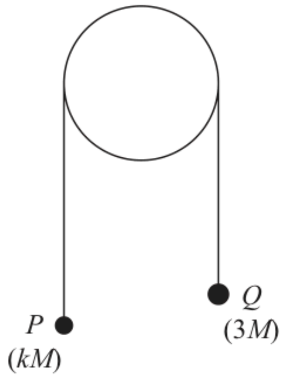

Figure 5

Two small balls, P and Q, have masses kM and $3M$ respectively, where $k < 3$

The balls are attached to the ends of a light inextensible string that passes over a fixed light smooth pulley.

The system is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 5.

The system is released from rest and, in the subsequent motion, P moves with an acceleration of magnitude $\frac{1}{5}g$

The balls are modelled as particles.

(a) Write down an equation of motion for P.

(2)

(b) Find the value of k.

(3)

Given that $M = 0.5\,\text{kg}$,

(c) find the magnitude of the force exerted on the pulley by the string while Q is moving downwards.

(3)

At the instant when the system is released, P is more than 2.5 m from the pulley and Q is 2.5 m above horizontal ground.
After hitting the ground, Q rebounds with a speed of $0.4\,\text{m}\,\text{s}^{-1}$

(d) Find the magnitude of the impulse received by Q when it hits the ground.

(5)

In the subsequent motion, P does not hit the pulley.

(e) Find the total time from when the balls are released until P first comes to rest.

(4)

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Worked Solution - Question 8

Newton's Second Law

Topic group

1. Part (a): Write equation of motion for P - Identify forces on P

Particle P (mass kM) moves upward with acceleration $\frac{g}{5}$: ; Tension T (upward) ; Weight kMg (downward)

2. Part (a): Write equation of motion for P - Apply Newton's Second Law

Taking upward as positive: $\sum F = ma$ $T - kMg = kM \times \frac{g}{5}$

3. Part (b): Find value of k - Write equation of motion for Q

Particle Q (mass $3M$) moves downward with acceleration $\frac{g}{5}$: Taking downward as positive for Q: $3Mg - T = 3M \times \frac{g}{5}$ $3Mg - T = \frac{3Mg}{5}$ $T = 3Mg - \frac{3Mg}{5}$ $T = \frac{12Mg}{5}$

4. Part (b): Find value of k - Substitute into equation for P

From part (a): $T - kMg = \frac{kMg}{5}$ $\frac{12Mg}{5} - kMg = \frac{kMg}{5}$ $\frac{12Mg}{5} = kMg + \frac{kMg}{5}$ $\frac{12Mg}{5} = \frac{5kMg+kMg}{5}$ $\frac{12Mg}{5} = \frac{6kMg}{5}$ $12 = 6k$ $k = 2$

5. Part (c): Find force on pulley while Q moving down - Identify tension in string

From part (b): $T = \frac{12Mg}{5}$ Given: $M = 0.5$ kg $T = \frac{12\times0.5\times g}{5} = \frac{6g}{5} = \frac{6\times9.8}{5}$ $= 11.76\text{ N}$

6. Part (c): Find force on pulley while Q moving down - Find force on pulley

The pulley experiences two tension forces: ; Horizontal tension from P: $T = 11.76$ N ; Vertical tension from Q: $T = 11.76$ N These are perpendicular, so use Pythagoras: $F = \sqrt{T^2 + T^2} = \sqrt{2T^2} = T\sqrt{2} = 11.76\sqrt{2} = 16.63\ldots$ $= 16.6\text{ N (3 sf)}$ Or: $F = \frac{6g\sqrt{2}}{5} = \frac{12g}{5} = 2.4g = 23.5$ N Wait, let me recalculate... Actually: $F = 2T$ (both tensions pull on pulley) $F = 2 \times \frac{12Mg}{5} = \frac{24Mg}{5}$ $= \frac{24\times0.5\times9.8}{5} = \frac{117.6}{5} = 23.5\text{ N (3 sf)}$ Or: $\frac{12g}{5} = 23.5$ N

7. Part (d): Find impulse when Q hits ground - Find velocity of Q just before hitting ground

Using SUVAT with: ; $u = 0$ (starts from rest) ; $a = \frac{g}{5} = \frac{9.8}{5} = 1.96$ m s^{-2} ; $s = 2.5$ m $v^2 = u^2 + 2as$ $v^2 = 0 + 2 \times \frac{g}{5} \times 2.5$ $v^2 = \frac{5g}{5}$ $v^2 = g$ $v = \sqrt{g} = \sqrt{9.8} = 3.130\ldots$ m s^{-1} Or exact: $v = \sqrt{g} = \sqrt{\frac{7g}{5}}$ Actually: $v^2 = 2 \times 1.96 \times 2.5 = 9.8$, so $v = \sqrt{9.8} = 3.13$ m s^{-1}

8. Part (d): Find impulse when Q hits ground - Calculate impulse

$\text{Impulse} = m(v_{\text{after}} - v_{\text{before}})$ Taking downward as positive before impact: ; Velocity before: $v_{\text{before}} = +3.13$ m s^{-1} (downward) ; Velocity after: $v_{\text{after}} = -0.4$ m s^{-1} (upward, rebounds) ; Mass of Q: $3M = 3 \times 0.5 = 1.5$ kg $I = 1.5 \times (-0.4 - 3.13) = 1.5 \times (-3.53) = -5.295\ldots$ $|I| = 5.3$ N s (2 sf) Or using exact values: $|I| = 1.5(\sqrt{g} + 0.4) = 1.5(3.13 + 0.4) = 5.3$ N s

9. Part (e): Find total time until P first comes to rest - Time for Q to fall (t_1)

Using $s = ut + \frac{1}{2}at^2$ with $u = 0$, $s = 2.5$ m, $a = \frac{g}{5}$: $2.5 = 0 + \frac{1}{2} \times \frac{g}{5} \times t_1^2$ $2.5 = \frac{gt_1^2}{10}$ $25 = gt_1^2$ $t_1^2 = \frac{25}{9.8} = 2.551\ldots$ $t_1 = \sqrt{\frac{25}{9.8}} = 1.597\ldots$ s Or exact: $t_1 = \sqrt{\frac{5\times5}{g}} = \frac{5}{\sqrt{g}}$ Better: $t_1 = \sqrt{\frac{5}{g/5}} = \sqrt{\frac{25}{g}}$ Actually: $2.5 = \frac{1}{2} \times 1.96 \times t_1^2$ $t_1^2 = \frac{5}{1.96} = 2.551$, so $t_1 = 1.597$ s

10. Part (e): Find total time until P first comes to rest - Velocity of P when Q hits ground

At time t_1, P has velocity: $v_P = 0 + \frac{g}{5} \times t_1 = \frac{9.8}{5} \times 1.597 = 1.96 \times 1.597$ $= 3.13 \text{ m s}^{-1}$ Or: $v_P = \sqrt{g}$ (same as Q)

11. Part (e): Find total time until P first comes to rest - Time for P to come to rest (t_2)

After Q hits ground, string goes slack. P continues upward under gravity alone. Using $v = u + at$ with $v = 0$, $u = 3.13$ m s^{-1}, $a = -g$: $0 = 3.13 - 9.8t_2$ $t_2 = \frac{3.13}{9.8} = 0.3194\ldots$ s Or: $t_2 = \frac{\sqrt{g}}{g} = \frac{1}{\sqrt{g}}$

12. Part (e): Find total time until P first comes to rest - Total time

$T = t_1 + t_2 = 1.597 + 0.319 = 1.916\ldots = 1.9 \text{ s (2 sf) or } 1.92 \text{ s (3 sf)}$

Final answer

$(a)\ T - kMg = \dfrac{kMg}{5}$. $(b)\ k = 2$. $(c)\ F = \dfrac{12g}{5}\ \mathrm{N} \approx 23.5\ \mathrm{N}$. $(d)\ |I| = 5.30\ \mathrm{N\,s}$. $(e)\ T = 1.92\ \mathrm{s}$

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PAST PAPER

WME01/01 October 2025

October 2025 | 7 questions | 75 marks

7
questions

75
marks

Answers
worked solution after each question

Question 1

1.

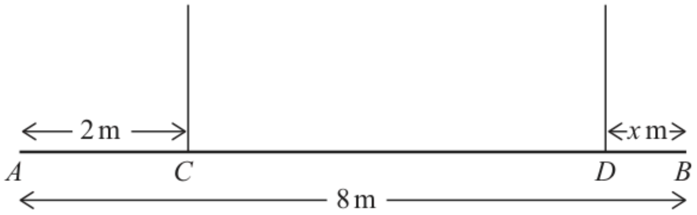

Figure 1

Figure 1 shows a sketch of a beam AB, with weight 240 N and length 8 m.

The beam is held in equilibrium in a horizontal position by two vertical ropes. The ropes are attached to the beam at the points C and D, where $AC = 2$ m and $DB = x$ metres.

The beam is modelled as a uniform rod and the ropes are modelled as light inextensible strings.

The tension in the rope at D is 90 N.

(a) Show that $x = \frac{2}{3}$

(3)

The rope at C will break if its tension exceeds 183 N. The rope at D cannot break.
A package of weight W newtons is now attached to the beam at A.
The beam remains horizontal and in equilibrium.

The package is modelled as a particle.

It is given that the rope at C does not break.

(b) Find the greatest possible value of W.

(4)

WME01/01 OCTOBER 2025

Worked Solution - Question 1

Moments

Topic group

1. Part (a): Show that x = 2/3 - Identify distances from C

From the diagram: ; Distance AC = 2 m ; Distance CD = 8 - 2 - x = (6-x) m ; Distance from A to center = 4 m, so distance from C to center = 4 - 2 = 2 m

2. Part (a): Show that x = 2/3 - Take moments about C

$\sum M_C = 0$ Clockwise moments = Anticlockwise moments $90 \times (6 - x) = 240 \times 2$ $90(6 - x) = 480$ $540 - 90x = 480$ $90x = 60$ $x = \frac{60}{90} = \frac{2}{3}$

3. Part (b): Find the greatest possible value of W - Apply vertical equilibrium

$\sum F_{\text{vertical}} = 0$ $T_C + T_D = 240 + W$ $183 + T_D = 240 + W$ $T_D = 57 + W$

4. Part (b): Find the greatest possible value of W - Take moments about C

$\sum M_C = 0$ Clockwise moments = Anticlockwise moments Distance CD = $6 - \frac{2}{3} = \frac{16}{3}$ m $T_D \times \frac{16}{3} + W \times 2 = 240 \times 2$ $(57 + W) \times \frac{16}{3} + 2W = 480$ $\frac{16(57+W)}{3} + 2W = 480$ $\frac{912+16W}{3} + 2W = 480$ Multiply through by 3: $912 + 16W + 6W = 1440$ $22W = 528$ $W = 24$

Final answer

$(a)\ x = \dfrac{2}{3}. \quad (b)\ W = 24\ \text{N}.$

Question 2

8 marks

Momentum, Impulse & Collisions

2.

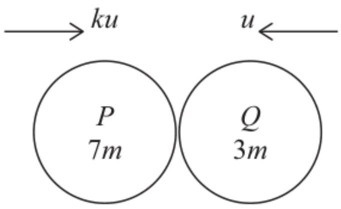

Figure 2

Particle P of mass $7m$ and particle Q of mass $3m$ are moving in opposite directions along the same straight line on a smooth horizontal surface. The particles collide directly.

Immediately **before** the collision, the speed of P is ku and the speed of Q is u, as shown in Figure 2.

Immediately **after** the collision, the speed of P is w and the speed of Q is $2w$.

The direction of motion of Q is reversed by the collision.

The impulse received by Q in the collision has magnitude $\frac{7}{2}mu$.

(a) Find w in terms of u.

(3)

(b) Find the two possible values of k.

(5)

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Worked Solution - Question 2

Momentum, Impulse & Collisions

Topic group

1. Part (a): Find w in terms of u - Apply impulse-momentum theorem to Q

$\text{Impulse} = \text{Final momentum} - \text{Initial momentum}$ For particle Q (mass = 3m): ; Initial velocity: $-u$ (moving left, negative) ; Final velocity: $+2w$ (moving right, positive) $\text{Impulse} = 3m(2w) - 3m(-u)$ $\frac{7mu}{2} = 6mw + 3mu$ $\frac{7mu}{2} = 3m(2w + u)$ $\frac{7u}{2} = 6w + 3u$ $6w = \frac{7u}{2} - 3u$ $6w = \frac{7u - 6u}{2}$ $6w = \frac{u}{2}$ $w = \frac{u}{12}$

2. Part (b): Find the two possible values of k - Apply conservation of momentum

$\text{Total momentum before} = \text{Total momentum after}$ Before collision: $\text{P momentum} = 7m \times ku = 7mku \text{ (right)}$ $\text{Q momentum} = 3m \times (-u) = -3mu \text{ (left)}$ $\text{Total} = 7mku - 3mu$ After collision: $\text{P momentum} = 7m \times w = 7mw \text{ (right)}$ $\text{Q momentum} = 3m \times 2w = 6mw \text{ (right)}$ $\text{Total} = 7mw + 6mw = 13mw$

3. Part (b): Find the two possible values of k - Set up equation

$7mku - 3mu = 13mw$ $7ku - 3u = 13w$

4. Part (b): Find the two possible values of k - Substitute $w = \frac{u}{12}$

$7ku - 3u = 13 \times \frac{u}{12}$ $7ku - 3u = \frac{13u}{12}$ $7ku = \frac{13u}{12} + 3u$ $7ku = \frac{13u + 36u}{12}$ $7ku = \frac{49u}{12}$ $7k = \frac{49}{12}$ $k = \frac{49}{84} = \frac{7}{12}$

5. Part (b): Find the two possible values of k - Consider both directions for P after collision

P could move either right (velocity = $+w$) or left (velocity = $-w$) Case 1: P moves right with velocity $+w$ Already solved: $k = \frac{7}{12}$ Case 2: P moves left with velocity $-w$ Total momentum after = $-7mw + 6mw = -mw$ $7mku - 3mu = -mw$ $7ku - 3u = -w$ $7ku - 3u = -\frac{u}{12}$ $7ku = 3u - \frac{u}{12}$ $7ku = \frac{36u - u}{12}$ $7ku = \frac{35u}{12}$ $k = \frac{35}{84} = \frac{5}{12}$

Final answer

(a) $w = \dfrac{u}{12}$. (b) $k = \dfrac{7}{12}$ or $k = \dfrac{5}{12}$.

Question 3

14 marks

Forces

3. [*In this question* **i** *and* **j** *are horizontal perpendicular unit vectors.*]

A particle P of mass 2 kg moves on a smooth horizontal surface under the action of two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, where $\mathbf{F}_1 = (-2\mathbf{i} + 3\mathbf{j})$ N and $\mathbf{F}_2 = (4\mathbf{i} + 2\mathbf{j})$ N.

(a) Find the acceleration of P.

(3)

At time $t = 0$, the velocity of P is $(3\mathbf{i} - 4\mathbf{j})\,\text{m s}^{-1}$

(b) Find the speed of P when $t = 3$ seconds.

(4)

An additional force, $\mathbf{F}_3 = (b\mathbf{i} + c\mathbf{j})$ N, is applied to P.

The resultant of $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ is equal to $\lambda(\mathbf{i} + \mathbf{j})$ N, where λ is a constant.

(c) Show that $b - c = 3$

(3)

The resultant of $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ has magnitude $10\sqrt{2}$ N.

(d) Find the two possible $\mathbf{F}_3$ forces.

(4)

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Worked Solution - Question 3

Forces

Topic group

1. Part (a): Find the acceleration of P - Find resultant force

$\mathbf{F}_{\text{resultant}} = \mathbf{F}_1 + \mathbf{F}_2$ $\mathbf{F}_{\text{resultant}} = (-2\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} + 2\mathbf{j})$
$= (-2+4)\mathbf{i} + (3+2)\mathbf{j} = 2\mathbf{i} + 5\mathbf{j}\text{ N}$

2. Part (a): Find the acceleration of P - Apply Newton's Second Law

$\mathbf{F} = m\mathbf{a}$ $2\mathbf{i} + 5\mathbf{j} = 2 \times \mathbf{a}$ $\mathbf{a} = \frac{2\mathbf{i}+5\mathbf{j}}{2}$ $\mathbf{a} = \mathbf{i} + 2.5\mathbf{j}\text{ m s}^{-2}$ Or: $\mathbf{a} = \mathbf{i} + \frac{5}{2}\mathbf{j}$ m s^{-2}

3. Part (b): Find the speed of P when t = 3 seconds - Find velocity at t = 3

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$ Given: ; $\mathbf{u} = 3\mathbf{i} - 4\mathbf{j}$ m s^{-1} ; $\mathbf{a} = \mathbf{i} + 2.5\mathbf{j}$ m s^{-2} ; $t = 3$ seconds
$\mathbf{v} = (3\mathbf{i} - 4\mathbf{j}) + (\mathbf{i} + 2.5\mathbf{j}) \times 3 = 3\mathbf{i} - 4\mathbf{j} + 3\mathbf{i} + 7.5\mathbf{j} = 6\mathbf{i} + 3.5\mathbf{j}\text{ m s}^{-1}$

4. Part (b): Find the speed of P when t = 3 seconds - Find the speed (magnitude of velocity)

$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$ $\text{Speed} = \sqrt{6^2 + 3.5^2} = \sqrt{36 + 12.25} = \sqrt{48.25} = 6.946\ldots$
$= 6.95\text{ m s}^{-1}\text{ (3 s.f.)}$

5. Part (c): Show that b - c = 3 - Find resultant of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$

$\mathbf{F}_{\text{resultant}} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = (-2\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} + 2\mathbf{j}) + (b\mathbf{i} + c\mathbf{j})$
$= (-2 + 4 + b)\mathbf{i} + (3 + 2 + c)\mathbf{j} = (2 + b)\mathbf{i} + (5 + c)\mathbf{j}$

6. Part (c): Show that b - c = 3 - Apply the condition

Given that resultant = $\lambda(\mathbf{i} + \mathbf{j})$, we can write: $(2 + b)\mathbf{i} + (5 + c)\mathbf{j} = \lambda\mathbf{i} + \lambda\mathbf{j}$

7. Part (c): Show that b - c = 3 - Compare coefficients

For this to be in the form $\lambda(\mathbf{i} + \mathbf{j})$, the coefficients must be equal: $2 + b = \lambda$
$5 + c = \lambda$ Therefore: $2 + b = 5 + c$ $b = 5 + c - 2$ $b = 3 + c$ $b - c = 3$

8. Part (d): Find the two possible $\mathbf{F}_3$ forces - Use the magnitude condition

Resultant = $(2+b)\mathbf{i}+(5+c)\mathbf{j}$ Since resultant = $\lambda(\mathbf{i}+\mathbf{j})$, we have $2+b=5+c=\lambda$ $|\mathbf{F}_{\text{resultant}}|=\lambda\sqrt{1^2+1^2}=\lambda\sqrt{2}$ Given: $|\mathbf{F}_{\text{resultant}}|=10\sqrt{2}$ $\lambda\sqrt{2}=10\sqrt{2}$ $\lambda=10 \quad \text{or} \quad \lambda=-10$ (The negative value comes from considering the opposite direction)

9. Part (d): Find the two possible $\mathbf{F}_3$ forces - Find b and c for each value of λ

Case 1: $\lambda=10$ $2+b=10 \quad \Rightarrow \quad b=8$ $5+c=10 \quad \Rightarrow \quad c=5$ Check: $b-c=8-5=3$ ✓ Therefore: $\mathbf{F}_3=8\mathbf{i}+5\mathbf{j}$ N Case 2: $\lambda=-10$ $2+b=-10 \quad \Rightarrow \quad b=-12$ $5+c=-10 \quad \Rightarrow \quad c=-15$ Check: $b-c=-12-(-15)=3$ ✓ Therefore: $\mathbf{F}_3=-12\mathbf{i}-15\mathbf{j}$ N

Final answer

$(a)\ \mathbf{a}=\mathbf{i}+2.5\mathbf{j}\ \mathrm{m\,s^{-2}}. \quad (b)\ \text{speed}=6.95\ \mathrm{m\,s^{-1}}. \quad (c)\ b-c=3. \quad (d)\ \mathbf{F}_3=(8\mathbf{i}+5\mathbf{j})\ \mathrm{N}\ \text{or}\ (-12\mathbf{i}-15\mathbf{j})\ \mathrm{N}$

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Question 4

9 marks

Constant Acceleration in 2D

4. The point A is 10 m above horizontal ground.
At time $t = 0$, a particle P is projected vertically upwards with speed $5\,\text{m}\,\text{s}^{-1}$ from A.
Particle P moves freely under gravity.

(a) Find the greatest height above A reached by P.
(3)

The point B is on the ground, vertically below A.
At time $t = 1$ second, a particle Q is projected vertically upwards with speed $7\,\text{m}\,\text{s}^{-1}$ from B.
Particle Q moves freely under gravity.

Particles P and Q collide at time $t = T$ seconds.

(b) Find the value of T.
(4)

(c) Find the speed of P at the instant immediately before the particles collide.
(2)

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Worked Solution - Question 4

Constant Acceleration in 2D

Topic group

1. Part (a): Find the greatest height above A reached by P - Identify the SUVAT variables

At maximum height, velocity = 0 ; $u = 5$ m s^{-1} (initial velocity) ; $v = 0$ m s^{-1} (at maximum height) ; $a = -9.8$ m s^{-2} (gravity, downward) ; $s = ?$ (displacement above A)

2. Part (a): Find the greatest height above A reached by P - Apply $v^2 = u^2 + 2as$

$v^2 = u^2 + 2as$ $0^2 = 5^2 + 2(-9.8)s$ $0 = 25 - 19.6s$ $19.6s = 25$ $s = \frac{25}{19.6}$
$s = 1.276\ldots$ $s = 1.28 \text{ m } (3 \text{ s.f.})$

3. Part (b): Find the value of T - Position of P at time t

Taking upward as positive, with origin at ground level: $s = s_0 + ut + \frac{1}{2}at^2$ For P (starting at A, 10 m above ground): $s_P = 10 + 5t + \frac{1}{2}(-9.8)t^2$
$s_P = 10 + 5t - 4.9t^2$

4. Part (b): Find the value of T - Position of Q at time t

Q starts at $t = 1$, so Q has been moving for $(t-1)$ seconds: For Q (starting at B, ground level): $s_Q = 0 + 7(t-1) + \frac{1}{2}(-9.8)(t-1)^2$
$s_Q = 7(t-1) - 4.9(t-1)^2$ $s_Q = 7t - 7 - 4.9(t^2 - 2t + 1)$
$s_Q = 7t - 7 - 4.9t^2 + 9.8t - 4.9$ $s_Q = -4.9t^2 + 16.8t - 11.9$

5. Part (b): Find the value of T - Set $s_P = s_Q$ at collision (when $t = T$)

$10 + 5T - 4.9T^2 = -4.9T^2 + 16.8T - 11.9$ $10 + 5T = 16.8T - 11.9$
$10 + 11.9 = 16.8T - 5T$ $21.9 = 11.8T$ $T = \frac{21.9}{11.8}$ $T = 1.855\ldots$
$T = 1.86 \text{ s } (3 \text{ s.f.})$

6. Part (c): Find the speed of P immediately before collision - Find velocity of P at time $T = 1.86$ s

$v = u + at$ For particle P: ; $u = 5$ m s^{-1} ; $a = -9.8$ m s^{-2} ; $t = 1.86$ s
$v_P = 5 + (-9.8)(1.86) = 5 - 18.228 = -13.228\ldots = -13.2 \text{ m s}^{-1}$ The negative sign indicates P is moving downward.

7. Part (c): Find the speed of P immediately before collision - Find speed

Speed = $|v|$ $\text{Speed} = |-13.2| = 13.2 \text{ m s}^{-1}$

Final answer

(a) maximum height above A $= 1.28$ m. (b) $T = 1.86$ s. (c) speed of P $= 13.2 \text{ m s}^{-1}$

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Question 5

13 marks

Resolving Forces, Inclined Planes

5.

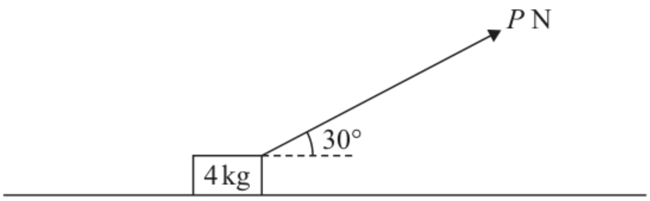

Figure 3

A box of mass 4 kg is placed on a rough horizontal surface.

A force of magnitude P newtons, acting at 30° to the horizontal, is applied to the box, as shown in Figure 3.

The coefficient of friction between the box and the surface is $\frac{2}{3}$

The box is modelled as a particle.

(a) Find the value of P when the box is on the point of sliding along the surface.

(6)

The value of P is now increased to 25 and the box moves along the surface.

Find

(b) the acceleration of the box,

(5)

(c) the speed of the box when it has moved 1.5 m.

(2)

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Worked Solution - Question 5

Resolving Forces, Inclined Planes

Topic group

1. Part (a): Find P when box is on point of sliding - Resolve vertically

$\uparrow: \sum F_{\text{vertical}} = 0$ $R + P\sin 30^\circ = mg$ $R + 0.5P = 4 \times 9.8$
$R = 39.2 - 0.5P \quad ...(1)$

2. Part (a): Find P when box is on point of sliding - Apply friction law

At limiting equilibrium (on point of sliding): $F = \mu R$ $F = \frac{2}{3} \times R$
$F = \frac{2}{3}(39.2 - 0.5P)$ $F = 26.133... - \frac{P}{3} \quad ...(2)$

3. Part (a): Find P when box is on point of sliding - Resolve horizontally

Box on point of sliding means $a = 0$: $\rightarrow: \sum F_{\text{horizontal}} = 0$ $P\cos 30^\circ = F$
$P \times \frac{\sqrt{3}}{2} = 26.133... - \frac{P}{3}$ $0.866P = 26.133 - 0.333P$
$0.866P + 0.333P = 26.133$ $1.199P = 26.133$ $P = 21.8 \text{ N (3 s.f.)}$

4. Part (b): Find acceleration when P = 25 N - Find normal reaction when P = 25 N

From equation (1) in part (a): $R = 39.2 - 0.5P$ $R = 39.2 - 0.5(25)$
$R = 39.2 - 12.5$ $R = 26.7 \text{ N}$

5. Part (b): Find acceleration when P = 25 N - Find friction force

Box is now moving, so: $F = \mu R$ $F = \frac{2}{3} \times 26.7$ $F = 17.8 \text{ N}$

6. Part (b): Find acceleration when P = 25 N - Resolve horizontally and apply F = ma

$\rightarrow: \sum F = ma$ $P\cos 30^\circ - F = ma$ $25 \times 0.866 - 17.8 = 4a$
$21.65 - 17.8 = 4a$ $3.85 = 4a$ $a = 0.9625$ $a = 0.96 \text{ m s}^{-2} \text{ (2 s.f.)}$

7. Part (c): Find speed when box has moved 1.5 m - Identify SUVAT variables

$u = 0$ m s^{-1} (starts from rest) ; $a = 0.9625$ m s^{-2} (from part b) ; $s = 1.5$ m ;
$v = ?$

8. Part (c): Find speed when box has moved 1.5 m - Apply $v^2 = u^2 + 2as$

$v^2 = u^2 + 2as$ $v^2 = 0^2 + 2(0.9625)(1.5)$ $v^2 = 2.8875$ $v = \sqrt{2.8875}$
$v = 1.699\ldots$ $v = 1.7\ \mathrm{m\ s^{-1}}$ (2 s.f.)

Final answer

$(a)\ P = 21.8\ \mathrm{N}$. $(b)\ a = 0.96\ \mathrm{m\,s^{-2}}$. $(c)\ v = 1.7\ \mathrm{m\,s^{-1}}$.

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Question 6

10 marks

Kinematics Graphs

6.

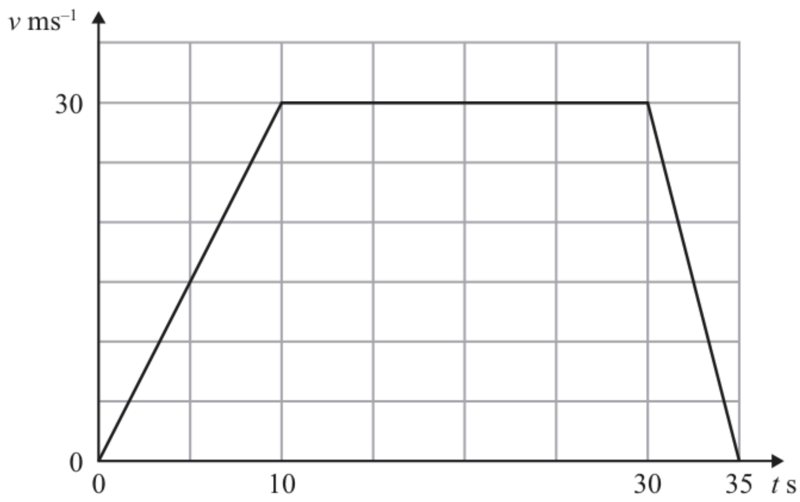

Figure 4

The point O lies on a straight horizontal road.
At time $t = 0$, a car leaves O and travels along the road.

The velocity-time graph in Figure 4 shows the velocity, $v\,\text{m}\,\text{s}^{-1}$, of the car at time t seconds for the first 35 seconds of its journey.

(a) Find

(i) the acceleration of the car for the period $0 \leqslant t \leqslant 10$

(ii) the deceleration of the car for the period $30 \leqslant t \leqslant 35$

(2)

(b) Sketch an acceleration-time graph for the car for the period $0 \leqslant t \leqslant 35$

(2)

(c) Find the distance travelled by the car for the period $0 \leqslant t \leqslant 35$

(2)

When $t = 5$, a motorcycle starts from rest at O.
The motorcycle travels along the same road as the car and in the same direction.

For the period $5 \leqslant t \leqslant 20$, the acceleration of the motorcycle is $A\,\text{m}\,\text{s}^{-2}$, where A is a positive constant.

The motorcycle catches up with the car when $t = 20$

(d) Find the value of A.

(4)

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Worked Solution - Question 6

Kinematics Graphs

Topic group

1. Part (a): Find accelerations

(i) Acceleration for $0 \leq t \leq 10$ $\text{Acceleration} = \frac{\Delta v}{\Delta t} = \text{gradient of v-t graph}$ $a = \frac{30-0}{10-0}$ $a = 3 \text{ m s}^{-2}$ (ii) Deceleration for $30 \leq t \leq 35$ $a = \frac{0-30}{35-30}$ $a = \frac{-30}{5}$ $a = -6 \text{ m s}^{-2}$ Deceleration = $|-6| = 6$ m s^{-2}

2. Part (b): Sketch acceleration-time graph

Analysis of each section: ; $0 \leq t \leq 10$: Constant acceleration = 3 m s^{-2} ; $10 < t \leq 30$: Constant velocity, so acceleration = 0 m s^{-2} ; $30 < t \leq 35$: Constant deceleration = -6 m s^{-2}

3. Part (c): Find distance travelled for $0 \leq t \leq 35$ - Calculate area under v-t graph

$\text{Distance} = \text{Area under v-t graph}$ The shape is a trapezium. We can split it into regions: Method 1: Trapezium formula $\text{Area} = \frac{1}{2}(a+b)h$ Where the parallel sides are the time intervals and height is velocity. Better to calculate as sum of shapes: Method 2: Sum of triangle + rectangle + triangle $\text{Area}_1 = \frac{1}{2} \times 10 \times 30 = 150 \text{ m}$ $\text{Area}_2 = 20 \times 30 = 600 \text{ m}$ $\text{Area}_3 = \frac{1}{2} \times 5 \times 30 = 75 \text{ m}$ $\text{Total distance} = 150 + 600 + 75 = 825 \text{ m}$

4. Part (d): Find value of A - Distance travelled by car (0 to 20 seconds)

Area under v-t graph from $t = 0$ to $t = 20$: $\text{Distance}_{\text{car}} = \text{Triangle} + \text{Rectangle} = \frac{1}{2}(10)(30) + (10)(30) = 150 + 300 = 450 \text{ m}$

5. Part (d): Find value of A - Distance travelled by motorcycle (5 to 20 seconds)

Motorcycle starts at $t = 5$ with constant acceleration A. Time of travel: $20 - 5 = 15$ seconds $s = ut + \frac{1}{2}at^2$ $\text{Distance}_{\text{motorcycle}} = 0 + \frac{1}{2}A(15)^2 = \frac{225A}{2} = 112.5A \text{ m}$

6. Part (d): Find value of A - Set distances equal

Motorcycle catches up with car when $t = 20$: $112.5A = 450$ $A = \frac{450}{112.5}$
$A = 4\ \mathrm{m\,s^{-2}}$

Final answer

$(a)(i)\ a = 3\ \mathrm{m\,s^{-2}},\ (ii)$ deceleration $= 6\ \mathrm{m\,s^{-2}}$. (b) correct acceleration-time graph with levels $3, 0, -6$. (c) distance $= 825$ m. $(d)\ A = 4\ \mathrm{m\,s^{-2}}$

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14 marks

Question 7

Resolving Forces, Inclined Planes

7.

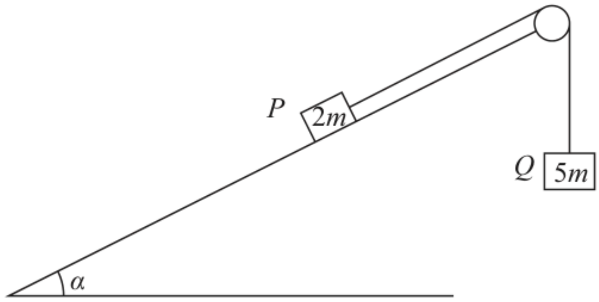

Figure 5

A block P of mass $2m$ is held at rest on a fixed rough plane.

The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{5}{12}$

One end of a light inextensible string is attached to P.
The string is parallel to a line of greatest slope of the plane and passes over a smooth light pulley which is fixed at the top of the plane.
The other end of the string is attached to a block Q of mass $5m$.
Block Q hangs vertically below the pulley, as shown in Figure 5.

The system is released from rest with the string taut and block P moves up the plane.

Immediately after the system is released, the tension in the string is T and the acceleration of the blocks is a.

The blocks are modelled as particles and air resistance is ignored.

(a) Write down an equation of motion for Q.

(2)

The coefficient of friction between P and the plane is $\frac{1}{8}$

(b) Find T in terms of m and g.

(7)

(c) State how the solution to part (b) uses the fact that the string is inextensible.

(1)

The magnitude of the force exerted on the pulley by the string is kmg.

(d) Find the value of k to 3 significant figures.

(4)

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Worked Solution - Question 7

Resolving Forces, Inclined Planes

Topic group

1. Part (a): Write equation of motion for Q

Forces on Q (mass = 5m): Taking downward as positive for Q: $\text{For Q: } \sum F = ma$
$5mg - T = 5ma$

2. Part (b): Find T in terms of m and g - Resolve perpendicular to plane

$\perp \text{ to plane: } \sum F = 0 \; R = 2mg\cos\alpha \; R = 2mg \times \frac{12}{13} \; R = \frac{24mg}{13}$

3. Part (b): Find T in terms of m and g - Find friction force

P is moving UP the plane, so friction acts DOWN the plane: $F = \mu R = \frac{1}{8} \times \frac{24mg}{13} = \frac{3mg}{13}$

4. Part (b): Find T in terms of m and g - Resolve parallel to plane

Taking up the plane as positive: $\parallel \text{ to plane: } \sum F = ma$
$T - 2mg\sin\alpha - F = 2ma \; T - 2mg \times \frac{5}{13} - \frac{3mg}{13} = 2ma$
$T - \frac{10mg}{13} - \frac{3mg}{13} = 2ma \; T - \frac{13mg}{13} = 2ma \; T - mg = 2ma \quad ...(1)$

5. Part (b): Find T in terms of m and g - Combine with equation for Q

From part (a): $5mg - T = 5ma$ ...(2) Add equations (1) and (2):
$(T - mg) + (5mg - T) = 2ma + 5ma \; 4mg = 7ma \; a = \frac{4g}{7}$

6. Part (b): Find T in terms of m and g - Substitute back to find T

From equation (1): $T = mg + 2ma = mg + 2m \times \frac{4g}{7} = mg + \frac{8mg}{7} = \frac{7mg+8mg}{7} = \frac{15mg}{7}$

7. Part (c): State how solution uses fact that string is inextensible

An inextensible string means the string cannot stretch. This has the following consequence: For an inextensible string over a pulley: The magnitude of acceleration of both connected particles must be equal. In this problem: $|a_P| = |a_Q| = a$ How this was used in part (b): When we wrote the equations of motion: ; For P: $T - mg = 2ma$; For Q: $5mg - T = 5ma$ We used the same acceleration a for both particles because the string is inextensible. If the string could stretch, the two particles could have different accelerations.

8. Part (d): Find value of k (force on pulley = kmg) - Identify the angle between tensions

The string from Q is vertical (downward). The string from P is along the plane at angle α to horizontal. Angle between the two strings = $90° + \alpha$

9. Part (d): Find value of k (force on pulley = kmg) - Use vector components

Let's take horizontal (right) and vertical (up) as positive directions. Tension from Q: ; Horizontal component: 0 ; Vertical component: $-T = -\frac{15mg}{7}$ (downward) Tension from P along plane: The plane makes angle α with horizontal, so: ; Horizontal component: $-T\cos\alpha = -\frac{15mg}{7} \times \frac{12}{13} = -\frac{180mg}{91}$; Vertical component: $T\sin\alpha = \frac{15mg}{7} \times \frac{5}{13} = \frac{75mg}{91}$

10. Part (d): Find value of k (force on pulley = kmg) - Find resultant

$R_x = 0 - \frac{180mg}{91} = -\frac{180mg}{91}$ $R_y = -\frac{15mg}{7} + \frac{75mg}{91}$ Convert $\frac{15mg}{7}$ to denominator 91: $\frac{15mg}{7} = \frac{195mg}{91}$ $R_y = -\frac{195mg}{91} + \frac{75mg}{91} = -\frac{120mg}{91}$

11. Part (d): Find value of k (force on pulley = kmg) - Find magnitude

$|R| = \sqrt{R_x^2 + R_y^2} = \sqrt{\left(\frac{180mg}{91}\right)^2 + \left(\frac{120mg}{91}\right)^2} = \frac{mg}{91}\sqrt{180^2 + 120^2}$ $= \frac{mg}{91}\sqrt{32400 + 14400} = \frac{mg}{91}\sqrt{46800} = \frac{mg}{91} \times 216.333\ldots = 2.377\ldots mg$ $= 2.38mg\ (3\text{ s.f.})$ Therefore: $k = 2.38$

Final answer

$(a)\ 5mg - T = 5ma. \quad (b)\ T = \frac{15mg}{7}. \quad (c)$ both particles have the same magnitude of acceleration. $\quad (d)\ k = 2.38$

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PAST PAPER

WME01/01 January 2026

January 2026 | 7 questions | 75 marks

7
questions

75
marks

Answers
worked solution after each question

Question 1

6 marks

Momentum, Impulse & Collisions

1. A particle P of mass $3m$ and a particle Q of mass $5m$ are on a smooth horizontal surface. The particles move towards each other in opposite directions along the same straight line and collide.

Immediately before the collision, the speed of P is $3u$ and the speed of Q is $4u$

The magnitude of the impulse exerted on P by Q in the collision is $\frac{33}{2}mu$

Find

(i) the speed of P immediately after the collision,

(ii) the speed of Q immediately after the collision.

(6)

WME01/01 JANUARY 2026

Worked Solution - Question 1

Momentum, Impulse & Collisions

Topic group

1. Choose a sign convention

Take the initial direction of P as positive. Then before impact, $v_P = 3u$ and $v_Q = -4u$. The impulse on P acts opposite to P\'s initial motion, so its signed impulse is $-\frac{33}{2}mu$.

2. Use impulse on P

For P, impulse equals change in momentum: $-\frac{33}{2}mu = 3m(v_P - 3u)$. Dividing by $3m$ gives $v_P - 3u = -\frac{11u}{2}$, so $v_P = -\frac{5u}{2}$.

3. Interpret the speed of P

The negative sign means P has reversed direction. Its speed is therefore $\left|-\frac{5u}{2}\right| = \frac{5u}{2}$.

4. Use the equal and opposite impulse on Q

Q receives impulse $+\frac{33}{2}mu$ in the positive direction. Hence $\frac{33}{2}mu = 5m(v_Q - (-4u)) = 5m(v_Q + 4u)$.

5. Interpret the speed of Q

Dividing by $5m$ gives $v_Q + 4u = \frac{33u}{10}$, so $v_Q = -\frac{7u}{10}$. Q is still moving in its original direction, and its speed is $\frac{7u}{10}$.

Final answer

(i) speed of P $= \frac{5u}{2}$. (ii) speed of Q $= \frac{7u}{10}$.

Question 2

10 marks

Kinematics Graphs

2.

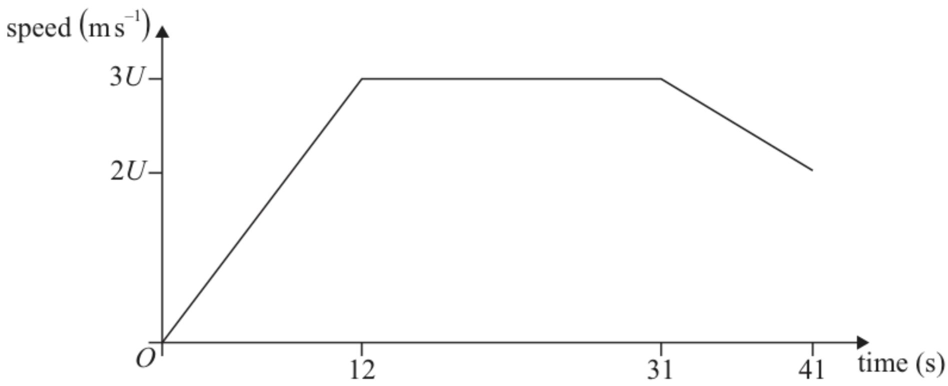

Figure 1

A particle P starts from rest at time $t = 0$ and moves in a straight line

- from $t = 0$ to $t = 12$ s, P accelerates uniformly until it reaches a speed of $3U$ m s^{-1}
- from $t = 12$ s to $t = 31$ s, P moves with constant speed $3U$ m s^{-1}
- from $t = 31$ s to $t = 41$ s, P decelerates uniformly until it has speed $2U$ m s^{-1}

as shown on the speed-time graph in Figure 1.

The distance travelled by P between $t = 0$ and $t = 41$ s is 600 m.

(a) Show that $U = 6$

(3)

(b) Find the distance travelled by P whilst moving from rest to a speed of 6 m s^{-1}

(2)

(c) Find the acceleration of P between $t = 0$ and $t = 12$ s.

(1)

(d) Find the deceleration of P between $t = 31$ s and $t = 41$ s.

(2)

(e) Sketch an acceleration-time graph to represent the motion of P from $t = 0$ to $t = 41$ s.

(2)

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Worked Solution - Question 2

Kinematics Graphs

Topic group

1. Use area under the speed-time graph

Distance is the area under a speed-time graph. The three areas are a triangle from 0 to 12, a rectangle from 12 to 31, and a trapezium from 31 to 41.

2. Show that U is 6

$\frac{1}{2}(12)(3U) + 19(3U) + \frac{1}{2}(3U + 2U)(10) = 600$. This gives $18U + 57U + 25U = 600$, so $100U = 600$ and $U = 6$.

3. Find the distance to speed 6

Since $U = 6$, the final speed $6\ \mathrm{m\,s^{-1}}$ occurs during the first section. The acceleration there is $\dfrac{18-0}{12} = 1.5\ \mathrm{m\,s^{-2}}$, so the time to reach 6 is 4 s. The distance is $\frac{1}{2}(4)(6) = 12\ \mathrm{m}$.

4. Find the acceleration and deceleration

From 0 to 12, $a = \dfrac{18}{12} = 1.5\ \mathrm{m\,s^{-2}}$. From 31 to 41, the speed falls from 18 to 12, so the deceleration is $\dfrac{18-12}{10} = 0.6\ \mathrm{m\,s^{-2}}$.

5. Describe the acceleration-time graph

The acceleration-time graph has three horizontal sections: 1.5 from $0 \le t \le 12$, 0 from $12 < t \le 31$, and -0.6 from $31 < t \le 41$.

Final answer

$(a)\ U = 6.\quad (b)\ 12\ \mathrm{m}.\quad (c)\ 1.5\ \mathrm{m\,s^{-2}}.\quad (d)\ 0.6\ \mathrm{m\,s^{-2}}.\quad (e)\ a(t) = 1.5, 0, -0.6$ on the three intervals.

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Question 3

8 marks

Forces

3. A particle moves on a smooth horizontal plane under the action of three horizontal forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$, where

$$\mathbf{F}_1 = (2p\mathbf{i} - 3p\mathbf{j})\text{N}$$

$$\mathbf{F}_2 = (q\mathbf{i} + \mathbf{j})\text{N}$$

$$\mathbf{F}_3 = (3\mathbf{i} - q\mathbf{j})\text{N}$$

where p and q are constants.

The resultant force acts in the direction of the vector $(4\mathbf{i} - 5\mathbf{j})$

(a) Show that

$$2p - q = 19$$

(4)

The mass of the particle is 0.5 kg.

Given that $p = 7$

(b) find the acceleration of the particle.

(4)

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Worked Solution - Question 3

Forces

Topic group

1. Add the three forces

The resultant force is
$(2p\mathbf{i} - 3p\mathbf{j}) + (q\mathbf{i} + \mathbf{j}) + (3\mathbf{i} - q\mathbf{j}) = (2p + q + 3)\mathbf{i} + (-3p + 1 - q)\mathbf{j}$.

2. Use the direction vector

The resultant acts in the direction $4\mathbf{i} - 5\mathbf{j}$, so the ratio of the components is the same: $\dfrac{-3p + 1 - q}{2p + q + 3} = \dfrac{-5}{4}$.

3. Show the required relation

Cross-multiplying gives $4(-3p + 1 - q) = -5(2p + q + 3)$. Therefore $-12p + 4 - 4q = -10p - 5q - 15$, so $2p - q = 19$.

4. Find q when p is 7

With $p = 7$, the relation gives $14 - q = 19$, hence $q = -5$.

5. Apply Newtons second law

Substitute $p = 7$ and $q = -5$ into the resultant: $\mathbf{F} = 12\mathbf{i} - 15\mathbf{j}$. Since the mass is 0.5 kg, $\mathbf{F} = m\mathbf{a}$ gives $12\mathbf{i} - 15\mathbf{j} = 0.5\mathbf{a}$.

6. Find the acceleration

Dividing by 0.5 gives $\mathbf{a} = 24\mathbf{i} - 30\mathbf{j}\ \mathrm{m\,s^{-2}}$.

Final answer

$(a)\ 2p - q = 19. \quad (b)\ \mathbf{a} = 24\mathbf{i} - 30\mathbf{j}\ \mathrm{m\,s^{-2}}$.

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9 marks

Question 4

Moments

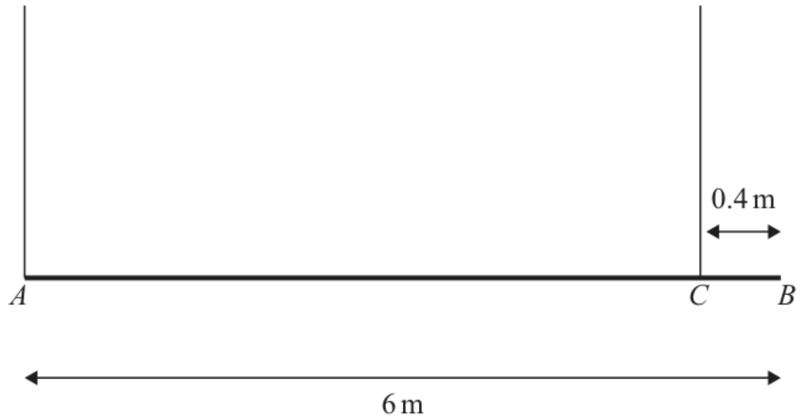

Figure 2

Figure 2 shows a uniform rod AB of length 6 m suspended by two light vertical ropes.

The first rope is attached to the rod at A.
The second rope is attached to the rod at the point C, where $CB = 0.4$ m.

The rod has mass 30 kg.

A particle of mass 20 kg is now attached to the rod at B.

The rod is in equilibrium in a horizontal position.

(a) Find

(i) the tension in the rope attached to the rod at C,
(ii) the tension in the rope attached to the rod at A.

(6)

The particle of mass 20 kg at B is removed and replaced by a particle of mass M kg.

The rod remains in equilibrium in a horizontal position.

(b) Find the exact maximum value of M.

(3)

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Worked Solution - Question 4

Moments

Topic group

1. Set the distances

The rod is 6 m long and C is 0.4 m from B, so $AC = 5.6$ m. The rod is uniform, so its weight $30g$ acts at the midpoint, 3 m from A. The 20 kg particle at B has weight $20g$.

2. Take moments about A

Moments about A remove the tension at A. Clockwise moments from the rod and particle equal the anticlockwise moment from the rope at C: $T_C(5.6) = 30g(3) + 20g(6)$.

3. Find the tension at C

$T_C(5.6) = 210g$, so $T_C = 37.5g = 367.5\text{ N}$. To suitable accuracy, $T_C = 368\text{ N}$, or 370 N to 2 significant figures.

4. Use vertical equilibrium

Vertically, $T_A + T_C = 30g + 20g = 50g$. Therefore $T_A = 50g - 37.5g = 12.5g = 122.5\text{ N}$, so $T_A = 123\text{ N}$, or 120 N to 2 significant figures.

5. Use the limiting case for maximum M

For the greatest possible mass at B, the rope at A is just slack, so $T_A = 0$. Take moments about C: the clockwise moment of Mg at B balances the anticlockwise moment of the rod weight.

6. Solve for M

The distance from C to B is 0.4 m and the rod weight acts $5.6 - 3 = 2.6$ m to the left of C. Hence $Mg(0.4) = 30g(2.6)$, so $M = \dfrac{78}{0.4} = 195$.

Final answer

$(a)(i)\ T_C = 368$ N or 370 N. $(a)(ii)\ T_A = 123$ N or 120 N. $(b)\ M = 195$

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Question 5

12 marks

Resolving Forces, Inclined Planes

5. The points O, A, B and C are on a rough horizontal surface where $OABC$ is a straight line.

A particle is projected horizontally from O and slides across the surface, passing through A, B and C in that order.

The particle passes through

- A with speed $U\,\text{m s}^{-1}$
- B with speed $\frac{U}{2}\,\text{m s}^{-1}$

The coefficient of friction between the particle and the surface is $\frac{1}{7}$

It takes 0.75 s for the particle to move from A to B.

(a) Show that $U = 2.1$ to 2 significant figures.

(6)

The particle passes through C with speed $\frac{U}{3}\,\text{m s}^{-1}$

(b) Find the time taken to move from B to C.

(3)

(c) Find the distance AC.

(3)

Worked Solution - Question 5

Resolving Forces, Inclined Planes

Topic group

1. Find the acceleration from friction

On a horizontal surface, $R = mg$ and the friction is $F = \mu R = \frac{1}{7}mg$. The friction is the only horizontal force, so the acceleration is opposite the motion with magnitude $\frac{F}{m} = \frac{g}{7} = 1.4\,\mathrm{m\,s^{-2}}$.

2. Use A to B to find U

From A to B, $u = U$, $v = \frac{U}{2}$, $a = -\frac{g}{7}$ and $t = 0.75$. Using $v = u + at$:
$\frac{U}{2} = U - \frac{g}{7}(0.75)$.

3. Show the value of U

With $g = 9.8$, $\frac{g}{7}(0.75) = 1.05$, so $\frac{U}{2} = U - 1.05$. Hence $U = 2.10\,\mathrm{m\,s^{-1}}$, which is $2.1\,\mathrm{m\,s^{-1}}$ to 2 significant figures.

4. Find the time from B to C

From B to C, the speed falls from $\frac{U}{2}$ to $\frac{U}{3}$ with the same acceleration -1.4. Thus $\frac{U}{3} = \frac{U}{2} - 1.4t$. With $U = 2.1$, this gives $0.7 = 1.05 - 1.4t$, so $t = 0.25$ s.

5. Find the distance AC

Use average speed on each section. $AB = \frac{U + U/2}{2}(0.75) = 1.18125\,\mathrm{m}$ and $BC = \frac{U/2 + U/3}{2}(0.25) = 0.21875\,\mathrm{m}$. Therefore
$AC = 1.18125 + 0.21875 = 1.4\,\mathrm{m}$.

Final answer

(a) $U = 2.1\ \mathrm{m\,s^{-1}}$ to 2 s.f.. (b) 0.25 s. (c) $AC = 1.4\ \mathrm{m}$.

Question 6

16 marks

Working with Vectors

6. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

At 9 am, a ship S leaves a harbour and moves with constant velocity $(10\mathbf{i} + 15\mathbf{j})\,\text{km h}^{-1}$

(a) Find the exact speed of S

(2)

The harbour is at the origin O.

At 9 am, a boat B leaves the point with position vector $(50\mathbf{i} + 10\mathbf{j})\,\text{km}$ and moves with constant velocity $(-8\mathbf{i} + 10\mathbf{j})\,\text{km h}^{-1}$

(b) Find the direction in which B is moving, giving your answer as a bearing to the nearest degree.

(3)

(c) Show that at time t hours after 9 am,

$$\overrightarrow{SB} = [(50 - 18t)\mathbf{i} + (10 - 5t)\mathbf{j}]\,\text{km}$$

(4)

(d) Show that B and S do not collide.

(3)

When B is north-east of S, the distance between S and B is d km.

(e) Find the value of d.

(4)

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Worked Solution - Question 6

Working with Vectors

Topic group

1. Find the speed of S

The velocity of S is $10\mathbf{i} + 15\mathbf{j}$. Its speed is the magnitude: $\sqrt{10^2 + 15^2} = \sqrt{325} = 5\sqrt{13}\,\mathrm{km\,h^{-1}}$.

2. Find the bearing of B

B has velocity $-8\mathbf{i} + 10\mathbf{j}$, so it moves west and north. The angle west of north satisfies $\tan\theta = \frac{8}{10}$, giving $\theta = 38.66°$. The bearing is $360° - 38.66° = 321°$ to the nearest degree.

3. Write both position vectors

At time t hours after 9 am, $\mathbf{r}_S = (10t)\mathbf{i} + (15t)\mathbf{j}$. Also $\mathbf{r}_B = (50\mathbf{i} + 10\mathbf{j}) + t(-8\mathbf{i} + 10\mathbf{j}) = (50 - 8t)\mathbf{i} + (10 + 10t)\mathbf{j}$.

4. Find the relative position vector

$\overrightarrow{SB} = \mathbf{r}_B - \mathbf{r}_S = [(50 - 8t) - 10t]\mathbf{i} + [(10 + 10t) - 15t]\mathbf{j} = (50 - 18t)\mathbf{i} + (10 -$

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5. Show they do not collide

For a collision, both components of $\overrightarrow{SB}$ must be zero at the same time. From $50 - 18t = 0$, $t = \frac{25}{9}$. From $10 - 5t = 0$, $t = 2$. These are different, so B and S do not collide.

6. Find the north-east or south-west instant

When the separation is along a $45°$ diagonal, the two components have equal magnitude. Using the given condition, set $50 - 18t = 10 - 5t$, so $40 = 13t$ and $t = \frac{40}{13}$.

7. Find the distance d

At $t = \frac{40}{13}$, both components are $-\frac{70}{13}$. Therefore

$d = \sqrt{\left(\frac{70}{13}\right)^2 + \left(\frac{70}{13}\right)^2} = \frac{70\sqrt{2}}{13} = 7.6\ \text{km}$ to 2 significant figures.

Final answer

$(a)\ 5\sqrt{13}\ \text{km h}^{-1}.\quad (b)\ 321°.\quad (c)\ \overrightarrow{SB} = (50 - 18t)\mathbf{i} + (10 - 5t)\mathbf{j}.\quad (d)$ no collision. $\quad (e)\ d = \frac{70\sqrt{2}}{13}\ \text{km} \approx 7.6\ \text{km}$

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Question 7

14 marks

Resolving Forces, Inclined Planes

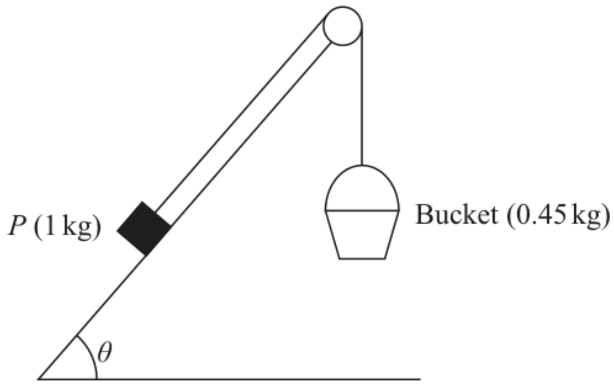

Figure 3

A rough plane is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{4}{3}$

One end of a light inextensible string is attached to a package P.
The string passes over a smooth pulley that is fixed at the top of the plane.
The string from P to the pulley lies along a line of greatest slope of the plane.
The other end of the string is attached to a bucket.
The package P has mass 1 kg and is **held** at rest on the plane.
The bucket has mass 0.45 kg and hangs vertically below the pulley with the string taut, as shown in Figure 3.

The coefficient of friction between P and the plane is μ

When a small block of mass 0.2 kg is placed in the bucket, the system is released and remains at rest in **limiting equilibrium**, with P on the point of slipping down the plane.

(a) Show that $\mu = \frac{1}{4}$

(7)

Additional identical blocks of mass 0.2 kg are added one at a time into the bucket until P starts to move **up** the plane.
At the instant when P starts to move up the plane, there are a **total** of n blocks in the bucket.

(b) Find

(i) the value of n

(ii) the magnitude of the initial acceleration of P

(7)

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Worked Solution - Question 7

Resolving Forces, Inclined Planes

Topic group

1. Find the trigonometric ratios

Since $\tan\theta = \frac{4}{3}$, use a 3-4-5 triangle: $\sin\theta = \frac{4}{5}$ and $\cos\theta = \frac{3}{5}$.

2. Use limiting equilibrium in part (a)

With one 0.2 kg block in the bucket, the bucket side has mass $0.45 + 0.2 = 0.65$ kg. Since the package is on the point of slipping down the plane, friction on P acts up the plane.

3. Find the friction force

The tension equals the bucket weight, so $T = 0.65g$. Along the plane for P in equilibrium, $T + F = g\sin\theta$. Hence $0.65g + F = \frac{4}{5}g$, so $F = \frac{3g}{20}$.

4. Find the normal reaction and mu

Perpendicular to the plane, $R = g\cos\theta = \frac{3}{5}g$. Since $F = \mu R$,

$\mu = \frac{F}{R} = \frac{3g/20}{3g/5} = \frac{1}{4}$.

5. Set the condition for upward motion

When P is about to move up the plane, friction acts down the plane. With n blocks in the bucket, the bucket-side mass is $0.45 + 0.2n$. The bucket weight must exceed the downslope resistance: $(0.45 + 0.2n)g > g\sin\theta + \mu R$.

6. Find n

Using $g\sin\theta = \frac{4}{5}g$ and $\mu R = \frac{1}{4} \cdot \frac{3}{5}g = \frac{3g}{20}$, the condition is $0.45 + 0.2n > \frac{4}{5} + \frac{3}{20} = 0.95$. Hence $0.2n > 0.50$, so $n > 2.5$ and the least integer is $n = 3$.

7. Apply Newtons second law after motion begins

For $n = 3$, the bucket-side mass is $0.45 + 0.6 = 1.05$ kg and the total moving mass is $1 + 1.05 = 2.05$ kg. The driving force is $1.05g - \frac{4}{5}g - \frac{3g}{20} = 0.10g$.

8. Find the acceleration

Using $F = ma$, $0.10g = 2.05a$, so $a = \frac{0.10g}{2.05} = \frac{2g}{41} = 0.478\ \mathrm{m\,s^{-2}}$ to 3 significant figures.

Final answer

$(a)\ \mu = \frac{1}{4}.\quad (b)(i)\ n = 3.\quad (b)(ii)\ a = \frac{2g}{41} = 0.478\ \mathrm{m\,s^{-2}}$ approximately

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