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Elite Mathematics
Edexcel International A Level

IAL MECHANICS 1 · WME01

WME01 Mechanics 1 Expertise With Answers

Questions 6 and above with worked-solution pages and cross-topic placements.

107
topic placements

2019–2026
18 exam sessions

Questions with worked solutions
checked by Dr Eslam

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TOPIC

Quantities, Units & Modelling

Every past-paper question on this topic, October 2019 to January 2026.

WME01/01 OCTOBER 2019

Question 6

12 marks | Quantities, Units & Modelling | Main topic: Kinematics Graphs

6. An athlete runs a 200 m race along a straight horizontal track.

In a model of the motion of the athlete, air resistance is ignored, the athlete starts from rest at time $t = 0$ seconds and moves with uniform acceleration $0.8\,\text{m s}^{-2}$ for T seconds, reaching a speed of $V\,\text{m s}^{-1}$. She then maintains this speed until she crosses the finishing line.

The total time from when the athlete starts to when she crosses the finishing line is 30 s.

(a) Sketch a speed-time graph for the model of the motion of the athlete from the instant when she starts to the instant when she crosses the finishing line.

(2)

(b) Write down an expression for V in terms of T.

(1)

(c) Show that $T^2 - kT + 500 = 0$, where k is a constant to be found.

(4)

(d) Hence find the value of T, justifying your answer carefully.

(3)

(e) Considering your speed-time graph or otherwise, state two ways, apart from including air resistance, in which the model could be made to be more realistic.

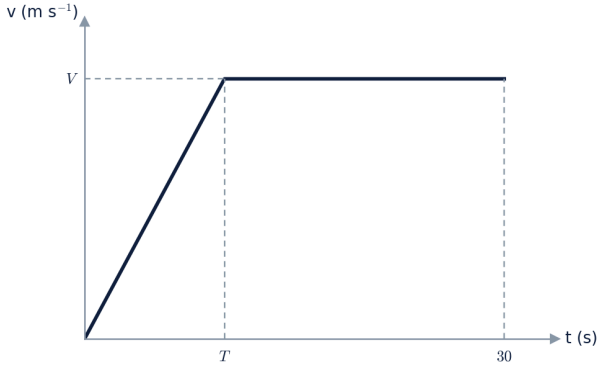
(2)

WORKED SOLUTION

Question 6 · WME01/01 October 2019

1 **Describe the speed-time graph**

The graph starts at $(0, 0)$, rises in a straight line with gradient 0.8 until time T and speed V, then stays horizontal at speed V until $t = 30$.



2 **Write V in terms of T**

Uniform acceleration gives $V = 0.8T$.

3 **Use area for the distance**

The distance is the area under the speed-time graph: $\frac{1}{2}TV + V(30 - T) = 200$.

4 **Substitute V**

Using $V = 0.8T$: $\frac{1}{2}T(0.8T) + (0.8T)(30 - T) = 200$.

5 **Form the quadratic**

$0.4T^2 + 24T - 0.8T^2 = 200$, so $-0.4T^2 + 24T - 200 = 0$. Multiplying by -2.5 gives $T^2 - 60T + 500 = 0$, so $k = 60$.

6 **Solve and choose the valid root**

$T^2 - 60T + 500 = (T - 10)(T - 50) = 0$, so $T = 10$ or 50. Since the whole race lasts 30 s, $T = 10$.

7 **Give two model improvements**

A more realistic model could include reaction time at the start, and could replace the instantaneous change from acceleration to constant speed with a smoother transition.

✓ **FINAL ANSWER**

(a) Speed-time graph rises linearly to V at $t = T$, then is horizontal until $t = 30$.

(b) $V = 0.8T$. (c) $T^2 - 60T + 500 = 0$, so $k = 60$.

(d) $T = 10$. (e) For example, include reaction time at the start and avoid the instantaneous change to constant speed.

WME01/01 MAY/JUNE 2021

Question 7

10 marks | Quantities, Units & Modelling | Main topic: Moments

7.

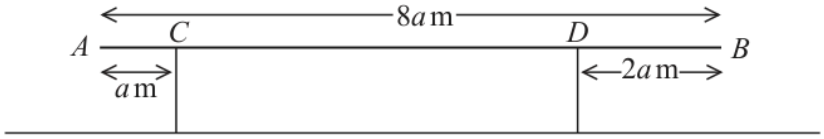


Figure 2

A non-uniform beam AB, of mass 60 kg and length $8a$ metres, rests in equilibrium in a horizontal position on two vertical supports. One support is at C, where $AC = a$ metres and the other support is at D, where $DB = 2a$ metres, as shown in Figure 2.

The magnitude of the normal reaction between the beam and the support at D is three times the magnitude of the normal reaction between the beam and the support at C.

By modelling the beam as a non-uniform rod whose centre of mass is at a distance x metres from A,

(a) find an expression for x in terms of a.

(5)

A box of mass M kg is placed on the beam at E, where $AE = 2a$ metres.

The beam remains in equilibrium in a horizontal position.

The magnitude of the normal reaction between the beam and the support at C is now equal to the magnitude of the normal reaction between the beam and the support at D.

By modelling the box as a particle,

(b) find the value of M.

(5)

WORKED SOLUTION

Question 7 · WME01/01 May/June 2021

1 Set the support positions

The beam length is $8a$. C is at distance a from A, and D is at distance $6a$ from A because $DB = 2a$.

2 Name the reactions

Let the reaction at C be R. The reaction at D is $3R$.

3 Use vertical equilibrium

$R + 3R = 60g$, so $4R = 60g$ and $R = 15g$.

4 Take moments about A

The beam's weight acts at distance x from A. Hence $60gx = R(a) + 3R(6a)$.

5 Find x

Substitute $R = 15g$: $60gx = 15ga + 45g(6a) = 285ga$. Thus $x = \frac{285a}{60} = \frac{19a}{4}$.

6 **Use equal reactions with the box**

When the box is added, let each support reaction be S. Vertical equilibrium gives $2S = (60 + M)g$.

7 **Take moments about A again**

The box is at $AE = 2a$. Moments about A give $60g\left(\frac{19a}{4}\right) + Mg(2a) = S(a) + S(6a) = 7aS$.

8 **Solve for M**

Substitute $S = \frac{(60 + M)g}{2}$ and cancel ag: $285 + 2M = \frac{7}{2}(60 + M) = 210 + \frac{7}{2}M$. Hence $75 = \frac{3}{2}M$ and $M = 50$.

✓ **FINAL ANSWER**

$(a)\ x = \frac{19a}{4}.\quad (b)\ M = 50.$

WME01/01 JANUARY 2023

Question 6

8 marks | Quantities, Units & Modelling | Main topic: Forces

6.

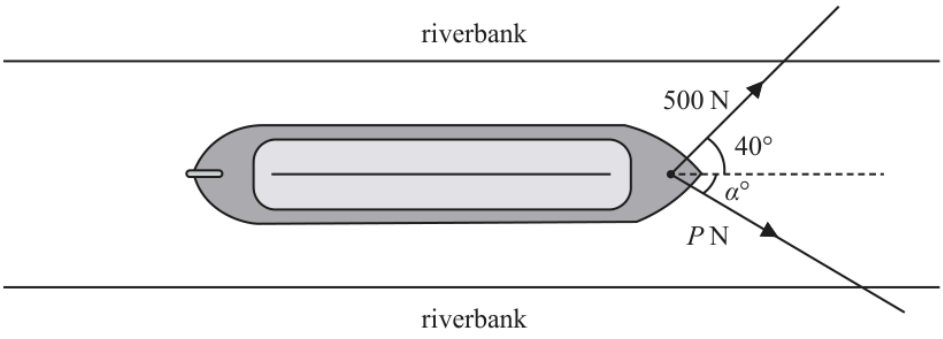


Figure 3

A boat is pulled along a river at a constant speed by two ropes.

The banks of the river are parallel and the boat travels horizontally in a straight line, parallel to the riverbanks.

- The tension in the first rope is 500 N acting at an angle of 40° to the direction of motion, as shown in Figure 3.
- The tension in the second rope is P newtons, acting at an angle of $\alpha°$ to the direction of motion, also shown in Figure 3.
- The resistance to motion of the boat as it moves through the water is a constant force of magnitude 900 N

The boat is modelled as a particle. The ropes are modelled as being light and lying in a horizontal plane.

Use the model to find

(i) the value of α

(ii) the value of P

(8)

WORKED SOLUTION

Question 6 · WME01/01 January 2023

1 Resolve perpendicular to the motion

The boat travels straight, so the sideways components balance: $500\sin 40° = P\sin\alpha$.

2 Resolve along the motion

The speed is constant, so the forward components balance the resistance: $500\cos 40° + P\cos\alpha = 900$.

3 Eliminate P

From the two equations, $P\sin\alpha = 500\sin 40°$ and $P\cos\alpha = 900 - 500\cos 40°$.

4 Find alpha

Divide the sine equation by the cosine equation: $\tan\alpha = \dfrac{500\sin 40°}{900 - 500\cos 40°}$. This gives $\alpha = 31.9°$, so $\alpha = 32°$ approximately.

5 Find P

Use $P\sin\alpha = 500\sin 40°$: $P = \dfrac{500\sin 40°}{\sin 31.9°} = 608.7\ldots$ N. Hence $P \approx 610$ N.

FINAL ANSWER

$(i)\ \alpha = 32°$ approximately. $\quad (ii)\ P = 610\ \text{N}$ approximately.

WME01/01 JANUARY 2023

Question 7

7 marks | Quantities, Units & Modelling | Main topic: Newton's Second Law

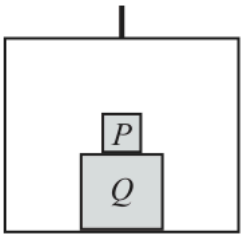


Figure 4

A simple lift operates by means of a vertical cable which is attached to the top of the lift.
The lift has mass m
A box Q is placed on the floor of the lift.
A box P is placed directly on top of box Q, as shown in Figure 4.

The cable is modelled as being light and inextensible and air resistance is modelled as being negligible.

The tension in the cable is $\frac{42mg}{5}$

The lift and its contents move vertically upwards with acceleration $\frac{2g}{5}$

Using the model,

(a) find, in terms of m, the combined mass of boxes P and Q

(4)

During the motion of the lift, the force exerted on box P by box Q is $\frac{14mg}{5}$

Using the model,

(b) find, in terms of m, the mass of box P

(3)

WORKED SOLUTION

Question 7 · WME01/01 January 2023

1 Define the box mass

Let the combined mass of boxes P and Q be M. The lift itself has mass m, so the total moving mass is $m + M$.

2 Apply Newtons second law to the whole system

Taking upward as positive, $\frac{42}{5}mg - (m+M)g = (m+M)\frac{2g}{5}$.

3 Find M

Move the weight term to the right: $\frac{42}{5}mg = (m+M)\left(g + \frac{2g}{5}\right) = \frac{7g}{5}(m+M)$. Hence $42m = 7(m+M)$ and $M = 5m$.

4 Let the mass of P be p

For box P alone, the upward force from Q is $\frac{14}{5}mg$, while its weight is pg. The acceleration is still $\frac{2g}{5}$ upward.

5 **Apply Newtons second law to P**
$\frac{14}{5}mg - pg = p\frac{2g}{5}$.

6 **Find p**
Divide by g: $\frac{14}{5}m = p + \frac{2p}{5} = \frac{7p}{5}$. Therefore $p = 2m$.

FINAL ANSWER
(a) combined mass of P and Q $= 5m$. (b) mass of P $= 2m$.

TOPIC

Working with Vectors

Every past-paper question on this topic, October 2019 to January 2026.

WME01/01 OCTOBER 2019

Question 7

7 marks | Working with Vectors

7. Two forces, $\mathbf{F}$ and $\mathbf{G}$, act on a particle. The force $\mathbf{F}$ has magnitude 4 N and acts in a direction with a bearing of 120° and the force $\mathbf{G}$ has magnitude 6 N and acts due north.

Given that $\mathbf{P} = 2\mathbf{F} + \mathbf{G}$, find

(i) the magnitude of $\mathbf{P}$

(ii) the direction of $\mathbf{P}$, giving your answer as a bearing to the nearest degree.

(7)

WORKED SOLUTION

Question 7 · WME01/01 October 2019

1 Resolve 2F

Since $\mathbf{F}$ has magnitude 4 N on a bearing of $120°$, $2\mathbf{F}$ has magnitude 8 N on the same bearing.

2 Write the components

$2\mathbf{F} = 8\sin 120°\mathbf{i} + 8\cos 120°\mathbf{j} = 4\sqrt{3}\mathbf{i} - 4\mathbf{j}$. Also $\mathbf{G} = 6\mathbf{j}$.

3 Find P

$\mathbf{P} = 2\mathbf{F} + \mathbf{G} = 4\sqrt{3}\mathbf{i} + 2\mathbf{j}$.

4 Find the magnitude

$|\mathbf{P}| = \sqrt{(4\sqrt{3})^2 + 2^2} = \sqrt{48 + 4} = \sqrt{52}$ N.

5 Find the bearing angle

The vector is in the north-east quadrant. The angle east of north satisfies $\tan\theta = \dfrac{4\sqrt{3}}{2} = 2\sqrt{3}$.

6 State the bearing

$\theta = 73.9°$ approximately, so the bearing is $074°$ to the nearest degree.

FINAL ANSWER

$(i)\ |\mathbf{P}| = \sqrt{52}\ \text{N} \approx 7.21\ \text{N}.$ $\quad (ii)\ \text{bearing} = 074°.$

WME01/01 OCTOBER 2019

Question 8

14 marks | Working with Vectors

8. [*In this question, the horizontal unit vectors* $\mathbf{i}$ *and* $\mathbf{j}$ *are directed due east and due north respectively and position vectors are given relative to a fixed origin* O.]

Two speedboats, A and B, are each moving with constant velocity. The velocity of A is $20\,\text{km h}^{-1}$ due west and the velocity of B is $40\,\text{km h}^{-1}$ on a bearing of 150°. The boats are modelled as particles.

At noon, the position vector of A is $60\mathbf{i}$ km and B is at the origin O. At time t hours after noon, the position vector of A is $\mathbf{r}$ km and the position vector of B is $\mathbf{s}$ km.

(a) Find the velocity of B in the form $(p\mathbf{i} + q\mathbf{j})\,\text{km h}^{-1}$

(3)

(b) Find expressions for $\mathbf{r}$ and $\mathbf{s}$ in terms of t, $\mathbf{i}$ and $\mathbf{j}$.

(3)

(c) Find the time, to the nearest minute, at which the distance between the boats is the same as it was at noon.

(8)

WORKED SOLUTION

Question 8 · WME01/01 October 2019

1 Resolve B's velocity

A bearing of $150°$ is $30°$ east of south. Therefore the east component is $40\sin 150° = 20$ and the north component is $40\cos 150° = -20\sqrt{3}$.

2 Write B's velocity

$\mathbf{v}_B = 20\mathbf{i} - 20\sqrt{3}\mathbf{j}\ \text{km h}^{-1}$.

3 Write A's position

Boat A starts at $60\mathbf{i}$ and travels due west at $20\ \text{km h}^{-1}$, so $\mathbf{r} = 60\mathbf{i} - 20t\mathbf{i} = (60 - 20t)\mathbf{i}$.

4 Write B's position

Boat B starts at O, so $\mathbf{s} = t(20\mathbf{i} - 20\sqrt{3}\mathbf{j}) = 20t\mathbf{i} - 20\sqrt{3}t\mathbf{j}$.

5 Find the separation vector

$\overrightarrow{AB} = \mathbf{s} - \mathbf{r} = (20t - (60 - 20t))\mathbf{i} - 20\sqrt{3}t\mathbf{j} = (40t - 60)\mathbf{i} - 20\sqrt{3}t\mathbf{j}$.

6 Use the noon distance

At noon the distance between the boats is 60 km. Set the later distance equal to 60: $(40t - 60)^2 + (-20\sqrt{3}t)^2 = 60^2$.

7 Solve for t

Expanding gives $2800t^2 - 4800t = 0$, so $t(2800t - 4800) = 0$. The later time is $t = \dfrac{4800}{2800} = \dfrac{12}{7}$ hours.

8 Convert to clock time

$\dfrac{12}{7}$ hours is 1 hour and $42.857\ldots$ minutes after noon, which rounds to $1{:}43$ pm, or $13{:}43$.

FINAL ANSWER

$(a)\ \mathbf{v}_B = 20\mathbf{i} - 20\sqrt{3}\mathbf{j}\ \text{km h}^{-1}.\quad (b)\ \mathbf{r} = (60 - 20t)\mathbf{i},\ \mathbf{s} = 20t\mathbf{i} - 20\sqrt{3}t\mathbf{j}.\quad (c)\ 13{:}43$, or $1{:}43\ \text{pm}$.

WME01/01 JANUARY 2020

Question 6

11 marks | Working with Vectors

6. A force $\mathbf{F}$ is given by $\mathbf{F} = (10\mathbf{i} + \mathbf{j})$ N.

(a) Find the exact value of the magnitude of $\mathbf{F}$.

(2)

(b) Find, in degrees, the size of the angle between the direction of $\mathbf{F}$ and the direction of the vector $(\mathbf{i} + \mathbf{j})$.

(4)

The resultant of the force $\mathbf{F}$ and the force $(-15\mathbf{i} + a\mathbf{j})$ N, where a is a constant, is parallel to, but in the opposite direction to, the vector $(2\mathbf{i} - 3\mathbf{j})$.

(c) Find the value of a.

(5)

WORKED SOLUTION

Question 6 · WME01/01 January 2020

1 Find the magnitude

For $\mathbf{F} = 10\mathbf{i} + \mathbf{j}$, $|\mathbf{F}| = \sqrt{10^2 + 1^2} = \sqrt{101}$ N.

2 Find the direction angle of F

The angle that $\mathbf{F}$ makes with the positive $\mathbf{i}$ direction is α, where $\tan\alpha = \frac{1}{10}$. Thus $\alpha = 5.71°$ approximately.

3 Compare with i plus j

The vector $\mathbf{i} + \mathbf{j}$ makes an angle of $45°$ with the positive $\mathbf{i}$ direction.

4 Find the angle between the vectors

So the required angle is $45° - 5.71° = 39.29°$, approximately $39.3°$.

5 Write the resultant

The resultant of $10\mathbf{i} + \mathbf{j}$ and $-15\mathbf{i} + a\mathbf{j}$ is $-5\mathbf{i} + (a+1)\mathbf{j}$.

6 Use the parallel opposite direction

Parallel but opposite to $2\mathbf{i} - 3\mathbf{j}$ means the resultant is a positive multiple of $-2\mathbf{i} + 3\mathbf{j}$.

7 Solve for a

Let $-5\mathbf{i} + (a+1)\mathbf{j} = \lambda(-2\mathbf{i} + 3\mathbf{j})$. From the $\mathbf{i}$ component, $-5 = -2\lambda$, so $\lambda = 2.5$. Then $a + 1 = 3(2.5) = 7.5$, hence $a = 6.5$.

FINAL ANSWER

(a) $|\mathbf{F}| = \sqrt{101}$ N. (b) $39.3°$ approximately. (c) $a = 6.5$.

WME01/01 JANUARY 2021

Question 6

12 marks | Working with Vectors

6. Two girls, Agatha and Brionie, are roller skating inside a large empty building. The girls are modelled as particles.

At time $t = 0$, Agatha is at the point with position vector $(11\mathbf{i} + 11\mathbf{j})$ m and Brionie is at the point with position vector $(7\mathbf{i} + 16\mathbf{j})$ m. The position vectors are given relative to the door, O, and $\mathbf{i}$ and $\mathbf{j}$ are horizontal perpendicular unit vectors.

Agatha skates with constant velocity $(3\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$

Brionie skates with constant velocity $(4\mathbf{i} - 2\mathbf{j})\,\text{m s}^{-1}$

(a) Find the position vector of Agatha at time t seconds.

(2)

At time $t = 6$ seconds, Agatha passes through the point P.

(b) Show that Brionie also passes through P and find the value of t when this occurs.

(4)

At time t seconds, Agatha is at the point A and Brionie is at the point B.

(c) Show that $\overrightarrow{AB} = [(t - 4)\mathbf{i} + (5 - t)\mathbf{j}]$ m

(2)

(d) Find the distance between the two girls when they are closest together.

(4)

WORKED SOLUTION

Question 6 · WME01/01 January 2021

1 Write Agatha position

$\mathbf{r}_A = (11\mathbf{i} + 11\mathbf{j}) + t(3\mathbf{i} - \mathbf{j}) = (11 + 3t)\mathbf{i} + (11 - t)\mathbf{j}$.

2 Find point P

At $t = 6$, Agatha is at $(11 + 18)\mathbf{i} + (11 - 6)\mathbf{j} = 29\mathbf{i} + 5\mathbf{j}$.

3 Write Brionie position

$\mathbf{r}_B = (7\mathbf{i} + 16\mathbf{j}) + t(4\mathbf{i} - 2\mathbf{j}) = (7 + 4t)\mathbf{i} + (16 - 2t)\mathbf{j}$.

4 Show Brionie passes P

Set $7 + 4t = 29$ to get $t = 5.5$. Also $16 - 2t = 5$ gives $t = 5.5$. So Brionie also passes through P at $t = 5.5$ s.

5 Find AB vector

$\overrightarrow{AB} = \mathbf{r}_B - \mathbf{r}_A = [7 + 4t - (11 + 3t)]\mathbf{i} + [16 - 2t - (11 - t)]\mathbf{j}$.

6 Simplify AB

So $\overrightarrow{AB} = (t - 4)\mathbf{i} + (5 - t)\mathbf{j}$ m.

7 **Minimise distance squared**

$AB^2 = (t-4)^2 + (5-t)^2 = 2(t-4.5)^2 + 0.5$.

8 **State the minimum distance**

The minimum occurs when $t = 4.5$, giving $AB^2 = 0.5$. Hence the minimum distance is $\sqrt{0.5} = 0.707\ldots$ m.

FINAL ANSWER

$(a)\ \mathbf{r}_A = (11+3t)\mathbf{i} + (11-t)\mathbf{j}$. $(b)\ t = 5.5$ s. $(c)\ \overrightarrow{AB} = (t-4)\mathbf{i} + (5-t)\mathbf{j}$. (d) minimum distance $= \sqrt{0.5}$ m ≈ 0.71 m.

WME01/01 OCTOBER 2021

Question 8

12 marks | Working with Vectors

8. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin.*]

At 7 am a ship leaves a port and moves with constant velocity. The position vector of the port is $(-2\mathbf{i} + 9\mathbf{j})$ km.

At 7.36 am the ship is at the point with position vector $(4\mathbf{i} + 6\mathbf{j})$ km.

(a) Show that the velocity of the ship is $(10\mathbf{i} - 5\mathbf{j})\,\text{km h}^{-1}$

(2)

(b) Find the position vector of the ship t hours after leaving port.

(2)

At 8.48 am a passenger on the ship notices that a lighthouse is due east of the ship.

At 9 am the same passenger notices that the lighthouse is now north east of the ship.

(c) Find the position vector of the lighthouse.

(4)

(d) Find the position vector of the ship when it is due south of the lighthouse.

(4)

(Total 12 marks)

WORKED SOLUTION

Question 8 · WME01/01 October 2021

1 Convert the time

From 7:00 to 7:36 is 36 minutes, which is 0.6 hours.

2 Find velocity

Displacement $= (4\mathbf{i}+6\mathbf{j}) - (-2\mathbf{i}+9\mathbf{j}) = 6\mathbf{i} - 3\mathbf{j}$. Divide by 0.6: $\mathbf{v} = 10\mathbf{i} - 5\mathbf{j}$.

3 Write the ship position

$\mathbf{r} = (-2\mathbf{i}+9\mathbf{j}) + t(10\mathbf{i} - 5\mathbf{j}) = (-2+10t)\mathbf{i} + (9-5t)\mathbf{j}$.

4 Use 8:48

At 8:48, $t = 1.8$. The ship position is $(-2+18)\mathbf{i} + (9-9)\mathbf{j} = 16\mathbf{i}$. The lighthouse is due east, so it has the same j-component, 0.

5 Use 9:00

At 9:00, $t = 2$. The ship position is $18\mathbf{i} - \mathbf{j}$. The lighthouse is north-east of the ship, so its east and north displacements from the ship are equal.

6 Find the lighthouse position

Since the lighthouse has j-component 0, from $18\mathbf{i} - \mathbf{j}$ it is 1 km north, so it is also 1 km east. Hence its position is $19\mathbf{i}$.

7 Find when the ship is due south

Due south of the lighthouse means the ship has i-component 19. Set $-2 + 10t = 19$, so $t = 2.1$.

8 **Find the ship position**

At $t = 2.1$, $\mathbf{r} = (-2 + 21)\mathbf{i} + (9 - 10.5)\mathbf{j} = 19\mathbf{i} - 1.5\mathbf{j}$.

FINAL ANSWER

$(a)\ \mathbf{v} = 10\mathbf{i} - 5\mathbf{j}\ \text{km h}^{-1}$. $(b)\ \mathbf{r} = (-2 + 10t)\mathbf{i} + (9 - 5t)\mathbf{j}$. (c) lighthouse $= 19\mathbf{i}$. (d) ship position $= 19\mathbf{i} - 1.5\mathbf{j}$.

WME01/01 JANUARY 2022

Question 6

12 marks | Working with Vectors

6. [*In this question* **i** *and* **j** *are horizontal unit vectors.*]

A particle P of mass 2 kg moves under the action of two forces, $(p\mathbf{i} + q\mathbf{j})$ N and $(2q\mathbf{i} + p\mathbf{j})$ N, where p and q are constants.

Given that the acceleration of P is $(\mathbf{i} - \mathbf{j})\,\text{m s}^{-2}$

(a) find the value of p and the value of q.

(5)

(b) Find the size of the angle between the direction of the acceleration and the vector $\mathbf{j}$.

(2)

At time $t = 0$, the velocity of P is $(3\mathbf{i} - 4\mathbf{j})\,\text{m s}^{-1}$

At $t = T$ seconds, P is moving in the direction of the vector $(11\mathbf{i} - 13\mathbf{j})$.

(c) Find the value of T.

(5)

WORKED SOLUTION

Question 6 · WME01/01 January 2022

1 Use F = ma

The total force is $(p\mathbf{i} + q\mathbf{j}) + (2q\mathbf{i} + p\mathbf{j}) = (p + 2q)\mathbf{i} + (p + q)\mathbf{j}$. Since the mass is 2 kg and acceleration is $\mathbf{i} - \mathbf{j}$, the total force is $2\mathbf{i} - 2\mathbf{j}$.

2 Equate components

$p + 2q = 2$ and $p + q = -2$.

3 Find p and q

Subtract the equations to get $q = 4$. Then $p + 4 = -2$, so $p = -6$.

4 Find the angle with j

The acceleration is $\mathbf{i} - \mathbf{j}$. Its dot product with $\mathbf{j}$ is -1, so the angle with the positive $\mathbf{j}$ direction is obtuse. Since the components are equal in magnitude, the angle is $135°$.

5 Write the velocity at time T

$\mathbf{v} = (3\mathbf{i} - 4\mathbf{j}) + T(\mathbf{i} - \mathbf{j}) = (3 + T)\mathbf{i} + (-4 - T)\mathbf{j}$.

6 Use the direction vector

The velocity is parallel to $11\mathbf{i} - 13\mathbf{j}$, so $\dfrac{3 + T}{11} = \dfrac{-4 - T}{-13}$.

7 Solve for T

$13(3 + T) = 11(4 + T)$, so $39 + 13T = 44 + 11T$. Hence $2T = 5$ and $T = 2.5$.

FINAL ANSWER

$(a)\ p = -6,\ q = 4.\quad (b)\ 135°.\quad (c)\ T = 2.5.$

WME01/01 JANUARY 2022

Question 8

14 marks | Working with Vectors

8. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin.*]

A ship A moves with constant velocity $(3\mathbf{i} - 10\mathbf{j})\,\text{km h}^{-1}$

At time t hours, the position vector of A is $\mathbf{r}$ km.

At time $t = 0$, A is at the point with position vector $(13\mathbf{i} + 5\mathbf{j})$ km.

(a) Find $\mathbf{r}$ in terms of t.

(2)

Another ship B moves with constant velocity $(15\mathbf{i} + 14\mathbf{j})\,\text{km h}^{-1}$

At time $t = 0$, B is at the point with position vector $(3\mathbf{i} - 5\mathbf{j})$ km.

(b) Show that, at time t hours,

$$\overrightarrow{AB} = \left[(12t - 10)\mathbf{i} + (24t - 10)\mathbf{j}\right]\text{km}$$

(4)

Given that the two ships do not change course,

(c) find the shortest distance between the two ships,

(6)

(d) find the bearing of ship B from ship A when the ships are closest.

(2)

(Total 14 marks)

WORKED SOLUTION

Question 8 · WME01/01 January 2022

1 Write the position of A

$\mathbf{r} = (13\mathbf{i} + 5\mathbf{j}) + t(3\mathbf{i} - 10\mathbf{j}) = (13 + 3t)\mathbf{i} + (5 - 10t)\mathbf{j}$.

2 Write the position of B

$\mathbf{s} = (3\mathbf{i} - 5\mathbf{j}) + t(15\mathbf{i} + 14\mathbf{j}) = (3 + 15t)\mathbf{i} + (-5 + 14t)\mathbf{j}$.

3 Find AB

$\overrightarrow{AB} = \mathbf{s} - \mathbf{r} = [3 + 15t - (13 + 3t)]\mathbf{i} + [-5 + 14t - (5 - 10t)]\mathbf{j}$.

4 Show the expression

This simplifies to $\overrightarrow{AB} = (12t - 10)\mathbf{i} + (24t - 10)\mathbf{j}$ km.

5 Use distance squared

The square of the distance is $d^2 = (12t - 10)^2 + (24t - 10)^2$.

6 Expand and minimise

$d^2 = 720t^2 - 720t + 200$. This quadratic is minimum at $t = \dfrac{720}{2 \cdot 720} = \dfrac{1}{2}$.

7 **Find the shortest distance**

At $t = \frac{1}{2}$, $\overrightarrow{AB} = (-4)\mathbf{i} + 2\mathbf{j}$, so the shortest distance is $\sqrt{(-4)^2 + 2^2} = \sqrt{20} = 2\sqrt{5}\ \text{km}$.

8 **Find the bearing**

At closest approach, B is west and north of A. The angle west of north satisfies $\tan\theta = \frac{4}{2} = 2$, so $\theta = 63.4°$. Bearing $= 360° - 63.4° = 296.6°$, which is $297°$.

✓ **FINAL ANSWER**

$(a)\ \mathbf{r} = (13 + 3t)\mathbf{i} + (5 - 10t)\mathbf{j}$. $(b)\ \overrightarrow{AB} = (12t - 10)\mathbf{i} + (24t - 10)\mathbf{j}$. $(c)\ 2\sqrt{5}\ \text{km} \approx 4.5\ \text{km}$. $(d)\ 297°$.

WME01/01 MAY/JUNE 2022

Question 6

6 marks | Working with Vectors

6. A particle P is moving with constant acceleration.

At time $t = 1$ second, P has velocity $(-\mathbf{i} + 4\mathbf{j})\,\mathrm{m\,s^{-1}}$

At time $t = 4$ seconds, P has velocity $(5\mathbf{i} - 8\mathbf{j})\,\mathrm{m\,s^{-1}}$

Find the speed of P at time $t = 3.5$ seconds.

(6)

WORKED SOLUTION

Question 6 · WME01/01 May/June 2022

1 Find the acceleration

The velocity changes from $-\mathbf{i} + 4\mathbf{j}$ at $t = 1$ to $5\mathbf{i} - 8\mathbf{j}$ at $t = 4$. The time difference is 3 s.

2 Calculate acceleration

$$\mathbf{a} = \frac{(5\mathbf{i} - 8\mathbf{j}) - (-\mathbf{i} + 4\mathbf{j})}{3} = \frac{6\mathbf{i} - 12\mathbf{j}}{3} = 2\mathbf{i} - 4\mathbf{j}.$$

3 Find velocity at t = 3.5

From $t = 1$ to $t = 3.5$ is 2.5 s, so $\mathbf{v} = (-\mathbf{i} + 4\mathbf{j}) + 2.5(2\mathbf{i} - 4\mathbf{j})$.

4 Simplify the velocity

$$\mathbf{v} = -\mathbf{i} + 4\mathbf{j} + 5\mathbf{i} - 10\mathbf{j} = 4\mathbf{i} - 6\mathbf{j}.$$

5 Find the speed

Speed $= \sqrt{4^2 + (-6)^2} = \sqrt{52} = 2\sqrt{13}\,\mathrm{m\,s^{-1}}$, about $7.2\,\mathrm{m\,s^{-1}}$.

FINAL ANSWER

$$\text{speed} = \sqrt{52} = 2\sqrt{13}\,\mathrm{m\,s^{-1}} \approx 7.2\,\mathrm{m\,s^{-1}}.$$

WME01/01 MAY/JUNE 2022

Question 8

17 marks | Working with Vectors

8. [*In this question,* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two boats, P and Q, are moving with constant velocities.

The velocity of P is $15\mathbf{i}\,\text{m s}^{-1}$ and the velocity of Q is $(20\mathbf{i} - 20\mathbf{j})\,\text{m s}^{-1}$

(a) Find the direction in which Q is travelling, giving your answer as a bearing.

(2)

The boats are modelled as particles.

At time $t = 0$, P is at the origin O and Q is at the point with position vector $200\mathbf{j}$ m.

At time t seconds, the position vector of P is $\mathbf{p}$ m and the position vector of Q is $\mathbf{q}$ m.

(b) Show that

$$\overrightarrow{PQ} = [5t\mathbf{i} + (200 - 20t)\mathbf{j}]\,\text{m}$$

(5)

(c) Find the bearing of P from Q when $t = 10$

(2)

(d) Find the distance between P and Q when Q is north east of P

(5)

(e) Find the times when P and Q are 200 m apart.

(3)

(Total 17 marks)

TOTAL FOR PAPER: 75 MARKS

WORKED SOLUTION

Question 8 · WME01/01 May/June 2022

1 Find the bearing of Q

Q has velocity $20\mathbf{i} - 20\mathbf{j}$, so it travels south-east with equal east and south components. Its bearing is $135°$.

2 Write P position

P starts at the origin and has velocity $15\mathbf{i}$, so $\mathbf{p} = 15t\mathbf{i}$.

3 Write Q position

Q starts at $200\mathbf{j}$ and has velocity $20\mathbf{i} - 20\mathbf{j}$, so $\mathbf{q} = 20t\mathbf{i} + (200 - 20t)\mathbf{j}$.

4 Show PQ

$\overrightarrow{PQ} = \mathbf{q} - \mathbf{p} = (20t - 15t)\mathbf{i} + (200 - 20t)\mathbf{j} = 5t\mathbf{i} + (200 - 20t)\mathbf{j}$.

5 Find the bearing at t = 10

At $t = 10$, $\overrightarrow{PQ} = 50\mathbf{i}$. This means Q is east of P, so P is west of Q. The bearing of P from Q is $270°$.

6 **Use the north-east condition**

Q is north-east of P when the i and j components of $\overrightarrow{PQ}$ are equal and positive. So $5t = 200 - 20t$.

7 **Find the distance then**

The equation gives $t = 8$. Then $\overrightarrow{PQ} = 40\mathbf{i} + 40\mathbf{j}$, so the distance is $\sqrt{40^2 + 40^2} = 40\sqrt{2}\ \text{m}$.

8 **Set the 200 m distance equation**

For distance 200 m, $(5t)^2 + (200 - 20t)^2 = 200^2$.

9 **Solve for t**

This simplifies to $425t^2 - 8000t = 0$, so $t(425t - 8000) = 0$. Hence $t = 0$ or $t = \dfrac{8000}{425} = \dfrac{320}{17}$ s.

FINAL ANSWER

$(a)\ 135°.\quad (b)\ \overrightarrow{PQ} = 5t\mathbf{i} + (200 - 20t)\mathbf{j}.\quad (c)\ 270°.\quad (d)\ 40\sqrt{2}\ \text{m}.\quad (e)\ t = 0\ \text{s or}\ \dfrac{320}{17}\ \text{s}.$

WME01/01 OCTOBER 2022

Question 6

9 marks | Working with Vectors

6. [*In this question,* **i** *and* **j** *are horizontal unit vectors.*]

A particle A of mass 0.5 kg is at rest on a smooth horizontal plane.

At time $t = 0$, two forces, $\mathbf{F}_1 = (-3\mathbf{i} + 2\mathbf{j})$ N and $\mathbf{F}_2 = (p\mathbf{i} + q\mathbf{j})$ N, where p and q are constants, are applied to A.

Given that A moves in the direction of the vector $(\mathbf{i} - 2\mathbf{j})$,

(a) show that $2p + q - 4 = 0$

(4)

Given that $p = 5$

(b) find the speed of A at time $t = 4$ seconds.

(5)

WORKED SOLUTION

Question 6 · WME01/01 October 2022

1 Find the resultant force

$\mathbf{F} = (-3\mathbf{i} + 2\mathbf{j}) + (p\mathbf{i} + q\mathbf{j}) = (p - 3)\mathbf{i} + (q + 2)\mathbf{j}$.

2 Use the direction of motion

Since the particle starts from rest, its motion is in the direction of the resultant force. This direction is $\mathbf{i} - 2\mathbf{j}$, so $\dfrac{q+2}{p-3} = -2$.

3 Show the relation

$q + 2 = -2(p - 3) = -2p + 6$, hence $2p + q - 4 = 0$.

4 Find q when p = 5

Substitute $p = 5$ into $2p + q - 4 = 0$: $10 + q - 4 = 0$, so $q = -6$.

5 Find the resultant force

The resultant force is $(5 - 3)\mathbf{i} + (-6 + 2)\mathbf{j} = 2\mathbf{i} - 4\mathbf{j}$ N.

6 Find acceleration

The mass is 0.5 kg, so $\mathbf{a} = \dfrac{2\mathbf{i} - 4\mathbf{j}}{0.5} = 4\mathbf{i} - 8\mathbf{j}$.

7 Find velocity at t = 4

Starting from rest, $\mathbf{v} = \mathbf{a}t = (4\mathbf{i} - 8\mathbf{j})4 = 16\mathbf{i} - 32\mathbf{j}$.

8 Find the speed

Speed $= \sqrt{16^2 + (-32)^2} = \sqrt{1280} = 16\sqrt{5}\ \mathrm{m\,s^{-1}}$, about $36\ \mathrm{m\,s^{-1}}$.

FINAL ANSWER

$(a)\ 2p + q - 4 = 0.$ $(b)\ \text{speed} = 16\sqrt{5}\ \mathrm{m\,s^{-1}} \approx 36\ \mathrm{m\,s^{-1}}$.

WME01/01 OCTOBER 2022

Question 8

16 marks | Working with Vectors

8. [*In this question,* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two ships, A and B, are moving with constant velocities.
The velocity of A is $(3\mathbf{i} + 12\mathbf{j})\,\text{km h}^{-1}$ and the velocity of B is $(p\mathbf{i} + q\mathbf{j})\,\text{km h}^{-1}$

(a) Find the speed of A.

(2)

The ships are modelled as particles.
At 12 noon, A is at the point with position vector $(-9\mathbf{i} + 6\mathbf{j})$ km and B is at the point with position vector $(16\mathbf{i} + 6\mathbf{j})$ km.

At time t hours after 12 noon,

$$\overrightarrow{AB} = [(25 - 12t)\mathbf{i} - 9t\mathbf{j}]\,\text{km}$$

(b) Find the value of p and the value of q.

(7)

(c) Find the bearing of A from B when the ships are 15 km apart, giving your answer to the nearest degree.

(7)

(Total 16 marks)

WORKED SOLUTION

Question 8 · WME01/01 October 2022

1 Find the speed of A
$|\mathbf{v}_A| = \sqrt{3^2 + 12^2} = \sqrt{153} = 3\sqrt{17}\ \text{km h}^{-1}$.

2 Write A position
At time t, $\mathbf{a} = (-9\mathbf{i} + 6\mathbf{j}) + t(3\mathbf{i} + 12\mathbf{j}) = (-9 + 3t)\mathbf{i} + (6 + 12t)\mathbf{j}$.

3 Write B position
$\mathbf{b} = (16\mathbf{i} + 6\mathbf{j}) + t(p\mathbf{i} + q\mathbf{j}) = (16 + pt)\mathbf{i} + (6 + qt)\mathbf{j}$.

4 Use AB vector
$\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = [25 + (p - 3)t]\mathbf{i} + [(q - 12)t]\mathbf{j}$.

5 Compare components
Given $\overrightarrow{AB} = (25 - 12t)\mathbf{i} - 9t\mathbf{j}$, compare coefficients: $p - 3 = -12$ and $q - 12 = -9$.

6 Find p and q
Therefore $p = -9$ and $q = 3$.

7 Find the time when the distance is 15
Use $|\overrightarrow{AB}| = 15$: $(25 - 12t)^2 + (-9t)^2 = 15^2$.

8 **Solve the distance equation**

This gives $225t^2 - 600t + 400 = 0$, so $(3t-4)^2 = 0$ and $t = \dfrac{4}{3}$.

9 **Find BA at this time**

At $t = \dfrac{4}{3}$, $\overrightarrow{AB} = 9\mathbf{i} - 12\mathbf{j}$. Therefore $\overrightarrow{BA} = -9\mathbf{i} + 12\mathbf{j}$.

10 **Convert to a bearing**

A is north-west of B. The angle west of north satisfies $\tan\theta = \dfrac{9}{12}$, so $\theta = 36.9°$. Bearing $= 360° - 36.9° = 323°$ to the nearest degree.

FINAL ANSWER

$(a)\ 3\sqrt{17}\ \text{km h}^{-1}$. $(b)\ p = -9,\ q = 3$. $(c)\ \text{bearing} = 323°$.

WME01/01 MAY/JUNE 2023

Question 8

9 marks | Working with Vectors

8.

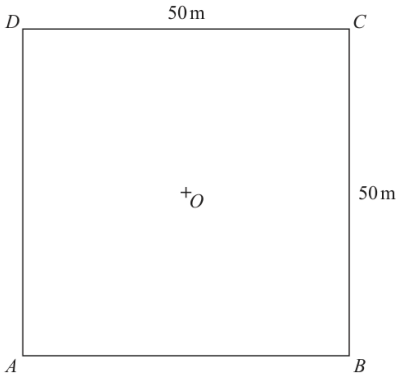


Figure 5

A square floor space $ABCD$, with centre O, is modelled as a flat horizontal surface measuring 50 m by 50 m, as shown in Figure 5.
The horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in the direction of $\overrightarrow{AB}$ and $\overrightarrow{AD}$ respectively.

All position vectors are given relative to O.

A small robot R is programmed to travel across the floor at a constant velocity.

- At time $t = 0$, R is at the point with position vector $(-2\mathbf{i} + \mathbf{j})$ m
- At time $t = 11$ s, R is at the point with position vector $(9\mathbf{i} + 23\mathbf{j})$ m
- At time t seconds, the position vector of R is $\mathbf{r}$ metres

(a) Find, in terms of t, $\mathbf{i}$ and $\mathbf{j}$, an expression for $\mathbf{r}$

(3)

A second robot S is at the point C.

- At time $t = 0$, S leaves C and moves with constant velocity $(-\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$
- At time t seconds, the position vector of S is $\mathbf{s}$ metres

(b) Write down, in terms of t, $\mathbf{i}$ and $\mathbf{j}$, an expression for $\mathbf{s}$

(1)

(c) Show that

$$\overrightarrow{SR} = [(2t - 27)\mathbf{i} + (3t - 24)\mathbf{j}]\text{ m}$$

(2)

(d) Find the time when the distance between R and S is a minimum.

(3)

WORKED SOLUTION

Question 8 · WME01/01 May/June 2023

1 Find the velocity of R

R moves from $-2\mathbf{i} + \mathbf{j}$ to $9\mathbf{i} + 23\mathbf{j}$ in 11 s. Its displacement is $11\mathbf{i} + 22\mathbf{j}$, so its velocity is $\mathbf{i} + 2\mathbf{j}$.

2 Write r

$\mathbf{r} = (-2\mathbf{i} + \mathbf{j}) + t(\mathbf{i} + 2\mathbf{j}) = (t - 2)\mathbf{i} + (2t + 1)\mathbf{j}$.

3 Write s

Point C has position vector $25\mathbf{i} + 25\mathbf{j}$. Since S moves with velocity $-\mathbf{i} - \mathbf{j}$, $\mathbf{s} = (25 - t)\mathbf{i} + (25 - t)\mathbf{j}$.

4 Find SR vector

$\overrightarrow{SR} = \mathbf{r} - \mathbf{s} = [(t - 2) - (25 - t)]\mathbf{i} + [(2t + 1) - (25 - t)]\mathbf{j}$.

5 Simplify SR

Therefore $\overrightarrow{SR} = (2t - 27)\mathbf{i} + (3t - 24)\mathbf{j}$ m.

6 **Minimise the distance squared**

The distance squared is $d^2 = (2t - 27)^2 + (3t - 24)^2$.

7 **Expand the quadratic**

$d^2 = 4t^2 - 108t + 729 + 9t^2 - 144t + 576 = 13t^2 - 252t + 1305$.

8 **Find the minimum time**

The quadratic is minimum at $t = \dfrac{252}{2 \cdot 13} = \dfrac{126}{13} = 9.69\ldots$ s.

FINAL ANSWER

$(a)\ \mathbf{r} = (t-2)\mathbf{i} + (2t+1)\mathbf{j}$. $(b)\ \mathbf{s} = (25-t)\mathbf{i} + (25-t)\mathbf{j}$. $(c)\ \overrightarrow{SR} = (2t-27)\mathbf{i} + (3t-24)\mathbf{j}$. $(d)\ t = \dfrac{126}{13}\ \text{s} \approx 9.7\ \text{s}$.

WME01/01 OCTOBER 2023

Question 6

15 marks | Working with Vectors

6. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

At 12:00, a ship P sets sail from a harbour with position vector $(15\mathbf{i} + 36\mathbf{j})$ km.
At 12:30, P is at the point with position vector $(20\mathbf{i} + 34\mathbf{j})$ km.

Given that P moves with constant velocity,

(a) show that the velocity of P is $(10\mathbf{i} - 4\mathbf{j})\,\text{km h}^{-1}$

(2)

At time t hours after 12:00, the position vector of P is $\mathbf{p}$ km.

(b) Find an expression for $\mathbf{p}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and t.

(2)

A second ship Q is also travelling at a constant velocity.

At time t hours after 12:00, the position vector of Q is given by $\mathbf{q}$ km, where

$$\mathbf{q} = (42 - 8t)\mathbf{i} + (9 + 14t)\mathbf{j}$$

Ships P and Q are modelled as particles.

If both ships maintained their course,

(c) (i) verify that they would collide at 13:30

(ii) find the position vector of the point at which the collision would occur.

(4)

At 12:30 Q changes speed and direction to avoid the collision.

Ship Q now travels due north with a constant speed of $15\,\text{km h}^{-1}$

Ship P maintains the same constant velocity throughout.

(d) Find the exact distance between P and Q at 14:30

(7)

WORKED SOLUTION

Question 6 · WME01/01 October 2023

1 Find the velocity of P

From 12:00 to 12:30, the displacement of P is $(20\mathbf{i} + 34\mathbf{j}) - (15\mathbf{i} + 36\mathbf{j}) = 5\mathbf{i} - 2\mathbf{j}$ km. The time is 0.5 hours, so $\mathbf{v}_P = \dfrac{5\mathbf{i} - 2\mathbf{j}}{0.5} = 10\mathbf{i} - 4\mathbf{j}$.

2 Write the position of P

At t hours after 12:00, $\mathbf{p} = (15\mathbf{i} + 36\mathbf{j}) + t(10\mathbf{i} - 4\mathbf{j}) = (15 + 10t)\mathbf{i} + (36 - 4t)\mathbf{j}$.

3 Verify the collision time

At 13:30, $t = 1.5$. Then $\mathbf{p} = (15 + 15)\mathbf{i} + (36 - 6)\mathbf{j} = 30\mathbf{i} + 30\mathbf{j}$. Also $\mathbf{q} = (42 - 8(1.5))\mathbf{i} + (9 + 14(1.5))\mathbf{j} = 30\mathbf{i} + 30\mathbf{j}$.

4 State the collision position

Since both position vectors are the same, the collision point has position vector $30\mathbf{i} + 30\mathbf{j}$ km.

5 Find P at 14:30

At 14:30, $t = 2.5$, so $\mathbf{p} = (15 + 25)\mathbf{i} + (36 - 10)\mathbf{j} = 40\mathbf{i} + 26\mathbf{j}$.

6 **Find Q after changing course**

At 12:30, $t = 0.5$, so $\mathbf{q} = 38\mathbf{i} + 16\mathbf{j}$. From 12:30 to 14:30 is 2 hours due north at $15\ \text{km h}^{-1}$, so the extra displacement is $30\mathbf{j}$. Thus Q is at $38\mathbf{i} + 46\mathbf{j}$.

7 **Find the distance**

The separation vector is $(40 - 38)\mathbf{i} + (26 - 46)\mathbf{j} = 2\mathbf{i} - 20\mathbf{j}$. The distance is $\sqrt{2^2 + 20^2} = \sqrt{404} = 2\sqrt{101}\ \text{km}$.

✓ **FINAL ANSWER**

$(a)\ \mathbf{v}_P = 10\mathbf{i} - 4\mathbf{j}\ \text{km h}^{-1}$. $(b)\ \mathbf{p} = (15 + 10t)\mathbf{i} + (36 - 4t)\mathbf{j}$. $(c)(ii)\ 30\mathbf{i} + 30\mathbf{j}$. $(d)\ 2\sqrt{101}\ \text{km}$.

WME01/01 JANUARY 2024

Question 7

11 marks | Working with Vectors

7. [*In this question* $\mathbf{i}$ *and* $\mathbf{j}$ *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin* O.]

At midnight, a ship S is at the point with position vector $(19\mathbf{i} + 22\mathbf{j})$ km

The ship travels with constant velocity $(12\mathbf{i} - 16\mathbf{j})\,\text{km h}^{-1}$

(a) Find the speed of S.

(2)

At time t hours after midnight, the position vector of S is $\mathbf{s}$ km.

(b) Find an expression for $\mathbf{s}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and t.

(2)

A lighthouse stands on a small rocky island. The lighthouse is modelled as being at the point with position vector $(26\mathbf{i} + 15\mathbf{j})$ km.

It is not safe for ships to be within 1.3 km of the lighthouse.

(c) (i) Find the value of t when S is closest to the lighthouse.

(ii) Hence determine whether it is safe for S to continue its course.

(7)

WORKED SOLUTION

Question 7 · WME01/01 January 2024

1 Find the speed

The velocity is $12\mathbf{i} - 16\mathbf{j}$, so the speed is $\sqrt{12^2 + (-16)^2} = 20\ \text{km h}^{-1}$.

2 Write the ship position vector

At time t hours after midnight, $\mathbf{s} = (19\mathbf{i} + 22\mathbf{j}) + t(12\mathbf{i} - 16\mathbf{j}) = (19 + 12t)\mathbf{i} + (22 - 16t)\mathbf{j}$.

3 Form the vector from lighthouse to ship

The lighthouse is at $26\mathbf{i} + 15\mathbf{j}$. Therefore $\overrightarrow{LS} = \mathbf{s} - \mathbf{l} = (12t - 7)\mathbf{i} + (7 - 16t)\mathbf{j}$.

4 Use distance squared

The square of the distance is $d^2 = (12t - 7)^2 + (7 - 16t)^2 = 400t^2 - 392t + 98$.

5 Find the time of closest approach

This quadratic is minimum when $t = \dfrac{392}{2 \cdot 400} = 0.49$.

6 Find the minimum distance

At $t = 0.49$, $d^2 = 1.96$, so $d = 1.4\ \text{km}$.

7 Decide if the course is safe

The unsafe distance is 1.3 km. Since $1.4 > 1.3$, it is safe for the ship to continue its course.

FINAL ANSWER

$(a)\ 20\ \text{km h}^{-1}.\quad (b)\ \mathbf{s} = (19 + 12t)\mathbf{i} + (22 - 16t)\mathbf{j}.\quad (c)(i)\ t = 0.49.\quad (c)(ii)\ \text{minimum distance} = 1.4\ \text{km},\ \text{so it is safe}.$

WME01/01 MAY/JUNE 2024

Question 7

13 marks | Working with Vectors

7. [*In this question, the horizontal unit vectors* **i** *and* **j** *are directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two speedboats, A and B, are each moving with constant velocity.

- the velocity of A is $40\,\text{km h}^{-1}$ due east
- the velocity of B is $20\,\text{km h}^{-1}$ on a bearing of angle α ($0° < \alpha < 90°$), where $\tan\alpha = \frac{4}{3}$

The boats are modelled as particles.

(a) Find, in terms of **i** and **j**, the velocity of B in km h^{-1}

(2)

At noon

- the position vector of A is $20\mathbf{j}\,\text{km}$
- the position vector of B is $(10\mathbf{i} + 5\mathbf{j})\,\text{km}$

At time t hours after noon

- the position vector of A is $\mathbf{r}\,\text{km}$, where $\mathbf{r} = 20\mathbf{j} + 40t\mathbf{i}$
- the position vector of B is $\mathbf{s}\,\text{km}$

(b) Find an expression for $\mathbf{s}$ in terms of t, **i** and **j**.

(2)

(c) Show that at time t hours after noon,

$$\overrightarrow{AB} = [(10 - 24t)\mathbf{i} + (12t - 15)\mathbf{j}]\,\text{km}$$

(2)

(d) Show that the boats will never collide.

(3)

(e) Find the distance between the boats when the bearing of B from A is 225°

(4)

WORKED SOLUTION

Question 7 · WME01/01 May/June 2024

1 Resolve the velocity of B

For a bearing angle α with $\tan\alpha = \frac{4}{3}$, use $\sin\alpha = \frac{4}{5}$ and $\cos\alpha = \frac{3}{5}$. Therefore $\mathbf{v}_B = 20\sin\alpha\mathbf{i} + 20\cos\alpha\mathbf{j} = 16\mathbf{i} + 12\mathbf{j}$.

2 Write the position of B

At noon, B is at $10\mathbf{i} + 5\mathbf{j}$. With constant velocity $16\mathbf{i} + 12\mathbf{j}$, $\mathbf{s} = (10 + 16t)\mathbf{i} + (5 + 12t)\mathbf{j}$.

3 Find AB vector

A has position vector $\mathbf{r} = 40t\mathbf{i} + 20\mathbf{j}$. Hence $\overrightarrow{AB} = \mathbf{s} - \mathbf{r} = (10 + 16t - 40t)\mathbf{i} + (5 + 12t - 20)\mathbf{j}$.

4 Show the printed expression

This simplifies to $\overrightarrow{AB} = (10 - 24t)\mathbf{i} + (12t - 15)\mathbf{j}$ km.

5 **Show the boats do not collide**

For collision, both components must be zero. From $10 - 24t = 0$, $t = \dfrac{5}{12}$. From $12t - 15 = 0$, $t = \dfrac{5}{4}$. These times are different, so the boats never collide.

6 **Use the bearing 225 degrees condition**

A bearing of $225°$ means B is south-west of A, so the i and j components of $\overrightarrow{AB}$ are equal and negative. Set $10 - 24t = 12t - 15$.

7 **Find the distance**

The equation gives $t = \dfrac{25}{36}$. Then each component is $-\dfrac{20}{3}$, so the distance is $\sqrt{\left(\dfrac{20}{3}\right)^2 + \left(\dfrac{20}{3}\right)^2} = \dfrac{20\sqrt{2}}{3}\ \text{km}$.

FINAL ANSWER

$(a)\ \mathbf{v}_B = 16\mathbf{i} + 12\mathbf{j}\ \text{km h}^{-1}$. $(b)\ \mathbf{s} = (10 + 16t)\mathbf{i} + (5 + 12t)\mathbf{j}$. $(c)\ \overrightarrow{AB} = (10 - 24t)\mathbf{i} + (12t - 15)\mathbf{j}$. (d) no collision. $(e)\ \dfrac{20\sqrt{2}}{3}\ \text{km} \approx 9.4\ \text{km}$.

WME01/01 MAY/JUNE 2025

Question 7

13 marks | Working with Vectors

7. [*In this question* **i** *and* **j** *are horizontal unit vectors.*]

Two boats, A and B, are each moving with constant acceleration.

At time t hours after noon, boat A has velocity $\mathbf{v}_A$ km h^{-1}, where

$$\mathbf{v}_A = 2\mathbf{i} + 3\mathbf{j} + (\mathbf{i} - 4\mathbf{j})t$$

(a) Find the magnitude of the acceleration of A.

(2)

When $t = 2$, the velocity of B is $(4\mathbf{i} + \mathbf{j})$ km h^{-1}

When $t = 5$, the velocity of B is $(\mathbf{i} - 5\mathbf{j})$ km h^{-1}

(b) Find the acceleration of B, giving your answer in terms of **i** and **j**.

(2)

(c) Find the velocity of B at time $t = 0$, giving your answer in terms of **i** and **j**.

(2)

At time T_1 hours after noon, both boats are moving with the same speed.

(d) Find the exact value of T_1

(4)

At time T_2 hours after noon, both boats are moving in the same direction.

(e) Show that $3T_2^2 + pT_2 + q = 0$, where p and q are integers to be found.

(3)

WORKED SOLUTION

Question 7 · WME01/01 May/June 2025

1 **Part (a): Find magnitude of acceleration of A - Identify acceleration from velocity function**

Given: $\mathbf{v}_A = 2\mathbf{i} + 3\mathbf{j} + (\mathbf{i} - 4\mathbf{j})t$

This is in form: $\mathbf{v} = \mathbf{u} + \mathbf{a}t$

Therefore:

; Initial velocity: $\mathbf{u}_A = 2\mathbf{i} + 3\mathbf{j}$
; Acceleration: $\mathbf{a}_A = \mathbf{i} - 4\mathbf{j}$ km h^{-2}

2 **Part (a): Find magnitude of acceleration of A - Find magnitude**

$|\mathbf{a}| = \sqrt{a_x^2 + a_y^2}$

$|\mathbf{a}_A| = \sqrt{1^2 + (-4)^2} = \sqrt{1 + 16} = \sqrt{17} = 4.123\ldots = 4.1 \text{ km h}^{-2} \text{ (2 sf)}$

3 **Part (b): Find acceleration of B - Use two velocity points to find acceleration**

Given:

; At $t = 2$: $\mathbf{v}_B = 4\mathbf{i} + \mathbf{j}$
; At $t = 5$: $\mathbf{v}_B = \mathbf{i} - 5\mathbf{j}$

$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t}$

$\mathbf{a}_B = \frac{(\mathbf{i}-5\mathbf{j})-(4\mathbf{i}+\mathbf{j})}{5-2} = \frac{(1-4)\mathbf{i}+(-5-1)\mathbf{j}}{3} = \frac{-3\mathbf{i}-6\mathbf{j}}{3} = -\mathbf{i} - 2\mathbf{j}\ \text{km h}^{-2}$

4 **Part (c): Find velocity of B at t = 0 - Use velocity equation backward**

$\mathbf{v}(t) = \mathbf{v}(t_0) + \mathbf{a}(t - t_0)$

From $t = 2$ back to $t = 0$:

$\mathbf{v}_B(0) = \mathbf{v}_B(2) + \mathbf{a}_B(0 - 2) = (4\mathbf{i} + \mathbf{j}) + (-\mathbf{i} - 2\mathbf{j})(-2) = 4\mathbf{i} + \mathbf{j} + 2\mathbf{i} + 4\mathbf{j} = 6\mathbf{i} + 5\mathbf{j}\ \text{km h}^{-1}$

5 **Part (d): Find exact value of T_1 when both have same speed - Express velocities at time t**

For boat A:
$\mathbf{v}_A(t) = 2\mathbf{i} + 3\mathbf{j} + (\mathbf{i} - 4\mathbf{j})t = (2 + t)\mathbf{i} + (3 - 4t)\mathbf{j}$

For boat B:
$\mathbf{v}_B(t) = (6\mathbf{i} + 5\mathbf{j}) + (-\mathbf{i} - 2\mathbf{j})t = (6 - t)\mathbf{i} + (5 - 2t)\mathbf{j}$

6 **Part (d): Find exact value of T_1 when both have same speed - Find magnitudes**

$|\mathbf{v}_A|^2 = (2 + t)^2 + (3 - 4t)^2 = 4 + 4t + t^2 + 9 - 24t + 16t^2 = 17t^2 - 20t + 13$

$|\mathbf{v}_B|^2 = (6 - t)^2 + (5 - 2t)^2 = 36 - 12t + t^2 + 25 - 20t + 4t^2 = 5t^2 - 32t + 61$

7 **Part (d): Find exact value of T_1 when both have same speed - Set magnitudes equal**

$17t^2 - 20t + 13 = 5t^2 - 32t + 61\ 12t^2 + 12t - 48 = 0\ t^2 + t - 4 = 0$

8 **Part (d): Find exact value of T_1 when both have same speed - Solve using quadratic formula**

$t = \frac{-1 \pm \sqrt{1+16}}{2} = \frac{-1 \pm \sqrt{17}}{2}$

Taking positive value:
$T_1 = \frac{-1+\sqrt{17}}{2}$

9 **Part (e): Show $3T_2^2 + pT_2 + q = 0$ - Condition for same direction**

Velocities parallel when:
$\frac{2+T_2}{6-T_2} = \frac{3-4T_2}{5-2T_2}$

10 **Part (e): Show $3T_2^2 + pT_2 + q = 0$ - Cross-multiply**

$(2 + T_2)(5 - 2T_2) = (3 - 4T_2)(6 - T_2)\ 10 - 4T_2 + 5T_2 - 2T_2^2 = 18 - 3T_2 - 24T_2 + 4T_2^2\ 10 + T_2 - 2T_2^2 = 18 - 27T_2 + 4T_2^2$
$0 = 4T_2^2 + 2T_2^2 - 27T_2 - T_2 + 18 - 10\ 0 = 6T_2^2 - 28T_2 + 8$

11 **Part (e): Show $3T_2^2 + pT_2 + q = 0$ - Divide by 2 to get required form**

$0 = 3T_2^2 - 14T_2 + 4$

Therefore: $p = -14$ and $q = 4$

FINAL ANSWER

$(a)\ |\mathbf{a}_A| = \sqrt{17}\ \text{km h}^{-2} \approx 4.1\ \text{km h}^{-2}$. $(b)\ \mathbf{a}_B = -\mathbf{i} - 2\mathbf{j}\ \text{km h}^{-2}$. $(c)\ \mathbf{v}_B(0) = 6\mathbf{i} + 5\mathbf{j}\ \text{km h}^{-1}$. $(d)\ T_1 = \frac{-1+\sqrt{17}}{2}$ hours. $(e)\ 3T_2^2 - 14T_2 + 4 = 0,\ p = -14,\ q = 4$

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WME01/01 JANUARY 2026

Question 6

16 marks | Working with Vectors

6. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

At 9 am, a ship S leaves a harbour and moves with constant velocity $(10\mathbf{i} + 15\mathbf{j})\,\text{km h}^{-1}$

(a) Find the exact speed of S

(2)

The harbour is at the origin O.

At 9 am, a boat B leaves the point with position vector $(50\mathbf{i} + 10\mathbf{j})$ km and moves with constant velocity $(-8\mathbf{i} + 10\mathbf{j})\,\text{km h}^{-1}$

(b) Find the direction in which B is moving, giving your answer as a bearing to the nearest degree.

(3)

(c) Show that at time t hours after 9 am,

$$\overrightarrow{SB} = [(50 - 18t)\mathbf{i} + (10 - 5t)\mathbf{j}]\,\text{km}$$

(4)

(d) Show that B and S do not collide.

(3)

When B is north-east of S, the distance between S and B is d km.

(e) Find the value of d.

(4)

WORKED SOLUTION

Question 6 · WME01/01 January 2026

1 Find the speed of S

The velocity of S is $10\mathbf{i} + 15\mathbf{j}$. Its speed is the magnitude: $\sqrt{10^2 + 15^2} = \sqrt{325} = 5\sqrt{13}\ \text{km h}^{-1}$.

2 Find the bearing of B

B has velocity $-8\mathbf{i} + 10\mathbf{j}$, so it moves west and north. The angle west of north satisfies $\tan\theta = \dfrac{8}{10}$, giving $\theta = 38.66°$. The bearing is $360° - 38.66° = 321°$ to the nearest degree.

3 Write both position vectors

At time t hours after 9 am, $\mathbf{r}_S = (10t)\mathbf{i} + (15t)\mathbf{j}$. Also $\mathbf{r}_B = (50\mathbf{i} + 10\mathbf{j}) + t(-8\mathbf{i} + 10\mathbf{j}) = (50 - 8t)\mathbf{i} + (10 + 10t)\mathbf{j}$.

4 Find the relative position vector

$\overrightarrow{SB} = \mathbf{r}_B - \mathbf{r}_S = [(50 - 8t) - 10t]\mathbf{i} + [(10 + 10t) - 15t]\mathbf{j} = (50 - 18t)\mathbf{i} + (10 - 5t)\mathbf{j}$.

5 Show they do not collide

For a collision, both components of $\overrightarrow{SB}$ must be zero at the same time. From $50 - 18t = 0$, $t = \dfrac{25}{9}$. From $10 - 5t = 0$, $t = 2$. These are different, so B and S do not collide.

6 **Find the north-east or south-west instant**

When the separation is along a 45° diagonal, the two components have equal magnitude. Using the given condition, set $50 - 18t = 10 - 5t$, so $40 = 13t$ and $t = \dfrac{40}{13}$.

7 **Find the distance d**

At $t = \dfrac{40}{13}$, both components are $-\dfrac{70}{13}$. Therefore $d = \sqrt{\left(\dfrac{70}{13}\right)^2 + \left(\dfrac{70}{13}\right)^2} = \dfrac{70\sqrt{2}}{13} = 7.6\ \text{km}$ to 2 significant figures.

FINAL ANSWER

$(a)\ 5\sqrt{13}\ \text{km h}^{-1}.$ $(b)\ 321^\circ.$ $(c)\ \overrightarrow{SB} = (50 - 18t)\mathbf{i} + (10 - 5t)\mathbf{j}.$ $(d)\ \text{no collision.}$ $(e)\ d = \dfrac{70\sqrt{2}}{13}\ \text{km} \approx 7.6\ \text{km}.$

WME01/01 OCTOBER 2019

Question 6

12 marks | Working with Vectors | Main topic: Kinematics Graphs

6. An athlete runs a 200 m race along a straight horizontal track.

In a model of the motion of the athlete, air resistance is ignored, the athlete starts from rest at time $t = 0$ seconds and moves with uniform acceleration $0.8\,\text{m s}^{-2}$ for T seconds, reaching a speed of $V\,\text{m s}^{-1}$. She then maintains this speed until she crosses the finishing line.

The total time from when the athlete starts to when she crosses the finishing line is 30 s.

(a) Sketch a speed-time graph for the model of the motion of the athlete from the instant when she starts to the instant when she crosses the finishing line.

(2)

(b) Write down an expression for V in terms of T.

(1)

(c) Show that $T^2 - kT + 500 = 0$, where k is a constant to be found.

(4)

(d) Hence find the value of T, justifying your answer carefully.

(3)

(e) Considering your speed-time graph or otherwise, state two ways, apart from including air resistance, in which the model could be made to be more realistic.

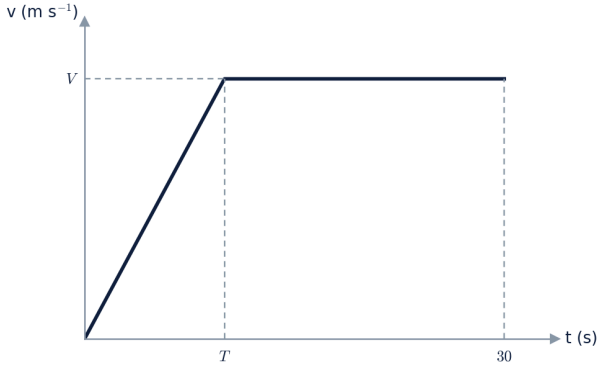
(2)

WORKED SOLUTION

Question 6 · WME01/01 October 2019

1 **Describe the speed-time graph**

The graph starts at $(0, 0)$, rises in a straight line with gradient 0.8 until time T and speed V, then stays horizontal at speed V until $t = 30$.



2 **Write V in terms of T**

Uniform acceleration gives $V = 0.8T$.

3 **Use area for the distance**

The distance is the area under the speed-time graph: $\frac{1}{2}TV + V(30 - T) = 200$.

4 **Substitute V**

Using $V = 0.8T$: $\frac{1}{2}T(0.8T) + (0.8T)(30 - T) = 200$.

5 **Form the quadratic**

$0.4T^2 + 24T - 0.8T^2 = 200$, so $-0.4T^2 + 24T - 200 = 0$. Multiplying by -2.5 gives $T^2 - 60T + 500 = 0$, so $k = 60$.

6 **Solve and choose the valid root**

$T^2 - 60T + 500 = (T - 10)(T - 50) = 0$, so $T = 10$ or 50. Since the whole race lasts 30 s, $T = 10$.

7 **Give two model improvements**

A more realistic model could include reaction time at the start, and could replace the instantaneous change from acceleration to constant speed with a smoother transition.

FINAL ANSWER

(a) Speed-time graph rises linearly to V at $t = T$, then is horizontal until $t = 30$.

(b) $V = 0.8T$. (c) $T^2 - 60T + 500 = 0$, so $k = 60$.

(d) $T = 10$. (e) For example, include reaction time at the start and avoid the instantaneous change to constant speed.

WME01/01 JANUARY 2023

Question 6

8 marks | Working with Vectors | Main topic: Forces

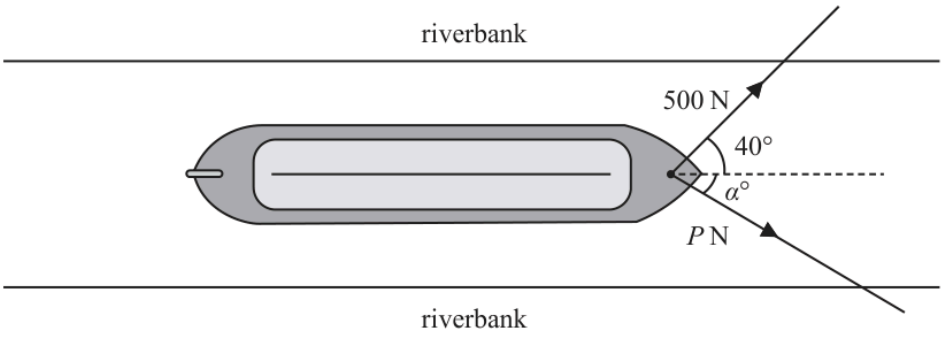


Figure 3

A boat is pulled along a river at a constant speed by two ropes.

The banks of the river are parallel and the boat travels horizontally in a straight line, parallel to the riverbanks.

- The tension in the first rope is 500 N acting at an angle of 40° to the direction of motion, as shown in Figure 3.
- The tension in the second rope is P newtons, acting at an angle of $\alpha°$ to the direction of motion, also shown in Figure 3.
- The resistance to motion of the boat as it moves through the water is a constant force of magnitude 900 N

The boat is modelled as a particle. The ropes are modelled as being light and lying in a horizontal plane.

Use the model to find

(i) the value of α

(ii) the value of P

(8)

WORKED SOLUTION

Question 6 · WME01/01 January 2023

1 Resolve perpendicular to the motion

The boat travels straight, so the sideways components balance: $500\sin 40° = P\sin\alpha$.

2 Resolve along the motion

The speed is constant, so the forward components balance the resistance: $500\cos 40° + P\cos\alpha = 900$.

3 Eliminate P

From the two equations, $P\sin\alpha = 500\sin 40°$ and $P\cos\alpha = 900 - 500\cos 40°$.

4 Find alpha

Divide the sine equation by the cosine equation: $\tan\alpha = \dfrac{500\sin 40°}{900 - 500\cos 40°}$. This gives $\alpha = 31.9°$, so $\alpha = 32°$ approximately.

5 Find P

Use $P\sin\alpha = 500\sin 40°$: $P = \dfrac{500\sin 40°}{\sin 31.9°} = 608.7\ldots$ N. Hence $P \approx 610$ N.

FINAL ANSWER

$(i)\ \alpha = 32°$ approximately. $(ii)\ P = 610$ N approximately.

TOPIC

Kinematics Graphs

Every past-paper question on this topic, October 2019 to January 2026.

WME01/01 OCTOBER 2019

Question 6

12 marks | Kinematics Graphs

6. An athlete runs a 200 m race along a straight horizontal track.

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The total time from when the athlete starts to when she crosses the finishing line is 30 s.

(a) Sketch a speed-time graph for the model of the motion of the athlete from the instant when she starts to the instant when she crosses the finishing line.

(2)

(b) Write down an expression for V in terms of T.

(1)

(c) Show that $T^2 - kT + 500 = 0$, where k is a constant to be found.

(4)

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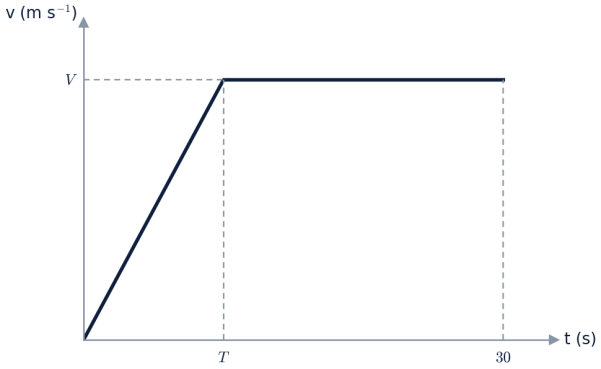
(2)

WORKED SOLUTION

Question 6 · WME01/01 October 2019

1 **Describe the speed-time graph**

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Uniform acceleration gives $V = 0.8T$.

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The distance is the area under the speed-time graph: $\frac{1}{2}TV + V(30 - T) = 200$.

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$0.4T^2 + 24T - 0.8T^2 = 200$, so $-0.4T^2 + 24T - 200 = 0$. Multiplying by -2.5 gives $T^2 - 60T + 500 = 0$, so $k = 60$.

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$T^2 - 60T + 500 = (T - 10)(T - 50) = 0$, so $T = 10$ or 50. Since the whole race lasts 30 s, $T = 10$.

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A more realistic model could include reaction time at the start, and could replace the instantaneous change from acceleration to constant speed with a smoother transition.

FINAL ANSWER

(a) Speed-time graph rises linearly to V at $t = T$, then is horizontal until $t = 30$.

(b) $V = 0.8T$. (c) $T^2 - 60T + 500 = 0$, so $k = 60$.

(d) $T = 10$. (e) For example, include reaction time at the start and avoid the instantaneous change to constant speed.

WME01/01 JANUARY 2021

Question 7

12 marks | Kinematics Graphs

7. A helicopter is hovering at rest above horizontal ground at the point H. A parachutist steps out of the helicopter and immediately falls vertically and freely under gravity from rest for 2.5 s. His parachute then opens and causes him to immediately decelerate at a constant rate of $3.9\,\text{m s}^{-2}$ for T seconds ($T < 6$), until his speed is reduced to $V\,\text{m s}^{-1}$. He then moves with this constant speed $V\,\text{m s}^{-1}$ until he hits the ground. While he is decelerating, he falls a distance of 73.75 m. The total time between the instant when he leaves H and the instant when he hits the ground is 20 s.

The parachutist is modelled as a particle.

(a) Find the speed of the parachutist at the instant when his parachute opens.

(1)

(b) Sketch a speed-time graph for the motion of the parachutist from the instant when he leaves H to the instant when he hits the ground.

(2)

(c) Find the value of T.

(5)

(d) Find, to the nearest metre, the height of the point H above the ground.

(4)

WORKED SOLUTION

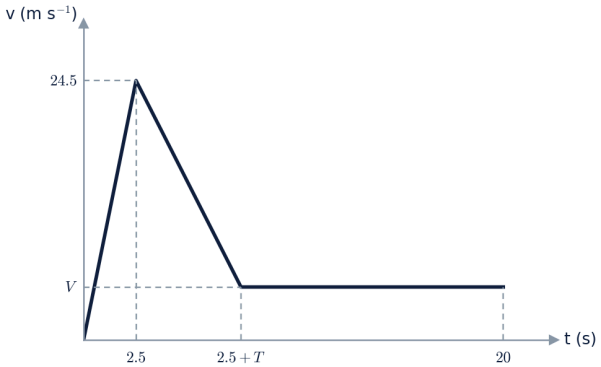
Question 7 · WME01/01 January 2021

1 **Find speed when parachute opens**

The parachutist falls freely from rest for 2.5 s, so $v = gt = 9.8(2.5) = 24.5\ \text{m s}^{-1}$.

2 **Describe the graph**

The speed-time graph starts at 0, rises linearly to 24.5 at $t = 2.5$, then decreases linearly with smaller gradient for T seconds, then stays horizontal at speed V until $t = 20$.



3 **Use the deceleration distance**

During the deceleration phase, initial speed is 24.5, acceleration is -3.9, time is T, and distance is 73.75.

4 **Form the equation for T**

$73.75 = 24.5T + \frac{1}{2}(-3.9)T^2 = 24.5T - 1.95T^2$.

5 **Choose the valid T**

Solving gives $T = 5$ or $T = 7.56\ldots$. Since $T < 6$, take $T = 5$.

6 **Find V**

$V = 24.5 - 3.9(5) = 5\ \mathrm{m\,s^{-1}}$.

7 **Find the total height**

Height is the total area under the speed-time graph: first triangle $\frac{1}{2}(2.5)(24.5) = 30.625$, deceleration distance 73.75, and final rectangle $(20 - 2.5 - 5)(5) = 62.5$.

8 **Round the height**

Total height $= 30.625 + 73.75 + 62.5 = 166.875$ m, which rounds to 167 m.

✓ **FINAL ANSWER**

$(a)\ 24.5\ \mathrm{m\,s^{-1}}$. (b) Speed-time graph rises to 24.5 at $t = 2.5$, falls to V at $t = 2.5 + T$, then is horizontal to $t = 20$.
$(c)\ T = 5$. $(d)\ H = 167\ \mathrm{m}$ to the nearest metre.

WME01/01 MAY/JUNE 2021

Question 8

13 marks | Kinematics Graphs

8. Two trams, tram A and tram B, run on parallel straight horizontal tracks. Initially the two trams are at rest in the depot and level with each other.

At time $t = 0$, tram A starts to move. Tram A moves with constant acceleration $2\,\text{m s}^{-2}$ for 5 seconds and then continues to move along the track at constant speed.

At time $t = 20$ seconds, tram B starts from rest and moves in the same direction as tram A. Tram B moves with constant acceleration $3\,\text{m s}^{-2}$ for 4 seconds and then continues to move along the track at constant speed.

The trams are modelled as particles.

(a) Sketch, on the same axes, a speed-time graph for the motion of tram A and a speed-time graph for the motion of tram B, from $t = 0$ to the instant when tram B overtakes tram A.

(3)

At the instant when the two trams are moving with the same speed, tram A is d metres in front of tram B.

(b) Find the value of d.

(5)

(c) Find the distance of the trams from the depot at the instant when tram B overtakes tram A.

(5)

(Total 13 marks)

WORKED SOLUTION

Question 8 · WME01/01 May/June 2021

1 Describe tram A graph

Tram A starts at $t = 0$, accelerates at $2\,\text{m s}^{-2}$ for 5 s, reaching speed $10\,\text{m s}^{-1}$, then continues at this constant speed.

2 Describe tram B graph

Tram B starts at $t = 20$, accelerates at $3\,\text{m s}^{-2}$ for 4 s, reaching speed $12\,\text{m s}^{-1}$, then continues at this constant speed. Its graph crosses A's distance later.

3 Find when speeds are equal

Tram A's constant speed is 10. Tram B reaches speed 10 after $\frac{10}{3}$ s of acceleration, so the time from $t = 0$ is $20 + \frac{10}{3} = \frac{70}{3}$ s.

4 Distance of A then

A travels $\frac{1}{2}(5)(10) = 25$ m in its acceleration phase, then $10\left(\frac{70}{3} - 5\right) = \frac{550}{3}$ m. Total $= \frac{625}{3}$ m.

5 Distance of B then

B is still accelerating from rest, so its distance is $\frac{1}{2}\left(\frac{10}{3}\right)(10) = \frac{50}{3}$ m.

6 **Find d**

$d = \dfrac{625}{3} - \dfrac{50}{3} = \dfrac{575}{3}$ m.

7 **Set overtake distances equal**

For $t > 24$, A's distance is $25 + 10(t-5) = 10t - 25$. B's distance is $\dfrac{1}{2}(4)(12) + 12(t-24) = 12t - 264$.

8 **Find the overtake time**

Set $10t - 25 = 12t - 264$. Then $2t = 239$ and $t = 119.5$ s.

9 **Find the distance from the depot**

Using A's distance, $10(119.5) - 25 = 1170$ m.

FINAL ANSWER

(a) Tram A reaches speed 10 at $t = 5$; tram B starts at $t = 20$, reaches speed 12 at $t = 24$, then later overtakes A.

$(b)\ d = \dfrac{575}{3}\ \text{m} \approx 192\ \text{m}.$ $(c)\ 1170\ \text{m}.$

WME01/01 MAY/JUNE 2022

Question 7

13 marks | Kinematics Graphs

7. Two small children, Ajaz and Beth, are running a 100 m race along a straight horizontal track.

They both start from rest, leaving the start line at the same time.

Ajaz accelerates at $0.8\,\text{m s}^{-2}$ up to a speed of $4\,\text{m s}^{-1}$ and then maintains this speed until he crosses the finish line.

Beth accelerates at $1\,\text{m s}^{-2}$ for T seconds and then maintains a constant speed until she crosses the finish line.

Ajaz and Beth cross the finish line at the same time.

(a) Sketch, on the same axes, a speed-time graph for each child, from the instant when they leave the start line to the instant when they cross the finish line.

(3)

(b) Find the time taken by Ajaz to complete the race.

(4)

(c) Find the value of T

(4)

(d) Find the difference in the speeds of the two children as they cross the finish line.

(2)

WORKED SOLUTION

Question 7 · WME01/01 May/June 2022

1 Describe Ajaz graph

Ajaz starts from rest, accelerates linearly to speed $4\,\text{m s}^{-1}$, then continues with a horizontal speed-time line at speed 4.

2 Describe Beth graph

Beth starts from rest, has a steeper straight acceleration line for T seconds, then a horizontal line. Both graphs finish at the same final time.

3 Find Ajaz acceleration time

Ajaz reaches speed 4 with acceleration 0.8, so the time is $\frac{4}{0.8} = 5$ s.

4 Find Ajaz total time

Distance during acceleration is $\frac{1}{2}(5)(4) = 10$ m. The remaining 90 m at speed 4 takes 22.5 s, so Ajaz's total time is 27.5 s.

5 Set Beth distance equation

Beth accelerates for T seconds to speed T, then runs at speed T until time 27.5. Her distance is $\frac{1}{2}T^2 + T(27.5 - T)$.

6 Solve for T

$100 = \frac{1}{2}T^2 + 27.5T - T^2 = 27.5T - \frac{1}{2}T^2$. Thus $T^2 - 55T + 200 = 0$. The valid root is $T = 3.915\ldots$ s.

7 Find the finish speeds

Ajaz finishes at $4\,\text{m s}^{-1}$. Beth's constant speed is $T = 3.915\ldots\ \text{m s}^{-1}$.

8 **Find the difference**

Difference $= 4 - 3.915\ldots = 0.085\ \text{m s}^{-1}$ approximately.

FINAL ANSWER

(a) Ajaz reaches speed 4 then runs horizontally; Beth has a steeper initial line to speed T, then horizontal, both ending at the same time.

(b) $27.5\ \text{s}$. (c) $T = 3.92\ \text{s}$ **approximately.** (d) $0.085\ \text{m s}^{-1}$ **approximately**.

Question 6

10 marks | Kinematics Graphs

6.

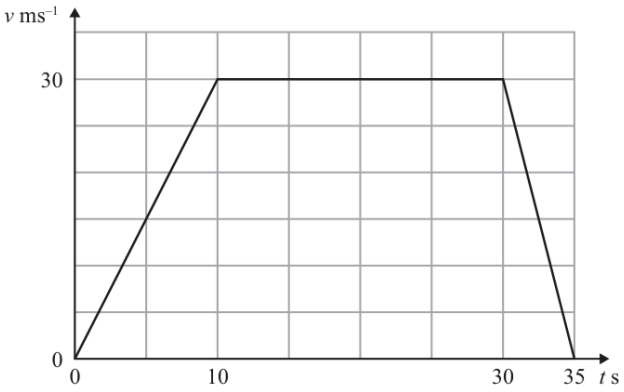


Figure 4

The point O lies on a straight horizontal road.
At time $t = 0$, a car leaves O and travels along the road.

The velocity-time graph in Figure 4 shows the velocity, $v\,\text{m s}^{-1}$, of the car at time t seconds for the first 35 seconds of its journey.

(a) Find

(i) the acceleration of the car for the period $0 \leqslant t \leqslant 10$

(ii) the deceleration of the car for the period $30 \leqslant t \leqslant 35$

(2)

(b) Sketch an acceleration-time graph for the car for the period $0 \leqslant t \leqslant 35$

(2)

(c) Find the distance travelled by the car for the period $0 \leqslant t \leqslant 35$

(2)

When $t = 5$, a motorcycle starts from rest at O.
The motorcycle travels along the same road as the car and in the same direction.

For the period $5 \leqslant t \leqslant 20$, the acceleration of the motorcycle is $A\,\text{m s}^{-2}$, where A is a positive constant.

The motorcycle catches up with the car when $t = 20$

(d) Find the value of A.

(4)

1 **Part (a): Find accelerations**

(i) Acceleration for $0 \leq t \leq 10$

$\text{Acceleration} = \frac{\Delta v}{\Delta t} = \text{gradient of v-t graph}$

$a = \frac{30-0}{10-0}\ a = 3\ \text{m s}^{-2}$

(ii) Deceleration for $30 \leq t \leq 35$

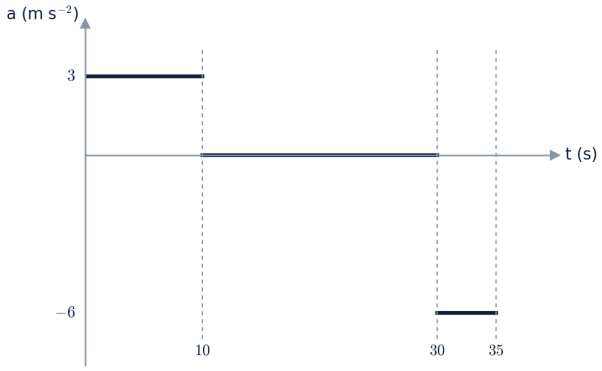
$a = \frac{0-30}{35-30}\ a = \frac{-30}{5}\ a = -6\ \text{m s}^{-2}$

Deceleration = $|-6| = 6$ m s^{-2}

2 **Part (b): Sketch acceleration-time graph**

Analysis of each section:

; $0 \leq t \leq 10$: Constant acceleration = 3 m s^{-2}
; $10 < t \leq 30$: Constant velocity, so acceleration = 0 m s^{-2}
; $30 < t \leq 35$: Constant deceleration = -6 m s^{-2}



3 **Part (c): Find distance travelled for $0 \leq t \leq 35$ - Calculate area under v-t graph**

$\text{Distance} = \text{Area under v-t graph}$

The shape is a trapezium. We can split it into regions:

Method 1: Trapezium formula

$\text{Area} = \frac{1}{2}(a+b)h$

Where the parallel sides are the time intervals and height is velocity.

Better to calculate as sum of shapes:

Method 2: Sum of triangle + rectangle + triangle

$\text{Area}_1 = \frac{1}{2} \times 10 \times 30 = 150\ \text{m}\ \text{Area}_2 = 20 \times 30 = 600\ \text{m}\ \text{Area}_3 = \frac{1}{2} \times 5 \times 30 = 75\ \text{m}$

$\text{Total distance} = 150 + 600 + 75 = 825\ \text{m}$

4 **Part (d): Find value of A - Distance travelled by car (0 to 20 seconds)**

Area under v-t graph from $t = 0$ to $t = 20$:

$$\text{Distance}_{\text{car}} = \text{Triangle} + \text{Rectangle} = \tfrac{1}{2}(10)(30) + (10)(30) = 150 + 300 = 450 \text{ m}$$

---

5 **Part (d): Find value of A - Distance travelled by motorcycle (5 to 20 seconds)**

Motorcycle starts at $t = 5$ with constant acceleration A.

Time of travel: $20 - 5 = 15$ seconds

$$s = ut + \tfrac{1}{2}at^2$$

$$\text{Distance}_{\text{motorcycle}} = 0 + \tfrac{1}{2}A(15)^2 = \tfrac{225A}{2} = 112.5A \text{ m}$$

---

6 **Part (d): Find value of A - Set distances equal**

Motorcycle catches up with car when $t = 20$:

$$112.5A = 450 \; A = \tfrac{450}{112.5} \; A = 4 \text{ m s}^{-2}$$

FINAL ANSWER

$(a)(i)\ a = 3 \text{ m s}^{-2},\ (ii)$ deceleration $= 6 \text{ m s}^{-2}$. (b) correct acceleration-time graph with levels $3, 0, -6$. (c) distance $= 825$ m. $(d)\ A = 4 \text{ m s}^{-2}$

.

WME01/01 OCTOBER 2019

Question 8

14 marks | Kinematics Graphs | Main topic: Working with Vectors

8. [*In this question, the horizontal unit vectors* **i** *and* **j** *are directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two speedboats, A and B, are each moving with constant velocity. The velocity of A is $20\,\text{km h}^{-1}$ due west and the velocity of B is $40\,\text{km h}^{-1}$ on a bearing of 150°. The boats are modelled as particles.

At noon, the position vector of A is $60\mathbf{i}$ km and B is at the origin O. At time t hours after noon, the position vector of A is $\mathbf{r}$ km and the position vector of B is $\mathbf{s}$ km.

(a) Find the velocity of B in the form $(p\mathbf{i} + q\mathbf{j})\,\text{km h}^{-1}$

(3)

(b) Find expressions for $\mathbf{r}$ and $\mathbf{s}$ in terms of t, $\mathbf{i}$ and $\mathbf{j}$.

(3)

(c) Find the time, to the nearest minute, at which the distance between the boats is the same as it was at noon.

(8)

WORKED SOLUTION

Question 8 · WME01/01 October 2019

1 Resolve B's velocity

A bearing of $150°$ is $30°$ east of south. Therefore the east component is $40\sin 150° = 20$ and the north component is $40\cos 150° = -20\sqrt{3}$.

2 Write B's velocity

$\mathbf{v}_B = 20\mathbf{i} - 20\sqrt{3}\mathbf{j}\ \text{km h}^{-1}$.

3 Write A's position

Boat A starts at $60\mathbf{i}$ and travels due west at $20\ \text{km h}^{-1}$, so $\mathbf{r} = 60\mathbf{i} - 20t\mathbf{i} = (60 - 20t)\mathbf{i}$.

4 Write B's position

Boat B starts at O, so $\mathbf{s} = t(20\mathbf{i} - 20\sqrt{3}\mathbf{j}) = 20t\mathbf{i} - 20\sqrt{3}t\mathbf{j}$.

5 Find the separation vector

$\overrightarrow{AB} = \mathbf{s} - \mathbf{r} = (20t - (60 - 20t))\mathbf{i} - 20\sqrt{3}t\mathbf{j} = (40t - 60)\mathbf{i} - 20\sqrt{3}t\mathbf{j}$.

6 Use the noon distance

At noon the distance between the boats is 60 km. Set the later distance equal to 60: $(40t - 60)^2 + (-20\sqrt{3}t)^2 = 60^2$.

7 Solve for t

Expanding gives $2800t^2 - 4800t = 0$, so $t(2800t - 4800) = 0$. The later time is $t = \dfrac{4800}{2800} = \dfrac{12}{7}$ hours.

8 Convert to clock time

$\dfrac{12}{7}$ hours is 1 hour and $42.857\ldots$ minutes after noon, which rounds to $1{:}43$ pm, or $13{:}43$.

FINAL ANSWER

$(a)\ \mathbf{v}_B = 20\mathbf{i} - 20\sqrt{3}\mathbf{j}\ \text{km h}^{-1}.\quad (b)\ \mathbf{r} = (60 - 20t)\mathbf{i},\ \mathbf{s} = 20t\mathbf{i} - 20\sqrt{3}t\mathbf{j}.\quad (c)\ 13{:}43$, or $1{:}43\ \text{pm}$.

WME01/01 OCTOBER 2021

Question 6

5 marks | Kinematics Graphs

Main topic: Constant Acceleration in 1D

6.

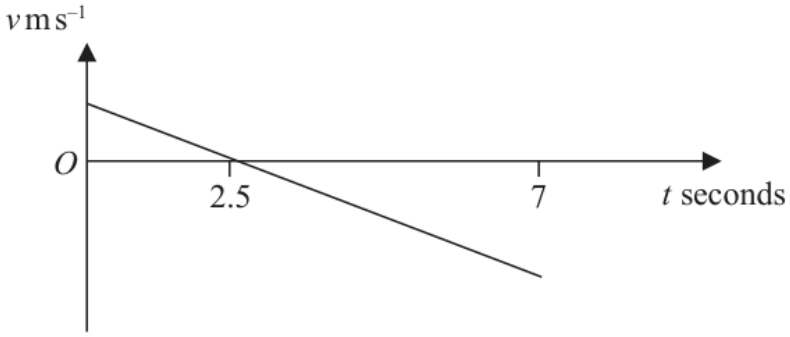


Figure 3

A small ball is thrown vertically upwards at time $t = 0$ from a point A which is above horizontal ground. The ball hits the ground 7 s later.

The ball is modelled as a particle moving freely under gravity.

The velocity-time graph shown in Figure 3 represents the motion of the ball for $0 \leqslant t \leqslant 7$

(a) Find the speed with which the ball is thrown.

(2)

(b) Find the height of A above the ground.

(3)

WORKED SOLUTION

Question 6 · WME01/01 October 2021

1 Use the velocity-time graph

The velocity becomes zero at $t = 2.5$ s. Since acceleration is $-g = -9.8\ \text{m s}^{-2}$, the initial velocity is $u = 9.8(2.5)$.

2 Find the initial speed

$u = 24.5\ \text{m s}^{-1}$.

3 Use displacement to the ground

From launch to $t = 7$, the displacement is the area under the velocity-time graph. Algebraically, $s = ut - \frac{1}{2}gt^2$ with $t = 7$.

4 Calculate displacement

$s = 24.5(7) - \frac{1}{2}(9.8)(7^2) = 171.5 - 240.1 = -68.6$ m.

5 State the height

The negative sign shows the ground is below A, so the height of A above the ground is $68.6\ \text{m}$.

FINAL ANSWER

$(a)\ 24.5\ \text{m s}^{-1}$. $(b)\ 68.6\ \text{m}$ approximately.

WME01/01 JANUARY 2024

Question 6

12 marks | Kinematics Graphs

Main topic: Constant Acceleration in 1D

6. A particle is projected vertically upwards from a point A with speed $24\,\text{m s}^{-1}$

The point A is 2.5 m vertically above the point B.

Point B lies on horizontal ground.

The particle moves freely under gravity until it hits the ground at B with speed $V\,\text{m s}^{-1}$

After hitting the ground the particle does not rebound.

(a) Find the value of V.

(3)

(b) Find the time taken for the particle to reach B.

(3)

The point C is 10 m vertically above A.

(c) Find the length of time for which the particle is above C.

(4)

(d) Sketch a speed-time graph for the motion of the particle from projection to the instant that it reaches B. (No further calculations are required.)

(2)

WORKED SOLUTION

Question 6 · WME01/01 January 2024

1 Find the impact speed

Take upward as positive from A. At B, the displacement is -2.5 m, with $u = 24$ and $a = -9.8$. Use $v^2 = u^2 + 2as$:
$V^2 = 24^2 + 2(-9.8)(-2.5) = 625$.

2 State V

$V = 25\,\text{m s}^{-1}$.

3 Find the time to reach B

The velocity at B is $-25\,\text{m s}^{-1}$. Using $v = u + at$, $-25 = 24 - 9.8t$, so $t = 5\,\text{s}$.

4 Set up the height C equation

C is 10 m above A. The particle is above C between the two times when $s = 10$: $10 = 24t - 4.9t^2$.

5 Solve for the two crossing times

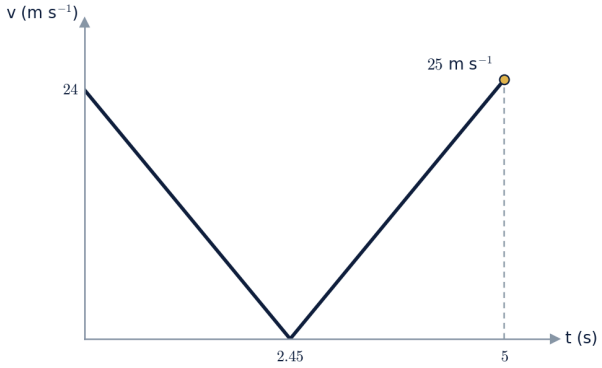
$4.9t^2 - 24t + 10 = 0$, giving $t = 0.459\ldots$ and $t = 4.438\ldots$.

6 Find the time above C

The length of time above C is $4.438\ldots - 0.459\ldots = 3.98\ldots\,\text{s}$, about $4.0\,\text{s}$.

7 **Describe the speed-time graph**

The graph starts at speed $\mathbf{24}$, decreases linearly to $\mathbf{0}$ at the highest point, then increases linearly to speed $\mathbf{25}$ at $\boldsymbol{t = 5}$. It is a V-shaped speed-time graph.



FINAL ANSWER

$(a)\ V = 25\ \mathrm{m\,s^{-1}}$. $(b)\ t = 5\ \mathrm{s}$. $(c)\ \mathbf{4.0\ s\ approximately}$. (d) Speed-time graph is V-shaped, from speed 24 to 0, then to speed 25 at t=5.

TOPIC

Constant Acceleration in 1D

Every past-paper question on this topic, October 2019 to January 2026.

WME01/01 OCTOBER 2021

Question 6

5 marks | Constant Acceleration in 1D

6.

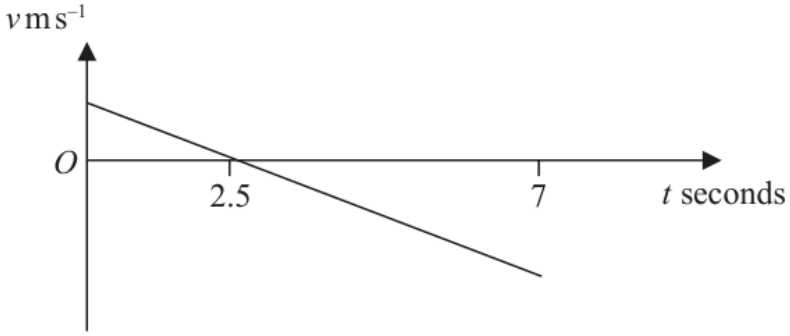


Figure 3

A small ball is thrown vertically upwards at time $t = 0$ from a point A which is above horizontal ground. The ball hits the ground 7 s later.

The ball is modelled as a particle moving freely under gravity.

The velocity-time graph shown in Figure 3 represents the motion of the ball for $0 \leqslant t \leqslant 7$

(a) Find the speed with which the ball is thrown.

(2)

(b) Find the height of A above the ground.

(3)

WORKED SOLUTION

Question 6 · WME01/01 October 2021

1 Use the velocity-time graph

The velocity becomes zero at $t = 2.5$ s. Since acceleration is $-g = -9.8\ \text{m s}^{-2}$, the initial velocity is $u = 9.8(2.5)$.

2 Find the initial speed

$u = 24.5\ \text{m s}^{-1}$.

3 Use displacement to the ground

From launch to $t = 7$, the displacement is the area under the velocity-time graph. Algebraically, $s = ut - \frac{1}{2}gt^2$ with $t = 7$.

4 Calculate displacement

$s = 24.5(7) - \frac{1}{2}(9.8)(7^2) = 171.5 - 240.1 = -68.6$ m.

5 State the height

The negative sign shows the ground is below A, so the height of A above the ground is $68.6\ \text{m}$.

FINAL ANSWER

$(a)\ 24.5\ \text{m s}^{-1}$. $(b)\ 68.6\ \text{m}$ approximately.

WME01/01 JANUARY 2024

Question 6

12 marks | Constant Acceleration in 1D

6. A particle is projected vertically upwards from a point A with speed $24\,\text{m s}^{-1}$

The point A is 2.5 m vertically above the point B.

Point B lies on horizontal ground.

The particle moves freely under gravity until it hits the ground at B with speed $V\,\text{m s}^{-1}$

After hitting the ground the particle does not rebound.

(a) Find the value of V.

(3)

(b) Find the time taken for the particle to reach B.

(3)

The point C is 10 m vertically above A.

(c) Find the length of time for which the particle is above C.

(4)

(d) Sketch a speed-time graph for the motion of the particle from projection to the instant that it reaches B. (No further calculations are required.)

(2)

WORKED SOLUTION

Question 6 · WME01/01 January 2024

1 Find the impact speed

Take upward as positive from A. At B, the displacement is -2.5 m, with $u = 24$ and $a = -9.8$. Use $v^2 = u^2 + 2as$: $V^2 = 24^2 + 2(-9.8)(-2.5) = 625$.

2 State V

$V = 25\ \text{m s}^{-1}$.

3 Find the time to reach B

The velocity at B is $-25\ \text{m s}^{-1}$. Using $v = u + at$, $-25 = 24 - 9.8t$, so $t = 5$ s.

4 Set up the height C equation

C is 10 m above A. The particle is above C between the two times when $s = 10$: $10 = 24t - 4.9t^2$.

5 Solve for the two crossing times

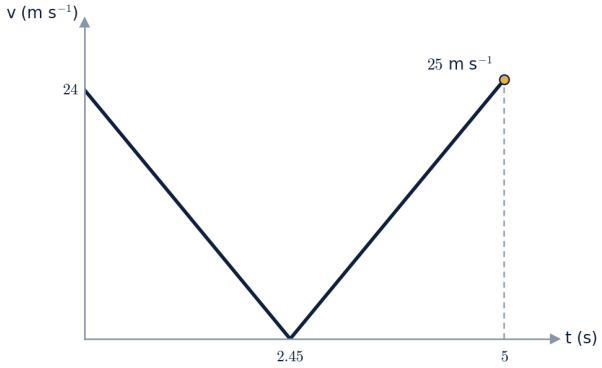
$4.9t^2 - 24t + 10 = 0$, giving $t = 0.459\ldots$ and $t = 4.438\ldots$.

6 Find the time above C

The length of time above C is $4.438\ldots - 0.459\ldots = 3.98\ldots$ s, about 4.0 s.

7 **Describe the speed-time graph**

The graph starts at speed $\mathbf{24}$, decreases linearly to $\mathbf{0}$ at the highest point, then increases linearly to speed $\mathbf{25}$ at $\boldsymbol{t = 5}$. It is a V-shaped speed-time graph.



FINAL ANSWER

$\boldsymbol{(a)\ V = 25\ \mathrm{m\,s^{-1}}.}$ $\boldsymbol{(b)\ t = 5\ \mathrm{s}.}$ $\boldsymbol{(c)\ 4.0\ \mathrm{s}}$ **approximately**. (d) Speed-time graph is V-shaped, from speed 24 to 0, then to speed 25 at t=5.

WME01/01 OCTOBER 2024

Question 7

11 marks | Constant Acceleration in 1D

7 At time $t = 0$, a small ball A is projected vertically upwards with speed $8\,\text{m s}^{-1}$ from a fixed point on horizontal ground.
The ball hits the ground again for the first time at time $t = T_1$ seconds.

Ball A is modelled as a particle moving freely under gravity.

(a) Show that $T_1 = 1.63$ to 3 significant figures.

(2)

After the first impact with the ground, A rebounds to a height of 2 m above the ground.

Given that the mass of A is 0.1 kg,

(b) find the magnitude of the impulse received by A as a result of its first impact with the ground.

(5)

At time $t = 1$ second, another small ball B is projected vertically upwards from another point on the ground with speed $5\,\text{m s}^{-1}$

Ball B is modelled as a particle moving freely under gravity.

At time $t = T_2$ seconds ($T_2 > 1$), A and B are at the same height above the ground for the first time.

(c) Find the value of T_2

(4)

WORKED SOLUTION

Question 7 · WME01/01 October 2024

1 Find the first flight time

For ball A, take upward as positive. Returning to the ground gives $0 = 8T_1 - 4.9T_1^2$. The non-zero solution is $T_1 = \dfrac{8}{4.9} = 1.63\,\text{s}$ to 3 significant figures.

2 Find the rebound speed

After impact, A rebounds to a maximum height of 2 m. At the top, $v = 0$, so $0 = u^2 - 2g(2)$. Hence $u = \sqrt{4g} = 6.261\,\text{m s}^{-1}$.

3 Use impulse as change in momentum

Just before the first impact, A has velocity $-8\,\text{m s}^{-1}$. Just after impact, it has velocity $+6.261\,\text{m s}^{-1}$. With mass 0.1 kg, $I = 0.1[6.261 - (-8)]$.

4 Calculate the impulse

$I = 1.426\ldots\,\text{N s}$, so the impulse is $1.43\,\text{N s}$ to 3 significant figures.

5 Write the height of A before first impact

For the first meeting with B, A is still in its first flight. Its height at time T_2 is $h_A = 8T_2 - 4.9T_2^2$.

6 Write the height of B

B starts at time $t = 1$, so at time T_2 it has been moving for $T_2 - 1$ seconds. Its height is $h_B = 5(T_2 - 1) - 4.9(T_2 - 1)^2$.

7 **Set the heights equal**

$8T_2 - 4.9T_2^2 = 5(T_2 - 1) - 4.9(T_2 - 1)^2$. Expanding and simplifying gives $6.8T_2 = 9.9$.

8 **Find T2**

$T_2 = \dfrac{9.9}{6.8} = 1.456\ldots$, so $T_2 = 1.46\ \text{s}$ to 3 significant figures.

✓ **FINAL ANSWER**

$(a)\ T_1 = 1.63\ \text{s}.\quad (b)\ I = 1.43\ \text{N s}.\quad (c)\ T_2 = 1.46\ \text{s}.$

WME01/01 JANUARY 2020

Question 7

18 marks | Constant Acceleration in 1D | Main topic: Newton's Second Law

7.

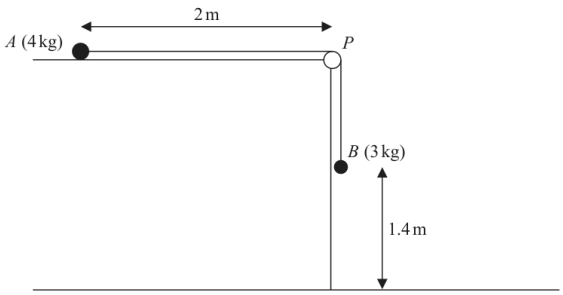


Figure 3

A particle A of mass 4 kg is held at rest on a rough horizontal table. Particle A is attached to one end of a string that passes over a pulley P. The pulley is fixed at the edge of the table. The other end of the string is attached to a particle B, of mass 3 kg, which hangs freely below P.

The part of the string from A to P is perpendicular to the edge of the table and A, P and B all lie in the same vertical plane.

The string is modelled as being light and inextensible and the pulley is modelled as being small, smooth and light.

The system is released from rest with the string taut. At the instant of release, A is 2 m from the edge of the table and B is 1.4 m above a horizontal floor, as shown in Figure 3.

After descending with constant acceleration for 2 seconds, B hits the floor and does not rebound.

(a) Show that the acceleration of A before B hits the floor is $0.7\,\text{m s}^{-2}$

(2)

(b) State which of the modelling assumptions you have used in order to answer part (a).

(1)

(c) Find the magnitude of the resultant force exerted on the pulley by the string.

(4)

The coefficient of friction between A and the table is μ.

(d) Find the value of μ.

(6)

(e) Determine, by calculation, whether or not A reaches the pulley.

(5)

(Total 18 marks)

WORKED SOLUTION

Question 7 · WME01/01 January 2020

1 Use B's descent

Before B hits the floor, it descends 1.4 m from rest in 2 s. Use $s = ut + \frac{1}{2}at^2$: $1.4 = \frac{1}{2}a(2)^2$. Hence $a = 0.7\ \text{m s}^{-2}$.

2 Name the modelling assumption

The string is inextensible, so A and B have the same magnitude of acceleration while the string is taut.

3 Find the tension

For B, taking downward as positive: $3g - T = 3(0.7)$. Thus $T = 3g - 2.1 = 27.3$ N.

4 Find the resultant force on the pulley

The pulley is pulled by two perpendicular tensions of magnitude T. Therefore the resultant is $\sqrt{T^2 + T^2} = T\sqrt{2} = 27.3\sqrt{2} = 38.6$ N approximately.

5 Find the friction on A

For A before B hits the floor, $T - F = 4(0.7)$. Hence $F = 27.3 - 2.8 = 24.5$ N.

6 **Find mu**

On the horizontal table, $R = 4g = 39.2$ N and $F = \mu R$. So $\mu = \dfrac{24.5}{39.2} = 0.625$.

7 **Find A's speed when B hits the floor**

At this instant, $v = at = 0.7(2) = 1.4\ \text{m s}^{-1}$. A has also moved 1.4 m towards the pulley.

8 **Find the extra stopping distance**

After B hits the floor, A is slowed only by friction. Its deceleration is $\mu g = 0.625(9.8) = 6.125\ \text{m s}^{-2}$. Use $0 = (1.4)^2 - 2(6.125)s$, giving $s = 0.16$ m.

9 **Compare with the distance to the pulley**

A's total distance is $1.4 + 0.16 = 1.56$ m, which is less than 2 m. Therefore A does not reach the pulley.

FINAL ANSWER

$(a)\ a = 0.7\ \text{m s}^{-2}$. (b) Inextensibility of the string.

$(c)\ 38.6\ \text{N}$. $(d)\ \mu = 0.625$. (e) A does not reach the pulley.

WME01/01 JANUARY 2021

Question 7

12 marks | Constant Acceleration in 1D | Main topic: Kinematics Graphs

7. A helicopter is hovering at rest above horizontal ground at the point H. A parachutist steps out of the helicopter and immediately falls vertically and freely under gravity from rest for 2.5 s. His parachute then opens and causes him to immediately decelerate at a constant rate of $3.9\,\text{m s}^{-2}$ for T seconds ($T < 6$), until his speed is reduced to $V\,\text{m s}^{-1}$. He then moves with this constant speed $V\,\text{m s}^{-1}$ until he hits the ground. While he is decelerating, he falls a distance of 73.75 m. The total time between the instant when he leaves H and the instant when he hits the ground is 20 s.

The parachutist is modelled as a particle.

(a) Find the speed of the parachutist at the instant when his parachute opens.

(1)

(b) Sketch a speed-time graph for the motion of the parachutist from the instant when he leaves H to the instant when he hits the ground.

(2)

(c) Find the value of T.

(5)

(d) Find, to the nearest metre, the height of the point H above the ground.

(4)

WORKED SOLUTION

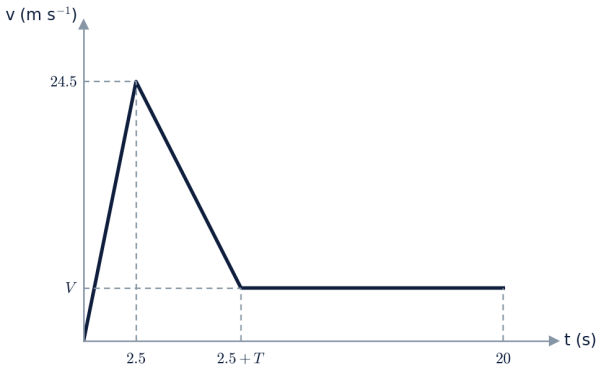
Question 7 · WME01/01 January 2021

1 Find speed when parachute opens

The parachutist falls freely from rest for $\mathbf{2.5}$ s, so $v = gt = 9.8(2.5) = 24.5\ \text{m s}^{-1}$.

2 Describe the graph

The speed-time graph starts at $\mathbf{0}$, rises linearly to $\mathbf{24.5}$ at $t = 2.5$, then decreases linearly with smaller gradient for T seconds, then stays horizontal at speed V until $t = 20$.



3 Use the deceleration distance

During the deceleration phase, initial speed is $\mathbf{24.5}$, acceleration is $\mathbf{-3.9}$, time is T, and distance is $\mathbf{73.75}$.

4 **Form the equation for T**

$73.75 = 24.5T + \frac{1}{2}(-3.9)T^2 = 24.5T - 1.95T^2$.

5 **Choose the valid T**

Solving gives $T = 5$ or $T = 7.56\ldots$. Since $T < 6$, take $T = 5$.

6 **Find V**

$V = 24.5 - 3.9(5) = 5\ \mathrm{m\,s^{-1}}$.

7 **Find the total height**

Height is the total area under the speed-time graph: first triangle $\frac{1}{2}(2.5)(24.5) = 30.625$, deceleration distance 73.75, and final rectangle $(20 - 2.5 - 5)(5) = 62.5$.

8 **Round the height**

Total height $= 30.625 + 73.75 + 62.5 = 166.875$ m, which rounds to 167 m.

✓ **FINAL ANSWER**

$(a)\ 24.5\ \mathrm{m\,s^{-1}}$. (b) Speed-time graph rises to 24.5 at $t = 2.5$, falls to V at $t = 2.5 + T$, then is horizontal to $t = 20$.
$(c)\ T = 5$. $(d)\ H = 167\ \mathrm{m}$ to the nearest metre.

WME01/01 JANUARY 2021

Question 8

17 marks | Constant Acceleration in 1D

Main topic: Resolving Forces, Inclined Planes

8.

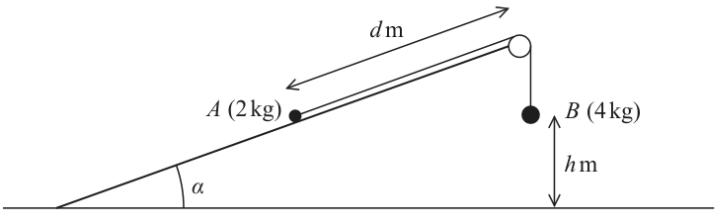


Figure 3

Two particles, A and B, have masses 2 kg and 4 kg respectively. The particles are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough plane. The plane is inclined to the horizontal ground at an angle α where $\tan\alpha = \frac{3}{4}$. The particle A is held at rest on the plane at a distance d metres from the pulley. The particle B hangs freely at rest, vertically below the pulley, at a distance h metres above the ground, as shown in Figure 3. The part of the string between A and the pulley is parallel to a line of greatest slope of the plane. The coefficient of friction between A and the plane is $\frac{1}{4}$

The system is released from rest with the string taut and B descends.

(a) Find the tension in the string as B descends.

(9)

On hitting the ground, B immediately comes to rest.

Given that A comes to rest before reaching the pulley,

(b) find, in terms of h, the range of possible values of d.

(7)

(c) State one physical factor, other than air resistance, that could be taken into account to make the model described above more realistic.

(1)

(Total 17 marks)

WORKED SOLUTION

Question 8 · WME01/01 January 2021

1 Use the trig ratios

Since $\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$.

2 Find friction on A

For A, $R = 2g\cos\alpha = \frac{8g}{5}$. With coefficient $\frac{1}{4}$, friction is $F = \frac{1}{4}R = \frac{2g}{5}$.

3 Equation for A

A moves up the plane. Taking up the plane as positive: $T - 2g\sin\alpha - F = 2a$.

4 Simplify A's equation

$T - 2g\cdot\frac{3}{5} - \frac{2g}{5} = 2a$, so $T - \frac{8g}{5} = 2a$.

5 Equation for B

B descends. Taking downward as positive: $4g - T = 4a$.

6 **Solve for T**

From A, $T = 2a + \frac{8g}{5}$. From B, $T = 4g - 4a$. Equating gives $6a = \frac{12g}{5}$, so $a = \frac{2g}{5}$. Then $T = 4g - 4\left(\frac{2g}{5}\right) = \frac{12g}{5}$ N.

7 **Find speed when B reaches the ground**

Before B hits the ground, both particles move distance h from rest with acceleration $\frac{2g}{5}$. Thus $v^2 = 2\left(\frac{2g}{5}\right)h = \frac{4gh}{5}$.

8 **Find extra distance after B stops**

After B hits the ground, A continues up the plane. The forces down the plane on A are $2g\sin\alpha + F = \frac{6g}{5} + \frac{2g}{5} = \frac{8g}{5}$, so its deceleration is $\frac{4g}{5}$.

9 **Calculate extra distance**

Let the extra distance be s. Then $0 = \frac{4gh}{5} - 2\left(\frac{4g}{5}\right)s$, so $s = \frac{h}{2}$.

10 **Find the range for d**

A travels total distance $h + \frac{h}{2} = \frac{3h}{2}$ before stopping. Since it comes to rest before reaching the pulley, $d > \frac{3h}{2}$.

11 **Name a more realistic factor**

A more realistic model could include friction at the pulley.

✓ **FINAL ANSWER**

$(a)\ T = \frac{12g}{5}\ \text{N} = 23.5\ \text{N}.$ $(b)\ d > \frac{3h}{2}.$ (c) A more realistic model could include friction at the pulley.

WME01/01 MAY/JUNE 2021

Question 6

13 marks | Constant Acceleration in 1D | Main topic: Resolving Forces, Inclined Planes

6. A fixed rough plane is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$

A particle of mass 6 kg is projected with speed 5 m s^{-1} from a point A on the plane, up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is $\frac{1}{4}$

(a) Find the magnitude of the frictional force acting on the particle as it moves up the plane.

(3)

The particle comes to instantaneous rest at the point B.

(b) Find the distance AB.

(5)

The particle now slides down the plane from B. At the instant when the particle passes through the point C on the plane, the speed of the particle is again 5 m s^{-1}

(c) Find the distance BC.

(5)

WORKED SOLUTION

Question 6 · WME01/01 May/June 2021

1 Use the trig ratios

Since $\tan\theta = \frac{5}{12}$, $\sin\theta = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$.

2 Find the normal reaction

$R = 6g\cos\theta = 6g \cdot \frac{12}{13} = \frac{72g}{13}$.

3 Find friction

With $\mu = \frac{1}{4}$, $F = \frac{1}{4}R = \frac{18g}{13} = 13.6\ \text{N}$ approximately.

4 Find acceleration while moving up

While moving up, both friction and the component of weight act down the plane. Taking up the plane as positive: $6a = -6g\sin\theta - F$.

5 Simplify the upward acceleration

$6a = -6g \cdot \frac{5}{13} - \frac{18g}{13} = -\frac{48g}{13}$, so $a = -\frac{8g}{13}$.

6 Find AB

Use $v^2 = u^2 + 2as$ with $u = 5$ and $v = 0$: $0 = 25 + 2\left(-\frac{8g}{13}\right)AB$. Hence $AB = 2.07\ldots\ \text{m}$.

7 Find acceleration while sliding down

When sliding down, friction acts up the plane. Taking down the plane as positive: $6a = 6g\sin\theta - F = \frac{30g}{13} - \frac{18g}{13} = \frac{12g}{13}$. Thus $a = \frac{2g}{13}$.

8 **Find BC**

Starting from rest at B and reaching speed 5 at C: $25 = 2\left(\frac{2g}{13}\right)BC$. Hence $BC = 8.29\ldots\ \text{m}$.

FINAL ANSWER

$(a)\ F = 13.6\ \text{N}.\quad (b)\ AB = 2.07\ \text{m} \approx 2.1\ \text{m}.\quad (c)\ BC = 8.29\ \text{m} \approx 8.3\ \text{m}.$

WME01/01 MAY/JUNE 2021

Question 7

10 marks | Constant Acceleration in 1D | Main topic: Moments

7.

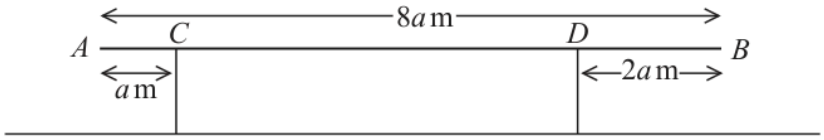


Figure 2

A non-uniform beam AB, of mass 60 kg and length $8a$ metres, rests in equilibrium in a horizontal position on two vertical supports. One support is at C, where $AC = a$ metres and the other support is at D, where $DB = 2a$ metres, as shown in Figure 2.

The magnitude of the normal reaction between the beam and the support at D is three times the magnitude of the normal reaction between the beam and the support at C.

By modelling the beam as a non-uniform rod whose centre of mass is at a distance x metres from A,

(a) find an expression for x in terms of a.

(5)

A box of mass M kg is placed on the beam at E, where $AE = 2a$ metres.

The beam remains in equilibrium in a horizontal position.

The magnitude of the normal reaction between the beam and the support at C is now equal to the magnitude of the normal reaction between the beam and the support at D.

By modelling the box as a particle,

(b) find the value of M.

(5)

WORKED SOLUTION

Question 7 · WME01/01 May/June 2021

1 Set the support positions

The beam length is $8a$. C is at distance a from A, and D is at distance $6a$ from A because $DB = 2a$.

2 Name the reactions

Let the reaction at C be R. The reaction at D is $3R$.

3 Use vertical equilibrium

$R + 3R = 60g$, so $4R = 60g$ and $R = 15g$.

4 Take moments about A

The beam's weight acts at distance x from A. Hence $60gx = R(a) + 3R(6a)$.

5 Find x

Substitute $R = 15g$: $60gx = 15ga + 45g(6a) = 285ga$. Thus $x = \frac{285a}{60} = \frac{19a}{4}$.

6 **Use equal reactions with the box**

When the box is added, let each support reaction be S. Vertical equilibrium gives $2S = (60 + M)g$.

7 **Take moments about A again**

The box is at $AE = 2a$. Moments about A give $60g\left(\frac{19a}{4}\right) + Mg(2a) = S(a) + S(6a) = 7aS$.

8 **Solve for M**

Substitute $S = \frac{(60 + M)g}{2}$ and cancel ag: $285 + 2M = \frac{7}{2}(60 + M) = 210 + \frac{7}{2}M$. Hence $75 = \frac{3}{2}M$ and $M = 50$.

✓ **FINAL ANSWER**

$(a)\ x = \frac{19a}{4}.\quad (b)\ M = 50.$

WME01/01 MAY/JUNE 2021

Question 8

13 marks | Constant Acceleration in 1D | Main topic: Kinematics Graphs

8. Two trams, tram A and tram B, run on parallel straight horizontal tracks. Initially the two trams are at rest in the depot and level with each other.

At time $t = 0$, tram A starts to move. Tram A moves with constant acceleration $2\,\text{m s}^{-2}$ for 5 seconds and then continues to move along the track at constant speed.

At time $t = 20$ seconds, tram B starts from rest and moves in the same direction as tram A. Tram B moves with constant acceleration $3\,\text{m s}^{-2}$ for 4 seconds and then continues to move along the track at constant speed.

The trams are modelled as particles.

(a) Sketch, on the same axes, a speed-time graph for the motion of tram A and a speed-time graph for the motion of tram B, from $t = 0$ to the instant when tram B overtakes tram A.

(3)

At the instant when the two trams are moving with the same speed, tram A is d metres in front of tram B.

(b) Find the value of d.

(5)

(c) Find the distance of the trams from the depot at the instant when tram B overtakes tram A.

(5)

(Total 13 marks)

WORKED SOLUTION

Question 8 · WME01/01 May/June 2021

1 Describe tram A graph

Tram A starts at $t = 0$, accelerates at $2\,\text{m s}^{-2}$ for 5 s, reaching speed $10\,\text{m s}^{-1}$, then continues at this constant speed.

2 Describe tram B graph

Tram B starts at $t = 20$, accelerates at $3\,\text{m s}^{-2}$ for 4 s, reaching speed $12\,\text{m s}^{-1}$, then continues at this constant speed. Its graph crosses A's distance later.

3 Find when speeds are equal

Tram A's constant speed is 10. Tram B reaches speed 10 after $\frac{10}{3}$ s of acceleration, so the time from $t = 0$ is $20 + \frac{10}{3} = \frac{70}{3}$ s.

4 Distance of A then

A travels $\frac{1}{2}(5)(10) = 25$ m in its acceleration phase, then $10\left(\frac{70}{3} - 5\right) = \frac{550}{3}$ m. Total $= \frac{625}{3}$ m.

5 Distance of B then

B is still accelerating from rest, so its distance is $\frac{1}{2}\left(\frac{10}{3}\right)(10) = \frac{50}{3}$ m.

6 **Find d**

$d = \dfrac{625}{3} - \dfrac{50}{3} = \dfrac{575}{3}$ m.

7 **Set overtake distances equal**

For $t > 24$, A's distance is $25 + 10(t-5) = 10t - 25$. B's distance is $\dfrac{1}{2}(4)(12) + 12(t-24) = 12t - 264$.

8 **Find the overtake time**

Set $10t - 25 = 12t - 264$. Then $2t = 239$ and $t = 119.5$ s.

9 **Find the distance from the depot**

Using A's distance, $10(119.5) - 25 = 1170$ m.

FINAL ANSWER

(a) Tram A reaches speed 10 at $t = 5$; tram B starts at $t = 20$, reaches speed 12 at $t = 24$, then later overtakes A.

$(b)\ d = \dfrac{575}{3}\ \text{m} \approx 192\ \text{m}.$ $(c)\ 1170\ \text{m}.$

WME01/01 MAY/JUNE 2022

Question 7

13 marks | Constant Acceleration in 1D | Main topic: Kinematics Graphs

7. Two small children, Ajaz and Beth, are running a 100 m race along a straight horizontal track.

They both start from rest, leaving the start line at the same time.

Ajaz accelerates at $0.8\,\text{m s}^{-2}$ up to a speed of $4\,\text{m s}^{-1}$ and then maintains this speed until he crosses the finish line.

Beth accelerates at $1\,\text{m s}^{-2}$ for T seconds and then maintains a constant speed until she crosses the finish line.

Ajaz and Beth cross the finish line at the same time.

(a) Sketch, on the same axes, a speed-time graph for each child, from the instant when they leave the start line to the instant when they cross the finish line.

(3)

(b) Find the time taken by Ajaz to complete the race.

(4)

(c) Find the value of T

(4)

(d) Find the difference in the speeds of the two children as they cross the finish line.

(2)

WORKED SOLUTION

Question 7 · WME01/01 May/June 2022

1 Describe Ajaz graph

Ajaz starts from rest, accelerates linearly to speed $4\,\text{m s}^{-1}$, then continues with a horizontal speed-time line at speed 4.

2 Describe Beth graph

Beth starts from rest, has a steeper straight acceleration line for T seconds, then a horizontal line. Both graphs finish at the same final time.

3 Find Ajaz acceleration time

Ajaz reaches speed 4 with acceleration 0.8, so the time is $\dfrac{4}{0.8} = 5$ s.

4 Find Ajaz total time

Distance during acceleration is $\frac{1}{2}(5)(4) = 10$ m. The remaining 90 m at speed 4 takes 22.5 s, so Ajaz's total time is 27.5 s.

5 Set Beth distance equation

Beth accelerates for T seconds to speed T, then runs at speed T until time 27.5. Her distance is $\frac{1}{2}T^2 + T(27.5 - T)$.

6 Solve for T

$100 = \frac{1}{2}T^2 + 27.5T - T^2 = 27.5T - \frac{1}{2}T^2$. Thus $T^2 - 55T + 200 = 0$. The valid root is $T = 3.915\ldots$ s.

7 Find the finish speeds

Ajaz finishes at $4\,\text{m s}^{-1}$. Beth's constant speed is $T = 3.915\ldots\,\text{m s}^{-1}$.

8 **Find the difference**

Difference $= 4 - 3.915\ldots = 0.085\ \text{m s}^{-1}$ approximately.

FINAL ANSWER

(a) Ajaz reaches speed 4 then runs horizontally; Beth has a steeper initial line to speed T, then horizontal, both ending at the same time.

(b) $27.5\ \text{s}$. (c) $T = 3.92\ \text{s}$ **approximately.** (d) $0.085\ \text{m s}^{-1}$ **approximately**.

WME01/01 JANUARY 2023

Question 7

7 marks | Constant Acceleration in 1D | Main topic: Newton's Second Law

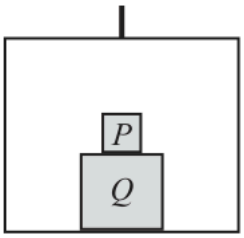


Figure 4

A simple lift operates by means of a vertical cable which is attached to the top of the lift.
The lift has mass m
A box Q is placed on the floor of the lift.
A box P is placed directly on top of box Q, as shown in Figure 4.

The cable is modelled as being light and inextensible and air resistance is modelled as being negligible.

The tension in the cable is $\frac{42mg}{5}$

The lift and its contents move vertically upwards with acceleration $\frac{2g}{5}$

Using the model,

(a) find, in terms of m, the combined mass of boxes P and Q

(4)

During the motion of the lift, the force exerted on box P by box Q is $\frac{14mg}{5}$

Using the model,

(b) find, in terms of m, the mass of box P

(3)

WORKED SOLUTION

Question 7 · WME01/01 January 2023

1 Define the box mass

Let the combined mass of boxes P and Q be M. The lift itself has mass m, so the total moving mass is $m + M$.

2 Apply Newtons second law to the whole system

Taking upward as positive, $\frac{42}{5}mg - (m+M)g = (m+M)\frac{2g}{5}$.

3 Find M

Move the weight term to the right: $\frac{42}{5}mg = (m+M)\left(g + \frac{2g}{5}\right) = \frac{7g}{5}(m+M)$. Hence $42m = 7(m+M)$ and $M = 5m$.

4 Let the mass of P be p

For box P alone, the upward force from Q is $\frac{14}{5}mg$, while its weight is pg. The acceleration is still $\frac{2g}{5}$ upward.

5 **Apply Newtons second law to P**

$\frac{14}{5}mg - pg = p\frac{2g}{5}$.

6 **Find p**

Divide by g: $\frac{14}{5}m = p + \frac{2p}{5} = \frac{7p}{5}$. Therefore $p = 2m$.

FINAL ANSWER

(a) combined mass of P and Q $= 5m$. (b) mass of P $= 2m$.

WME01/01 JANUARY 2023

Question 8

15 marks | Constant Acceleration in 1D

Main topic: Resolving Forces, Inclined Planes

8.

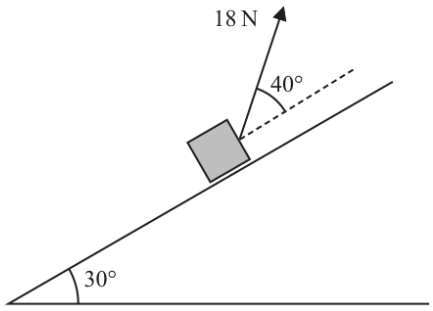


Figure 5

A parcel of mass 2 kg is pulled up a rough inclined plane by the action of a constant force.

The force has magnitude 18 N and acts at an angle of 40° to the plane.

The line of action of the force lies in a vertical plane containing a line of greatest slope of the inclined plane.

The plane is inclined at an angle of 30° to the horizontal, as shown in Figure 5.

The coefficient of friction between the plane and the parcel is 0.3

The parcel is modelled as a particle P

(a) Find the acceleration of P

(8)

The points A and B lie on a line of greatest slope of the plane, where $AB = 5\,\text{m}$ and B is above A. Particle P passes through A with speed $2\,\text{m s}^{-1}$ in the direction AB.

(b) Find the speed of P as it passes through B.

(3)

The force of 18 N is removed at the instant P passes through B. As a result, P comes to rest at the point C.

(c) Determine whether P will remain at rest at C. You must show all stages of your working clearly.

(4)

WORKED SOLUTION

Question 8 · WME01/01 January 2023

1 Resolve perpendicular to the plane

The force of 18 N is angled away from the plane, so $R + 18\sin 40° = 2g\cos 30°$. Hence $R = 2g\cos 30° - 18\sin 40°$.

2 Use friction

The parcel moves up the plane, so friction acts down the plane. With $\mu = 0.3$, $F = 0.3R$.

3 Resolve parallel to the plane

Taking up the plane as positive: $18\cos 40° - F - 2g\sin 30° = 2a$.

4 Substitute for R and F

$18\cos 40° - 0.3(2g\cos 30° - 18\sin 40°) - 2g\sin 30° = 2a$.

5 Find the acceleration

Using $g = 9.8$, this gives $a = 1.18\ldots\ \text{m s}^{-2}$ up the plane.

6 **Find the speed at B**

From A to B, $u = 2$, $s = 5$ and $a = 1.18\ldots$. Use $v^2 = u^2 + 2as$: $v^2 = 2^2 + 2(1.18\ldots)(5)$.

7 **State the speed**

$v = 3.98\ldots\ \text{m s}^{-1}$, about $4.0\ \text{m s}^{-1}$.

8 **Check rest after the force is removed**

At C, with the pulling force removed, $R = 2g\cos 30°$. The maximum friction is $0.3R = 0.3(2g\cos 30°) = 5.09\ldots\ \text{N}$.

9 **Compare with the downslope weight component**

The component of weight down the plane is $2g\sin 30° = 9.8\ \text{N}$. Since $9.8 > 5.09\ldots$, friction is not large enough to hold P at rest, so P will not remain at rest at C.

✓ **FINAL ANSWER**

$(a)\ a = 1.18\ \text{m s}^{-2}$ **up the plane.** $(b)\ v = 3.98\ \text{m s}^{-1} \approx 4.0\ \text{m s}^{-1}$. (c) P will not remain at rest at C.

WME01/01 MAY/JUNE 2023

Question 7

11 marks | Constant Acceleration in 1D | Main topic: Newton's Second Law

7.

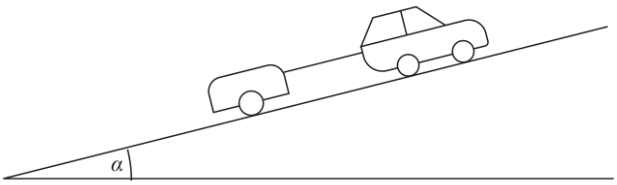


Figure 4

A car of mass 1200 kg is towing a trailer of mass 600 kg up a straight road, as shown in Figure 4.

The road is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{12}$

The driving force produced by the engine of the car is 3000 N.

The car moves with acceleration $0.75\,\text{m s}^{-2}$

The non-gravitational resistance to motion of

- the **car** is modelled as a constant force of magnitude $2R$ newtons
- the **trailer** is modelled as a constant force of magnitude R newtons

The car and the trailer are modelled as particles.
The tow bar between the car and trailer is modelled as a light rod that is parallel to the direction of motion.

Using the model,

(a) show that the value of R is 60

(4)

(b) find the tension in the tow bar.

(3)

When the car and trailer are moving at a speed of $12\,\text{m s}^{-1}$, the tow bar breaks.

Given that the non-gravitational resistance to motion of the trailer remains unchanged,

(c) use the model to find the further distance moved by the trailer before it first comes to rest.

(4)

WORKED SOLUTION

Question 7 · WME01/01 May/June 2023

1 Use the whole system

For car plus trailer, the total mass is 1800 kg and the total resistance is $2R + R = 3R$. Taking up the slope as positive: $3000 - 1800g\sin\alpha - 3R = 1800(0.75)$.

2 Show R is 60

Since $\sin\alpha = \frac{1}{12}$, $1800g\sin\alpha = 1800 \cdot 9.8 \cdot \frac{1}{12} = 1470$. So $3000 - 1470 - 3R = 1350$, giving $180 = 3R$ and $R = 60$ N.

3 Apply Newtons second law to the trailer

For the trailer while it is being towed, $T - 600g\sin\alpha - R = 600(0.75)$.

4 Find the tow-bar tension

$T - 600 \cdot 9.8 \cdot \frac{1}{12} - 60 = 450$, so $T - 490 - 60 = 450$ and $T = 1000$ N.

5 **Find the trailer acceleration after the break**

After the tow bar breaks, the trailer is still moving up the slope, but the component of weight and resistance act down the slope. Taking up the slope as positive: $-600g\sin\alpha - 60 = 600a$.

6 **Simplify the acceleration**

$-490 - 60 = 600a$, so $a = -\dfrac{550}{600} = -\dfrac{11}{12}\ \mathrm{m\,s^{-2}}$.

7 **Find the stopping distance**

With initial speed $12\ \mathrm{m\,s^{-1}}$ and final speed 0, use $v^2 = u^2 + 2as$: $0 = 12^2 + 2\left(-\dfrac{11}{12}\right)d$. Hence $d = \dfrac{864}{11} = 78.5\ldots\ \mathrm{m}$.

FINAL ANSWER

$(a)\ R = 60\ \mathrm{N}$. $(b)\ T = 1000\ \mathrm{N}$. $(c)\ d = 78.5\ \mathrm{m}$ approximately.

WME01/01 JANUARY 2024

Question 8

12 marks | Constant Acceleration in 1D | Main topic: Resolving Forces, Inclined Planes

8.

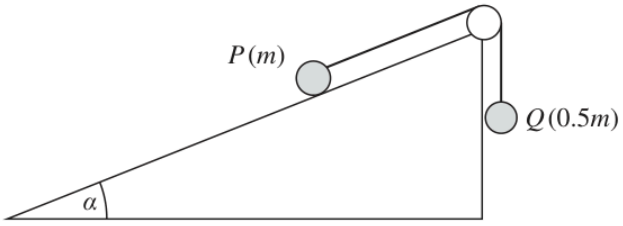


Figure 5

A fixed rough plane is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{5}{12}$

A small smooth pulley is fixed at the top of the plane.

One end of a light inextensible string is attached to a particle P which is at rest on the plane. The string passes over the pulley and the other end of the string is attached to a particle Q which hangs vertically below the pulley, as shown in Figure 5.

Particle P has mass m and particle Q has mass $0.5m$

The string from P to the pulley lies along a line of greatest slope of the plane.

The coefficient of friction between P and the plane is μ.

The system is in **limiting equilibrium** with the string taut and P is on the point of slipping **up** the plane.

(a) Find the value of μ.

(8)

The string breaks and P begins to move down the plane.

When particle P has travelled a distance of 0.8 m down the plane, the speed of P is $V\,\text{m s}^{-1}$

(b) Find the value of V.

(4)

WORKED SOLUTION

Question 8 · WME01/01 January 2024

1 Use the trig ratios

Given $\tan\alpha = \frac{5}{12}$, use $\sin\alpha = \frac{5}{13}$ and $\cos\alpha = \frac{12}{13}$.

2 Find the tension

Q has mass $0.5m$ and the system is in equilibrium, so the tension is $T = 0.5mg$.

3 Resolve P parallel to the plane

P is on the point of slipping up the plane, so friction acts down the plane. Along the plane, $T = mg\sin\alpha + F$.

4 Find the friction force

$F = T - mg\sin\alpha = \frac{1}{2}mg - \frac{5}{13}mg = \frac{3}{26}mg$.

5 **Find the normal reaction and mu**

$R = mg\cos\alpha = \dfrac{12}{13}mg$. Since $F = \mu R$, $\mu = \dfrac{3mg/26}{12mg/13} = \dfrac{1}{8}$.

6 **Find acceleration after the string breaks**

P moves down the plane, so friction acts up the plane. The resultant down the plane is $mg\sin\alpha - F = \dfrac{5}{13}mg - \dfrac{1}{8}\cdot\dfrac{12}{13}mg = \dfrac{7}{26}mg$.

7 **Apply Newtons second law**

As P has mass m, $a = \dfrac{7g}{26}$ down the plane.

8 **Find V**

From rest over 0.8 m, $V^2 = 2\left(\dfrac{7g}{26}\right)(0.8)$. With $g = 9.8$, $V = 2.05\ \text{m s}^{-1}$ to 3 significant figures.

FINAL ANSWER

$(a)\ \mu = \dfrac{1}{8}$. $(b)\ V = 2.05\ \text{m s}^{-1}$ approximately.

WME01/01 MAY/JUNE 2024

Question 6

12 marks | Constant Acceleration in 1D

Main topic: Resolving Forces, Inclined Planes

6.

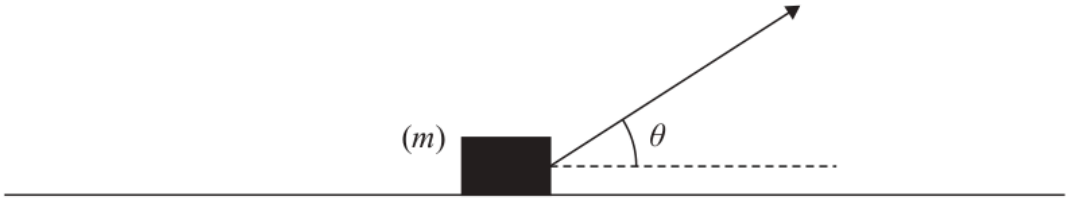


Figure 3

A box of mass m lies on a rough horizontal plane. The box is pulled along the plane in a straight line at **constant speed** by a light rope. The rope is inclined at an angle θ to the plane, as shown in Figure 3.

The coefficient of friction between the box and the plane is $\frac{1}{3}$

The box is modelled as a particle.

Given that $\tan\theta = \frac{3}{4}$

(a) find, in terms of m and g, the tension in the rope.

(7)

The rope is now removed and the box is placed at rest on the plane.
The box is then projected horizontally along the plane with speed u.

The box is again modelled as a particle.

When the box has moved a distance d along the plane, the speed of the box is $\frac{1}{2}u$.

(b) Find d in terms of u and g.

(5)

WORKED SOLUTION

Question 6 · WME01/01 May/June 2024

1 Use the trig ratios

Given $\tan\theta = \frac{3}{4}$, use $\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$.

2 Resolve vertically

The box moves at constant speed, so it is in equilibrium. Vertically, $R + T\sin\theta = mg$, hence $R = mg - T\sin\theta$.

3 Resolve horizontally

Horizontally, the tension component balances friction: $T\cos\theta = F$.

4 Use friction

With $\mu = \frac{1}{3}$, $F = \frac{1}{3}R$. Therefore $T\cos\theta = \frac{1}{3}(mg - T\sin\theta)$.

5 Find T

Substitute $\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$: $\frac{4}{5}T = \frac{1}{3}\left(mg - \frac{3}{5}T\right)$. This gives $\frac{4}{5}T = \frac{mg}{3} - \frac{1}{5}T$, so $T = \frac{mg}{3}$.

6 **Find the deceleration when the rope is removed**

On the horizontal plane without the rope, $R = mg$ and friction is $\frac{1}{3}mg$. Hence the acceleration is $-\frac{g}{3}$.

7 **Use SUVAT to find d**

When the speed has fallen from u to $\frac{1}{2}u$, use $v^2 = u^2 + 2as$: $\left(\frac{u}{2}\right)^2 = u^2 + 2\left(-\frac{g}{3}\right)d$.

8 **Solve for d**

$\frac{u^2}{4} = u^2 - \frac{2gd}{3}$, so $\frac{2gd}{3} = \frac{3u^2}{4}$ and $d = \frac{9u^2}{8g}$.

FINAL ANSWER

$(a)\ T = \frac{mg}{3}$. $(b)\ d = \frac{9u^2}{8g}$.

WME01/01 OCTOBER 2024

Question 6

12 marks | Constant Acceleration in 1D | Main topic: Resolving Forces, Inclined Planes

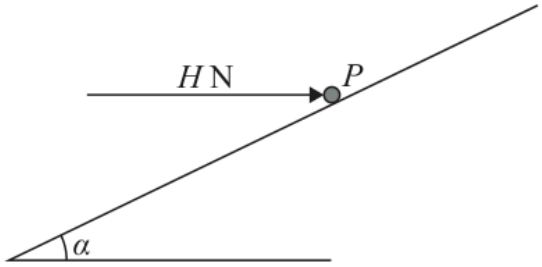


Figure 4

A particle P of mass 5 kg lies on the surface of a rough plane.

The plane is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$

The particle is held in equilibrium by a horizontal force of magnitude H newtons, as shown in Figure 4.

The horizontal force acts in a vertical plane containing a line of greatest slope of the inclined plane.

The coefficient of friction between the particle and the plane is $\frac{1}{4}$

(a) Find the smallest possible value of H.

(6)

The horizontal force is now removed, and P starts to slide down the slope.

In the first T seconds after P is released from rest, P slides 1.5 m down the slope.

(b) Find the value of T.

(6)

WORKED SOLUTION

Question 6 · WME01/01 October 2024

1 Use the triangle for alpha

From $\tan \alpha = \frac{3}{4}$, use $\sin \alpha = \frac{3}{5}$ and $\cos \alpha = \frac{4}{5}$.

2 Resolve perpendicular to the plane

The horizontal force has a component into the plane, so $R = 5g\cos\alpha + H\sin\alpha = 4g + \frac{3}{5}H$.

3 Resolve parallel for the smallest H

For the smallest H, the particle is on the point of sliding down, so friction acts up the plane. Parallel to the plane, $H\cos\alpha + F = 5g\sin\alpha$.

4 Use friction to find H

With $F = \frac{1}{4}R$, the parallel equation becomes $\frac{4}{5}H + \frac{1}{4}\left(4g + \frac{3}{5}H\right) = 3g$. Hence $0.95H + g = 3g$, so $H = \frac{40g}{19} = 20.6\ \text{N}$.

5 **Find the acceleration after H is removed**

When H is removed, $R = 5g\cos\alpha = 4g$ and friction is $\frac{1}{4}R = g$. The component of weight down the plane is $5g\sin\alpha = 3g$, so the resultant down the plane is $2g$.

6 **Apply Newtons second law**

$2g = 5a$, so $a = \dfrac{2g}{5} = 3.92\ \text{m s}^{-2}$ down the plane.

7 **Find T from the displacement**

From rest, the particle moves 1.5 m with acceleration $2g/5$. Using $s = \frac{1}{2}at^2$: $1.5 = \frac{1}{2}\left(\frac{2g}{5}\right)T^2 = \frac{g}{5}T^2$. Thus $T = \sqrt{\dfrac{7.5}{9.8}} = 0.875\ \text{s}$.

FINAL ANSWER

$(a)\ H = \dfrac{40g}{19}\ \text{N} \approx 20.6\ \text{N}.$ $\quad (b)\ T = 0.875\ \text{s}.$

WME01/01 JANUARY 2025

Question 7

15 marks | Constant Acceleration in 1D | Main topic: Newton's Second Law

7.

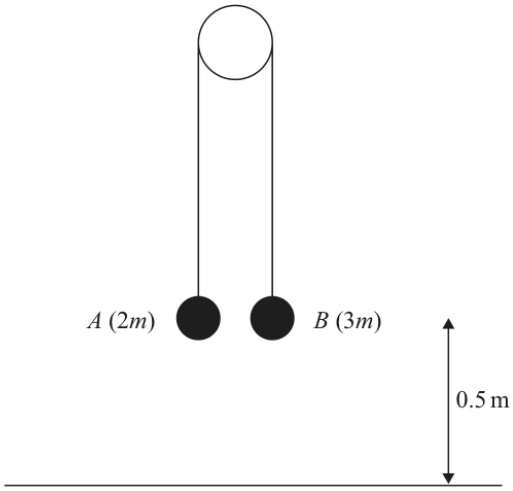


Figure 4

One end of a light inextensible string is attached to a particle A of mass $2m$.
The other end is attached to a particle B of mass $3m$.
The string passes over a small smooth fixed pulley.
The string is taut and both straight parts of the string are vertical.
Both particles are held at a distance 0.5 m above a horizontal surface, as shown in Figure 4.

The system is released from rest and B moves downwards.

(a) Find the tension in the string in terms of m and g.

(5)

(b) Find the speed of B at the instant it strikes the surface.

(4)

In the subsequent motion, A does not reach the pulley and B does not rebound after it strikes the surface.

(c) Find the time from the instant when the system is released from rest to the instant when A first reaches a height of 1.06 m above the surface.

(6)

WORKED SOLUTION

Question 7 · WME01/01 January 2025

1 Write the equations of motion

B moves down and A moves up with the same acceleration a. For A: $T - 2mg = 2ma$. For B: $3mg - T = 3ma$.

2 Find the acceleration

Adding the two equations gives $mg = 5ma$, so $a = \dfrac{g}{5}$.

3 Find the tension

Use $T - 2mg = 2ma$: $T = 2mg + 2m\left(\dfrac{g}{5}\right) = \dfrac{12mg}{5}$.

4 Find the speed when B hits the surface

B starts from rest and moves 0.5 m with acceleration $g/5$. Using $v^2 = u^2 + 2as$, $v^2 = 2\left(\dfrac{g}{5}\right)(0.5) = \dfrac{g}{5} = 1.96$.

5 Calculate the impact speed

$v = \sqrt{1.96} = 1.4\ \text{m s}^{-1}$.

6 **Find the first time interval**

The time until B hits the surface is found from $0.5 = \frac{1}{2}\left(\frac{g}{5}\right)t_1^2$. With $g = 9.8$, $t_1 = \frac{5}{7} = 0.714285\ldots$ s.

7 **Find the extra time after B stops**

After B hits the surface, A continues upwards freely under gravity. It is then at height 1.00 m and must rise another 0.06 m with initial speed $1.4\ \text{m s}^{-1}$. So $0.06 = 1.4t_2 - 4.9t_2^2$.

8 **Find the total time**

Solving $0.06 = 1.4t_2 - 4.9t_2^2$ gives the first positive time $t_2 = 0.0525\ldots$ s. Therefore the total time is $0.714285\ldots + 0.0525\ldots = 0.767$ s.

✓ **FINAL ANSWER**

$(a)\ T = \frac{12mg}{5}$. $(b)\ v = 1.4\ \text{m s}^{-1}$. $(c)\ t = 0.767$ s approximately.

WME01/01 MAY/JUNE 2025

Question 6

11 marks | Constant Acceleration in 1D

Main topic: Resolving Forces, Inclined Planes

6.

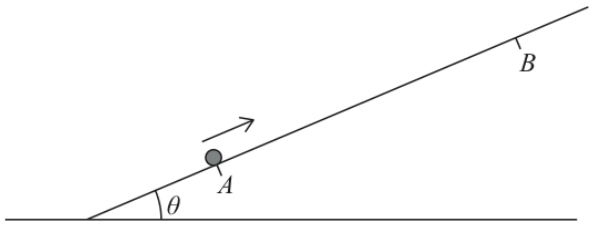


Figure 4

Figure 4 shows a rough plane inclined at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$

The points A and B lie on a line of greatest slope of the plane, with B above A.

A package P of mass m is projected up the plane from A towards B.

The coefficient of friction between the plane and the package is $\frac{1}{4}$

The package P is modelled as a particle.

(a) Show that the **deceleration** of P, as it moves from A to B, is $\frac{4}{5}g$ **(6)**

The package P comes to rest at B.

Given that P is projected from A with speed $U\,\text{m s}^{-1}$ and that $AB = 1.5\,\text{m}$,

(b) find the value of U. **(2)**

On reaching B, P is held at rest there by a force of magnitude X newtons acting up the plane in the direction AB.

Given that the mass of P is $2\,\text{kg}$,

(c) find the smallest possible value of X. **(3)**

WORKED SOLUTION

Question 6 · WME01/01 May/June 2025

1 **Part (a): Show deceleration = $\frac{4g}{5}$ - Find exact trig values**

Given: $\tan\theta = \frac{3}{4}$

Using 3-4-5 right triangle:
$\sin\theta = \frac{3}{5}, \quad \cos\theta = \frac{4}{5}$

2 **Part (a): Show deceleration = $\frac{4g}{5}$ - Resolve perpendicular to plane**

$\perp: \sum F = 0$

$R = mg\cos\theta\, R = \frac{4mg}{5}$

3 **Part (a): Show deceleration = $\frac{4g}{5}$ - Calculate friction force**

Package moving UP, so friction acts DOWN the plane:

$F = \mu R = \frac{1}{4} \times \frac{4mg}{5} = \frac{mg}{5}$

4 **Part (a): Show deceleration = $\frac{4g}{5}$ - Resolve parallel to plane**

Taking up the plane as positive:

$\parallel: \sum F = ma$

$-mg\sin\theta - F = ma \; -mg \times \frac{3}{5} - \frac{mg}{5} = ma \; -\frac{3mg}{5} - \frac{mg}{5} = ma \; -\frac{4mg}{5} = ma \; a = -\frac{4g}{5}$

Deceleration = $\left|-\frac{4g}{5}\right| = \frac{4g}{5}$

---

5 **Part (b): Find initial speed U - Identify SUVAT variables**

Package projected up plane, comes to rest at B:

; $u = U$ m s^{-1} (unknown)
; $v = 0$ m s^{-1} (comes to rest)
; $a = -\frac{4g}{5}$ m s^{-2} (from part a)
; $s = 1.5$ m

---

6 **Part (b): Find initial speed U - Apply $v^2 = u^2 + 2as$**

$v^2 = u^2 + 2as$

$0^2 = U^2 + 2\left(-\frac{4g}{5}\right)(1.5) \; 0 = U^2 - 2 \times \frac{4\times9.8}{5} \times 1.5 \; 0 = U^2 - \frac{8\times9.8\times1.5}{5} \; 0 = U^2 - 23.52 \; U^2 = 23.52 \; U = 4.85 \text{ m s}^{-1}$

Or using exact form: $U = \sqrt{2 \times \frac{4g}{5} \times 1.5} = \sqrt{\frac{12g}{5}}$

---

7 **Part (c): Find smallest value of X - Analyze forces when package at rest at B**

Package held at rest on slope by force X up the plane.

Without X, package would slide down (component down > friction up).

For MINIMUM X, friction acts UP the plane (at limiting friction).

---

8 **Part (c): Find smallest value of X - Resolve perpendicular to plane**

Mass = 2 kg, so weight = $2g$ N

$R = 2g\cos\theta = 2g \times \frac{4}{5} = \frac{8g}{5}$ N

---

9 **Part (c): Find smallest value of X - Calculate friction**

Friction acts UP the plane (limiting):

$F = \mu R = \frac{1}{4} \times \frac{8g}{5} = \frac{2g}{5}$ N

---

10 **Part (c): Find smallest value of X - Resolve parallel to plane**

For equilibrium (package at rest):

$\parallel: \sum F = 0$

Taking up plane as positive:

$X + F - 2g\sin\theta = 0 \; X + \frac{2g}{5} - 2g \times \frac{3}{5} = 0 \; X + \frac{2g}{5} - \frac{6g}{5} = 0 \; X = \frac{6g}{5} - \frac{2g}{5} \; X = \frac{4g}{5} \; X = \frac{4\times9.8}{5} \; X = 7.84$ N

✓ **FINAL ANSWER**

(a) **deceleration** $= \frac{4g}{5}$, as required.

(b) $U = 4.85 \text{ m s}^{-1}$ (about 4.8 m s^{-1}).

(c) $X = 7.84$ N (about 7.8 N).

Question 6

10 marks | Constant Acceleration in 1D | Main topic: Kinematics Graphs

6.

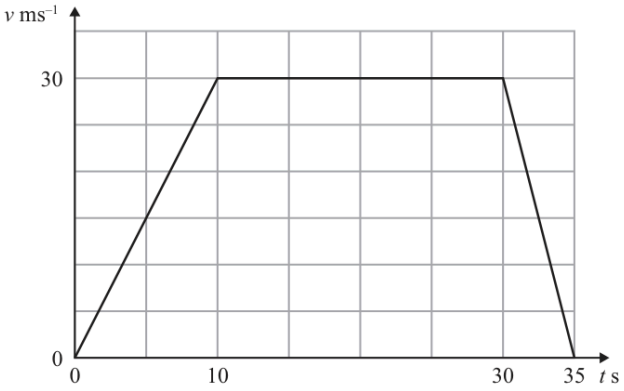


Figure 4

The point O lies on a straight horizontal road.
At time $t = 0$, a car leaves O and travels along the road.

The velocity-time graph in Figure 4 shows the velocity, $v\,\text{m s}^{-1}$, of the car at time t seconds for the first 35 seconds of its journey.

(a) Find

(i) the acceleration of the car for the period $0 \leqslant t \leqslant 10$

(ii) the deceleration of the car for the period $30 \leqslant t \leqslant 35$

(2)

(b) Sketch an acceleration-time graph for the car for the period $0 \leqslant t \leqslant 35$

(2)

(c) Find the distance travelled by the car for the period $0 \leqslant t \leqslant 35$

(2)

When $t = 5$, a motorcycle starts from rest at O.
The motorcycle travels along the same road as the car and in the same direction.

For the period $5 \leqslant t \leqslant 20$, the acceleration of the motorcycle is $A\,\text{m s}^{-2}$, where A is a positive constant.

The motorcycle catches up with the car when $t = 20$

(d) Find the value of A.

(4)

1 **Part (a): Find accelerations**

(i) Acceleration for $0 \leq t \leq 10$

$\text{Acceleration} = \frac{\Delta v}{\Delta t} = \text{gradient of v-t graph}$

$a = \frac{30-0}{10-0}$ $a = 3 \text{ m s}^{-2}$

(ii) Deceleration for $30 \leq t \leq 35$

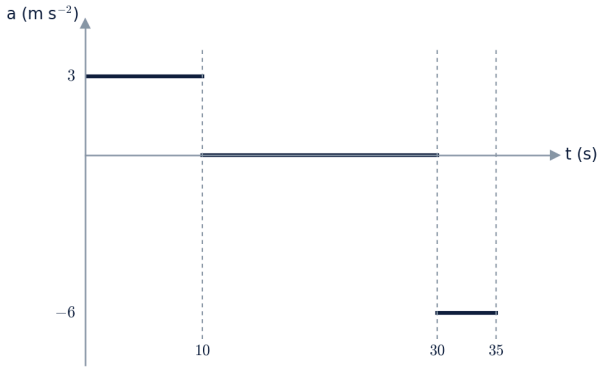
$a = \frac{0-30}{35-30}$ $a = \frac{-30}{5}$ $a = -6 \text{ m s}^{-2}$

Deceleration = $|-6| = 6$ m s^{-2}

2 **Part (b): Sketch acceleration-time graph**

Analysis of each section:

; $0 \leq t \leq 10$: Constant acceleration = 3 m s^{-2}
; $10 < t \leq 30$: Constant velocity, so acceleration = 0 m s^{-2}
; $30 < t \leq 35$: Constant deceleration = -6 m s^{-2}



3 **Part (c): Find distance travelled for $0 \leq t \leq 35$ - Calculate area under v-t graph**

$\text{Distance} = \text{Area under v-t graph}$

The shape is a trapezium. We can split it into regions:

Method 1: Trapezium formula

$\text{Area} = \frac{1}{2}(a+b)h$

Where the parallel sides are the time intervals and height is velocity.

Better to calculate as sum of shapes:

Method 2: Sum of triangle + rectangle + triangle

$\text{Area}_1 = \frac{1}{2} \times 10 \times 30 = 150 \text{ m}$ $\text{Area}_2 = 20 \times 30 = 600 \text{ m}$ $\text{Area}_3 = \frac{1}{2} \times 5 \times 30 = 75 \text{ m}$

$\text{Total distance} = 150 + 600 + 75 = 825 \text{ m}$

4 **Part (d): Find value of A - Distance travelled by car (0 to 20 seconds)**

Area under v-t graph from $t = 0$ to $t = 20$:

$$\text{Distance}_{\text{car}} = \text{Triangle} + \text{Rectangle} = \tfrac{1}{2}(10)(30) + (10)(30) = 150 + 300 = 450 \text{ m}$$

5 **Part (d): Find value of A - Distance travelled by motorcycle (5 to 20 seconds)**

Motorcycle starts at $t = 5$ with constant acceleration A.

Time of travel: $20 - 5 = 15$ seconds

$$s = ut + \tfrac{1}{2}at^2$$

$$\text{Distance}_{\text{motorcycle}} = 0 + \tfrac{1}{2}A(15)^2 = \tfrac{225A}{2} = 112.5A \text{ m}$$

6 **Part (d): Find value of A - Set distances equal**

Motorcycle catches up with car when $t = 20$:

$$112.5A = 450 \; A = \tfrac{450}{112.5} \; A = 4 \text{ m s}^{-2}$$

✓ **FINAL ANSWER**

$(a)(i)\ a = 3 \text{ m s}^{-2},\ (ii)$ deceleration $= 6 \text{ m s}^{-2}$. (b) correct acceleration-time graph with levels $3, 0, -6$. (c) distance $= 825$ m. $(d)\ A = 4 \text{ m s}^{-2}$.

TOPIC

Constant Acceleration in 2D

Every past-paper question on this topic, October 2019 to January 2026.

WME01/01 JANUARY 2021

Question 6

12 marks | Constant Acceleration in 2D | Main topic: Working with Vectors

6. Two girls, Agatha and Brionie, are roller skating inside a large empty building. The girls are modelled as particles.

At time $t = 0$, Agatha is at the point with position vector $(11\mathbf{i} + 11\mathbf{j})$ m and Brionie is at the point with position vector $(7\mathbf{i} + 16\mathbf{j})$ m. The position vectors are given relative to the door, O, and $\mathbf{i}$ and $\mathbf{j}$ are horizontal perpendicular unit vectors.

Agatha skates with constant velocity $(3\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$

Brionie skates with constant velocity $(4\mathbf{i} - 2\mathbf{j})\,\text{m s}^{-1}$

(a) Find the position vector of Agatha at time t seconds.

(2)

At time $t = 6$ seconds, Agatha passes through the point P.

(b) Show that Brionie also passes through P and find the value of t when this occurs.

(4)

At time t seconds, Agatha is at the point A and Brionie is at the point B.

(c) Show that $\overrightarrow{AB} = [(t - 4)\mathbf{i} + (5 - t)\mathbf{j}]$ m

(2)

(d) Find the distance between the two girls when they are closest together.

(4)

WORKED SOLUTION

Question 6 · WME01/01 January 2021

1 Write Agatha position

$\mathbf{r}_A = (11\mathbf{i} + 11\mathbf{j}) + t(3\mathbf{i} - \mathbf{j}) = (11 + 3t)\mathbf{i} + (11 - t)\mathbf{j}$.

2 Find point P

At $t = 6$, Agatha is at $(11 + 18)\mathbf{i} + (11 - 6)\mathbf{j} = 29\mathbf{i} + 5\mathbf{j}$.

3 Write Brionie position

$\mathbf{r}_B = (7\mathbf{i} + 16\mathbf{j}) + t(4\mathbf{i} - 2\mathbf{j}) = (7 + 4t)\mathbf{i} + (16 - 2t)\mathbf{j}$.

4 Show Brionie passes P

Set $7 + 4t = 29$ to get $t = 5.5$. Also $16 - 2t = 5$ gives $t = 5.5$. So Brionie also passes through P at $t = 5.5$ s.

5 Find AB vector

$\overrightarrow{AB} = \mathbf{r}_B - \mathbf{r}_A = [7 + 4t - (11 + 3t)]\mathbf{i} + [16 - 2t - (11 - t)]\mathbf{j}$.

6 Simplify AB

So $\overrightarrow{AB} = (t - 4)\mathbf{i} + (5 - t)\mathbf{j}$ m.

7 **Minimise distance squared**

$AB^2 = (t-4)^2 + (5-t)^2 = 2(t-4.5)^2 + 0.5$.

8 **State the minimum distance**

The minimum occurs when $t = 4.5$, giving $AB^2 = 0.5$. Hence the minimum distance is $\sqrt{0.5} = 0.707\ldots$ m.

FINAL ANSWER

$(a)\ \mathbf{r}_A = (11+3t)\mathbf{i} + (11-t)\mathbf{j}$. $(b)\ t = 5.5$ s. $(c)\ \overrightarrow{AB} = (t-4)\mathbf{i} + (5-t)\mathbf{j}$. (d) minimum distance $= \sqrt{0.5}$ m ≈ 0.71 m.

WME01/01 OCTOBER 2021

Question 8

12 marks | Constant Acceleration in 2D | Main topic: Working with Vectors

8. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin.*]

At 7 am a ship leaves a port and moves with constant velocity. The position vector of the port is $(-2\mathbf{i} + 9\mathbf{j})$ km.

At 7.36 am the ship is at the point with position vector $(4\mathbf{i} + 6\mathbf{j})$ km.

(a) Show that the velocity of the ship is $(10\mathbf{i} - 5\mathbf{j})\,\text{km h}^{-1}$

(2)

(b) Find the position vector of the ship t hours after leaving port.

(2)

At 8.48 am a passenger on the ship notices that a lighthouse is due east of the ship.

At 9 am the same passenger notices that the lighthouse is now north east of the ship.

(c) Find the position vector of the lighthouse.

(4)

(d) Find the position vector of the ship when it is due south of the lighthouse.

(4)

(Total 12 marks)

WORKED SOLUTION

Question 8 · WME01/01 October 2021

1 Convert the time

From 7:00 to 7:36 is 36 minutes, which is 0.6 hours.

2 Find velocity

Displacement $= (4\mathbf{i}+6\mathbf{j}) - (-2\mathbf{i}+9\mathbf{j}) = 6\mathbf{i} - 3\mathbf{j}$. Divide by 0.6: $\mathbf{v} = 10\mathbf{i} - 5\mathbf{j}$.

3 Write the ship position

$\mathbf{r} = (-2\mathbf{i}+9\mathbf{j}) + t(10\mathbf{i} - 5\mathbf{j}) = (-2+10t)\mathbf{i} + (9-5t)\mathbf{j}$.

4 Use 8:48

At 8:48, $t = 1.8$. The ship position is $(-2+18)\mathbf{i} + (9-9)\mathbf{j} = 16\mathbf{i}$. The lighthouse is due east, so it has the same j-component, 0.

5 Use 9:00

At 9:00, $t = 2$. The ship position is $18\mathbf{i} - \mathbf{j}$. The lighthouse is north-east of the ship, so its east and north displacements from the ship are equal.

6 Find the lighthouse position

Since the lighthouse has j-component 0, from $18\mathbf{i} - \mathbf{j}$ it is 1 km north, so it is also 1 km east. Hence its position is $19\mathbf{i}$.

7 Find when the ship is due south

Due south of the lighthouse means the ship has i-component 19. Set $-2 + 10t = 19$, so $t = 2.1$.

8 **Find the ship position**

At $t = 2.1$, $\mathbf{r} = (-2 + 21)\mathbf{i} + (9 - 10.5)\mathbf{j} = 19\mathbf{i} - 1.5\mathbf{j}$.

FINAL ANSWER

$(a)\ \mathbf{v} = 10\mathbf{i} - 5\mathbf{j}\ \text{km h}^{-1}$. $(b)\ \mathbf{r} = (-2 + 10t)\mathbf{i} + (9 - 5t)\mathbf{j}$. (c) lighthouse $= 19\mathbf{i}$. (d) ship position $= 19\mathbf{i} - 1.5\mathbf{j}$.

WME01/01 JANUARY 2022

Question 6

12 marks | Constant Acceleration in 2D | Main topic: Working with Vectors

6. [*In this question* **i** *and* **j** *are horizontal unit vectors.*]

A particle P of mass 2 kg moves under the action of two forces, $(p\mathbf{i} + q\mathbf{j})$N and $(2q\mathbf{i} + p\mathbf{j})$N, where p and q are constants.

Given that the acceleration of P is $(\mathbf{i} - \mathbf{j})\,\mathrm{m\,s^{-2}}$

(a) find the value of p and the value of q.

(5)

(b) Find the size of the angle between the direction of the acceleration and the vector $\mathbf{j}$.

(2)

At time $t = 0$, the velocity of P is $(3\mathbf{i} - 4\mathbf{j})\,\mathrm{m\,s^{-1}}$

At $t = T$ seconds, P is moving in the direction of the vector $(11\mathbf{i} - 13\mathbf{j})$.

(c) Find the value of T.

(5)

WORKED SOLUTION

Question 6 · WME01/01 January 2022

1 Use F = ma

The total force is $(p\mathbf{i} + q\mathbf{j}) + (2q\mathbf{i} + p\mathbf{j}) = (p + 2q)\mathbf{i} + (p + q)\mathbf{j}$. Since the mass is 2 kg and acceleration is $\mathbf{i} - \mathbf{j}$, the total force is $2\mathbf{i} - 2\mathbf{j}$.

2 Equate components

$p + 2q = 2$ and $p + q = -2$.

3 Find p and q

Subtract the equations to get $q = 4$. Then $p + 4 = -2$, so $p = -6$.

4 Find the angle with j

The acceleration is $\mathbf{i} - \mathbf{j}$. Its dot product with $\mathbf{j}$ is -1, so the angle with the positive $\mathbf{j}$ direction is obtuse. Since the components are equal in magnitude, the angle is $135°$.

5 Write the velocity at time T

$\mathbf{v} = (3\mathbf{i} - 4\mathbf{j}) + T(\mathbf{i} - \mathbf{j}) = (3 + T)\mathbf{i} + (-4 - T)\mathbf{j}$.

6 Use the direction vector

The velocity is parallel to $11\mathbf{i} - 13\mathbf{j}$, so $\dfrac{3 + T}{11} = \dfrac{-4 - T}{-13}$.

7 Solve for T

$13(3 + T) = 11(4 + T)$, so $39 + 13T = 44 + 11T$. Hence $2T = 5$ and $T = 2.5$.

FINAL ANSWER

$(a)\ p = -6,\ q = 4.\quad (b)\ 135°.\quad (c)\ T = 2.5.$

WME01/01 JANUARY 2022

Question 8

14 marks | Constant Acceleration in 2D | Main topic: Working with Vectors

8. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin.*]

A ship A moves with constant velocity $(3\mathbf{i} - 10\mathbf{j})\,\text{km h}^{-1}$

At time t hours, the position vector of A is $\mathbf{r}$ km.

At time $t = 0$, A is at the point with position vector $(13\mathbf{i} + 5\mathbf{j})$ km.

(a) Find $\mathbf{r}$ in terms of t.

(2)

Another ship B moves with constant velocity $(15\mathbf{i} + 14\mathbf{j})\,\text{km h}^{-1}$

At time $t = 0$, B is at the point with position vector $(3\mathbf{i} - 5\mathbf{j})$ km.

(b) Show that, at time t hours,

$$\overrightarrow{AB} = [(12t - 10)\mathbf{i} + (24t - 10)\mathbf{j}]\,\text{km}$$

(4)

Given that the two ships do not change course,

(c) find the shortest distance between the two ships,

(6)

(d) find the bearing of ship B from ship A when the ships are closest.

(2)

(Total 14 marks)

WORKED SOLUTION

Question 8 · WME01/01 January 2022

1 Write the position of A

$\mathbf{r} = (13\mathbf{i} + 5\mathbf{j}) + t(3\mathbf{i} - 10\mathbf{j}) = (13 + 3t)\mathbf{i} + (5 - 10t)\mathbf{j}$.

2 Write the position of B

$\mathbf{s} = (3\mathbf{i} - 5\mathbf{j}) + t(15\mathbf{i} + 14\mathbf{j}) = (3 + 15t)\mathbf{i} + (-5 + 14t)\mathbf{j}$.

3 Find AB

$\overrightarrow{AB} = \mathbf{s} - \mathbf{r} = [3 + 15t - (13 + 3t)]\mathbf{i} + [-5 + 14t - (5 - 10t)]\mathbf{j}$.

4 Show the expression

This simplifies to $\overrightarrow{AB} = (12t - 10)\mathbf{i} + (24t - 10)\mathbf{j}$ km.

5 Use distance squared

The square of the distance is $d^2 = (12t - 10)^2 + (24t - 10)^2$.

6 **Expand and minimise**

$d^2 = 720t^2 - 720t + 200$. This quadratic is minimum at $t = \dfrac{720}{2 \cdot 720} = \dfrac{1}{2}$.

7 **Find the shortest distance**

At $t = \dfrac{1}{2}$, $\overrightarrow{AB} = (-4)\mathbf{i} + 2\mathbf{j}$, so the shortest distance is $\sqrt{(-4)^2 + 2^2} = \sqrt{20} = 2\sqrt{5}\ \text{km}$.

8 **Find the bearing**

At closest approach, B is west and north of A. The angle west of north satisfies $\tan\theta = \dfrac{4}{2} = 2$, so $\theta = 63.4°$. Bearing $= 360° - 63.4° = 296.6°$, which is $297°$.

FINAL ANSWER

$(a)\ \mathbf{r} = (13 + 3t)\mathbf{i} + (5 - 10t)\mathbf{j}.$ $(b)\ \overrightarrow{AB} = (12t - 10)\mathbf{i} + (24t - 10)\mathbf{j}.$ $(c)\ 2\sqrt{5}\ \text{km} \approx 4.5\ \text{km}.$ $(d)\ 297°.$

Question 6

6 marks | Constant Acceleration in 2D | Main topic: Working with Vectors

6. A particle P is moving with constant acceleration.

At time $t = 1$ second, P has velocity $(-\mathbf{i} + 4\mathbf{j})\,\mathrm{m\,s^{-1}}$

At time $t = 4$ seconds, P has velocity $(5\mathbf{i} - 8\mathbf{j})\,\mathrm{m\,s^{-1}}$

Find the speed of P at time $t = 3.5$ seconds.

(6)

WORKED SOLUTION

Question 6 · WME01/01 May/June 2022

1 Find the acceleration

The velocity changes from $-\mathbf{i} + 4\mathbf{j}$ at $t = 1$ to $5\mathbf{i} - 8\mathbf{j}$ at $t = 4$. The time difference is 3 s.

2 Calculate acceleration

$$\mathbf{a} = \frac{(5\mathbf{i} - 8\mathbf{j}) - (-\mathbf{i} + 4\mathbf{j})}{3} = \frac{6\mathbf{i} - 12\mathbf{j}}{3} = 2\mathbf{i} - 4\mathbf{j}.$$

3 Find velocity at t = 3.5

From $t = 1$ to $t = 3.5$ is 2.5 s, so $\mathbf{v} = (-\mathbf{i} + 4\mathbf{j}) + 2.5(2\mathbf{i} - 4\mathbf{j})$.

4 Simplify the velocity

$\mathbf{v} = -\mathbf{i} + 4\mathbf{j} + 5\mathbf{i} - 10\mathbf{j} = 4\mathbf{i} - 6\mathbf{j}$.

5 Find the speed

Speed $= \sqrt{4^2 + (-6)^2} = \sqrt{52} = 2\sqrt{13}\,\mathrm{m\,s^{-1}}$, about $7.2\,\mathrm{m\,s^{-1}}$.

FINAL ANSWER

$\text{speed} = \sqrt{52} = 2\sqrt{13}\,\mathrm{m\,s^{-1}} \approx 7.2\,\mathrm{m\,s^{-1}}$.

Question 8

17 marks | Constant Acceleration in 2D | Main topic: Working with Vectors

8. [*In this question,* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two boats, P and Q, are moving with constant velocities.

The velocity of P is $15\mathbf{i}\,\text{m s}^{-1}$ and the velocity of Q is $(20\mathbf{i} - 20\mathbf{j})\,\text{m s}^{-1}$

(a) Find the direction in which Q is travelling, giving your answer as a bearing.

(2)

The boats are modelled as particles.

At time $t = 0$, P is at the origin O and Q is at the point with position vector $200\mathbf{j}$ m.

At time t seconds, the position vector of P is $\mathbf{p}$ m and the position vector of Q is $\mathbf{q}$ m.

(b) Show that

$$\overrightarrow{PQ} = [5t\mathbf{i} + (200 - 20t)\mathbf{j}]\,\text{m}$$

(5)

(c) Find the bearing of P from Q when $t = 10$

(2)

(d) Find the distance between P and Q when Q is north east of P

(5)

(e) Find the times when P and Q are 200 m apart.

(3)

(Total 17 marks)

TOTAL FOR PAPER: 75 MARKS

WORKED SOLUTION

Question 8 · WME01/01 May/June 2022

1 Find the bearing of Q

Q has velocity $20\mathbf{i} - 20\mathbf{j}$, so it travels south-east with equal east and south components. Its bearing is $135°$.

2 Write P position

P starts at the origin and has velocity $15\mathbf{i}$, so $\mathbf{p} = 15t\mathbf{i}$.

3 Write Q position

Q starts at $200\mathbf{j}$ and has velocity $20\mathbf{i} - 20\mathbf{j}$, so $\mathbf{q} = 20t\mathbf{i} + (200 - 20t)\mathbf{j}$.

4 Show PQ

$\overrightarrow{PQ} = \mathbf{q} - \mathbf{p} = (20t - 15t)\mathbf{i} + (200 - 20t)\mathbf{j} = 5t\mathbf{i} + (200 - 20t)\mathbf{j}$.

5 Find the bearing at t = 10

At $t = 10$, $\overrightarrow{PQ} = 50\mathbf{i}$. This means Q is east of P, so P is west of Q. The bearing of P from Q is $270°$.

6 **Use the north-east condition**

Q is north-east of P when the i and j components of $\overrightarrow{PQ}$ are equal and positive. So $5t = 200 - 20t$.

7 **Find the distance then**

The equation gives $t = 8$. Then $\overrightarrow{PQ} = 40\mathbf{i} + 40\mathbf{j}$, so the distance is $\sqrt{40^2 + 40^2} = 40\sqrt{2}\ \text{m}$.

8 **Set the 200 m distance equation**

For distance 200 m, $(5t)^2 + (200 - 20t)^2 = 200^2$.

9 **Solve for t**

This simplifies to $425t^2 - 8000t = 0$, so $t(425t - 8000) = 0$. Hence $t = 0$ or $t = \dfrac{8000}{425} = \dfrac{320}{17}$ s.

FINAL ANSWER

$(a)\ 135°.\quad (b)\ \overrightarrow{PQ} = 5t\mathbf{i} + (200 - 20t)\mathbf{j}.\quad (c)\ 270°.\quad (d)\ 40\sqrt{2}\ \text{m}.\quad (e)\ t = 0\ \text{s or}\ \dfrac{320}{17}\ \text{s}.$

WME01/01 OCTOBER 2022

Question 8

16 marks | Constant Acceleration in 2D | Main topic: Working with Vectors

8. [*In this question,* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two ships, A and B, are moving with constant velocities.
The velocity of A is $(3\mathbf{i} + 12\mathbf{j})\,\text{km h}^{-1}$ and the velocity of B is $(p\mathbf{i} + q\mathbf{j})\,\text{km h}^{-1}$

(a) Find the speed of A.

(2)

The ships are modelled as particles.
At 12 noon, A is at the point with position vector $(-9\mathbf{i} + 6\mathbf{j})$ km and B is at the point with position vector $(16\mathbf{i} + 6\mathbf{j})$ km.

At time t hours after 12 noon,

$$\overrightarrow{AB} = [(25 - 12t)\mathbf{i} - 9t\mathbf{j}]\,\text{km}$$

(b) Find the value of p and the value of q.

(7)

(c) Find the bearing of A from B when the ships are 15 km apart, giving your answer to the nearest degree.

(7)

(Total 16 marks)

WORKED SOLUTION

Question 8 · WME01/01 October 2022

1 Find the speed of A

$|\mathbf{v}_A| = \sqrt{3^2 + 12^2} = \sqrt{153} = 3\sqrt{17}\ \text{km h}^{-1}$.

2 Write A position

At time t, $\mathbf{a} = (-9\mathbf{i} + 6\mathbf{j}) + t(3\mathbf{i} + 12\mathbf{j}) = (-9 + 3t)\mathbf{i} + (6 + 12t)\mathbf{j}$.

3 Write B position

$\mathbf{b} = (16\mathbf{i} + 6\mathbf{j}) + t(p\mathbf{i} + q\mathbf{j}) = (16 + pt)\mathbf{i} + (6 + qt)\mathbf{j}$.

4 Use AB vector

$\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = [25 + (p - 3)t]\mathbf{i} + [(q - 12)t]\mathbf{j}$.

5 Compare components

Given $\overrightarrow{AB} = (25 - 12t)\mathbf{i} - 9t\mathbf{j}$, compare coefficients: $p - 3 = -12$ and $q - 12 = -9$.

6 Find p and q

Therefore $p = -9$ and $q = 3$.

7 Find the time when the distance is 15

Use $|\overrightarrow{AB}| = 15$: $(25 - 12t)^2 + (-9t)^2 = 15^2$.

8 **Solve the distance equation**

This gives $225t^2 - 600t + 400 = 0$, so $(3t - 4)^2 = 0$ and $t = \dfrac{4}{3}$.

9 **Find BA at this time**

At $t = \dfrac{4}{3}$, $\overrightarrow{AB} = 9\mathbf{i} - 12\mathbf{j}$. Therefore $\overrightarrow{BA} = -9\mathbf{i} + 12\mathbf{j}$.

10 **Convert to a bearing**

A is north-west of B. The angle west of north satisfies $\tan\theta = \dfrac{9}{12}$, so $\theta = 36.9°$. Bearing $= 360° - 36.9° = 323°$ to the nearest degree.

FINAL ANSWER

$(a)\ 3\sqrt{17}\ \text{km h}^{-1}$. $(b)\ p = -9,\ q = 3$. $(c)\ \text{bearing} = 323°$.

WME01/01 MAY/JUNE 2023

Question 8

9 marks | Constant Acceleration in 2D | Main topic: Working with Vectors

8.

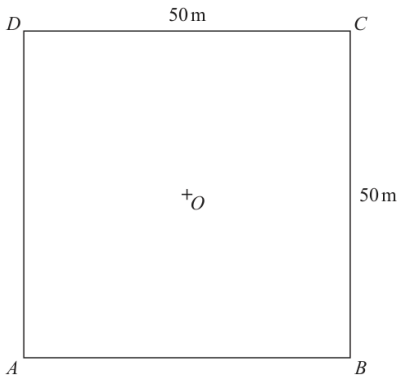


Figure 5

A square floor space $ABCD$, with centre O, is modelled as a flat horizontal surface measuring 50 m by 50 m, as shown in Figure 5.
The horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in the direction of $\overrightarrow{AB}$ and $\overrightarrow{AD}$ respectively.

All position vectors are given relative to O.

A small robot R is programmed to travel across the floor at a constant velocity.

- At time $t = 0$, R is at the point with position vector $(-2\mathbf{i} + \mathbf{j})$ m
- At time $t = 11$ s, R is at the point with position vector $(9\mathbf{i} + 23\mathbf{j})$ m
- At time t seconds, the position vector of R is $\mathbf{r}$ metres

(a) Find, in terms of t, $\mathbf{i}$ and $\mathbf{j}$, an expression for $\mathbf{r}$

(3)

A second robot S is at the point C.

- At time $t = 0$, S leaves C and moves with constant velocity $(-\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$
- At time t seconds, the position vector of S is $\mathbf{s}$ metres

(b) Write down, in terms of t, $\mathbf{i}$ and $\mathbf{j}$, an expression for $\mathbf{s}$

(1)

(c) Show that

$$\overrightarrow{SR} = [(2t - 27)\mathbf{i} + (3t - 24)\mathbf{j}]\text{ m}$$

(2)

(d) Find the time when the distance between R and S is a minimum.

(3)

WORKED SOLUTION

Question 8 · WME01/01 May/June 2023

1 Find the velocity of R

R moves from $-2\mathbf{i} + \mathbf{j}$ to $9\mathbf{i} + 23\mathbf{j}$ in 11 s. Its displacement is $11\mathbf{i} + 22\mathbf{j}$, so its velocity is $\mathbf{i} + 2\mathbf{j}$.

2 Write r

$\mathbf{r} = (-2\mathbf{i} + \mathbf{j}) + t(\mathbf{i} + 2\mathbf{j}) = (t - 2)\mathbf{i} + (2t + 1)\mathbf{j}$.

3 Write s

Point C has position vector $25\mathbf{i} + 25\mathbf{j}$. Since S moves with velocity $-\mathbf{i} - \mathbf{j}$, $\mathbf{s} = (25 - t)\mathbf{i} + (25 - t)\mathbf{j}$.

4 Find SR vector

$\overrightarrow{SR} = \mathbf{r} - \mathbf{s} = [(t - 2) - (25 - t)]\mathbf{i} + [(2t + 1) - (25 - t)]\mathbf{j}$.

5 Simplify SR

Therefore $\overrightarrow{SR} = (2t - 27)\mathbf{i} + (3t - 24)\mathbf{j}$ m.

6 **Minimise the distance squared**

The distance squared is $d^2 = (2t - 27)^2 + (3t - 24)^2$.

7 **Expand the quadratic**

$d^2 = 4t^2 - 108t + 729 + 9t^2 - 144t + 576 = 13t^2 - 252t + 1305$.

8 **Find the minimum time**

The quadratic is minimum at $t = \dfrac{252}{2 \cdot 13} = \dfrac{126}{13} = 9.69\ldots$ s.

FINAL ANSWER

$(a)\ \mathbf{r} = (t - 2)\mathbf{i} + (2t + 1)\mathbf{j}.$ $(b)\ \mathbf{s} = (25 - t)\mathbf{i} + (25 - t)\mathbf{j}.$ $(c)\ \overrightarrow{SR} = (2t - 27)\mathbf{i} + (3t - 24)\mathbf{j}.$ $(d)\ t = \dfrac{126}{13}\ \text{s} \approx 9.7\ \text{s}.$

WME01/01 OCTOBER 2023

Question 6

15 marks | Constant Acceleration in 2D | Main topic: Working with Vectors

6. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

At 12:00, a ship P sets sail from a harbour with position vector $(15\mathbf{i} + 36\mathbf{j})$ km.
At 12:30, P is at the point with position vector $(20\mathbf{i} + 34\mathbf{j})$ km.

Given that P moves with constant velocity,

(a) show that the velocity of P is $(10\mathbf{i} - 4\mathbf{j})\,\text{km h}^{-1}$

(2)

At time t hours after 12:00, the position vector of P is $\mathbf{p}$ km.

(b) Find an expression for $\mathbf{p}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and t.

(2)

A second ship Q is also travelling at a constant velocity.

At time t hours after 12:00, the position vector of Q is given by $\mathbf{q}$ km, where

$$\mathbf{q} = (42 - 8t)\mathbf{i} + (9 + 14t)\mathbf{j}$$

Ships P and Q are modelled as particles.

If both ships maintained their course,

(c) (i) verify that they would collide at 13:30

(ii) find the position vector of the point at which the collision would occur.

(4)

At 12:30 Q changes speed and direction to avoid the collision.

Ship Q now travels due north with a constant speed of $15\,\text{km h}^{-1}$

Ship P maintains the same constant velocity throughout.

(d) Find the exact distance between P and Q at 14:30

(7)

WORKED SOLUTION

Question 6 · WME01/01 October 2023

1 Find the velocity of P

From 12:00 to 12:30, the displacement of P is $(20\mathbf{i} + 34\mathbf{j}) - (15\mathbf{i} + 36\mathbf{j}) = 5\mathbf{i} - 2\mathbf{j}$ km. The time is 0.5 hours, so $\mathbf{v}_P = \dfrac{5\mathbf{i} - 2\mathbf{j}}{0.5} = 10\mathbf{i} - 4\mathbf{j}$.

2 Write the position of P

At t hours after 12:00, $\mathbf{p} = (15\mathbf{i} + 36\mathbf{j}) + t(10\mathbf{i} - 4\mathbf{j}) = (15 + 10t)\mathbf{i} + (36 - 4t)\mathbf{j}$.

3 Verify the collision time

At 13:30, $t = 1.5$. Then $\mathbf{p} = (15 + 15)\mathbf{i} + (36 - 6)\mathbf{j} = 30\mathbf{i} + 30\mathbf{j}$. Also $\mathbf{q} = (42 - 8(1.5))\mathbf{i} + (9 + 14(1.5))\mathbf{j} = 30\mathbf{i} + 30\mathbf{j}$.

4 State the collision position

Since both position vectors are the same, the collision point has position vector $30\mathbf{i} + 30\mathbf{j}$ km.

5 Find P at 14:30

At 14:30, $t = 2.5$, so $\mathbf{p} = (15 + 25)\mathbf{i} + (36 - 10)\mathbf{j} = 40\mathbf{i} + 26\mathbf{j}$.

6 **Find Q after changing course**

At 12:30, $t = 0.5$, so $\mathbf{q} = 38\mathbf{i} + 16\mathbf{j}$. From 12:30 to 14:30 is 2 hours due north at $15\ \text{km h}^{-1}$, so the extra displacement is $30\mathbf{j}$. Thus Q is at $38\mathbf{i} + 46\mathbf{j}$.

7 **Find the distance**

The separation vector is $(40 - 38)\mathbf{i} + (26 - 46)\mathbf{j} = 2\mathbf{i} - 20\mathbf{j}$. The distance is $\sqrt{2^2 + 20^2} = \sqrt{404} = 2\sqrt{101}\ \text{km}$.

✓ **FINAL ANSWER**

$(a)\ \mathbf{v}_P = 10\mathbf{i} - 4\mathbf{j}\ \text{km h}^{-1}$. $(b)\ \mathbf{p} = (15 + 10t)\mathbf{i} + (36 - 4t)\mathbf{j}$. $(c)(ii)\ 30\mathbf{i} + 30\mathbf{j}$. $(d)\ 2\sqrt{101}\ \text{km}$.

WME01/01 JANUARY 2024

Question 7

11 marks | Constant Acceleration in 2D | Main topic: Working with Vectors

7. [*In this question* $\mathbf{i}$ *and* $\mathbf{j}$ *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin* O.]

At midnight, a ship S is at the point with position vector $(19\mathbf{i} + 22\mathbf{j})$ km

The ship travels with constant velocity $(12\mathbf{i} - 16\mathbf{j})\,\text{km h}^{-1}$

(a) Find the speed of S.

(2)

At time t hours after midnight, the position vector of S is $\mathbf{s}$ km.

(b) Find an expression for $\mathbf{s}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and t.

(2)

A lighthouse stands on a small rocky island. The lighthouse is modelled as being at the point with position vector $(26\mathbf{i} + 15\mathbf{j})$ km.

It is not safe for ships to be within 1.3 km of the lighthouse.

(c) (i) Find the value of t when S is closest to the lighthouse.

(ii) Hence determine whether it is safe for S to continue its course.

(7)

WORKED SOLUTION

Question 7 · WME01/01 January 2024

1 Find the speed

The velocity is $12\mathbf{i} - 16\mathbf{j}$, so the speed is $\sqrt{12^2 + (-16)^2} = 20\ \text{km h}^{-1}$.

2 Write the ship position vector

At time t hours after midnight, $\mathbf{s} = (19\mathbf{i} + 22\mathbf{j}) + t(12\mathbf{i} - 16\mathbf{j}) = (19 + 12t)\mathbf{i} + (22 - 16t)\mathbf{j}$.

3 Form the vector from lighthouse to ship

The lighthouse is at $26\mathbf{i} + 15\mathbf{j}$. Therefore $\overrightarrow{LS} = \mathbf{s} - \mathbf{l} = (12t - 7)\mathbf{i} + (7 - 16t)\mathbf{j}$.

4 Use distance squared

The square of the distance is $d^2 = (12t - 7)^2 + (7 - 16t)^2 = 400t^2 - 392t + 98$.

5 Find the time of closest approach

This quadratic is minimum when $t = \dfrac{392}{2 \cdot 400} = 0.49$.

6 Find the minimum distance

At $t = 0.49$, $d^2 = 1.96$, so $d = 1.4\ \text{km}$.

7 Decide if the course is safe

The unsafe distance is 1.3 km. Since $1.4 > 1.3$, it is safe for the ship to continue its course.

FINAL ANSWER

$(a)\ 20\ \text{km h}^{-1}.\quad (b)\ \mathbf{s} = (19 + 12t)\mathbf{i} + (22 - 16t)\mathbf{j}.\quad (c)(i)\ t = 0.49.\quad (c)(ii)\ \text{minimum distance} = 1.4\ \text{km},\ \text{so it is safe}.$

WME01/01 MAY/JUNE 2024

Question 7

13 marks | Constant Acceleration in 2D | Main topic: Working with Vectors

7. [*In this question, the horizontal unit vectors* **i** *and* **j** *are directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two speedboats, A and B, are each moving with constant velocity.

- the velocity of A is $40\,\text{km h}^{-1}$ due east
- the velocity of B is $20\,\text{km h}^{-1}$ on a bearing of angle α ($0° < \alpha < 90°$), where $\tan\alpha = \frac{4}{3}$

The boats are modelled as particles.

(a) Find, in terms of **i** and **j**, the velocity of B in km h^{-1}

(2)

At noon

- the position vector of A is $20\mathbf{j}\,\text{km}$
- the position vector of B is $(10\mathbf{i} + 5\mathbf{j})\,\text{km}$

At time t hours after noon

- the position vector of A is $\mathbf{r}\,\text{km}$, where $\mathbf{r} = 20\mathbf{j} + 40t\mathbf{i}$
- the position vector of B is $\mathbf{s}\,\text{km}$

(b) Find an expression for $\mathbf{s}$ in terms of t, **i** and **j**.

(2)

(c) Show that at time t hours after noon,

$$\overrightarrow{AB} = [(10 - 24t)\mathbf{i} + (12t - 15)\mathbf{j}]\,\text{km}$$

(2)

(d) Show that the boats will never collide.

(3)

(e) Find the distance between the boats when the bearing of B from A is 225°

(4)

WORKED SOLUTION

Question 7 · WME01/01 May/June 2024

1 Resolve the velocity of B

For a bearing angle α with $\tan\alpha = \frac{4}{3}$, use $\sin\alpha = \frac{4}{5}$ and $\cos\alpha = \frac{3}{5}$. Therefore $\mathbf{v}_B = 20\sin\alpha\mathbf{i} + 20\cos\alpha\mathbf{j} = 16\mathbf{i} + 12\mathbf{j}$.

2 Write the position of B

At noon, B is at $10\mathbf{i} + 5\mathbf{j}$. With constant velocity $16\mathbf{i} + 12\mathbf{j}$, $\mathbf{s} = (10 + 16t)\mathbf{i} + (5 + 12t)\mathbf{j}$.

3 Find AB vector

A has position vector $\mathbf{r} = 40t\mathbf{i} + 20\mathbf{j}$. Hence $\overrightarrow{AB} = \mathbf{s} - \mathbf{r} = (10 + 16t - 40t)\mathbf{i} + (5 + 12t - 20)\mathbf{j}$.

4 Show the printed expression

This simplifies to $\overrightarrow{AB} = (10 - 24t)\mathbf{i} + (12t - 15)\mathbf{j}$ km.

5 **Show the boats do not collide**

For collision, both components must be zero. From $10 - 24t = 0$, $t = \dfrac{5}{12}$. From $12t - 15 = 0$, $t = \dfrac{5}{4}$. These times are different, so the boats never collide.

6 **Use the bearing 225 degrees condition**

A bearing of $225°$ means B is south-west of A, so the i and j components of $\overrightarrow{AB}$ are equal and negative. Set $10 - 24t = 12t - 15$.

7 **Find the distance**

The equation gives $t = \dfrac{25}{36}$. Then each component is $-\dfrac{20}{3}$, so the distance is $\sqrt{\left(\dfrac{20}{3}\right)^2 + \left(\dfrac{20}{3}\right)^2} = \dfrac{20\sqrt{2}}{3}\ \text{km}$.

FINAL ANSWER

$(a)\ \mathbf{v}_B = 16\mathbf{i} + 12\mathbf{j}\ \text{km h}^{-1}$. $(b)\ \mathbf{s} = (10 + 16t)\mathbf{i} + (5 + 12t)\mathbf{j}$. $(c)\ \overrightarrow{AB} = (10 - 24t)\mathbf{i} + (12t - 15)\mathbf{j}$. (d) no collision. $(e)\ \dfrac{20\sqrt{2}}{3}\ \text{km} \approx 9.4\ \text{km}$.

WME01/01 MAY/JUNE 2025

Question 7

13 marks | Constant Acceleration in 2D | Main topic: Working with Vectors

7. [*In this question* **i** *and* **j** *are horizontal unit vectors.*]

Two boats, A and B, are each moving with constant acceleration.

At time t hours after noon, boat A has velocity $\mathbf{v}_A$ km h^{-1}, where

$$\mathbf{v}_A = 2\mathbf{i} + 3\mathbf{j} + (\mathbf{i} - 4\mathbf{j})t$$

(a) Find the magnitude of the acceleration of A.

(2)

When $t = 2$, the velocity of B is $(4\mathbf{i} + \mathbf{j})$ km h^{-1}

When $t = 5$, the velocity of B is $(\mathbf{i} - 5\mathbf{j})$ km h^{-1}

(b) Find the acceleration of B, giving your answer in terms of **i** and **j**.

(2)

(c) Find the velocity of B at time $t = 0$, giving your answer in terms of **i** and **j**.

(2)

At time T_1 hours after noon, both boats are moving with the same speed.

(d) Find the exact value of T_1

(4)

At time T_2 hours after noon, both boats are moving in the same direction.

(e) Show that $3T_2^2 + pT_2 + q = 0$, where p and q are integers to be found.

(3)

WORKED SOLUTION

Question 7 · WME01/01 May/June 2025

1 **Part (a): Find magnitude of acceleration of A - Identify acceleration from velocity function**

Given: $\mathbf{v}_A = 2\mathbf{i} + 3\mathbf{j} + (\mathbf{i} - 4\mathbf{j})t$

This is in form: $\mathbf{v} = \mathbf{u} + \mathbf{a}t$

Therefore:

; Initial velocity: $\mathbf{u}_A = 2\mathbf{i} + 3\mathbf{j}$
; Acceleration: $\mathbf{a}_A = \mathbf{i} - 4\mathbf{j}$ km h^{-2}

2 **Part (a): Find magnitude of acceleration of A - Find magnitude**

$|\mathbf{a}| = \sqrt{a_x^2 + a_y^2}$

$|\mathbf{a}_A| = \sqrt{1^2 + (-4)^2} = \sqrt{1 + 16} = \sqrt{17} = 4.123\ldots = 4.1 \text{ km h}^{-2} \text{ (2 sf)}$

3 **Part (b): Find acceleration of B - Use two velocity points to find acceleration**

Given:

; At $t = 2$: $\mathbf{v}_B = 4\mathbf{i} + \mathbf{j}$
; At $t = 5$: $\mathbf{v}_B = \mathbf{i} - 5\mathbf{j}$

$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t}$

$\mathbf{a}_B = \frac{(\mathbf{i}-5\mathbf{j})-(4\mathbf{i}+\mathbf{j})}{5-2} = \frac{(1-4)\mathbf{i}+(-5-1)\mathbf{j}}{3} = \frac{-3\mathbf{i}-6\mathbf{j}}{3} = -\mathbf{i} - 2\mathbf{j}\ \text{km h}^{-2}$

---

4 **Part (c): Find velocity of B at t = 0 - Use velocity equation backward**

$\mathbf{v}(t) = \mathbf{v}(t_0) + \mathbf{a}(t - t_0)$

From $t = 2$ back to $t = 0$:

$\mathbf{v}_B(0) = \mathbf{v}_B(2) + \mathbf{a}_B(0 - 2) = (4\mathbf{i} + \mathbf{j}) + (-\mathbf{i} - 2\mathbf{j})(-2) = 4\mathbf{i} + \mathbf{j} + 2\mathbf{i} + 4\mathbf{j} = 6\mathbf{i} + 5\mathbf{j}\ \text{km h}^{-1}$

---

5 **Part (d): Find exact value of T_1 when both have same speed - Express velocities at time t**

For boat A:
$\mathbf{v}_A(t) = 2\mathbf{i} + 3\mathbf{j} + (\mathbf{i} - 4\mathbf{j})t = (2 + t)\mathbf{i} + (3 - 4t)\mathbf{j}$

For boat B:
$\mathbf{v}_B(t) = (6\mathbf{i} + 5\mathbf{j}) + (-\mathbf{i} - 2\mathbf{j})t = (6 - t)\mathbf{i} + (5 - 2t)\mathbf{j}$

---

6 **Part (d): Find exact value of T_1 when both have same speed - Find magnitudes**

$|\mathbf{v}_A|^2 = (2 + t)^2 + (3 - 4t)^2 = 4 + 4t + t^2 + 9 - 24t + 16t^2 = 17t^2 - 20t + 13$

$|\mathbf{v}_B|^2 = (6 - t)^2 + (5 - 2t)^2 = 36 - 12t + t^2 + 25 - 20t + 4t^2 = 5t^2 - 32t + 61$

---

7 **Part (d): Find exact value of T_1 when both have same speed - Set magnitudes equal**

$17t^2 - 20t + 13 = 5t^2 - 32t + 61\ 12t^2 + 12t - 48 = 0\ t^2 + t - 4 = 0$

---

8 **Part (d): Find exact value of T_1 when both have same speed - Solve using quadratic formula**

$t = \frac{-1 \pm \sqrt{1+16}}{2} = \frac{-1 \pm \sqrt{17}}{2}$

Taking positive value:
$T_1 = \frac{-1+\sqrt{17}}{2}$

---

9 **Part (e): Show $3T_2^2 + pT_2 + q = 0$ - Condition for same direction**

Velocities parallel when:
$\frac{2+T_2}{6-T_2} = \frac{3-4T_2}{5-2T_2}$

---

10 **Part (e): Show $3T_2^2 + pT_2 + q = 0$ - Cross-multiply**

$(2 + T_2)(5 - 2T_2) = (3 - 4T_2)(6 - T_2)\ 10 - 4T_2 + 5T_2 - 2T_2^2 = 18 - 3T_2 - 24T_2 + 4T_2^2\ 10 + T_2 - 2T_2^2 = 18 - 27T_2 + 4T_2^2$
$0 = 4T_2^2 + 2T_2^2 - 27T_2 - T_2 + 18 - 10\ 0 = 6T_2^2 - 28T_2 + 8$

---

11 **Part (e): Show $3T_2^2 + pT_2 + q = 0$ - Divide by 2 to get required form**

$0 = 3T_2^2 - 14T_2 + 4$

Therefore: $p = -14$ and $q = 4$

✓ **FINAL ANSWER**

$(a)\ |\mathbf{a}_A| = \sqrt{17}\,\text{km h}^{-2} \approx 4.1\,\text{km h}^{-2}$. $(b)\ \mathbf{a}_B = -\mathbf{i} - 2\mathbf{j}\,\text{km h}^{-2}$. $(c)\ \mathbf{v}_B(0) = 6\mathbf{i} + 5\mathbf{j}\,\text{km h}^{-1}$. $(d)\ T_1 = \frac{-1+\sqrt{17}}{2}$ hours. $(e)\ 3T_2^2 - 14T_2 + 4 = 0,\ p = -14,\ q = 4$
.

WME01/01 JANUARY 2026

Question 6

16 marks | Constant Acceleration in 2D | Main topic: Working with Vectors

6. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

At 9 am, a ship S leaves a harbour and moves with constant velocity $(10\mathbf{i} + 15\mathbf{j})\,\text{km h}^{-1}$

(a) Find the exact speed of S

(2)

The harbour is at the origin O.

At 9 am, a boat B leaves the point with position vector $(50\mathbf{i} + 10\mathbf{j})$ km and moves with constant velocity $(-8\mathbf{i} + 10\mathbf{j})\,\text{km h}^{-1}$

(b) Find the direction in which B is moving, giving your answer as a bearing to the nearest degree.

(3)

(c) Show that at time t hours after 9 am,

$$\overrightarrow{SB} = [(50 - 18t)\mathbf{i} + (10 - 5t)\mathbf{j}]\,\text{km}$$

(4)

(d) Show that B and S do not collide.

(3)

When B is north-east of S, the distance between S and B is d km.

(e) Find the value of d.

(4)

WORKED SOLUTION

Question 6 · WME01/01 January 2026

1 Find the speed of S

The velocity of S is $10\mathbf{i} + 15\mathbf{j}$. Its speed is the magnitude: $\sqrt{10^2 + 15^2} = \sqrt{325} = 5\sqrt{13}\ \text{km h}^{-1}$.

2 Find the bearing of B

B has velocity $-8\mathbf{i} + 10\mathbf{j}$, so it moves west and north. The angle west of north satisfies $\tan\theta = \dfrac{8}{10}$, giving $\theta = 38.66°$. The bearing is $360° - 38.66° = 321°$ to the nearest degree.

3 Write both position vectors

At time t hours after 9 am, $\mathbf{r}_S = (10t)\mathbf{i} + (15t)\mathbf{j}$. Also $\mathbf{r}_B = (50\mathbf{i} + 10\mathbf{j}) + t(-8\mathbf{i} + 10\mathbf{j}) = (50 - 8t)\mathbf{i} + (10 + 10t)\mathbf{j}$.

4 Find the relative position vector

$\overrightarrow{SB} = \mathbf{r}_B - \mathbf{r}_S = [(50 - 8t) - 10t]\mathbf{i} + [(10 + 10t) - 15t]\mathbf{j} = (50 - 18t)\mathbf{i} + (10 - 5t)\mathbf{j}$.

5 Show they do not collide

For a collision, both components of $\overrightarrow{SB}$ must be zero at the same time. From $50 - 18t = 0$, $t = \dfrac{25}{9}$. From $10 - 5t = 0$, $t = 2$. These are different, so B and S do not collide.

6 **Find the north-east or south-west instant**

When the separation is along a $45°$ diagonal, the two components have equal magnitude. Using the given condition, set $50 - 18t = 10 - 5t$, so $40 = 13t$ and $t = \dfrac{40}{13}$.

7 **Find the distance d**

At $t = \dfrac{40}{13}$, both components are $-\dfrac{70}{13}$. Therefore $d = \sqrt{\left(\dfrac{70}{13}\right)^2 + \left(\dfrac{70}{13}\right)^2} = \dfrac{70\sqrt{2}}{13} = 7.6\ \text{km}$ to 2 significant figures.

FINAL ANSWER

$(a)\ 5\sqrt{13}\ \text{km h}^{-1}.$ $(b)\ 321°.$ $(c)\ \overrightarrow{SB} = (50 - 18t)\mathbf{i} + (10 - 5t)\mathbf{j}.$ $(d)\ \text{no collision.}$ $(e)\ d = \dfrac{70\sqrt{2}}{13}\ \text{km} \approx 7.6\ \text{km}.$

TOPIC

Forces

Every past-paper question on this topic, October 2019 to January 2026.

WME01/01 JANUARY 2023

Question 6

8 marks | Forces

6.

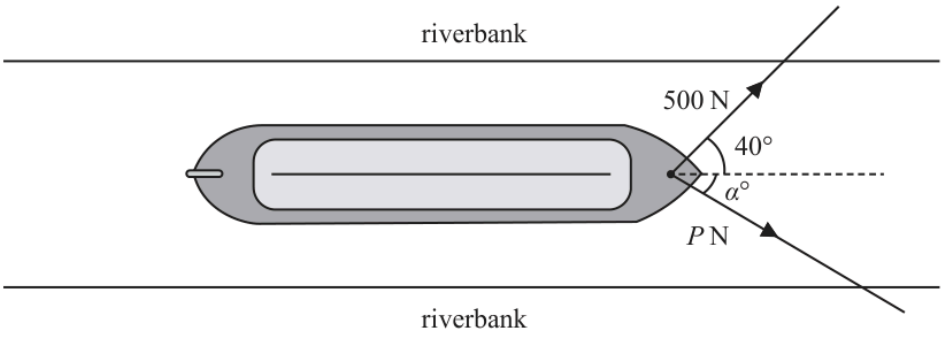


Figure 3

A boat is pulled along a river at a constant speed by two ropes.

The banks of the river are parallel and the boat travels horizontally in a straight line, parallel to the riverbanks.

- The tension in the first rope is 500 N acting at an angle of 40° to the direction of motion, as shown in Figure 3.
- The tension in the second rope is P newtons, acting at an angle of $\alpha°$ to the direction of motion, also shown in Figure 3.
- The resistance to motion of the boat as it moves through the water is a constant force of magnitude 900 N

The boat is modelled as a particle. The ropes are modelled as being light and lying in a horizontal plane.

Use the model to find

(i) the value of α

(ii) the value of P

(8)

WORKED SOLUTION

Question 6 · WME01/01 January 2023

1 Resolve perpendicular to the motion

The boat travels straight, so the sideways components balance: $500 \sin 40° = P \sin \alpha$.

2 Resolve along the motion

The speed is constant, so the forward components balance the resistance: $500 \cos 40° + P \cos \alpha = 900$.

3 Eliminate P

From the two equations, $P \sin \alpha = 500 \sin 40°$ and $P \cos \alpha = 900 - 500 \cos 40°$.

4 Find alpha

Divide the sine equation by the cosine equation: $\tan \alpha = \dfrac{500 \sin 40°}{900 - 500 \cos 40°}$. This gives $\alpha = 31.9°$, so $\alpha = 32°$ approximately.

5 Find P

Use $P \sin \alpha = 500 \sin 40°$: $P = \dfrac{500 \sin 40°}{\sin 31.9°} = 608.7\ldots$ N. Hence $P \approx 610$ N.

FINAL ANSWER

$(i)\ \alpha = 32°$ approximately. $\quad (ii)\ P = 610\ \text{N}$ approximately.

WME01/01 MAY/JUNE 2023

Question 6

7 marks | Forces

6.

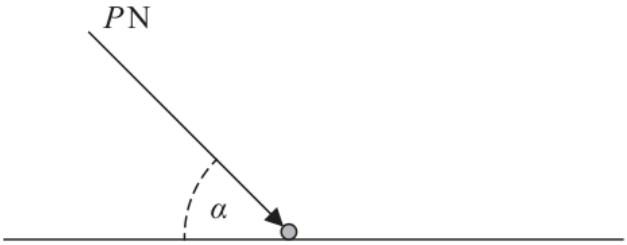


Figure 3

A particle of weight W newtons lies at rest on a rough horizontal surface, as shown in Figure 3.
A force of magnitude P newtons is applied to the particle.

The force acts at an angle α to the horizontal, where $\tan\alpha = \frac{4}{3}$

The coefficient of friction between the particle and the surface is $\frac{1}{4}$

Given that the particle does not move, show that

$$P \leqslant \frac{5W}{8}$$

(7)

WORKED SOLUTION

Question 6 · WME01/01 May/June 2023

1 Use the trig ratios

Given $\tan\alpha = \frac{4}{3}$, use $\sin\alpha = \frac{4}{5}$ and $\cos\alpha = \frac{3}{5}$.

2 Resolve vertically

The applied force increases the normal reaction in this model, so $R = W + P\sin\alpha$.

3 Resolve horizontally

For the particle not to move, friction must be able to balance the horizontal component of the applied force, so $P\cos\alpha \leq F_{\max}$.

4 Use limiting friction

Since $\mu = \frac{1}{4}$, $F_{\max} = \frac{1}{4}R = \frac{1}{4}(W + P\sin\alpha)$.

5 Substitute the exact trig values

Thus $P\cdot\frac{3}{5} \leq \frac{1}{4}\left(W + P\cdot\frac{4}{5}\right)$.

6 Rearrange the inequality

Multiplying by 20 gives $12P \leq 5W + 4P$. Hence $8P \leq 5W$, so $P \leq \frac{5}{8}W$.

FINAL ANSWER

$$P \leq \frac{5}{8}W.$$

WME01/01 OCTOBER 2019

Question 7

7 marks | Forces | Main topic: Working with Vectors

7. Two forces, $\mathbf{F}$ and $\mathbf{G}$, act on a particle. The force $\mathbf{F}$ has magnitude 4 N and acts in a direction with a bearing of 120° and the force $\mathbf{G}$ has magnitude 6 N and acts due north.

Given that $\mathbf{P} = 2\mathbf{F} + \mathbf{G}$, find

(i) the magnitude of $\mathbf{P}$

(ii) the direction of $\mathbf{P}$, giving your answer as a bearing to the nearest degree.

(7)

WORKED SOLUTION

Question 7 · WME01/01 October 2019

1 Resolve 2F

Since $\mathbf{F}$ has magnitude 4 N on a bearing of $120°$, $2\mathbf{F}$ has magnitude 8 N on the same bearing.

2 Write the components

$2\mathbf{F} = 8\sin 120°\mathbf{i} + 8\cos 120°\mathbf{j} = 4\sqrt{3}\mathbf{i} - 4\mathbf{j}$. Also $\mathbf{G} = 6\mathbf{j}$.

3 Find P

$\mathbf{P} = 2\mathbf{F} + \mathbf{G} = 4\sqrt{3}\mathbf{i} + 2\mathbf{j}$.

4 Find the magnitude

$|\mathbf{P}| = \sqrt{(4\sqrt{3})^2 + 2^2} = \sqrt{48 + 4} = \sqrt{52}$ N.

5 Find the bearing angle

The vector is in the north-east quadrant. The angle east of north satisfies $\tan\theta = \dfrac{4\sqrt{3}}{2} = 2\sqrt{3}$.

6 State the bearing

$\theta = 73.9°$ approximately, so the bearing is $074°$ to the nearest degree.

FINAL ANSWER

$(i)\ |\mathbf{P}| = \sqrt{52}\ \text{N} \approx 7.21\ \text{N}.$ $(ii)\ \text{bearing} = 074°.$

WME01/01 JANUARY 2020

Question 7

18 marks | Forces | Main topic: Newton's Second Law

7.

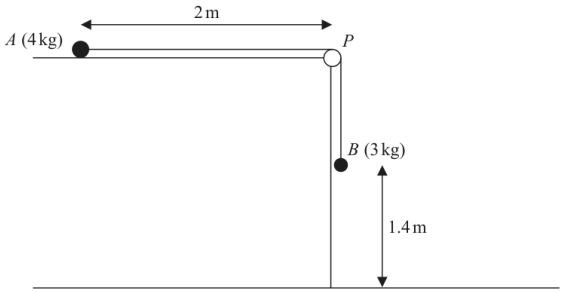


Figure 3

A particle A of mass 4 kg is held at rest on a rough horizontal table. Particle A is attached to one end of a string that passes over a pulley P. The pulley is fixed at the edge of the table. The other end of the string is attached to a particle B, of mass 3 kg, which hangs freely below P.

The part of the string from A to P is perpendicular to the edge of the table and A, P and B all lie in the same vertical plane.

The string is modelled as being light and inextensible and the pulley is modelled as being small, smooth and light.

The system is released from rest with the string taut. At the instant of release, A is 2 m from the edge of the table and B is 1.4 m above a horizontal floor, as shown in Figure 3.

After descending with constant acceleration for 2 seconds, B hits the floor and does not rebound.

(a) Show that the acceleration of A before B hits the floor is $0.7\,\text{m s}^{-2}$

(2)

(b) State which of the modelling assumptions you have used in order to answer part (a).

(1)

(c) Find the magnitude of the resultant force exerted on the pulley by the string.

(4)

The coefficient of friction between A and the table is μ.

(d) Find the value of μ.

(6)

(e) Determine, by calculation, whether or not A reaches the pulley.

(5)

(Total 18 marks)

WORKED SOLUTION

Question 7 · WME01/01 January 2020

1 Use B's descent

Before B hits the floor, it descends 1.4 m from rest in 2 s. Use $s = ut + \frac{1}{2}at^2$: $1.4 = \frac{1}{2}a(2)^2$. Hence $a = 0.7\ \text{m s}^{-2}$.

2 Name the modelling assumption

The string is inextensible, so A and B have the same magnitude of acceleration while the string is taut.

3 Find the tension

For B, taking downward as positive: $3g - T = 3(0.7)$. Thus $T = 3g - 2.1 = 27.3$ N.

4 Find the resultant force on the pulley

The pulley is pulled by two perpendicular tensions of magnitude T. Therefore the resultant is $\sqrt{T^2 + T^2} = T\sqrt{2} = 27.3\sqrt{2} = 38.6$ N approximately.

5 Find the friction on A

For A before B hits the floor, $T - F = 4(0.7)$. Hence $F = 27.3 - 2.8 = 24.5$ N.

6 **Find mu**

On the horizontal table, $R = 4g = 39.2$ N and $F = \mu R$. So $\mu = \dfrac{24.5}{39.2} = 0.625$.

7 **Find A's speed when B hits the floor**

At this instant, $v = at = 0.7(2) = 1.4\ \text{m s}^{-1}$. A has also moved 1.4 m towards the pulley.

8 **Find the extra stopping distance**

After B hits the floor, A is slowed only by friction. Its deceleration is $\mu g = 0.625(9.8) = 6.125\ \text{m s}^{-2}$. Use $0 = (1.4)^2 - 2(6.125)s$, giving $s = 0.16$ m.

9 **Compare with the distance to the pulley**

A's total distance is $1.4 + 0.16 = 1.56$ m, which is less than 2 m. Therefore A does not reach the pulley.

FINAL ANSWER

$(a)\ a = 0.7\ \text{m s}^{-2}$. (b) Inextensibility of the string.

$(c)\ 38.6\ \text{N}$. $(d)\ \mu = 0.625$. (e) A does not reach the pulley.

WME01/01 JANUARY 2025

Question 6

12 marks | Forces

Main topic: Resolving Forces, Inclined Planes

6.

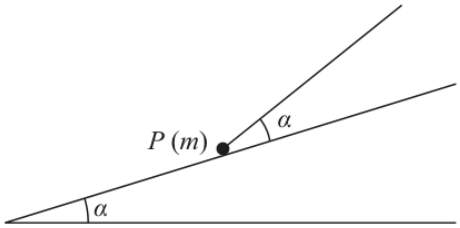


Figure 3

A particle P of mass m is held in equilibrium on a fixed rough inclined plane by a light inextensible string.

The plane is inclined at an angle α to the horizontal, where $\alpha < 45°$
The string is inclined to the plane at angle α, as shown in Figure 3.

The string lies in a vertical plane that contains a line of greatest slope of the inclined plane.

When the tension in the string is $0.75\,mg$, P is on the point of moving up the plane.

(a) Find an expression for the magnitude of the frictional force acting on P, giving your answer in terms of m, g and α.

(3)

The coefficient of friction between P and the plane is $\frac{1}{2}$

(b) Show that

$$\tan\alpha = \frac{2}{5}$$

(6)

The string breaks.

(c) Determine whether P remains at rest. You must justify your reasoning.

(3)

WORKED SOLUTION

Question 6 · WME01/01 January 2025

1 Resolve parallel to the plane

P is on the point of moving up the plane, so friction acts down the plane. The tension has component $0.75mg\cos\alpha$ up the plane, and the weight component is $mg\sin\alpha$ down the plane.

2 Find the frictional force

For equilibrium parallel to the plane, $0.75mg\cos\alpha = mg\sin\alpha + F$. Hence $F = 0.75mg\cos\alpha - mg\sin\alpha$.

3 Resolve perpendicular to the plane

The string has a component away from the plane, so $R = mg\cos\alpha - 0.75mg\sin\alpha$.

4 Use the coefficient of friction

Given $\mu = \frac{1}{2}$, limiting friction gives $F = \frac{1}{2}R$. Therefore $0.75mg\cos\alpha - mg\sin\alpha = \frac{1}{2}(mg\cos\alpha - 0.75mg\sin\alpha)$.

5 **Show the tangent result**

Divide by mg and collect terms: $0.75\cos\alpha - \sin\alpha = 0.5\cos\alpha - 0.375\sin\alpha$. Thus $0.25\cos\alpha = 0.625\sin\alpha$, so $\tan\alpha = \dfrac{0.25}{0.625} = \dfrac{2}{5}$.

6 **Check what happens when the string breaks**

Without the string, $R = mg\cos\alpha$ and the maximum friction is $\dfrac{1}{2}mg\cos\alpha$. The component of weight down the plane is $mg\sin\alpha$.

7 **Compare friction with the downslope component**

Since $\tan\alpha = \dfrac{2}{5} < \dfrac{1}{2}$, we have $mg\sin\alpha < \dfrac{1}{2}mg\cos\alpha$. The available friction is large enough to hold P, so P remains at rest.

FINAL ANSWER

$(a)\ F = 0.75mg\cos\alpha - mg\sin\alpha.$ $(b)\ \tan\alpha = \dfrac{2}{5}.$ $(c)\ P$ remains at rest.

TOPIC

Newton's Second Law

Every past-paper question on this topic, October 2019 to January 2026.

WME01/01 JANUARY 2020

Question 7

18 marks | Newton's Second Law

7.

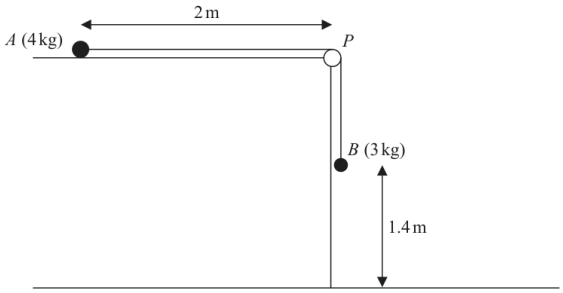


Figure 3

A particle A of mass 4 kg is held at rest on a rough horizontal table. Particle A is attached to one end of a string that passes over a pulley P. The pulley is fixed at the edge of the table. The other end of the string is attached to a particle B, of mass 3 kg, which hangs freely below P.

The part of the string from A to P is perpendicular to the edge of the table and A, P and B all lie in the same vertical plane.

The string is modelled as being light and inextensible and the pulley is modelled as being small, smooth and light.

The system is released from rest with the string taut. At the instant of release, A is 2 m from the edge of the table and B is 1.4 m above a horizontal floor, as shown in Figure 3.

After descending with constant acceleration for 2 seconds, B hits the floor and does not rebound.

(a) Show that the acceleration of A before B hits the floor is $0.7\,\text{m s}^{-2}$

(2)

(b) State which of the modelling assumptions you have used in order to answer part (a).

(1)

(c) Find the magnitude of the resultant force exerted on the pulley by the string.

(4)

The coefficient of friction between A and the table is μ.

(d) Find the value of μ.

(6)

(e) Determine, by calculation, whether or not A reaches the pulley.

(5)

(Total 18 marks)

WORKED SOLUTION

Question 7 · WME01/01 January 2020

1 Use B's descent

Before B hits the floor, it descends 1.4 m from rest in 2 s. Use $s = ut + \frac{1}{2}at^2$: $1.4 = \frac{1}{2}a(2)^2$. Hence $a = 0.7\ \text{m s}^{-2}$.

2 Name the modelling assumption

The string is inextensible, so A and B have the same magnitude of acceleration while the string is taut.

3 Find the tension

For B, taking downward as positive: $3g - T = 3(0.7)$. Thus $T = 3g - 2.1 = 27.3$ N.

4 Find the resultant force on the pulley

The pulley is pulled by two perpendicular tensions of magnitude T. Therefore the resultant is $\sqrt{T^2 + T^2} = T\sqrt{2} = 27.3\sqrt{2} = 38.6$ N approximately.

5 Find the friction on A

For A before B hits the floor, $T - F = 4(0.7)$. Hence $F = 27.3 - 2.8 = 24.5$ N.

6 **Find mu**

On the horizontal table, $R = 4g = 39.2$ N and $F = \mu R$. So $\mu = \dfrac{24.5}{39.2} = 0.625$.

7 **Find A's speed when B hits the floor**

At this instant, $v = at = 0.7(2) = 1.4\ \text{m s}^{-1}$. A has also moved 1.4 m towards the pulley.

8 **Find the extra stopping distance**

After B hits the floor, A is slowed only by friction. Its deceleration is $\mu g = 0.625(9.8) = 6.125\ \text{m s}^{-2}$. Use $0 = (1.4)^2 - 2(6.125)s$, giving $s = 0.16$ m.

9 **Compare with the distance to the pulley**

A's total distance is $1.4 + 0.16 = 1.56$ m, which is less than 2 m. Therefore A does not reach the pulley.

FINAL ANSWER

$(a)\ a = 0.7\ \text{m s}^{-2}$. (b) Inextensibility of the string.

$(c)\ 38.6\ \text{N}$. $(d)\ \mu = 0.625$. (e) A does not reach the pulley.

WME01/01 OCTOBER 2022

Question 7

13 marks | Newton's Second Law

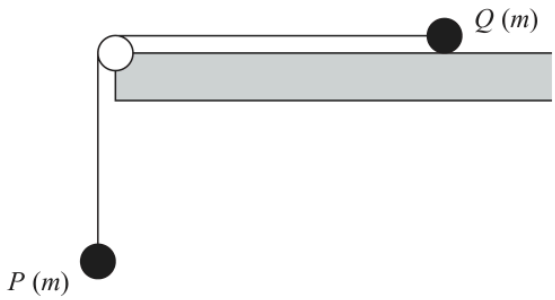


Figure 4

A particle P of mass m is attached to one end of a light inextensible string. Another particle Q, also of mass m, is attached to the other end of the string. The string passes over a small smooth pulley which is fixed at the edge of a rough horizontal table. Particle Q is held at rest on the table and particle P hangs vertically below the pulley with the string taut, as shown in Figure 4.

The pulley, P and Q all lie in the same vertical plane.

The coefficient of friction between Q and the table is μ, where $\mu < 1$

Particle Q is released from rest.

The tension in the string before Q hits the pulley is kmg, where k is a constant.

(a) Find k in terms of μ.

(7)

Given that Q is initially a distance d from the pulley,

(b) find, in terms of d, g and μ, the time taken by Q, after release, to reach the pulley.

(4)

(c) Describe what would happen if $\mu \geqslant 1$, giving a reason for your answer.

(2)

WORKED SOLUTION

Question 7 · WME01/01 October 2022

1 Write the friction on Q

For Q on the horizontal table, $R = mg$, so the friction is $F = \mu mg$.

2 Apply Newtons second law to P

P moves downward and Q moves towards the pulley. For P, taking downward as positive: $mg - T = ma$.

3 Apply Newtons second law to Q

For Q, taking motion towards the pulley as positive: $T - \mu mg = ma$.

4 Use T = kmg

From P, $a = g(1 - k)$. From Q, $a = g(k - \mu)$.

5 Find k

Equate the two accelerations: $1 - k = k - \mu$. Hence $2k = 1 + \mu$ and $k = \dfrac{1 + \mu}{2}$.

6 **Find the acceleration**

$a = g(1-k) = g\left(1 - \dfrac{1+\mu}{2}\right) = \dfrac{g(1-\mu)}{2}$.

7 **Use motion from rest**

Q starts from rest and travels distance d, so $d = \dfrac{1}{2}at^2 = \dfrac{1}{2} \cdot \dfrac{g(1-\mu)}{2}t^2$.

8 **Find the time**

$d = \dfrac{g(1-\mu)t^2}{4}$, so $t = \sqrt{\dfrac{4d}{g(1-\mu)}}$.

9 **Consider mu at least 1**

If $\mu \geq 1$, the maximum available friction on Q is at least mg, so it can balance the pull caused by the hanging particle. The system would not start moving.

✓ **FINAL ANSWER**

$(a)\ k = \dfrac{1+\mu}{2}$. $(b)\ t = \sqrt{\dfrac{4d}{g(1-\mu)}}$. (c) If $\mu \geq 1$, the system would not move because friction can balance the pull from P.

WME01/01 JANUARY 2023

Question 7

7 marks | Newton's Second Law

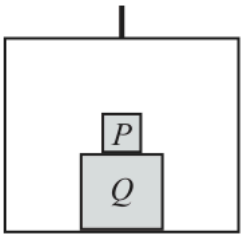


Figure 4

A simple lift operates by means of a vertical cable which is attached to the top of the lift.
The lift has mass m
A box Q is placed on the floor of the lift.
A box P is placed directly on top of box Q, as shown in Figure 4.

The cable is modelled as being light and inextensible and air resistance is modelled as being negligible.

The tension in the cable is $\frac{42mg}{5}$

The lift and its contents move vertically upwards with acceleration $\frac{2g}{5}$

Using the model,

(a) find, in terms of m, the combined mass of boxes P and Q

(4)

During the motion of the lift, the force exerted on box P by box Q is $\frac{14mg}{5}$

Using the model,

(b) find, in terms of m, the mass of box P

(3)

WORKED SOLUTION

Question 7 · WME01/01 January 2023

1 Define the box mass

Let the combined mass of boxes P and Q be M. The lift itself has mass m, so the total moving mass is $m + M$.

2 Apply Newtons second law to the whole system

Taking upward as positive, $\frac{42}{5}mg - (m+M)g = (m+M)\frac{2g}{5}$.

3 Find M

Move the weight term to the right: $\frac{42}{5}mg = (m+M)\left(g + \frac{2g}{5}\right) = \frac{7g}{5}(m+M)$. Hence $42m = 7(m+M)$ and $M = 5m$.

4 Let the mass of P be p

For box P alone, the upward force from Q is $\frac{14}{5}mg$, while its weight is pg. The acceleration is still $\frac{2g}{5}$ upward.

5 **Apply Newtons second law to P**

$\frac{14}{5}mg - pg = p\frac{2g}{5}$.

6 **Find p**

Divide by g: $\frac{14}{5}m = p + \frac{2p}{5} = \frac{7p}{5}$. Therefore $p = 2m$.

FINAL ANSWER

(a) combined mass of P and Q $= 5m$. (b) mass of P $= 2m$.

WME01/01 MAY/JUNE 2023

Question 7

11 marks | Newton's Second Law

7.

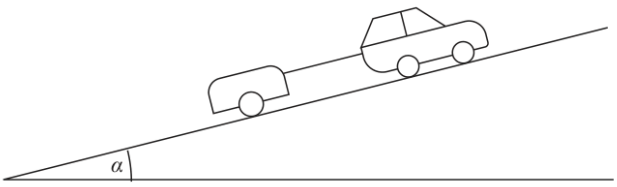


Figure 4

A car of mass 1200 kg is towing a trailer of mass 600 kg up a straight road, as shown in Figure 4.

The road is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{12}$

The driving force produced by the engine of the car is 3000 N.

The car moves with acceleration $0.75\,\text{m s}^{-2}$

The non-gravitational resistance to motion of

- the **car** is modelled as a constant force of magnitude $2R$ newtons
- the **trailer** is modelled as a constant force of magnitude R newtons

The car and the trailer are modelled as particles.
The tow bar between the car and trailer is modelled as a light rod that is parallel to the direction of motion.

Using the model,

(a) show that the value of R is 60

(4)

(b) find the tension in the tow bar.

(3)

When the car and trailer are moving at a speed of $12\,\text{m s}^{-1}$, the tow bar breaks.

Given that the non-gravitational resistance to motion of the trailer remains unchanged,

(c) use the model to find the further distance moved by the trailer before it first comes to rest.

(4)

WORKED SOLUTION

Question 7 · WME01/01 May/June 2023

1 Use the whole system

For car plus trailer, the total mass is 1800 kg and the total resistance is $2R + R = 3R$. Taking up the slope as positive: $3000 - 1800g\sin\alpha - 3R = 1800(0.75)$.

2 Show R is 60

Since $\sin\alpha = \frac{1}{12}$, $1800g\sin\alpha = 1800 \cdot 9.8 \cdot \frac{1}{12} = 1470$. So $3000 - 1470 - 3R = 1350$, giving $180 = 3R$ and $R = 60\ \text{N}$.

3 Apply Newtons second law to the trailer

For the trailer while it is being towed, $T - 600g\sin\alpha - R = 600(0.75)$.

4 Find the tow-bar tension

$T - 600 \cdot 9.8 \cdot \frac{1}{12} - 60 = 450$, so $T - 490 - 60 = 450$ and $T = 1000\ \text{N}$.

5 **Find the trailer acceleration after the break**

After the tow bar breaks, the trailer is still moving up the slope, but the component of weight and resistance act down the slope. Taking up the slope as positive: $-600g\sin\alpha - 60 = 600a$.

6 **Simplify the acceleration**

$-490 - 60 = 600a$, so $a = -\dfrac{550}{600} = -\dfrac{11}{12}\ \text{m s}^{-2}$.

7 **Find the stopping distance**

With initial speed $12\ \text{m s}^{-1}$ and final speed 0, use $v^2 = u^2 + 2as$: $0 = 12^2 + 2\left(-\dfrac{11}{12}\right)d$. Hence $d = \dfrac{864}{11} = 78.5\ldots\ \text{m}$.

FINAL ANSWER

$(a)\ R = 60\ \text{N}$. $\quad (b)\ T = 1000\ \text{N}$. $\quad (c)\ d = 78.5\ \text{m}$ **approximately**.

WME01/01 JANUARY 2025

Question 7

15 marks | Newton's Second Law

7.

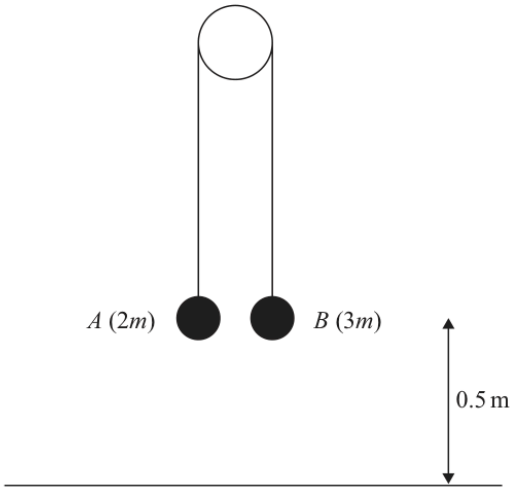


Figure 4

One end of a light inextensible string is attached to a particle A of mass $2m$.
The other end is attached to a particle B of mass $3m$.
The string passes over a small smooth fixed pulley.
The string is taut and both straight parts of the string are vertical.
Both particles are held at a distance 0.5 m above a horizontal surface, as shown in Figure 4.

The system is released from rest and B moves downwards.

(a) Find the tension in the string in terms of m and g.

(5)

(b) Find the speed of B at the instant it strikes the surface.

(4)

In the subsequent motion, A does not reach the pulley and B does not rebound after it strikes the surface.

(c) Find the time from the instant when the system is released from rest to the instant when A first reaches a height of 1.06 m above the surface.

(6)

WORKED SOLUTION

Question 7 · WME01/01 January 2025

1 Write the equations of motion

B moves down and A moves up with the same acceleration a. For A: $T - 2mg = 2ma$. For B: $3mg - T = 3ma$.

2 Find the acceleration

Adding the two equations gives $mg = 5ma$, so $a = \frac{g}{5}$.

3 Find the tension

Use $T - 2mg = 2ma$: $T = 2mg + 2m\left(\frac{g}{5}\right) = \frac{12mg}{5}$.

4 Find the speed when B hits the surface

B starts from rest and moves 0.5 m with acceleration $g/5$. Using $v^2 = u^2 + 2as$, $v^2 = 2\left(\frac{g}{5}\right)(0.5) = \frac{g}{5} = 1.96$.

5 Calculate the impact speed

$v = \sqrt{1.96} = 1.4\ \text{m s}^{-1}$.

6 **Find the first time interval**

The time until B hits the surface is found from $0.5 = \frac{1}{2}\left(\frac{g}{5}\right)t_1^2$. With $g = 9.8$, $t_1 = \frac{5}{7} = 0.714285\ldots$ s.

7 **Find the extra time after B stops**

After B hits the surface, A continues upwards freely under gravity. It is then at height 1.00 m and must rise another 0.06 m with initial speed $1.4\ \text{m s}^{-1}$. So $0.06 = 1.4t_2 - 4.9t_2^2$.

8 **Find the total time**

Solving $0.06 = 1.4t_2 - 4.9t_2^2$ gives the first positive time $t_2 = 0.0525\ldots$ s. Therefore the total time is $0.714285\ldots + 0.0525\ldots = 0.767$ s.

✓ **FINAL ANSWER**

$(a)\ T = \frac{12mg}{5}$. $(b)\ v = 1.4\ \text{m s}^{-1}$. $(c)\ t = 0.767$ s approximately.

WME01/01 MAY/JUNE 2025

Question 8

17 marks | Newton's Second Law

8.

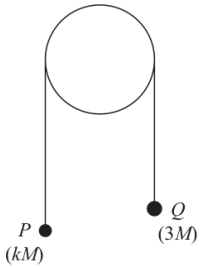


Figure 5

Two small balls, P and Q, have masses kM and $3M$ respectively, where $k < 3$

The balls are attached to the ends of a light inextensible string that passes over a fixed light smooth pulley.

The system is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 5.

The system is released from rest and, in the subsequent motion, P moves with an acceleration of magnitude $\frac{1}{5}g$

The balls are modelled as particles.

(a) Write down an equation of motion for P.

(2)

(b) Find the value of k.

(3)

Given that $M = 0.5\,\text{kg}$,

(c) find the magnitude of the force exerted on the pulley by the string while Q is moving downwards.

(3)

At the instant when the system is released, P is more than 2.5 m from the pulley and Q is 2.5 m above horizontal ground.
After hitting the ground, Q rebounds with a speed of $0.4\,\text{m s}^{-1}$

(d) Find the magnitude of the impulse received by Q when it hits the ground.

(5)

In the subsequent motion, P does not hit the pulley.

(e) Find the total time from when the balls are released until P first comes to rest.

(4)

WORKED SOLUTION

Question 8 · WME01/01 May/June 2025

1 **Part (a): Write equation of motion for P - Identify forces on P**

Particle P (mass $\boldsymbol{kM}$) moves upward with acceleration $\frac{g}{5}$:

; Tension T (upward)
; Weight $\boldsymbol{kMg}$ (downward)

2 **Part (a): Write equation of motion for P - Apply Newton's Second Law**

Taking upward as positive:

$$\sum F = ma$$

$$T - kMg = kM \times \frac{g}{5}$$

3 **Part (b): Find value of k - Write equation of motion for Q**

Particle Q (mass $3M$) moves downward with acceleration $\frac{g}{5}$:

Taking downward as positive for Q:

$$3Mg - T = 3M \times \frac{g}{5} \; 3Mg - T = \frac{3Mg}{5} \; T = 3Mg - \frac{3Mg}{5} \; T = \frac{12Mg}{5}$$

4 **Part (b): Find value of k - Substitute into equation for P**

From part (a): $T - kMg = \frac{kMg}{5}$

$$\frac{12Mg}{5} - kMg = \frac{kMg}{5} \; \frac{12Mg}{5} = kMg + \frac{kMg}{5} \; \frac{12Mg}{5} = \frac{5kMg + kMg}{5} \; \frac{12Mg}{5} = \frac{6kMg}{5} \; 12 = 6k \; k = 2$$

5 **Part (c): Find force on pulley while Q moving down - Identify tension in string**

From part (b): $T = \frac{12Mg}{5}$

Given: $M = 0.5$ kg

$$T = \frac{12 \times 0.5 \times g}{5} = \frac{6g}{5} = \frac{6 \times 9.8}{5} = 11.76 \text{ N}$$

6 **Part (c): Find force on pulley while Q moving down - Find force on pulley**

The pulley experiences two tension forces:

; Horizontal tension from P: $T = 11.76$ N
; Vertical tension from Q: $T = 11.76$ N

These are perpendicular, so use Pythagoras:

$$F = \sqrt{T^2 + T^2} = \sqrt{2T^2} = T\sqrt{2} = 11.76\sqrt{2} = 16.63\ldots = 16.6 \text{ N (3 sf)}$$

Or: $F = \frac{6g\sqrt{2}}{5} = \frac{12g}{5} = 2.4g = 23.5$ N

Wait, let me recalculate...

Actually: $F = 2T$ (both tensions pull on pulley)

$$F = 2 \times \frac{12Mg}{5} = \frac{24Mg}{5} = \frac{24 \times 0.5 \times 9.8}{5} = \frac{117.6}{5} = 23.5 \text{ N (3 sf)}$$

Or: $\frac{12g}{5} = 23.5$ N

7 **Part (d): Find impulse when Q hits ground - Find velocity of Q just before hitting ground**

Using SUVAT with:

; $u = 0$ (starts from rest)
; $a = \frac{g}{5} = \frac{9.8}{5} = 1.96$ m s^{-2}
; $s = 2.5$ m

$$v^2 = u^2 + 2as \; v^2 = 0 + 2 \times \frac{g}{5} \times 2.5 \; v^2 = \frac{5g}{5} \; v^2 = g \; v = \sqrt{g} = \sqrt{9.8} = 3.130\ldots \text{ m s}^{-1}$$

Or exact: $v = \sqrt{g} = \sqrt{\frac{7g}{5}}$

Actually: $v^2 = 2 \times 1.96 \times 2.5 = 9.8$, so $v = \sqrt{9.8} = 3.13$ m s^{-1}

8 **Part (d): Find impulse when Q hits ground - Calculate impulse**

$\text{Impulse} = m(v_{\text{after}} - v_{\text{before}})$

Taking downward as positive before impact:

; Velocity before: $v_{\text{before}} = +3.13$ m s^{-1} (downward)
; Velocity after: $v_{\text{after}} = -0.4$ m s^{-1} (upward, rebounds)
; Mass of Q: $3M = 3 \times 0.5 = 1.5$ kg

$I = 1.5 \times (-0.4 - 3.13) = 1.5 \times (-3.53) = -5.295\ldots \, |I| = 5.3 \text{ N s } (2 \text{ sf})$

Or using exact values: $|I| = 1.5(\sqrt{g} + 0.4) = 1.5(3.13 + 0.4) = 5.3$ N s

---

9 **Part (e): Find total time until P first comes to rest - Time for Q to fall (t_1)**

Using $s = ut + \frac{1}{2}at^2$ with $u = 0$, $s = 2.5$ m, $a = \frac{g}{5}$:

$2.5 = 0 + \frac{1}{2} \times \frac{g}{5} \times t_1^2 \, 2.5 = \frac{gt_1^2}{10} \, 25 = gt_1^2 \, t_1^2 = \frac{25}{9.8} = 2.551\ldots \, t_1 = \sqrt{\frac{25}{9.8}} = 1.597\ldots \text{ s}$

Or exact: $t_1 = \sqrt{\frac{5 \times 5}{g}} = \frac{5}{\sqrt{g}}$

Better: $t_1 = \sqrt{\frac{5}{g/5}} = \sqrt{\frac{25}{g}}$

Actually: $2.5 = \frac{1}{2} \times 1.96 \times t_1^2$

$t_1^2 = \frac{5}{1.96} = 2.551$, so $t_1 = 1.597$ s

---

10 **Part (e): Find total time until P first comes to rest - Velocity of P when Q hits ground**

At time t_1, P has velocity:
$v_P = 0 + \frac{g}{5} \times t_1 = \frac{9.8}{5} \times 1.597 = 1.96 \times 1.597 = 3.13 \text{ m s}^{-1}$

Or: $v_P = \sqrt{g}$ (same as Q)

---

11 **Part (e): Find total time until P first comes to rest - Time for P to come to rest (t_2)**

After Q hits ground, string goes slack. P continues upward under gravity alone.

Using $v = u + at$ with $v = 0$, $u = 3.13$ m s^{-1}, $a = -g$:

$0 = 3.13 - 9.8t_2 \, t_2 = \frac{3.13}{9.8} = 0.3194\ldots \text{ s}$

Or: $t_2 = \frac{\sqrt{g}}{g} = \frac{1}{\sqrt{g}}$

---

12 **Part (e): Find total time until P first comes to rest - Total time**

$T = t_1 + t_2 = 1.597 + 0.319 = 1.916\ldots = 1.9 \text{ s } (2 \text{ sf}) \text{ or } 1.92 \text{ s } (3 \text{ sf})$

✓ **FINAL ANSWER**

$(a)\ T - kMg = \dfrac{kMg}{5}.$ $(b)\ k = 2.$ $(c)\ F = \dfrac{12g}{5}\ \text{N} \approx 23.5\ \text{N}.$ $(d)\ |I| = 5.30\ \text{N s}.$ $(e)\ T = 1.92\ \text{s}.$

WME01/01 OCTOBER 2019

Question 6

12 marks | Newton's Second Law | Main topic: Kinematics Graphs

6. An athlete runs a 200 m race along a straight horizontal track.

In a model of the motion of the athlete, air resistance is ignored, the athlete starts from rest at time $t = 0$ seconds and moves with uniform acceleration $0.8\,\text{m s}^{-2}$ for T seconds, reaching a speed of $V\,\text{m s}^{-1}$. She then maintains this speed until she crosses the finishing line.

The total time from when the athlete starts to when she crosses the finishing line is 30 s.

(a) Sketch a speed-time graph for the model of the motion of the athlete from the instant when she starts to the instant when she crosses the finishing line.

(2)

(b) Write down an expression for V in terms of T.

(1)

(c) Show that $T^2 - kT + 500 = 0$, where k is a constant to be found.

(4)

(d) Hence find the value of T, justifying your answer carefully.

(3)

(e) Considering your speed-time graph or otherwise, state two ways, apart from including air resistance, in which the model could be made to be more realistic.

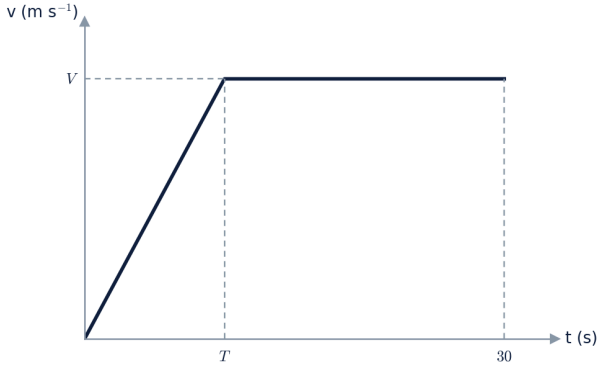
(2)

WORKED SOLUTION

Question 6 · WME01/01 October 2019

1 **Describe the speed-time graph**

The graph starts at $(0, 0)$, rises in a straight line with gradient 0.8 until time T and speed V, then stays horizontal at speed V until $t = 30$.



2 **Write V in terms of T**

Uniform acceleration gives $V = 0.8T$.

3 **Use area for the distance**

The distance is the area under the speed-time graph: $\frac{1}{2}TV + V(30 - T) = 200$.

4 **Substitute V**

Using $V = 0.8T$: $\frac{1}{2}T(0.8T) + (0.8T)(30 - T) = 200$.

5 **Form the quadratic**

$0.4T^2 + 24T - 0.8T^2 = 200$, so $-0.4T^2 + 24T - 200 = 0$. Multiplying by -2.5 gives $T^2 - 60T + 500 = 0$, so $k = 60$.

6 **Solve and choose the valid root**

$T^2 - 60T + 500 = (T - 10)(T - 50) = 0$, so $T = 10$ or 50. Since the whole race lasts 30 s, $T = 10$.

7 **Give two model improvements**

A more realistic model could include reaction time at the start, and could replace the instantaneous change from acceleration to constant speed with a smoother transition.

FINAL ANSWER

(a) Speed-time graph rises linearly to V at $t = T$, then is horizontal until $t = 30$.

(b) $V = 0.8T$. (c) $T^2 - 60T + 500 = 0$, so $k = 60$.

(d) $T = 10$. (e) For example, include reaction time at the start and avoid the instantaneous change to constant speed.

WME01/01 JANUARY 2021

Question 8

17 marks | Newton's Second Law | Main topic: Resolving Forces, Inclined Planes

8.

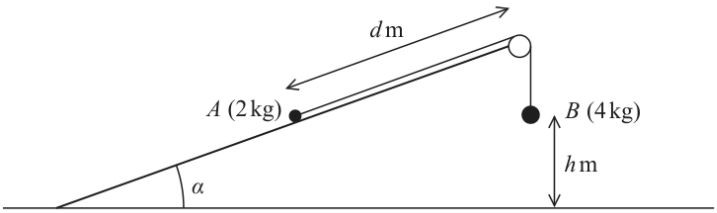


Figure 3

Two particles, A and B, have masses 2 kg and 4 kg respectively. The particles are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough plane. The plane is inclined to the horizontal ground at an angle α where $\tan\alpha = \frac{3}{4}$. The particle A is held at rest on the plane at a distance d metres from the pulley. The particle B hangs freely at rest, vertically below the pulley, at a distance h metres above the ground, as shown in Figure 3. The part of the string between A and the pulley is parallel to a line of greatest slope of the plane. The coefficient of friction between A and the plane is $\frac{1}{4}$

The system is released from rest with the string taut and B descends.

(a) Find the tension in the string as B descends.

(9)

On hitting the ground, B immediately comes to rest.

Given that A comes to rest before reaching the pulley,

(b) find, in terms of h, the range of possible values of d.

(7)

(c) State one physical factor, other than air resistance, that could be taken into account to make the model described above more realistic.

(1)

(Total 17 marks)

WORKED SOLUTION

Question 8 · WME01/01 January 2021

1 Use the trig ratios

Since $\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$.

2 Find friction on A

For A, $R = 2g\cos\alpha = \frac{8g}{5}$. With coefficient $\frac{1}{4}$, friction is $F = \frac{1}{4}R = \frac{2g}{5}$.

3 Equation for A

A moves up the plane. Taking up the plane as positive: $T - 2g\sin\alpha - F = 2a$.

4 Simplify A's equation

$T - 2g\cdot\frac{3}{5} - \frac{2g}{5} = 2a$, so $T - \frac{8g}{5} = 2a$.

5 Equation for B

B descends. Taking downward as positive: $4g - T = 4a$.

6 **Solve for T**

From A, $T = 2a + \frac{8g}{5}$. From B, $T = 4g - 4a$. Equating gives $6a = \frac{12g}{5}$, so $a = \frac{2g}{5}$. Then $T = 4g - 4\left(\frac{2g}{5}\right) = \frac{12g}{5}$ N.

7 **Find speed when B reaches the ground**

Before B hits the ground, both particles move distance h from rest with acceleration $\frac{2g}{5}$. Thus $v^2 = 2\left(\frac{2g}{5}\right)h = \frac{4gh}{5}$.

8 **Find extra distance after B stops**

After B hits the ground, A continues up the plane. The forces down the plane on A are $2g\sin\alpha + F = \frac{6g}{5} + \frac{2g}{5} = \frac{8g}{5}$, so its deceleration is $\frac{4g}{5}$.

9 **Calculate extra distance**

Let the extra distance be s. Then $0 = \frac{4gh}{5} - 2\left(\frac{4g}{5}\right)s$, so $s = \frac{h}{2}$.

10 **Find the range for d**

A travels total distance $h + \frac{h}{2} = \frac{3h}{2}$ before stopping. Since it comes to rest before reaching the pulley, $d > \frac{3h}{2}$.

11 **Name a more realistic factor**

A more realistic model could include friction at the pulley.

✓ **FINAL ANSWER**

$(a)\ T = \frac{12g}{5}\ \text{N} = 23.5\ \text{N}.$ $(b)\ d > \frac{3h}{2}.$ (c) A more realistic model could include friction at the pulley.

WME01/01 OCTOBER 2021

Question 7

14 marks | Newton's Second Law | Main topic: Resolving Forces, Inclined Planes

7.

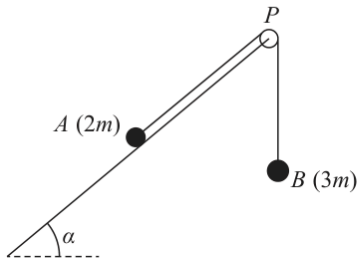


Figure 4

One end of a light inextensible string is attached to a particle A of mass $2m$. The other end of the string is attached to a particle B of mass $3m$. The string passes over a small, smooth, light pulley P which is fixed at the top of a rough inclined plane. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

Particle A is held at rest on the plane with the string taut and B hanging freely below P, as shown in Figure 4. The section of the string AP is parallel to a line of greatest slope of the plane.

The coefficient of friction between A and the plane is $\frac{1}{2}$

Particle A is released and begins to move up the plane.

For the motion before A reaches the pulley,

(a) (i) write down an equation of motion for A,

(ii) write down an equation of motion for B,

(4)

(b) find, in terms of g, the acceleration of A,

(5)

(c) find the magnitude of the force exerted on the pulley by the string.

(4)

(d) State how you have used the information that P is a smooth pulley.

(1)

WORKED SOLUTION

Question 7 · WME01/01 October 2021

1 Equation for A

A moves up the plane, so friction and the component of weight act down the plane. Taking up the plane as positive:
$T - 2mg\sin\alpha - F = 2ma$.

2 Equation for B

B moves downward. Taking downward as positive: $3mg - T = 3ma$.

3 Find friction on A

Since $\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$. The normal reaction on A is $R = 2mg\cos\alpha = \frac{8mg}{5}$, so $F = \frac{1}{2}R = \frac{4mg}{5}$.

4 Simplify A's equation

$T - 2mg\cdot\frac{3}{5} - \frac{4mg}{5} = 2ma$, so $T - 2mg = 2ma$.

5 **Solve with B's equation**

From B, $T = 3mg - 3ma$. Substitute into $T - 2mg = 2ma$: $3mg - 3ma - 2mg = 2ma$. Thus $mg = 5ma$ and $a = \frac{g}{5}$.

6 **Find the tension**

Using $3mg - T = 3m\left(\frac{g}{5}\right)$ gives $T = 3mg - \frac{3mg}{5} = \frac{12mg}{5}$.

7 **Find the pulley resultant**

The two tensions are one vertical and one along the plane. Components of the resultant are $T\cos\alpha$ horizontally and $T + T\sin\alpha$ vertically.

8 **Calculate the magnitude**

Resultant $= T\sqrt{\cos^2\alpha + (1+\sin\alpha)^2} = T\sqrt{(\frac{4}{5})^2 + (\frac{8}{5})^2} = \frac{4\sqrt{5}}{5}T$. With $T = \frac{12mg}{5}$, this is $\frac{48\sqrt{5}}{25}mg$.

9 **Use the smooth pulley information**

The pulley is smooth, so the tension is the same on both sides of the pulley.

✓ **FINAL ANSWER**

$(a)(i)\ T - 2mg\sin\alpha - F = 2ma.$ $(a)(ii)\ 3mg - T = 3ma.$ $(b)\ a = \frac{g}{5}.$ $(c)\ \text{resultant} = \frac{48\sqrt{5}}{25}mg.$ (d) The tension is the same on both sides of the pulley.

WME01/01 JANUARY 2022

Question 6

12 marks | Newton's Second Law | Main topic: Working with Vectors

6. [*In this question* **i** *and* **j** *are horizontal unit vectors.*]

A particle P of mass 2 kg moves under the action of two forces, $(p\mathbf{i} + q\mathbf{j})$N and $(2q\mathbf{i} + p\mathbf{j})$N, where p and q are constants.

Given that the acceleration of P is $(\mathbf{i} - \mathbf{j})\,\mathrm{m\,s^{-2}}$

(a) find the value of p and the value of q.

(5)

(b) Find the size of the angle between the direction of the acceleration and the vector $\mathbf{j}$.

(2)

At time $t = 0$, the velocity of P is $(3\mathbf{i} - 4\mathbf{j})\,\mathrm{m\,s^{-1}}$

At $t = T$ seconds, P is moving in the direction of the vector $(11\mathbf{i} - 13\mathbf{j})$.

(c) Find the value of T.

(5)

WORKED SOLUTION

Question 6 · WME01/01 January 2022

1 Use F = ma

The total force is $(p\mathbf{i} + q\mathbf{j}) + (2q\mathbf{i} + p\mathbf{j}) = (p + 2q)\mathbf{i} + (p + q)\mathbf{j}$. Since the mass is 2 kg and acceleration is $\mathbf{i} - \mathbf{j}$, the total force is $2\mathbf{i} - 2\mathbf{j}$.

2 Equate components

$p + 2q = 2$ and $p + q = -2$.

3 Find p and q

Subtract the equations to get $q = 4$. Then $p + 4 = -2$, so $p = -6$.

4 Find the angle with j

The acceleration is $\mathbf{i} - \mathbf{j}$. Its dot product with $\mathbf{j}$ is -1, so the angle with the positive $\mathbf{j}$ direction is obtuse. Since the components are equal in magnitude, the angle is $135°$.

5 Write the velocity at time T

$\mathbf{v} = (3\mathbf{i} - 4\mathbf{j}) + T(\mathbf{i} - \mathbf{j}) = (3 + T)\mathbf{i} + (-4 - T)\mathbf{j}$.

6 Use the direction vector

The velocity is parallel to $11\mathbf{i} - 13\mathbf{j}$, so $\dfrac{3 + T}{11} = \dfrac{-4 - T}{-13}$.

7 Solve for T

$13(3 + T) = 11(4 + T)$, so $39 + 13T = 44 + 11T$. Hence $2T = 5$ and $T = 2.5$.

FINAL ANSWER

$(a)\ p = -6,\ q = 4.$ $(b)\ 135°.$ $(c)\ T = 2.5.$

WME01/01 JANUARY 2022

Question 7

13 marks | Newton's Second Law

Main topic: Resolving Forces, Inclined Planes

7.

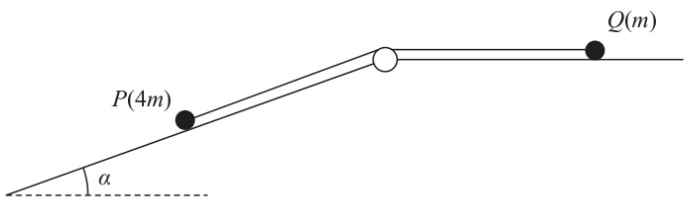


Figure 4

A particle P of mass $4m$ lies on the surface of a fixed rough inclined plane.

The plane is inclined to the horizontal at an angle α where $\tan\alpha = \frac{3}{4}$

The particle P is attached to one end of a light inextensible string.

The string passes over a small smooth pulley that is fixed at the top of the plane. The other end of the string is attached to a particle Q of mass m which lies on a smooth horizontal plane.

The string lies along the horizontal plane and in the vertical plane that contains the pulley and a line of greatest slope of the inclined plane.

The system is released from rest with the string taut, as shown in Figure 4, and P moves down the plane.

The coefficient of friction between P and the plane is $\frac{1}{4}$

For the motion before Q reaches the pulley

(a) write down an equation of motion for Q,

(1)

(b) find, in terms of m and g, the tension in the string,

(7)

(c) find the magnitude of the force exerted on the pulley by the string.

(4)

(d) State where in your working you have used the information that the string is light.

(1)

WORKED SOLUTION

Question 7 · WME01/01 January 2022

1 Write the equation for Q

Q lies on a smooth horizontal plane. The only horizontal force on Q is the tension, so $T = ma$.

2 Use the trig ratios

Since $\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$.

3 Find friction on P

For P, $R = 4mg\cos\alpha = 4mg \cdot \frac{4}{5} = \frac{16mg}{5}$. With coefficient $\frac{1}{4}$, friction is $F = \frac{1}{4}R = \frac{4mg}{5}$.

4 Apply Newtons second law to P

P moves down the plane. Taking down the plane as positive: $4mg\sin\alpha - T - F = 4ma$.

5 Substitute values

$4mg \cdot \frac{3}{5} - T - \frac{4mg}{5} = 4ma$, so $\frac{8mg}{5} - T = 4ma$.

6 **Use T = ma**

From Q, $a = \dfrac{T}{m}$. Substitute into the equation for P: $\dfrac{8mg}{5} - T = 4T$.

7 **Find T**

$\dfrac{8mg}{5} = 5T$, so $T = \dfrac{8}{25}mg$.

8 **Find the pulley resultant**

The pulley is pulled by two tensions of magnitude T: one horizontal and one along the inclined string. Their resultant has components $T - T\cos\alpha$ horizontally and $T\sin\alpha$ vertically.

9 **Calculate the resultant**

Magnitude $= \sqrt{(T - T\cos\alpha)^2 + (T\sin\alpha)^2} = T\sqrt{(1 - \frac{4}{5})^2 + (\frac{3}{5})^2} = \dfrac{T}{5}\sqrt{10}$. With $T = \dfrac{8}{25}mg$, the resultant is $\dfrac{8\sqrt{10}}{125}mg$.

10 **Use the light-string model**

The string is light, so the tension can be treated as the same on the P side and the Q side of the pulley.

✓ **FINAL ANSWER**

$(a)\ T = ma.$ $(b)\ T = \dfrac{8}{25}mg.$ $(c)\ \text{resultant} = \dfrac{8\sqrt{10}}{125}mg.$ (d) Same tension was used on both sides of the pulley.

Question 6

9 marks | Newton's Second Law | Main topic: Working with Vectors

6. [*In this question,* **i** *and* **j** *are horizontal unit vectors.*]

A particle A of mass 0.5 kg is at rest on a smooth horizontal plane.

At time $t = 0$, two forces, $\mathbf{F}_1 = (-3\mathbf{i} + 2\mathbf{j})$ N and $\mathbf{F}_2 = (p\mathbf{i} + q\mathbf{j})$ N, where p and q are constants, are applied to A.

Given that A moves in the direction of the vector $(\mathbf{i} - 2\mathbf{j})$,

(a) show that $2p + q - 4 = 0$

(4)

Given that $p = 5$

(b) find the speed of A at time $t = 4$ seconds.

(5)

WORKED SOLUTION

Question 6 · WME01/01 October 2022

1 Find the resultant force

$\mathbf{F} = (-3\mathbf{i} + 2\mathbf{j}) + (p\mathbf{i} + q\mathbf{j}) = (p - 3)\mathbf{i} + (q + 2)\mathbf{j}$.

2 Use the direction of motion

Since the particle starts from rest, its motion is in the direction of the resultant force. This direction is $\mathbf{i} - 2\mathbf{j}$, so $\dfrac{q+2}{p-3} = -2$.

3 Show the relation

$q + 2 = -2(p - 3) = -2p + 6$, hence $2p + q - 4 = 0$.

4 Find q when p = 5

Substitute $p = 5$ into $2p + q - 4 = 0$: $10 + q - 4 = 0$, so $q = -6$.

5 Find the resultant force

The resultant force is $(5 - 3)\mathbf{i} + (-6 + 2)\mathbf{j} = 2\mathbf{i} - 4\mathbf{j}$ N.

6 Find acceleration

The mass is 0.5 kg, so $\mathbf{a} = \dfrac{2\mathbf{i} - 4\mathbf{j}}{0.5} = 4\mathbf{i} - 8\mathbf{j}$.

7 Find velocity at t = 4

Starting from rest, $\mathbf{v} = \mathbf{a}t = (4\mathbf{i} - 8\mathbf{j})4 = 16\mathbf{i} - 32\mathbf{j}$.

8 Find the speed

Speed $= \sqrt{16^2 + (-32)^2} = \sqrt{1280} = 16\sqrt{5}\ \mathrm{m\,s^{-1}}$, about $36\ \mathrm{m\,s^{-1}}$.

FINAL ANSWER

$(a)\ 2p + q - 4 = 0.$ $(b)\ \text{speed} = 16\sqrt{5}\ \mathrm{m\,s^{-1}} \approx 36\ \mathrm{m\,s^{-1}}$.

WME01/01 JANUARY 2023

Question 8

15 marks | Newton's Second Law

Main topic: Resolving Forces, Inclined Planes

8.

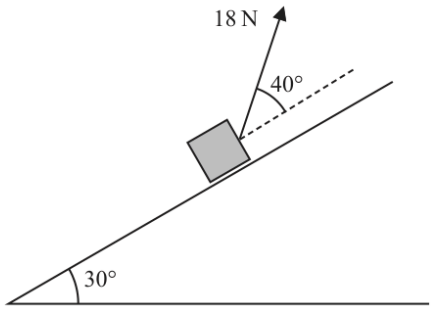


Figure 5

A parcel of mass 2 kg is pulled up a rough inclined plane by the action of a constant force.

The force has magnitude 18 N and acts at an angle of 40° to the plane.

The line of action of the force lies in a vertical plane containing a line of greatest slope of the inclined plane.

The plane is inclined at an angle of 30° to the horizontal, as shown in Figure 5.

The coefficient of friction between the plane and the parcel is 0.3

The parcel is modelled as a particle P

(a) Find the acceleration of P

(8)

The points A and B lie on a line of greatest slope of the plane, where $AB = 5\,\text{m}$ and B is above A. Particle P passes through A with speed $2\,\text{m s}^{-1}$ in the direction AB.

(b) Find the speed of P as it passes through B.

(3)

The force of 18 N is removed at the instant P passes through B. As a result, P comes to rest at the point C.

(c) Determine whether P will remain at rest at C. You must show all stages of your working clearly.

(4)

WORKED SOLUTION

Question 8 · WME01/01 January 2023

1 Resolve perpendicular to the plane

The force of 18 N is angled away from the plane, so $R + 18\sin 40° = 2g\cos 30°$. Hence $R = 2g\cos 30° - 18\sin 40°$.

2 Use friction

The parcel moves up the plane, so friction acts down the plane. With $\mu = 0.3$, $F = 0.3R$.

3 Resolve parallel to the plane

Taking up the plane as positive: $18\cos 40° - F - 2g\sin 30° = 2a$.

4 Substitute for R and F

$18\cos 40° - 0.3(2g\cos 30° - 18\sin 40°) - 2g\sin 30° = 2a$.

5 Find the acceleration

Using $g = 9.8$, this gives $a = 1.18\ldots\ \text{m s}^{-2}$ up the plane.

6 **Find the speed at B**

From A to B, $u = 2$, $s = 5$ and $a = 1.18\ldots$. Use $v^2 = u^2 + 2as$: $v^2 = 2^2 + 2(1.18\ldots)(5)$.

7 **State the speed**

$v = 3.98\ldots\ \text{m s}^{-1}$, about $4.0\ \text{m s}^{-1}$.

8 **Check rest after the force is removed**

At C, with the pulling force removed, $R = 2g\cos 30°$. The maximum friction is $0.3R = 0.3(2g\cos 30°) = 5.09\ldots\ \text{N}$.

9 **Compare with the downslope weight component**

The component of weight down the plane is $2g\sin 30° = 9.8\ \text{N}$. Since $9.8 > 5.09\ldots$, friction is not large enough to hold P at rest, so P will not remain at rest at C.

✓ **FINAL ANSWER**

$(a)\ a = 1.18\ \text{m s}^{-2}$ **up the plane.** $(b)\ v = 3.98\ \text{m s}^{-1} \approx 4.0\ \text{m s}^{-1}$**.** (c) P will not remain at rest at C.

WME01/01 MAY/JUNE 2023

Question 6

7 marks | Newton's Second Law | Main topic: Forces

6.

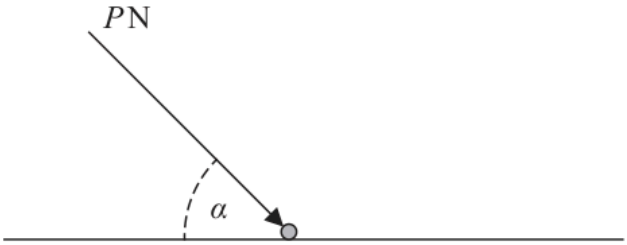


Figure 3

A particle of weight W newtons lies at rest on a rough horizontal surface, as shown in Figure 3.
A force of magnitude P newtons is applied to the particle.

The force acts at an angle α to the horizontal, where $\tan\alpha = \frac{4}{3}$

The coefficient of friction between the particle and the surface is $\frac{1}{4}$

Given that the particle does not move, show that

$$P \leqslant \frac{5W}{8}$$

(7)

WORKED SOLUTION

Question 6 · WME01/01 May/June 2023

1 Use the trig ratios

Given $\tan\alpha = \frac{4}{3}$, use $\sin\alpha = \frac{4}{5}$ and $\cos\alpha = \frac{3}{5}$.

2 Resolve vertically

The applied force increases the normal reaction in this model, so $R = W + P\sin\alpha$.

3 Resolve horizontally

For the particle not to move, friction must be able to balance the horizontal component of the applied force, so $P\cos\alpha \leq F_{\max}$.

4 Use limiting friction

Since $\mu = \frac{1}{4}$, $F_{\max} = \frac{1}{4}R = \frac{1}{4}(W + P\sin\alpha)$.

5 Substitute the exact trig values

Thus $P\cdot\frac{3}{5} \leq \frac{1}{4}\left(W + P\cdot\frac{4}{5}\right)$.

6 Rearrange the inequality

Multiplying by 20 gives $12P \leq 5W + 4P$. Hence $8P \leq 5W$, so $P \leq \frac{5}{8}W$.

FINAL ANSWER

$$P \leq \frac{5}{8}W.$$

WME01/01 OCTOBER 2023

Question 7

13 marks | Newton's Second Law

Main topic: Resolving Forces, Inclined Planes

7.

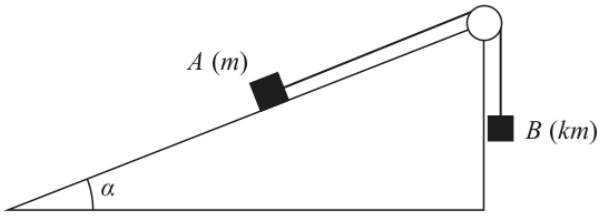


Figure 4

Figure 4 shows a block A of mass m held at rest on a rough plane.
The plane is inclined at an angle α to the horizontal and the coefficient of friction between the block and the plane is μ.

One end of a light inextensible string is now attached to A. The string passes over a small smooth pulley which is fixed at the top of the plane.
The other end of the string is attached to a block B of mass km.
Block B hangs vertically below the pulley, with the string taut.

The string from A to the pulley lies along a line of greatest slope of the plane.

Both A and B are modelled as particles.

When the system is released from rest, A moves up the plane and the tension in the string is $\frac{4mg}{3}$

Given that $\mu = \frac{1}{3}$ and $\tan\alpha = \frac{7}{24}$

(a) (i) find the magnitude of the acceleration of A, giving your answer in terms of g,

(ii) find the value of k.

(9)

(b) Find the magnitude of the resultant force exerted on the pulley by the string, giving your answer in terms of m and g.

(4)

WORKED SOLUTION

Question 7 · WME01/01 October 2023

1 Use the trig ratios

Given $\tan\alpha = \frac{7}{24}$, use $\sin\alpha = \frac{7}{25}$ and $\cos\alpha = \frac{24}{25}$.

2 Find friction on A

For block A, $R = mg\cos\alpha = \frac{24mg}{25}$. Since $\mu = \frac{1}{3}$, friction is $F = \frac{1}{3}R = \frac{8mg}{25}$.

3 Apply Newtons second law to A

A moves up the plane, so friction and the component of weight act down the plane. Taking up the plane as positive:
$T - mg\sin\alpha - F = ma$.

4 Find the acceleration

Substitute $T = \frac{4}{3}mg$, $mg\sin\alpha = \frac{7mg}{25}$ and $F = \frac{8mg}{25}$: $ma = \frac{4}{3}mg - \frac{15mg}{25} = \left(\frac{4}{3} - \frac{3}{5}\right)mg = \frac{11mg}{15}$. Hence $a = \frac{11g}{15}$.

5 **Apply Newtons second law to B**

Block B moves downwards with the same acceleration. Taking downward as positive: $kmg - T = kma$.

6 **Find k**

Substitute $T = \frac{4}{3}mg$ and $a = \frac{11g}{15}$: $kmg - \frac{4}{3}mg = km\frac{11g}{15}$. Divide by mg to get $k - \frac{4}{3} = \frac{11k}{15}$, so $15k - 20 = 11k$ and $k = 5$.

7 **Resolve the force on the pulley**

The pulley is pulled by two tensions of magnitude T. One is vertical and the other is along the plane. The horizontal component of the resultant is $T\cos\alpha$ and the vertical component is $T + T\sin\alpha$.

8 **Find the resultant**

The magnitude is $T\sqrt{\cos^2\alpha + (1+\sin\alpha)^2}$. With $\sin\alpha = \frac{7}{25}$ and $\cos\alpha = \frac{24}{25}$, this is $T\sqrt{\frac{576}{625} + \frac{1024}{625}} = \frac{8T}{5}$. Since $T = \frac{4}{3}mg$, the resultant is $\frac{32}{15}mg$.

FINAL ANSWER

$(a)(i)\ a = \frac{11g}{15}.\quad (a)(ii)\ k = 5.\quad (b)\ \text{resultant} = \frac{32}{15}mg.$

WME01/01 MAY/JUNE 2024

Question 8

15 marks | Newton's Second Law

Main topic: Resolving Forces, Inclined Planes

8.

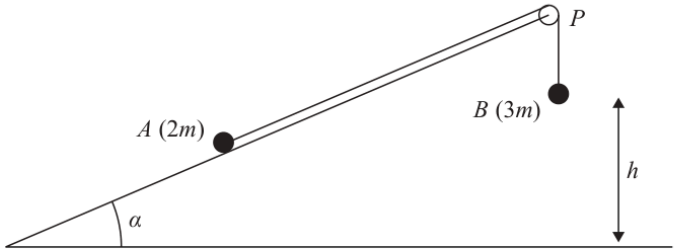


Figure 4

One end of a light inextensible string is attached to a particle A of mass $2m$. The other end of the string is attached to a particle B of mass $3m$. Particle A is held at rest on a rough plane which is inclined to horizontal ground at an angle α, where $\tan\alpha = \frac{5}{12}$

The string passes over a small smooth pulley P which is fixed at the top of the plane. Particle B hangs vertically below P with the string taut, at a height h above the ground, as shown in Figure 4.

The part of the string between A and P lies along a line of greatest slope of the plane. The two particles, the string and the pulley all lie in the same vertical plane.

The coefficient of friction between A and the plane is $\frac{11}{36}$

The particle A is released from rest and begins to move up the plane.

(a) Show that the frictional force acting on A as it moves up the plane is $\frac{22mg}{39}$ **(3)**

(b) Write down an equation of motion for B. **(2)**

(c) Show that the acceleration of A immediately after its release is $\frac{1}{3}g$ **(4)**

In the subsequent motion, A comes to rest before it reaches the pulley.

(d) Find, in terms of h, the total distance travelled by A from when it was released from rest to when it first comes to rest again. **(6)**

WORKED SOLUTION

Question 8 · WME01/01 May/June 2024

1 Use the trig ratios

Here $\tan\alpha = \frac{5}{12}$, so $\sin\alpha = \frac{5}{13}$ and $\cos\alpha = \frac{12}{13}$.

2 Show the frictional force on A

For A, $R = 2mg\cos\alpha = 2mg \cdot \frac{12}{13} = \frac{24mg}{13}$. Since $\mu = \frac{11}{36}$, $F = \mu R = \frac{11}{36} \cdot \frac{24mg}{13} = \frac{22mg}{39}$.

3 Write the equation for B

B moves down as A moves up. Taking downward as positive for B: $3mg - T = 3ma$.

4 Write the equation for A

Taking up the plane as positive for A: $T - 2mg\sin\alpha - F = 2ma$. Substituting $\sin\alpha = \frac{5}{13}$ and $F = \frac{22mg}{39}$ gives $T - \frac{10mg}{13} - \frac{22mg}{39} = 2ma$.

5 **Show the acceleration**

From B, $T = 3mg - 3ma$. Substitute into A equation: $3mg - 3ma - \frac{10mg}{13} - \frac{22mg}{39} = 2ma$. The constant part is $\frac{5mg}{3}$, so $\frac{5mg}{3} = 5ma$ and $a = \frac{g}{3}$.

6 **Find the speed when B reaches the ground**

While B descends distance h, A moves distance h up the plane from rest with acceleration $g/3$. Thus $v^2 = 2\left(\frac{g}{3}\right)h = \frac{2gh}{3}$.

7 **Find the later deceleration of A**

After B reaches the ground, the string no longer pulls A. The forces down the plane on A are $2mg\sin\alpha + F = \frac{10mg}{13} + \frac{22mg}{39} = \frac{4mg}{3}$. Since A has mass $2m$, its deceleration is $\frac{2g}{3}$.

8 **Find the extra distance and total distance**

Let the extra distance be d. Using $0 = v^2 - 2\left(\frac{2g}{3}\right)d$ with $v^2 = \frac{2gh}{3}$ gives $d = \frac{h}{2}$. Total distance travelled by A is $h + \frac{h}{2} = \frac{3h}{2}$.

FINAL ANSWER

$(a)\ F = \frac{22}{39}mg.\quad (b)\ 3mg - T = 3ma.\quad (c)\ a = \frac{g}{3}.\quad (d)\ \text{total distance} = \frac{3h}{2}.$

WME01/01 OCTOBER 2025

Question 7

14 marks | Newton's Second Law | Main topic: Resolving Forces, Inclined Planes

7.

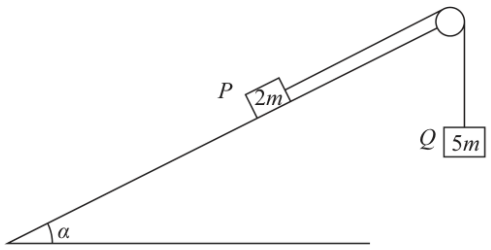


Figure 5

A block P of mass $2m$ is held at rest on a fixed rough plane.

The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{5}{12}$

One end of a light inextensible string is attached to P.
The string is parallel to a line of greatest slope of the plane and passes over a smooth light pulley which is fixed at the top of the plane.
The other end of the string is attached to a block Q of mass $5m$.
Block Q hangs vertically below the pulley, as shown in Figure 5.

The system is released from rest with the string taut and block P moves up the plane.

Immediately after the system is released, the tension in the string is T and the acceleration of the blocks is a.

The blocks are modelled as particles and air resistance is ignored.

(a) Write down an equation of motion for Q.

(2)

The coefficient of friction between P and the plane is $\frac{1}{8}$

(b) Find T in terms of m and g.

(7)

(c) State how the solution to part (b) uses the fact that the string is inextensible.

(1)

The magnitude of the force exerted on the pulley by the string is kmg.

(d) Find the value of k to 3 significant figures.

(4)

WORKED SOLUTION

Question 7 · WME01/01 October 2025

1 **Part (a): Write equation of motion for Q**

Forces on Q (mass = 5m):

Taking downward as positive for Q:

$\text{For Q: } \sum F = ma$

$5mg - T = 5ma$

2 **Part (b): Find T in terms of m and g - Resolve perpendicular to plane**

$\perp$ to plane: $\sum F = 0$

$R = 2mg\cos\alpha \; R = 2mg \times \frac{12}{13} \; R = \frac{24mg}{13}$

3 **Part (b): Find T in terms of m and g - Find friction force**

P is moving UP the plane, so friction acts DOWN the plane:

$F = \mu R = \frac{1}{8} \times \frac{24mg}{13} = \frac{3mg}{13}$

4 **Part (b): Find T in terms of m and g - Resolve parallel to plane**

Taking up the plane as positive:

$\parallel$ to plane: $\sum F = ma$

$T - 2mg\sin\alpha - F = 2ma\ T - 2mg \times \frac{5}{13} - \frac{3mg}{13} = 2ma\ T - \frac{10mg}{13} - \frac{3mg}{13} = 2ma\ T - \frac{13mg}{13} = 2ma\ T - mg = 2ma \quad ...(1)$

5 **Part (b): Find T in terms of m and g - Combine with equation for Q**

From part (a): $5mg - T = 5ma$ ...(2)

Add equations (1) and (2):

$(T - mg) + (5mg - T) = 2ma + 5ma\ 4mg = 7ma\ a = \frac{4g}{7}$

6 **Part (b): Find T in terms of m and g - Substitute back to find T**

From equation (1):

$T = mg + 2ma = mg + 2m \times \frac{4g}{7} = mg + \frac{8mg}{7} = \frac{7mg+8mg}{7} = \frac{15mg}{7}$

7 **Part (c): State how solution uses fact that string is inextensible**

An inextensible string means the string cannot stretch.

This has the following consequence:

For an inextensible string over a pulley:

The magnitude of acceleration of both connected particles must be equal.

In this problem: $|a_P| = |a_Q| = a$

How this was used in part (b):

When we wrote the equations of motion:

; For P: $T - mg = 2ma$
; For Q: $5mg - T = 5ma$

We used the same acceleration a for both particles because the string is inextensible.

If the string could stretch, the two particles could have different accelerations.

8 **Part (d): Find value of k (force on pulley = kmg) - Identify the angle between tensions**

The string from Q is vertical (downward).
The string from P is along the plane at angle α to horizontal.

Angle between the two strings = $90° + \alpha$

9 **Part (d): Find value of k (force on pulley = kmg) - Use vector components**

Let's take horizontal (right) and vertical (up) as positive directions.

Tension from Q:

; Horizontal component: 0
; Vertical component: $-T = -\frac{15mg}{7}$ (downward)

Tension from P along plane:
The plane makes angle α with horizontal, so:

; Horizontal component: $-T\cos\alpha = -\frac{15mg}{7} \times \frac{12}{13} = -\frac{180mg}{91}$
; Vertical component: $T\sin\alpha = \frac{15mg}{7} \times \frac{5}{13} = \frac{75mg}{91}$

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10 **Part (d): Find value of k (force on pulley = kmg) - Find resultant**

$R_x = 0 - \frac{180mg}{91} = -\frac{180mg}{91}$ $R_y = -\frac{15mg}{7} + \frac{75mg}{91}$
Convert $\frac{15mg}{7}$ to denominator 91: $\frac{15mg}{7} = \frac{195mg}{91}$
$R_y = -\frac{195mg}{91} + \frac{75mg}{91} = -\frac{120mg}{91}$

---

11 **Part (d): Find value of k (force on pulley = kmg) - Find magnitude**

$|R| = \sqrt{R_x^2 + R_y^2} = \sqrt{\left(\frac{180mg}{91}\right)^2 + \left(\frac{120mg}{91}\right)^2} = \frac{mg}{91}\sqrt{180^2 + 120^2} = \frac{mg}{91}\sqrt{32400 + 14400} = \frac{mg}{91}\sqrt{46800} = \frac{mg}{91} \times 216.333\ldots$
$= 2.377\ldots mg = 2.38mg$ (3 s.f.)
Therefore: $k = 2.38$

FINAL ANSWER

$(a)\ 5mg - T = 5ma.$ $(b)\ T = \frac{15mg}{7}.$ (c) both particles have the same magnitude of acceleration. $(d)\ k = 2.38.$

WME01/01 JANUARY 2026

Question 7

14 marks | Newton's Second Law

Main topic: Resolving Forces, Inclined Planes

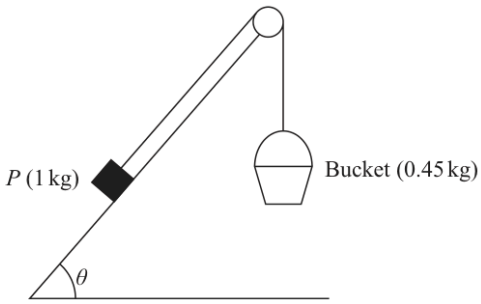


Figure 3

A rough plane is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{4}{3}$

One end of a light inextensible string is attached to a package P.
The string passes over a smooth pulley that is fixed at the top of the plane.
The string from P to the pulley lies along a line of greatest slope of the plane.
The other end of the string is attached to a bucket.
The package P has mass 1 kg and is **held** at rest on the plane.
The bucket has mass 0.45 kg and hangs vertically below the pulley with the string taut, as shown in Figure 3.

The coefficient of friction between P and the plane is μ

When a small block of mass 0.2 kg is placed in the bucket, the system is released and remains at rest in **limiting equilibrium**, with P on the point of slipping down the plane.

(a) Show that $\mu = \frac{1}{4}$

(7)

Additional identical blocks of mass 0.2 kg are added one at a time into the bucket until P starts to move **up** the plane.
At the instant when P starts to move up the plane, there are a **total** of n blocks in the bucket.

(b) Find

(i) the value of n

(ii) the magnitude of the initial acceleration of P

(7)

WORKED SOLUTION

Question 7 · WME01/01 January 2026

1 Find the trigonometric ratios

Since $\tan\theta = \frac{4}{3}$, use a 3-4-5 triangle: $\sin\theta = \frac{4}{5}$ and $\cos\theta = \frac{3}{5}$.

2 Use limiting equilibrium in part (a)

With one 0.2 kg block in the bucket, the bucket side has mass $0.45 + 0.2 = 0.65$ kg. Since the package is on the point of slipping down the plane, friction on P acts up the plane.

3 Find the friction force

The tension equals the bucket weight, so $T = 0.65g$. Along the plane for P in equilibrium, $T + F = g\sin\theta$. Hence $0.65g + F = \frac{4}{5}g$, so $F = \frac{3g}{20}$.

4 Find the normal reaction and mu

Perpendicular to the plane, $R = g\cos\theta = \frac{3}{5}g$. Since $F = \mu R$, $\mu = \frac{F}{R} = \frac{3g/20}{3g/5} = \frac{1}{4}$.

5 **Set the condition for upward motion**

When P is about to move up the plane, friction acts down the plane. With n blocks in the bucket, the bucket-side mass is $0.45 + 0.2n$. The bucket weight must exceed the downslope resistance: $(0.45 + 0.2n)g > g\sin\theta + \mu R$.

6 **Find n**

Using $g\sin\theta = \frac{4}{5}g$ and $\mu R = \frac{1}{4} \cdot \frac{3}{5}g = \frac{3g}{20}$, the condition is $0.45 + 0.2n > \frac{4}{5} + \frac{3}{20} = 0.95$. Hence $0.2n > 0.50$, so $n > 2.5$ and the least integer is $n = 3$.

7 **Apply Newtons second law after motion begins**

For $n = 3$, the bucket-side mass is $0.45 + 0.6 = 1.05$ kg and the total moving mass is $1 + 1.05 = 2.05$ kg. The driving force is $1.05g - \frac{4}{5}g - \frac{3g}{20} = 0.10g$.

8 **Find the acceleration**

Using $F = ma$, $0.10g = 2.05a$, so $a = \frac{0.10g}{2.05} = \frac{2g}{41} = 0.478\ \mathrm{m\,s^{-2}}$ to 3 significant figures.

FINAL ANSWER

$(a)\ \mu = \frac{1}{4}.\quad (b)(i)\ n = 3.\quad (b)(ii)\ a = \frac{2g}{41} = 0.478\ \mathrm{m\,s^{-2}}$ approximately.

Dr Eslam Ahmed · eliteigcse.com

TOPIC

Resolving Forces, Inclined Planes

Every past-paper question on this topic, October 2019 to January 2026.

Question 8

17 marks | Resolving Forces, Inclined Planes

8.

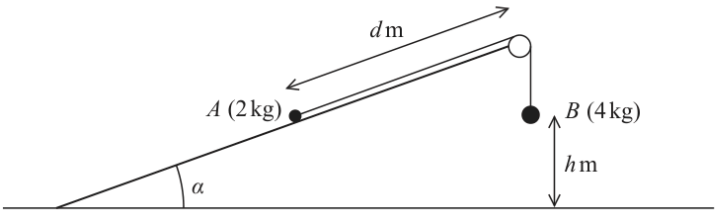


Figure 3

Two particles, A and B, have masses 2 kg and 4 kg respectively. The particles are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough plane. The plane is inclined to the horizontal ground at an angle α where $\tan\alpha = \frac{3}{4}$. The particle A is held at rest on the plane at a distance d metres from the pulley. The particle B hangs freely at rest, vertically below the pulley, at a distance h metres above the ground, as shown in Figure 3. The part of the string between A and the pulley is parallel to a line of greatest slope of the plane. The coefficient of friction between A and the plane is $\frac{1}{4}$

The system is released from rest with the string taut and B descends.

(a) Find the tension in the string as B descends.

(9)

On hitting the ground, B immediately comes to rest.

Given that A comes to rest before reaching the pulley,

(b) find, in terms of h, the range of possible values of d.

(7)

(c) State one physical factor, other than air resistance, that could be taken into account to make the model described above more realistic.

(1)

(Total 17 marks)

WORKED SOLUTION

Question 8 · WME01/01 January 2021

1 Use the trig ratios

Since $\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$.

2 Find friction on A

For A, $R = 2g\cos\alpha = \frac{8g}{5}$. With coefficient $\frac{1}{4}$, friction is $F = \frac{1}{4}R = \frac{2g}{5}$.

3 Equation for A

A moves up the plane. Taking up the plane as positive: $T - 2g\sin\alpha - F = 2a$.

4 Simplify A's equation

$T - 2g\cdot\frac{3}{5} - \frac{2g}{5} = 2a$, so $T - \frac{8g}{5} = 2a$.

5 Equation for B

B descends. Taking downward as positive: $4g - T = 4a$.

6 **Solve for T**

From A, $T = 2a + \frac{8g}{5}$. From B, $T = 4g - 4a$. Equating gives $6a = \frac{12g}{5}$, so $a = \frac{2g}{5}$. Then $T = 4g - 4\left(\frac{2g}{5}\right) = \frac{12g}{5}$ N.

7 **Find speed when B reaches the ground**

Before B hits the ground, both particles move distance h from rest with acceleration $\frac{2g}{5}$. Thus $v^2 = 2\left(\frac{2g}{5}\right)h = \frac{4gh}{5}$.

8 **Find extra distance after B stops**

After B hits the ground, A continues up the plane. The forces down the plane on A are $2g\sin\alpha + F = \frac{6g}{5} + \frac{2g}{5} = \frac{8g}{5}$, so its deceleration is $\frac{4g}{5}$.

9 **Calculate extra distance**

Let the extra distance be s. Then $0 = \frac{4gh}{5} - 2\left(\frac{4g}{5}\right)s$, so $s = \frac{h}{2}$.

10 **Find the range for d**

A travels total distance $h + \frac{h}{2} = \frac{3h}{2}$ before stopping. Since it comes to rest before reaching the pulley, $d > \frac{3h}{2}$.

11 **Name a more realistic factor**

A more realistic model could include friction at the pulley.

✓ **FINAL ANSWER**

$(a)\ T = \frac{12g}{5}\ \text{N} = 23.5\ \text{N}.$ $(b)\ d > \frac{3h}{2}.$ (c) A more realistic model could include friction at the pulley.

WME01/01 MAY/JUNE 2021

Question 6

13 marks | Resolving Forces, Inclined Planes

6. A fixed rough plane is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$

A particle of mass 6 kg is projected with speed 5 m s^{-1} from a point A on the plane, up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is $\frac{1}{4}$

(a) Find the magnitude of the frictional force acting on the particle as it moves up the plane.

(3)

The particle comes to instantaneous rest at the point B.

(b) Find the distance AB.

(5)

The particle now slides down the plane from B. At the instant when the particle passes through the point C on the plane, the speed of the particle is again 5 m s^{-1}

(c) Find the distance BC.

(5)

WORKED SOLUTION

Question 6 · WME01/01 May/June 2021

1 Use the trig ratios

Since $\tan\theta = \frac{5}{12}$, $\sin\theta = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$.

2 Find the normal reaction

$R = 6g\cos\theta = 6g \cdot \frac{12}{13} = \frac{72g}{13}$.

3 Find friction

With $\mu = \frac{1}{4}$, $F = \frac{1}{4}R = \frac{18g}{13} = 13.6\ \text{N}$ approximately.

4 Find acceleration while moving up

While moving up, both friction and the component of weight act down the plane. Taking up the plane as positive: $6a = -6g\sin\theta - F$.

5 Simplify the upward acceleration

$6a = -6g \cdot \frac{5}{13} - \frac{18g}{13} = -\frac{48g}{13}$, so $a = -\frac{8g}{13}$.

6 Find AB

Use $v^2 = u^2 + 2as$ with $u = 5$ and $v = 0$: $0 = 25 + 2\left(-\frac{8g}{13}\right)AB$. Hence $AB = 2.07\ldots\ \text{m}$.

7 Find acceleration while sliding down

When sliding down, friction acts up the plane. Taking down the plane as positive: $6a = 6g\sin\theta - F = \frac{30g}{13} - \frac{18g}{13} = \frac{12g}{13}$. Thus $a = \frac{2g}{13}$.

8 **Find BC**

Starting from rest at B and reaching speed 5 at C: $25 = 2\left(\frac{2g}{13}\right)BC$. Hence $BC = 8.29\ldots\ \text{m}$.

FINAL ANSWER

$(a)\ F = 13.6\ \text{N}.\quad (b)\ AB = 2.07\ \text{m} \approx 2.1\ \text{m}.\quad (c)\ BC = 8.29\ \text{m} \approx 8.3\ \text{m}.$

WME01/01 OCTOBER 2021

Question 7

14 marks | Resolving Forces, Inclined Planes

7.

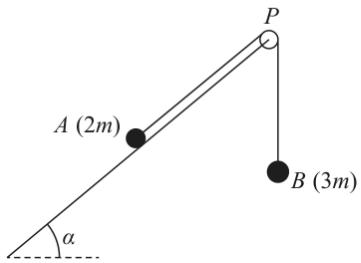


Figure 4

One end of a light inextensible string is attached to a particle A of mass $2m$. The other end of the string is attached to a particle B of mass $3m$. The string passes over a small, smooth, light pulley P which is fixed at the top of a rough inclined plane. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

Particle A is held at rest on the plane with the string taut and B hanging freely below P, as shown in Figure 4. The section of the string AP is parallel to a line of greatest slope of the plane.

The coefficient of friction between A and the plane is $\frac{1}{2}$

Particle A is released and begins to move up the plane.

For the motion before A reaches the pulley,

(a) (i) write down an equation of motion for A,

(ii) write down an equation of motion for B,

(4)

(b) find, in terms of g, the acceleration of A,

(5)

(c) find the magnitude of the force exerted on the pulley by the string.

(4)

(d) State how you have used the information that P is a smooth pulley.

(1)

WORKED SOLUTION

Question 7 · WME01/01 October 2021

1 Equation for A

A moves up the plane, so friction and the component of weight act down the plane. Taking up the plane as positive:
$T - 2mg\sin\alpha - F = 2ma$.

2 Equation for B

B moves downward. Taking downward as positive: $3mg - T = 3ma$.

3 Find friction on A

Since $\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$. The normal reaction on A is $R = 2mg\cos\alpha = \frac{8mg}{5}$, so $F = \frac{1}{2}R = \frac{4mg}{5}$.

4 Simplify A's equation

$T - 2mg\cdot\frac{3}{5} - \frac{4mg}{5} = 2ma$, so $T - 2mg = 2ma$.

5 **Solve with B's equation**

From B, $T = 3mg - 3ma$. Substitute into $T - 2mg = 2ma$: $3mg - 3ma - 2mg = 2ma$. Thus $mg = 5ma$ and $a = \frac{g}{5}$.

6 **Find the tension**

Using $3mg - T = 3m\left(\frac{g}{5}\right)$ gives $T = 3mg - \frac{3mg}{5} = \frac{12mg}{5}$.

7 **Find the pulley resultant**

The two tensions are one vertical and one along the plane. Components of the resultant are $T\cos\alpha$ horizontally and $T + T\sin\alpha$ vertically.

8 **Calculate the magnitude**

Resultant $= T\sqrt{\cos^2\alpha + (1+\sin\alpha)^2} = T\sqrt{(\frac{4}{5})^2 + (\frac{8}{5})^2} = \frac{4\sqrt{5}}{5}T$. With $T = \frac{12mg}{5}$, this is $\frac{48\sqrt{5}}{25}mg$.

9 **Use the smooth pulley information**

The pulley is smooth, so the tension is the same on both sides of the pulley.

✓ **FINAL ANSWER**

$(a)(i)\ T - 2mg\sin\alpha - F = 2ma.$ $(a)(ii)\ 3mg - T = 3ma.$ $(b)\ a = \frac{g}{5}.$ $(c)\ \text{resultant} = \frac{48\sqrt{5}}{25}mg.$ (d) The tension is the same on both sides of the pulley.

WME01/01 JANUARY 2022

Question 7

13 marks | Resolving Forces, Inclined Planes

7.

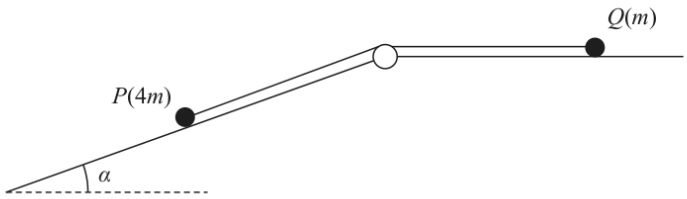


Figure 4

A particle P of mass $4m$ lies on the surface of a fixed rough inclined plane.

The plane is inclined to the horizontal at an angle α where $\tan\alpha = \frac{3}{4}$

The particle P is attached to one end of a light inextensible string.

The string passes over a small smooth pulley that is fixed at the top of the plane. The other end of the string is attached to a particle Q of mass m which lies on a smooth horizontal plane.

The string lies along the horizontal plane and in the vertical plane that contains the pulley and a line of greatest slope of the inclined plane.

The system is released from rest with the string taut, as shown in Figure 4, and P moves down the plane.

The coefficient of friction between P and the plane is $\frac{1}{4}$

For the motion before Q reaches the pulley

(a) write down an equation of motion for Q,

(1)

(b) find, in terms of m and g, the tension in the string,

(7)

(c) find the magnitude of the force exerted on the pulley by the string.

(4)

(d) State where in your working you have used the information that the string is light.

(1)

WORKED SOLUTION

Question 7 · WME01/01 January 2022

1 Write the equation for Q

Q lies on a smooth horizontal plane. The only horizontal force on Q is the tension, so $T = ma$.

2 Use the trig ratios

Since $\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$.

3 Find friction on P

For P, $R = 4mg\cos\alpha = 4mg \cdot \frac{4}{5} = \frac{16mg}{5}$. With coefficient $\frac{1}{4}$, friction is $F = \frac{1}{4}R = \frac{4mg}{5}$.

4 Apply Newtons second law to P

P moves down the plane. Taking down the plane as positive: $4mg\sin\alpha - T - F = 4ma$.

5 Substitute values

$4mg \cdot \frac{3}{5} - T - \frac{4mg}{5} = 4ma$, so $\frac{8mg}{5} - T = 4ma$.

6 **Use T = ma**

From Q, $a = \dfrac{T}{m}$. Substitute into the equation for P: $\dfrac{8mg}{5} - T = 4T$.

7 **Find T**

$\dfrac{8mg}{5} = 5T$, so $T = \dfrac{8}{25}mg$.

8 **Find the pulley resultant**

The pulley is pulled by two tensions of magnitude T: one horizontal and one along the inclined string. Their resultant has components $T - T\cos\alpha$ horizontally and $T\sin\alpha$ vertically.

9 **Calculate the resultant**

Magnitude $= \sqrt{(T - T\cos\alpha)^2 + (T\sin\alpha)^2} = T\sqrt{(1 - \frac{4}{5})^2 + (\frac{3}{5})^2} = \dfrac{T}{5}\sqrt{10}$. With $T = \dfrac{8}{25}mg$, the resultant is $\dfrac{8\sqrt{10}}{125}mg$.

10 **Use the light-string model**

The string is light, so the tension can be treated as the same on the P side and the Q side of the pulley.

✓ **FINAL ANSWER**

$(a)\ T = ma.$ $(b)\ T = \dfrac{8}{25}mg.$ $(c)\ \text{resultant} = \dfrac{8\sqrt{10}}{125}mg.$ (d) Same tension was used on both sides of the pulley.

WME01/01 JANUARY 2023

Question 8

15 marks | Resolving Forces, Inclined Planes

8.

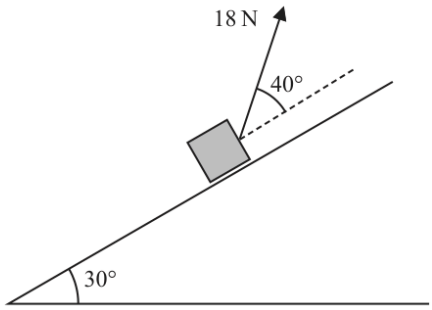


Figure 5

A parcel of mass 2 kg is pulled up a rough inclined plane by the action of a constant force.

The force has magnitude 18 N and acts at an angle of 40° to the plane.

The line of action of the force lies in a vertical plane containing a line of greatest slope of the inclined plane.

The plane is inclined at an angle of 30° to the horizontal, as shown in Figure 5.

The coefficient of friction between the plane and the parcel is 0.3

The parcel is modelled as a particle P

(a) Find the acceleration of P

(8)

The points A and B lie on a line of greatest slope of the plane, where $AB = 5\,\text{m}$ and B is above A. Particle P passes through A with speed $2\,\text{m s}^{-1}$ in the direction AB.

(b) Find the speed of P as it passes through B.

(3)

The force of 18 N is removed at the instant P passes through B. As a result, P comes to rest at the point C.

(c) Determine whether P will remain at rest at C. You must show all stages of your working clearly.

(4)

WORKED SOLUTION

Question 8 · WME01/01 January 2023

1 Resolve perpendicular to the plane

The force of 18 N is angled away from the plane, so $R + 18\sin 40° = 2g\cos 30°$. Hence $R = 2g\cos 30° - 18\sin 40°$.

2 Use friction

The parcel moves up the plane, so friction acts down the plane. With $\mu = 0.3$, $F = 0.3R$.

3 Resolve parallel to the plane

Taking up the plane as positive: $18\cos 40° - F - 2g\sin 30° = 2a$.

4 Substitute for R and F

$18\cos 40° - 0.3(2g\cos 30° - 18\sin 40°) - 2g\sin 30° = 2a$.

5 Find the acceleration

Using $g = 9.8$, this gives $a = 1.18\ldots\ \text{m s}^{-2}$ up the plane.

6 **Find the speed at B**

From A to B, $u = 2$, $s = 5$ and $a = 1.18\ldots$. Use $v^2 = u^2 + 2as$: $v^2 = 2^2 + 2(1.18\ldots)(5)$.

7 **State the speed**

$v = 3.98\ldots\ \text{m s}^{-1}$, about $4.0\ \text{m s}^{-1}$.

8 **Check rest after the force is removed**

At C, with the pulling force removed, $R = 2g\cos 30°$. The maximum friction is $0.3R = 0.3(2g\cos 30°) = 5.09\ldots\ \text{N}$.

9 **Compare with the downslope weight component**

The component of weight down the plane is $2g\sin 30° = 9.8\ \text{N}$. Since $9.8 > 5.09\ldots$, friction is not large enough to hold P at rest, so P will not remain at rest at C.

✓ **FINAL ANSWER**

$(a)\ a = 1.18\ \text{m s}^{-2}$ **up the plane.** $(b)\ v = 3.98\ \text{m s}^{-1} \approx 4.0\ \text{m s}^{-1}$. (c) P will not remain at rest at C.

WME01/01 OCTOBER 2023

Question 7

13 marks | Resolving Forces, Inclined Planes

7.

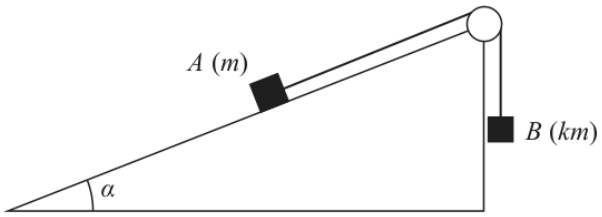


Figure 4

Figure 4 shows a block A of mass m held at rest on a rough plane.
The plane is inclined at an angle α to the horizontal and the coefficient of friction between the block and the plane is μ.

One end of a light inextensible string is now attached to A. The string passes over a small smooth pulley which is fixed at the top of the plane.
The other end of the string is attached to a block B of mass km.
Block B hangs vertically below the pulley, with the string taut.

The string from A to the pulley lies along a line of greatest slope of the plane.

Both A and B are modelled as particles.

When the system is released from rest, A moves up the plane and the tension in the string is $\frac{4mg}{3}$

Given that $\mu = \frac{1}{3}$ and $\tan\alpha = \frac{7}{24}$

(a) (i) find the magnitude of the acceleration of A, giving your answer in terms of g,

(ii) find the value of k.

(9)

(b) Find the magnitude of the resultant force exerted on the pulley by the string, giving your answer in terms of m and g.

(4)

WORKED SOLUTION

Question 7 · WME01/01 October 2023

1 Use the trig ratios

Given $\tan\alpha = \frac{7}{24}$, use $\sin\alpha = \frac{7}{25}$ and $\cos\alpha = \frac{24}{25}$.

2 Find friction on A

For block A, $R = mg\cos\alpha = \frac{24mg}{25}$. Since $\mu = \frac{1}{3}$, friction is $F = \frac{1}{3}R = \frac{8mg}{25}$.

3 Apply Newtons second law to A

A moves up the plane, so friction and the component of weight act down the plane. Taking up the plane as positive:

$T - mg\sin\alpha - F = ma$.

4 Find the acceleration

Substitute $T = \frac{4}{3}mg$, $mg\sin\alpha = \frac{7mg}{25}$ and $F = \frac{8mg}{25}$: $ma = \frac{4}{3}mg - \frac{15mg}{25} = \left(\frac{4}{3} - \frac{3}{5}\right)mg = \frac{11mg}{15}$. Hence $a = \frac{11g}{15}$.

5 **Apply Newtons second law to B**

Block B moves downwards with the same acceleration. Taking downward as positive: $kmg - T = kma$.

6 **Find k**

Substitute $T = \frac{4}{3}mg$ and $a = \frac{11g}{15}$: $kmg - \frac{4}{3}mg = km\frac{11g}{15}$. Divide by mg to get $k - \frac{4}{3} = \frac{11k}{15}$, so $15k - 20 = 11k$ and $k = 5$.

7 **Resolve the force on the pulley**

The pulley is pulled by two tensions of magnitude T. One is vertical and the other is along the plane. The horizontal component of the resultant is $T\cos\alpha$ and the vertical component is $T + T\sin\alpha$.

8 **Find the resultant**

The magnitude is $T\sqrt{\cos^2\alpha + (1+\sin\alpha)^2}$. With $\sin\alpha = \frac{7}{25}$ and $\cos\alpha = \frac{24}{25}$, this is $T\sqrt{\frac{576}{625} + \frac{1024}{625}} = \frac{8T}{5}$. Since $T = \frac{4}{3}mg$, the resultant is $\frac{32}{15}mg$.

FINAL ANSWER

$(a)(i)\ a = \frac{11g}{15}.$ $(a)(ii)\ k = 5.$ $(b)\ \text{resultant} = \frac{32}{15}mg.$

WME01/01 JANUARY 2024

Question 8

12 marks | Resolving Forces, Inclined Planes

8.

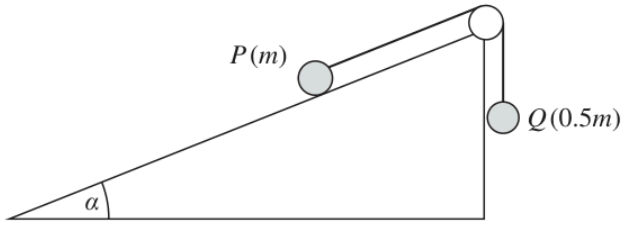


Figure 5

A fixed rough plane is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{5}{12}$

A small smooth pulley is fixed at the top of the plane.

One end of a light inextensible string is attached to a particle P which is at rest on the plane. The string passes over the pulley and the other end of the string is attached to a particle Q which hangs vertically below the pulley, as shown in Figure 5.

Particle P has mass m and particle Q has mass $0.5m$

The string from P to the pulley lies along a line of greatest slope of the plane.

The coefficient of friction between P and the plane is μ.

The system is in **limiting equilibrium** with the string taut and P is on the point of slipping **up** the plane.

(a) Find the value of μ.

(8)

The string breaks and P begins to move down the plane.

When particle P has travelled a distance of 0.8 m down the plane, the speed of P is $V\,\text{m s}^{-1}$

(b) Find the value of V.

(4)

WORKED SOLUTION

Question 8 · WME01/01 January 2024

1 Use the trig ratios

Given $\tan\alpha = \frac{5}{12}$, use $\sin\alpha = \frac{5}{13}$ and $\cos\alpha = \frac{12}{13}$.

2 Find the tension

Q has mass $0.5m$ and the system is in equilibrium, so the tension is $T = 0.5mg$.

3 Resolve P parallel to the plane

P is on the point of slipping up the plane, so friction acts down the plane. Along the plane, $T = mg\sin\alpha + F$.

4 Find the friction force

$F = T - mg\sin\alpha = \frac{1}{2}mg - \frac{5}{13}mg = \frac{3}{26}mg$.

5 Find the normal reaction and mu

$R = mg\cos\alpha = \frac{12}{13}mg$. Since $F = \mu R$, $\mu = \frac{3mg/26}{12mg/13} = \frac{1}{8}$.

6 **Find acceleration after the string breaks**

P moves down the plane, so friction acts up the plane. The resultant down the plane is $mg\sin\alpha - F = \frac{5}{13}mg - \frac{1}{8}\cdot\frac{12}{13}mg = \frac{7}{26}mg$.

7 **Apply Newtons second law**

As P has mass m, $a = \frac{7g}{26}$ down the plane.

8 **Find V**

From rest over 0.8 m, $V^2 = 2\left(\frac{7g}{26}\right)(0.8)$. With $g = 9.8$, $V = 2.05\ \mathrm{m\,s^{-1}}$ to 3 significant figures.

FINAL ANSWER

$(a)\ \mu = \frac{1}{8}$. $(b)\ V = 2.05\ \mathrm{m\,s^{-1}}$ approximately.

WME01/01 MAY/JUNE 2024

Question 6

12 marks | Resolving Forces, Inclined Planes

6.

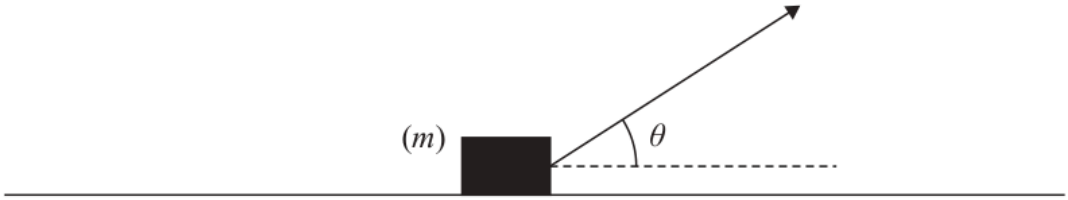


Figure 3

A box of mass m lies on a rough horizontal plane. The box is pulled along the plane in a straight line at **constant speed** by a light rope. The rope is inclined at an angle θ to the plane, as shown in Figure 3.

The coefficient of friction between the box and the plane is $\frac{1}{3}$

The box is modelled as a particle.

Given that $\tan\theta = \frac{3}{4}$

(a) find, in terms of m and g, the tension in the rope.

(7)

The rope is now removed and the box is placed at rest on the plane.
The box is then projected horizontally along the plane with speed u.

The box is again modelled as a particle.

When the box has moved a distance d along the plane, the speed of the box is $\frac{1}{2}u$.

(b) Find d in terms of u and g.

(5)

WORKED SOLUTION

Question 6 · WME01/01 May/June 2024

1 Use the trig ratios

Given $\tan\theta = \frac{3}{4}$, use $\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$.

2 Resolve vertically

The box moves at constant speed, so it is in equilibrium. Vertically, $R + T\sin\theta = mg$, hence $R = mg - T\sin\theta$.

3 Resolve horizontally

Horizontally, the tension component balances friction: $T\cos\theta = F$.

4 Use friction

With $\mu = \frac{1}{3}$, $F = \frac{1}{3}R$. Therefore $T\cos\theta = \frac{1}{3}(mg - T\sin\theta)$.

5 Find T

Substitute $\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$: $\frac{4}{5}T = \frac{1}{3}\left(mg - \frac{3}{5}T\right)$. This gives $\frac{4}{5}T = \frac{mg}{3} - \frac{1}{5}T$, so $T = \frac{mg}{3}$.

6 **Find the deceleration when the rope is removed**

On the horizontal plane without the rope, $R = mg$ and friction is $\frac{1}{3}mg$. Hence the acceleration is $-\frac{g}{3}$.

7 **Use SUVAT to find d**

When the speed has fallen from u to $\frac{1}{2}u$, use $v^2 = u^2 + 2as$: $\left(\frac{u}{2}\right)^2 = u^2 + 2\left(-\frac{g}{3}\right)d$.

8 **Solve for d**

$\frac{u^2}{4} = u^2 - \frac{2gd}{3}$, so $\frac{2gd}{3} = \frac{3u^2}{4}$ and $d = \frac{9u^2}{8g}$.

FINAL ANSWER

$(a)\ T = \frac{mg}{3}$. $(b)\ d = \frac{9u^2}{8g}$.

WME01/01 MAY/JUNE 2024

Question 8

15 marks | Resolving Forces, Inclined Planes

8.

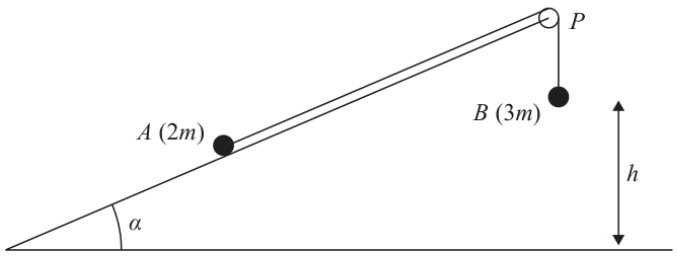


Figure 4

One end of a light inextensible string is attached to a particle A of mass $2m$. The other end of the string is attached to a particle B of mass $3m$. Particle A is held at rest on a rough plane which is inclined to horizontal ground at an angle α, where $\tan\alpha = \frac{5}{12}$

The string passes over a small smooth pulley P which is fixed at the top of the plane. Particle B hangs vertically below P with the string taut, at a height h above the ground, as shown in Figure 4.

The part of the string between A and P lies along a line of greatest slope of the plane. The two particles, the string and the pulley all lie in the same vertical plane.

The coefficient of friction between A and the plane is $\frac{11}{36}$

The particle A is released from rest and begins to move up the plane.

(a) Show that the frictional force acting on A as it moves up the plane is $\frac{22mg}{39}$ **(3)**

(b) Write down an equation of motion for B. **(2)**

(c) Show that the acceleration of A immediately after its release is $\frac{1}{3}g$ **(4)**

In the subsequent motion, A comes to rest before it reaches the pulley.

(d) Find, in terms of h, the total distance travelled by A from when it was released from rest to when it first comes to rest again. **(6)**

WORKED SOLUTION

Question 8 · WME01/01 May/June 2024

1 Use the trig ratios

Here $\tan\alpha = \frac{5}{12}$, so $\sin\alpha = \frac{5}{13}$ and $\cos\alpha = \frac{12}{13}$.

2 Show the frictional force on A

For A, $R = 2mg\cos\alpha = 2mg \cdot \frac{12}{13} = \frac{24mg}{13}$. Since $\mu = \frac{11}{36}$, $F = \mu R = \frac{11}{36} \cdot \frac{24mg}{13} = \frac{22mg}{39}$.

3 Write the equation for B

B moves down as A moves up. Taking downward as positive for B: $3mg - T = 3ma$.

4 Write the equation for A

Taking up the plane as positive for A: $T - 2mg\sin\alpha - F = 2ma$. Substituting $\sin\alpha = \frac{5}{13}$ and $F = \frac{22mg}{39}$ gives $T - \frac{10mg}{13} - \frac{22mg}{39} = 2ma$.

5 **Show the acceleration**

From B, $T = 3mg - 3ma$. Substitute into A equation: $3mg - 3ma - \frac{10mg}{13} - \frac{22mg}{39} = 2ma$. The constant part is $\frac{5mg}{3}$, so $\frac{5mg}{3} = 5ma$ and $a = \frac{g}{3}$.

6 **Find the speed when B reaches the ground**

While B descends distance h, A moves distance h up the plane from rest with acceleration $g/3$. Thus $v^2 = 2\left(\frac{g}{3}\right)h = \frac{2gh}{3}$.

7 **Find the later deceleration of A**

After B reaches the ground, the string no longer pulls A. The forces down the plane on A are $2mg\sin\alpha + F = \frac{10mg}{13} + \frac{22mg}{39} = \frac{4mg}{3}$. Since A has mass $2m$, its deceleration is $\frac{2g}{3}$.

8 **Find the extra distance and total distance**

Let the extra distance be d. Using $0 = v^2 - 2\left(\frac{2g}{3}\right)d$ with $v^2 = \frac{2gh}{3}$ gives $d = \frac{h}{2}$. Total distance travelled by A is $h + \frac{h}{2} = \frac{3h}{2}$.

FINAL ANSWER

$(a)\ F = \frac{22}{39}mg.\quad (b)\ 3mg - T = 3ma.\quad (c)\ a = \frac{g}{3}.\quad (d)\ \text{total distance} = \frac{3h}{2}.$

WME01/01 OCTOBER 2024

Question 6

12 marks | Resolving Forces, Inclined Planes

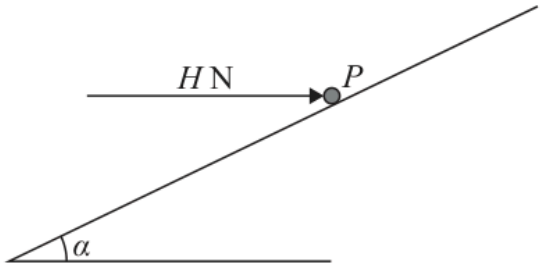


Figure 4

A particle P of mass 5 kg lies on the surface of a rough plane.

The plane is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$

The particle is held in equilibrium by a horizontal force of magnitude H newtons, as shown in Figure 4.

The horizontal force acts in a vertical plane containing a line of greatest slope of the inclined plane.

The coefficient of friction between the particle and the plane is $\frac{1}{4}$

(a) Find the smallest possible value of H.

(6)

The horizontal force is now removed, and P starts to slide down the slope.

In the first T seconds after P is released from rest, P slides 1.5 m down the slope.

(b) Find the value of T.

(6)

WORKED SOLUTION

Question 6 · WME01/01 October 2024

1 Use the triangle for alpha

From $\tan \alpha = \frac{3}{4}$, use $\sin \alpha = \frac{3}{5}$ and $\cos \alpha = \frac{4}{5}$.

2 Resolve perpendicular to the plane

The horizontal force has a component into the plane, so $R = 5g\cos\alpha + H\sin\alpha = 4g + \frac{3}{5}H$.

3 Resolve parallel for the smallest H

For the smallest H, the particle is on the point of sliding down, so friction acts up the plane. Parallel to the plane, $H\cos\alpha + F = 5g\sin\alpha$.

4 Use friction to find H

With $F = \frac{1}{4}R$, the parallel equation becomes $\frac{4}{5}H + \frac{1}{4}\left(4g + \frac{3}{5}H\right) = 3g$. Hence $0.95H + g = 3g$, so $H = \frac{40g}{19} = 20.6\ \text{N}$.

5 **Find the acceleration after H is removed**

When H is removed, $R = 5g\cos\alpha = 4g$ and friction is $\frac{1}{4}R = g$. The component of weight down the plane is $5g\sin\alpha = 3g$, so the resultant down the plane is $2g$.

6 **Apply Newtons second law**

$2g = 5a$, so $a = \dfrac{2g}{5} = 3.92\ \text{m s}^{-2}$ down the plane.

7 **Find T from the displacement**

From rest, the particle moves 1.5 m with acceleration $2g/5$. Using $s = \frac{1}{2}at^2$: $1.5 = \frac{1}{2}\left(\frac{2g}{5}\right)T^2 = \frac{g}{5}T^2$. Thus $T = \sqrt{\dfrac{7.5}{9.8}} = 0.875\ \text{s}$.

FINAL ANSWER

$(a)\ H = \dfrac{40g}{19}\ \text{N} \approx 20.6\ \text{N}.$ $\quad (b)\ T = 0.875\ \text{s}.$

WME01/01 JANUARY 2025

Question 6

12 marks | Resolving Forces, Inclined Planes

6.

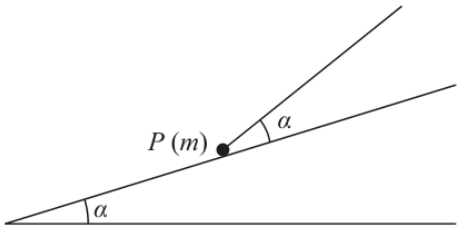


Figure 3

A particle P of mass m is held in equilibrium on a fixed rough inclined plane by a light inextensible string.

The plane is inclined at an angle α to the horizontal, where $\alpha < 45°$
The string is inclined to the plane at angle α, as shown in Figure 3.

The string lies in a vertical plane that contains a line of greatest slope of the inclined plane.

When the tension in the string is $0.75\,mg$, P is on the point of moving up the plane.

(a) Find an expression for the magnitude of the frictional force acting on P, giving your answer in terms of m, g and α.

(3)

The coefficient of friction between P and the plane is $\frac{1}{2}$

(b) Show that

$$\tan\alpha = \frac{2}{5}$$

(6)

The string breaks.

(c) Determine whether P remains at rest. You must justify your reasoning.

(3)

WORKED SOLUTION

Question 6 · WME01/01 January 2025

1 Resolve parallel to the plane

P is on the point of moving up the plane, so friction acts down the plane. The tension has component $0.75mg\cos\alpha$ up the plane, and the weight component is $mg\sin\alpha$ down the plane.

2 Find the frictional force

For equilibrium parallel to the plane, $0.75mg\cos\alpha = mg\sin\alpha + F$. Hence $F = 0.75mg\cos\alpha - mg\sin\alpha$.

3 Resolve perpendicular to the plane

The string has a component away from the plane, so $R = mg\cos\alpha - 0.75mg\sin\alpha$.

4 Use the coefficient of friction

Given $\mu = \frac{1}{2}$, limiting friction gives $F = \frac{1}{2}R$. Therefore $0.75mg\cos\alpha - mg\sin\alpha = \frac{1}{2}(mg\cos\alpha - 0.75mg\sin\alpha)$.

5 **Show the tangent result**

Divide by mg and collect terms: $0.75\cos\alpha - \sin\alpha = 0.5\cos\alpha - 0.375\sin\alpha$. Thus $0.25\cos\alpha = 0.625\sin\alpha$, so $\tan\alpha = \dfrac{0.25}{0.625} = \dfrac{2}{5}$.

6 **Check what happens when the string breaks**

Without the string, $R = mg\cos\alpha$ and the maximum friction is $\dfrac{1}{2}mg\cos\alpha$. The component of weight down the plane is $mg\sin\alpha$.

7 **Compare friction with the downslope component**

Since $\tan\alpha = \dfrac{2}{5} < \dfrac{1}{2}$, we have $mg\sin\alpha < \dfrac{1}{2}mg\cos\alpha$. The available friction is large enough to hold P, so P remains at rest.

FINAL ANSWER

$(a)\ F = 0.75mg\cos\alpha - mg\sin\alpha.$ $(b)\ \tan\alpha = \dfrac{2}{5}.$ $(c)\ P$ remains at rest.

WME01/01 MAY/JUNE 2025

Question 6

11 marks | Resolving Forces, Inclined Planes

6.

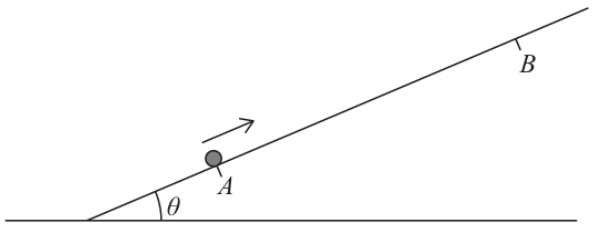


Figure 4

Figure 4 shows a rough plane inclined at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$

The points A and B lie on a line of greatest slope of the plane, with B above A.

A package P of mass m is projected up the plane from A towards B.

The coefficient of friction between the plane and the package is $\frac{1}{4}$

The package P is modelled as a particle.

(a) Show that the **deceleration** of P, as it moves from A to B, is $\frac{4}{5}g$ **(6)**

The package P comes to rest at B.

Given that P is projected from A with speed $U\,\text{m s}^{-1}$ and that $AB = 1.5\,\text{m}$,

(b) find the value of U. **(2)**

On reaching B, P is held at rest there by a force of magnitude X newtons acting up the plane in the direction AB.

Given that the mass of P is $2\,\text{kg}$,

(c) find the smallest possible value of X. **(3)**

WORKED SOLUTION

Question 6 · WME01/01 May/June 2025

1 **Part (a): Show deceleration = $\frac{4g}{5}$ - Find exact trig values**

Given: $\tan\theta = \frac{3}{4}$

Using 3-4-5 right triangle:

$\sin\theta = \frac{3}{5}, \quad \cos\theta = \frac{4}{5}$

2 **Part (a): Show deceleration = $\frac{4g}{5}$ - Resolve perpendicular to plane**

$\perp: \sum F = 0$

$R = mg\cos\theta \; R = \frac{4mg}{5}$

3 **Part (a): Show deceleration = $\frac{4g}{5}$ - Calculate friction force**

Package moving UP, so friction acts DOWN the plane:

$F = \mu R = \frac{1}{4} \times \frac{4mg}{5} = \frac{mg}{5}$

4 **Part (a): Show deceleration = $\frac{4g}{5}$ - Resolve parallel to plane**

Taking up the plane as positive:

$\parallel: \sum F = ma$

$-mg\sin\theta - F = ma \; -mg \times \frac{3}{5} - \frac{mg}{5} = ma \; -\frac{3mg}{5} - \frac{mg}{5} = ma \; -\frac{4mg}{5} = ma \; a = -\frac{4g}{5}$

Deceleration = $\left|-\frac{4g}{5}\right| = \frac{4g}{5}$

---

5 **Part (b): Find initial speed U - Identify SUVAT variables**

Package projected up plane, comes to rest at B:

; $u = U$ m s^{-1} (unknown)
; $v = 0$ m s^{-1} (comes to rest)
; $a = -\frac{4g}{5}$ m s^{-2} (from part a)
; $s = 1.5$ m

---

6 **Part (b): Find initial speed U - Apply $v^2 = u^2 + 2as$**

$v^2 = u^2 + 2as$

$0^2 = U^2 + 2\left(-\frac{4g}{5}\right)(1.5) \; 0 = U^2 - 2 \times \frac{4\times9.8}{5} \times 1.5 \; 0 = U^2 - \frac{8\times9.8\times1.5}{5} \; 0 = U^2 - 23.52 \; U^2 = 23.52 \; U = 4.85 \text{ m s}^{-1}$

Or using exact form: $U = \sqrt{2 \times \frac{4g}{5} \times 1.5} = \sqrt{\frac{12g}{5}}$

---

7 **Part (c): Find smallest value of X - Analyze forces when package at rest at B**

Package held at rest on slope by force X up the plane.

Without X, package would slide down (component down > friction up).

For MINIMUM X, friction acts UP the plane (at limiting friction).

---

8 **Part (c): Find smallest value of X - Resolve perpendicular to plane**

Mass = 2 kg, so weight = $2g$ N

$R = 2g\cos\theta = 2g \times \frac{4}{5} = \frac{8g}{5}$ N

---

9 **Part (c): Find smallest value of X - Calculate friction**

Friction acts UP the plane (limiting):

$F = \mu R = \frac{1}{4} \times \frac{8g}{5} = \frac{2g}{5}$ N

---

10 **Part (c): Find smallest value of X - Resolve parallel to plane**

For equilibrium (package at rest):

$\parallel: \sum F = 0$

Taking up plane as positive:

$X + F - 2g\sin\theta = 0 \; X + \frac{2g}{5} - 2g \times \frac{3}{5} = 0 \; X + \frac{2g}{5} - \frac{6g}{5} = 0 \; X = \frac{6g}{5} - \frac{2g}{5} \; X = \frac{4g}{5} \; X = \frac{4\times9.8}{5} \; X = 7.84$ N

✓ **FINAL ANSWER**

(a) **deceleration** $= \frac{4g}{5}$, as required.

(b) $U = 4.85 \text{ m s}^{-1}$ (about 4.8 m s^{-1}).

(c) $X = 7.84$ N (about 7.8 N).

WME01/01 OCTOBER 2025

Question 7

14 marks | Resolving Forces, Inclined Planes

7.

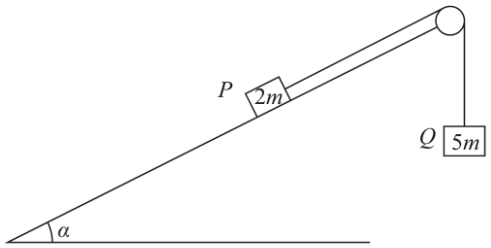


Figure 5

A block P of mass $2m$ is held at rest on a fixed rough plane.

The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{5}{12}$

One end of a light inextensible string is attached to P.
The string is parallel to a line of greatest slope of the plane and passes over a smooth light pulley which is fixed at the top of the plane.
The other end of the string is attached to a block Q of mass $5m$.
Block Q hangs vertically below the pulley, as shown in Figure 5.

The system is released from rest with the string taut and block P moves up the plane.

Immediately after the system is released, the tension in the string is T and the acceleration of the blocks is a.

The blocks are modelled as particles and air resistance is ignored.

(a) Write down an equation of motion for Q.

(2)

The coefficient of friction between P and the plane is $\frac{1}{8}$

(b) Find T in terms of m and g.

(7)

(c) State how the solution to part (b) uses the fact that the string is inextensible.

(1)

The magnitude of the force exerted on the pulley by the string is kmg.

(d) Find the value of k to 3 significant figures.

(4)

WORKED SOLUTION

Question 7 · WME01/01 October 2025

1 **Part (a): Write equation of motion for Q**

Forces on Q (mass = 5m):

Taking downward as positive for Q:

For Q: $\sum F = ma$

$5mg - T = 5ma$

2 **Part (b): Find T in terms of m and g - Resolve perpendicular to plane**

$\perp$ to plane: $\sum F = 0$

$R = 2mg\cos\alpha \; R = 2mg \times \frac{12}{13} \; R = \frac{24mg}{13}$

3 **Part (b): Find T in terms of m and g - Find friction force**

P is moving UP the plane, so friction acts DOWN the plane:

$F = \mu R = \frac{1}{8} \times \frac{24mg}{13} = \frac{3mg}{13}$

4 **Part (b): Find T in terms of m and g - Resolve parallel to plane**

Taking up the plane as positive:

$\parallel$ to plane: $\sum F = ma$

$T - 2mg\sin\alpha - F = 2ma\ T - 2mg \times \frac{5}{13} - \frac{3mg}{13} = 2ma\ T - \frac{10mg}{13} - \frac{3mg}{13} = 2ma\ T - \frac{13mg}{13} = 2ma\ T - mg = 2ma \quad ...(1)$

5 **Part (b): Find T in terms of m and g - Combine with equation for Q**

From part (a): $5mg - T = 5ma$ ...(2)

Add equations (1) and (2):

$(T - mg) + (5mg - T) = 2ma + 5ma\ 4mg = 7ma\ a = \frac{4g}{7}$

6 **Part (b): Find T in terms of m and g - Substitute back to find T**

From equation (1):

$T = mg + 2ma = mg + 2m \times \frac{4g}{7} = mg + \frac{8mg}{7} = \frac{7mg+8mg}{7} = \frac{15mg}{7}$

7 **Part (c): State how solution uses fact that string is inextensible**

An inextensible string means the string cannot stretch.

This has the following consequence:

For an inextensible string over a pulley:

The magnitude of acceleration of both connected particles must be equal.

In this problem: $|a_P| = |a_Q| = a$

How this was used in part (b):

When we wrote the equations of motion:

; For P: $T - mg = 2ma$
; For Q: $5mg - T = 5ma$

We used the same acceleration a for both particles because the string is inextensible.

If the string could stretch, the two particles could have different accelerations.

8 **Part (d): Find value of k (force on pulley = kmg) - Identify the angle between tensions**

The string from Q is vertical (downward).
The string from P is along the plane at angle α to horizontal.

Angle between the two strings = $90° + \alpha$

9 **Part (d): Find value of k (force on pulley = kmg) - Use vector components**

Let's take horizontal (right) and vertical (up) as positive directions.

Tension from Q:

; Horizontal component: $\mathbf{0}$
; Vertical component: $-T = -\frac{15mg}{7}$ (downward)

Tension from P along plane:
The plane makes angle α with horizontal, so:

; Horizontal component: $-T\cos\alpha = -\frac{15mg}{7} \times \frac{12}{13} = -\frac{180mg}{91}$
; Vertical component: $T\sin\alpha = \frac{15mg}{7} \times \frac{5}{13} = \frac{75mg}{91}$

---

10 **Part (d): Find value of k (force on pulley = kmg) - Find resultant**

$R_x = 0 - \frac{180mg}{91} = -\frac{180mg}{91}$ $R_y = -\frac{15mg}{7} + \frac{75mg}{91}$
Convert $\frac{15mg}{7}$ to denominator 91: $\frac{15mg}{7} = \frac{195mg}{91}$
$R_y = -\frac{195mg}{91} + \frac{75mg}{91} = -\frac{120mg}{91}$

---

11 **Part (d): Find value of k (force on pulley = kmg) - Find magnitude**

$|R| = \sqrt{R_x^2 + R_y^2} = \sqrt{\left(\frac{180mg}{91}\right)^2 + \left(\frac{120mg}{91}\right)^2} = \frac{mg}{91}\sqrt{180^2 + 120^2} = \frac{mg}{91}\sqrt{32400 + 14400} = \frac{mg}{91}\sqrt{46800} = \frac{mg}{91} \times 216.333\ldots$
$= 2.377\ldots mg = 2.38mg$ (3 s.f.)
Therefore: $k = 2.38$

FINAL ANSWER

$(a)\ 5mg - T = 5ma.$ $(b)\ T = \frac{15mg}{7}.$ (c) both particles have the same magnitude of acceleration. $(d)\ k = 2.38.$

WME01/01 JANUARY 2026

Question 7

14 marks | Resolving Forces, Inclined Planes

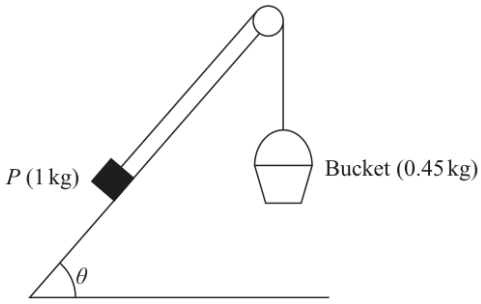


Figure 3

A rough plane is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{4}{3}$

One end of a light inextensible string is attached to a package P.
The string passes over a smooth pulley that is fixed at the top of the plane.
The string from P to the pulley lies along a line of greatest slope of the plane.
The other end of the string is attached to a bucket.
The package P has mass 1 kg and is **held** at rest on the plane.
The bucket has mass 0.45 kg and hangs vertically below the pulley with the string taut, as shown in Figure 3.

The coefficient of friction between P and the plane is μ

When a small block of mass 0.2 kg is placed in the bucket, the system is released and remains at rest in **limiting equilibrium**, with P on the point of slipping down the plane.

(a) Show that $\mu = \frac{1}{4}$

(7)

Additional identical blocks of mass 0.2 kg are added one at a time into the bucket until P starts to move **up** the plane.
At the instant when P starts to move up the plane, there are a **total** of n blocks in the bucket.

(b) Find

(i) the value of n

(ii) the magnitude of the initial acceleration of P

(7)

WORKED SOLUTION

Question 7 · WME01/01 January 2026

1 Find the trigonometric ratios

Since $\tan\theta = \frac{4}{3}$, use a 3-4-5 triangle: $\sin\theta = \frac{4}{5}$ and $\cos\theta = \frac{3}{5}$.

2 Use limiting equilibrium in part (a)

With one 0.2 kg block in the bucket, the bucket side has mass $0.45 + 0.2 = 0.65$ kg. Since the package is on the point of slipping down the plane, friction on P acts up the plane.

3 Find the friction force

The tension equals the bucket weight, so $T = 0.65g$. Along the plane for P in equilibrium, $T + F = g\sin\theta$. Hence $0.65g + F = \frac{4}{5}g$, so $F = \frac{3g}{20}$.

4 Find the normal reaction and mu

Perpendicular to the plane, $R = g\cos\theta = \frac{3}{5}g$. Since $F = \mu R$, $\mu = \frac{F}{R} = \frac{3g/20}{3g/5} = \frac{1}{4}$.

5 **Set the condition for upward motion**

When P is about to move up the plane, friction acts down the plane. With n blocks in the bucket, the bucket-side mass is $0.45 + 0.2n$. The bucket weight must exceed the downslope resistance: $(0.45 + 0.2n)g > g\sin\theta + \mu R$.

6 **Find n**

Using $g\sin\theta = \frac{4}{5}g$ and $\mu R = \frac{1}{4} \cdot \frac{3}{5}g = \frac{3g}{20}$, the condition is $0.45 + 0.2n > \frac{4}{5} + \frac{3}{20} = 0.95$. Hence $0.2n > 0.50$, so $n > 2.5$ and the least integer is $n = 3$.

7 **Apply Newtons second law after motion begins**

For $n = 3$, the bucket-side mass is $0.45 + 0.6 = 1.05$ kg and the total moving mass is $1 + 1.05 = 2.05$ kg. The driving force is $1.05g - \frac{4}{5}g - \frac{3g}{20} = 0.10g$.

8 **Find the acceleration**

Using $F = ma$, $0.10g = 2.05a$, so $a = \frac{0.10g}{2.05} = \frac{2g}{41} = 0.478\ \mathrm{m\,s^{-2}}$ to 3 significant figures.

FINAL ANSWER

$(a)\ \mu = \frac{1}{4}.\quad (b)(i)\ n = 3.\quad (b)(ii)\ a = \frac{2g}{41} = 0.478\ \mathrm{m\,s^{-2}}$ approximately.

WME01/01 MAY/JUNE 2021

Question 7

10 marks | Resolving Forces, Inclined Planes | Main topic: Moments

7.

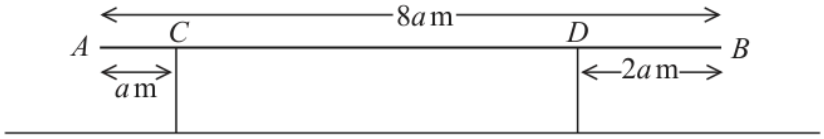


Figure 2

A non-uniform beam AB, of mass 60 kg and length $8a$ metres, rests in equilibrium in a horizontal position on two vertical supports. One support is at C, where $AC = a$ metres and the other support is at D, where $DB = 2a$ metres, as shown in Figure 2.

The magnitude of the normal reaction between the beam and the support at D is three times the magnitude of the normal reaction between the beam and the support at C.

By modelling the beam as a non-uniform rod whose centre of mass is at a distance x metres from A,

(a) find an expression for x in terms of a.

(5)

A box of mass M kg is placed on the beam at E, where $AE = 2a$ metres.

The beam remains in equilibrium in a horizontal position.

The magnitude of the normal reaction between the beam and the support at C is now equal to the magnitude of the normal reaction between the beam and the support at D.

By modelling the box as a particle,

(b) find the value of M.

(5)

WORKED SOLUTION

Question 7 · WME01/01 May/June 2021

1 Set the support positions

The beam length is $8a$. C is at distance a from A, and D is at distance $6a$ from A because $DB = 2a$.

2 Name the reactions

Let the reaction at C be R. The reaction at D is $3R$.

3 Use vertical equilibrium

$R + 3R = 60g$, so $4R = 60g$ and $R = 15g$.

4 Take moments about A

The beam's weight acts at distance x from A. Hence $60gx = R(a) + 3R(6a)$.

5 Find x

Substitute $R = 15g$: $60gx = 15ga + 45g(6a) = 285ga$. Thus $x = \frac{285a}{60} = \frac{19a}{4}$.

6 **Use equal reactions with the box**

When the box is added, let each support reaction be S. Vertical equilibrium gives $2S = (60 + M)g$.

7 **Take moments about A again**

The box is at $AE = 2a$. Moments about A give $60g\left(\frac{19a}{4}\right) + Mg(2a) = S(a) + S(6a) = 7aS$.

8 **Solve for M**

Substitute $S = \frac{(60 + M)g}{2}$ and cancel ag: $285 + 2M = \frac{7}{2}(60 + M) = 210 + \frac{7}{2}M$. Hence $75 = \frac{3}{2}M$ and $M = 50$.

✓ **FINAL ANSWER**

$(a)\ x = \frac{19a}{4}.\quad (b)\ M = 50.$

WME01/01 OCTOBER 2022

Question 7

13 marks | Resolving Forces, Inclined Planes | Main topic: Newton's Second Law

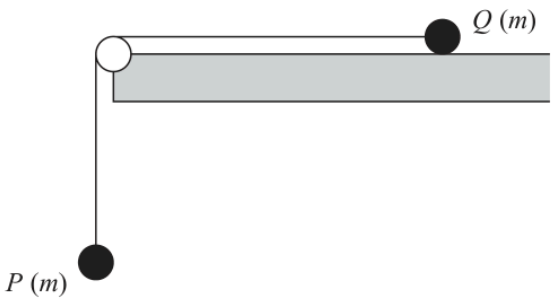


Figure 4

A particle P of mass m is attached to one end of a light inextensible string. Another particle Q, also of mass m, is attached to the other end of the string. The string passes over a small smooth pulley which is fixed at the edge of a rough horizontal table. Particle Q is held at rest on the table and particle P hangs vertically below the pulley with the string taut, as shown in Figure 4.

The pulley, P and Q all lie in the same vertical plane.

The coefficient of friction between Q and the table is μ, where $\mu < 1$

Particle Q is released from rest.

The tension in the string before Q hits the pulley is kmg, where k is a constant.

(a) Find k in terms of μ.

(7)

Given that Q is initially a distance d from the pulley,

(b) find, in terms of d, g and μ, the time taken by Q, after release, to reach the pulley.

(4)

(c) Describe what would happen if $\mu \geqslant 1$, giving a reason for your answer.

(2)

WORKED SOLUTION

Question 7 · WME01/01 October 2022

1 Write the friction on Q

For Q on the horizontal table, $R = mg$, so the friction is $F = \mu mg$.

2 Apply Newtons second law to P

P moves downward and Q moves towards the pulley. For P, taking downward as positive: $mg - T = ma$.

3 Apply Newtons second law to Q

For Q, taking motion towards the pulley as positive: $T - \mu mg = ma$.

4 Use T = kmg

From P, $a = g(1 - k)$. From Q, $a = g(k - \mu)$.

5 Find k

Equate the two accelerations: $1 - k = k - \mu$. Hence $2k = 1 + \mu$ and $k = \dfrac{1 + \mu}{2}$.

6 **Find the acceleration**

$a = g(1-k) = g\left(1 - \dfrac{1+\mu}{2}\right) = \dfrac{g(1-\mu)}{2}$.

7 **Use motion from rest**

Q starts from rest and travels distance d, so $d = \dfrac{1}{2}at^2 = \dfrac{1}{2} \cdot \dfrac{g(1-\mu)}{2}t^2$.

8 **Find the time**

$d = \dfrac{g(1-\mu)t^2}{4}$, so $t = \sqrt{\dfrac{4d}{g(1-\mu)}}$.

9 **Consider mu at least 1**

If $\mu \geq 1$, the maximum available friction on Q is at least mg, so it can balance the pull caused by the hanging particle. The system would not start moving.

FINAL ANSWER

$(a)\ k = \dfrac{1+\mu}{2}$. $(b)\ t = \sqrt{\dfrac{4d}{g(1-\mu)}}$. (c) If $\mu \geq 1$, the system would not move because friction can balance the pull from P.

WME01/01 MAY/JUNE 2023

Question 7

11 marks | Resolving Forces, Inclined Planes | Main topic: Newton's Second Law

7.

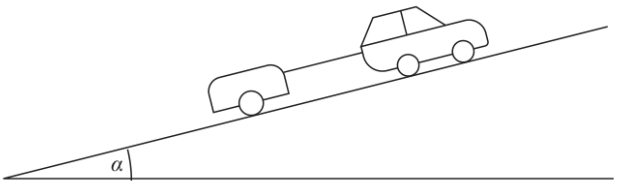


Figure 4

A car of mass 1200 kg is towing a trailer of mass 600 kg up a straight road, as shown in Figure 4.

The road is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{12}$

The driving force produced by the engine of the car is 3000 N.

The car moves with acceleration $0.75\,\text{m s}^{-2}$

The non-gravitational resistance to motion of

- the **car** is modelled as a constant force of magnitude $2R$ newtons
- the **trailer** is modelled as a constant force of magnitude R newtons

The car and the trailer are modelled as particles.
The tow bar between the car and trailer is modelled as a light rod that is parallel to the direction of motion.

Using the model,

(a) show that the value of R is 60

(4)

(b) find the tension in the tow bar.

(3)

When the car and trailer are moving at a speed of $12\,\text{m s}^{-1}$, the tow bar breaks.

Given that the non-gravitational resistance to motion of the trailer remains unchanged,

(c) use the model to find the further distance moved by the trailer before it first comes to rest.

(4)

WORKED SOLUTION

Question 7 · WME01/01 May/June 2023

1 Use the whole system

For car plus trailer, the total mass is 1800 kg and the total resistance is $2R + R = 3R$. Taking up the slope as positive: $3000 - 1800g\sin\alpha - 3R = 1800(0.75)$.

2 Show R is 60

Since $\sin\alpha = \frac{1}{12}$, $1800g\sin\alpha = 1800 \cdot 9.8 \cdot \frac{1}{12} = 1470$. So $3000 - 1470 - 3R = 1350$, giving $180 = 3R$ and $R = 60$ N.

3 Apply Newtons second law to the trailer

For the trailer while it is being towed, $T - 600g\sin\alpha - R = 600(0.75)$.

4 Find the tow-bar tension

$T - 600 \cdot 9.8 \cdot \frac{1}{12} - 60 = 450$, so $T - 490 - 60 = 450$ and $T = 1000$ N.

5 **Find the trailer acceleration after the break**

After the tow bar breaks, the trailer is still moving up the slope, but the component of weight and resistance act down the slope. Taking up the slope as positive: $-600g\sin\alpha - 60 = 600a$.

6 **Simplify the acceleration**

$-490 - 60 = 600a$, so $a = -\dfrac{550}{600} = -\dfrac{11}{12}\ \text{m s}^{-2}$.

7 **Find the stopping distance**

With initial speed $12\ \text{m s}^{-1}$ and final speed 0, use $v^2 = u^2 + 2as$: $0 = 12^2 + 2\left(-\dfrac{11}{12}\right)d$. Hence $d = \dfrac{864}{11} = 78.5\ldots\ \text{m}$.

FINAL ANSWER

$(a)\ R = 60\ \text{N}$. $(b)\ T = 1000\ \text{N}$. $(c)\ d = 78.5\ \text{m}$ approximately.

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TOPIC

Momentum, Impulse & Collisions

Every past-paper question on this topic, October 2019 to January 2026.

WME01/01 JANUARY 2024

Question 6

12 marks | Momentum, Impulse & Collisions | Main topic: Constant Acceleration in 1D

6. A particle is projected vertically upwards from a point A with speed $24\,\text{m s}^{-1}$

The point A is 2.5 m vertically above the point B.

Point B lies on horizontal ground.

The particle moves freely under gravity until it hits the ground at B with speed $V\,\text{m s}^{-1}$

After hitting the ground the particle does not rebound.

(a) Find the value of V.

(3)

(b) Find the time taken for the particle to reach B.

(3)

The point C is 10 m vertically above A.

(c) Find the length of time for which the particle is above C.

(4)

(d) Sketch a speed-time graph for the motion of the particle from projection to the instant that it reaches B. (No further calculations are required.)

(2)

WORKED SOLUTION

Question 6 · WME01/01 January 2024

1 Find the impact speed

Take upward as positive from A. At B, the displacement is -2.5 m, with $u = 24$ and $a = -9.8$. Use $v^2 = u^2 + 2as$:
$V^2 = 24^2 + 2(-9.8)(-2.5) = 625$.

2 State V

$V = 25\ \text{m s}^{-1}$.

3 Find the time to reach B

The velocity at B is $-25\ \text{m s}^{-1}$. Using $v = u + at$, $-25 = 24 - 9.8t$, so $t = 5$ s.

4 Set up the height C equation

C is 10 m above A. The particle is above C between the two times when $s = 10$: $10 = 24t - 4.9t^2$.

5 Solve for the two crossing times

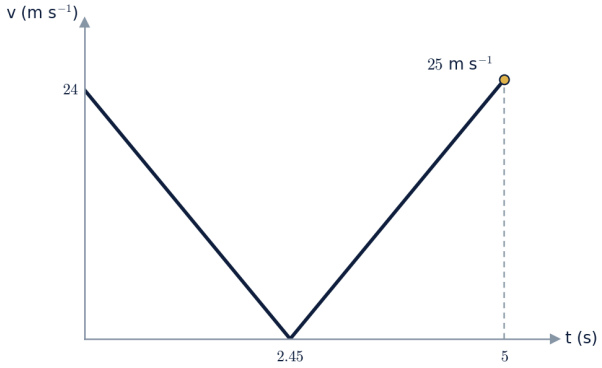
$4.9t^2 - 24t + 10 = 0$, giving $t = 0.459\ldots$ and $t = 4.438\ldots$.

6 Find the time above C

The length of time above C is $4.438\ldots - 0.459\ldots = 3.98\ldots$ s, about 4.0 s.

7 **Describe the speed-time graph**

The graph starts at speed $\mathbf{24}$, decreases linearly to $\mathbf{0}$ at the highest point, then increases linearly to speed $\mathbf{25}$ at $\boldsymbol{t = 5}$. It is a V-shaped speed-time graph.



FINAL ANSWER

$\boldsymbol{(a)\ V = 25\ \mathrm{m\,s^{-1}}.}$ $\boldsymbol{(b)\ t = 5\ \mathrm{s}.}$ $\boldsymbol{(c)\ 4.0\ \mathrm{s}}$ **approximately**. (d) Speed-time graph is V-shaped, from speed 24 to 0, then to speed 25 at t=5.

WME01/01 OCTOBER 2024

Question 7

11 marks | Momentum, Impulse & Collisions | Main topic: Constant Acceleration in 1D

7 At time $t = 0$, a small ball A is projected vertically upwards with speed $8\,\text{m s}^{-1}$ from a fixed point on horizontal ground.
The ball hits the ground again for the first time at time $t = T_1$ seconds.

Ball A is modelled as a particle moving freely under gravity.

(a) Show that $T_1 = 1.63$ to 3 significant figures.

(2)

After the first impact with the ground, A rebounds to a height of $2\,\text{m}$ above the ground.

Given that the mass of A is $0.1\,\text{kg}$,

(b) find the magnitude of the impulse received by A as a result of its first impact with the ground.

(5)

At time $t = 1$ second, another small ball B is projected vertically upwards from another point on the ground with speed $5\,\text{m s}^{-1}$

Ball B is modelled as a particle moving freely under gravity.

At time $t = T_2$ seconds ($T_2 > 1$), A and B are at the same height above the ground for the first time.

(c) Find the value of T_2

(4)

WORKED SOLUTION

Question 7 · WME01/01 October 2024

1 Find the first flight time

For ball A, take upward as positive. Returning to the ground gives $0 = 8T_1 - 4.9T_1^2$. The non-zero solution is $T_1 = \dfrac{8}{4.9} = 1.63\,\text{s}$ to 3 significant figures.

2 Find the rebound speed

After impact, A rebounds to a maximum height of 2 m. At the top, $v = 0$, so $0 = u^2 - 2g(2)$. Hence $u = \sqrt{4g} = 6.261\,\text{m s}^{-1}$.

3 Use impulse as change in momentum

Just before the first impact, A has velocity $-8\,\text{m s}^{-1}$. Just after impact, it has velocity $+6.261\,\text{m s}^{-1}$. With mass 0.1 kg, $I = 0.1[6.261 - (-8)]$.

4 Calculate the impulse

$I = 1.426\ldots\,\text{N s}$, so the impulse is $1.43\,\text{N s}$ to 3 significant figures.

5 Write the height of A before first impact

For the first meeting with B, A is still in its first flight. Its height at time T_2 is $h_A = 8T_2 - 4.9T_2^2$.

6 Write the height of B

B starts at time $t = 1$, so at time T_2 it has been moving for $T_2 - 1$ seconds. Its height is $h_B = 5(T_2 - 1) - 4.9(T_2 - 1)^2$.

7 **Set the heights equal**

$8T_2 - 4.9T_2^2 = 5(T_2 - 1) - 4.9(T_2 - 1)^2$. Expanding and simplifying gives $6.8T_2 = 9.9$.

8 **Find T2**

$T_2 = \dfrac{9.9}{6.8} = 1.456\ldots$, so $T_2 = 1.46\ \text{s}$ to 3 significant figures.

FINAL ANSWER

$(a)\ T_1 = 1.63\ \text{s}.\quad (b)\ I = 1.43\ \text{N s}.\quad (c)\ T_2 = 1.46\ \text{s}.$

WME01/01 MAY/JUNE 2025

Question 8

17 marks | Momentum, Impulse & Collisions | Main topic: Newton's Second Law

8.

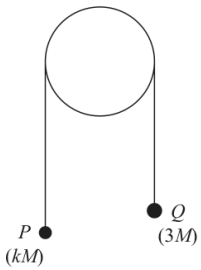


Figure 5

Two small balls, P and Q, have masses kM and $3M$ respectively, where $k < 3$

The balls are attached to the ends of a light inextensible string that passes over a fixed light smooth pulley.

The system is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 5.

The system is released from rest and, in the subsequent motion, P moves with an acceleration of magnitude $\frac{1}{5}g$

The balls are modelled as particles.

(a) Write down an equation of motion for P.

(2)

(b) Find the value of k.

(3)

Given that $M = 0.5\,\text{kg}$,

(c) find the magnitude of the force exerted on the pulley by the string while Q is moving downwards.

(3)

At the instant when the system is released, P is more than 2.5 m from the pulley and Q is 2.5 m above horizontal ground.
After hitting the ground, Q rebounds with a speed of $0.4\,\text{m s}^{-1}$

(d) Find the magnitude of the impulse received by Q when it hits the ground.

(5)

In the subsequent motion, P does not hit the pulley.

(e) Find the total time from when the balls are released until P first comes to rest.

(4)

WORKED SOLUTION

Question 8 · WME01/01 May/June 2025

1 **Part (a): Write equation of motion for P - Identify forces on P**

Particle P (mass $\boldsymbol{kM}$) moves upward with acceleration $\frac{g}{5}$:

; Tension T (upward)
; Weight $\boldsymbol{kMg}$ (downward)

2 **Part (a): Write equation of motion for P - Apply Newton's Second Law**

Taking upward as positive:

$$\sum F = ma$$

$$T - kMg = kM \times \frac{g}{5}$$

3 **Part (b): Find value of k - Write equation of motion for Q**

Particle Q (mass $3M$) moves downward with acceleration $\frac{g}{5}$:

Taking downward as positive for Q:

$$3Mg - T = 3M \times \frac{g}{5} \; 3Mg - T = \frac{3Mg}{5} \; T = 3Mg - \frac{3Mg}{5} \; T = \frac{12Mg}{5}$$

4 **Part (b): Find value of k - Substitute into equation for P**

From part (a): $T - kMg = \frac{kMg}{5}$

$$\frac{12Mg}{5} - kMg = \frac{kMg}{5} \; \frac{12Mg}{5} = kMg + \frac{kMg}{5} \; \frac{12Mg}{5} = \frac{5kMg + kMg}{5} \; \frac{12Mg}{5} = \frac{6kMg}{5} \; 12 = 6k \; k = 2$$

5 **Part (c): Find force on pulley while Q moving down - Identify tension in string**

From part (b): $T = \frac{12Mg}{5}$

Given: $M = 0.5$ kg

$$T = \frac{12 \times 0.5 \times g}{5} = \frac{6g}{5} = \frac{6 \times 9.8}{5} = 11.76 \text{ N}$$

6 **Part (c): Find force on pulley while Q moving down - Find force on pulley**

The pulley experiences two tension forces:

; Horizontal tension from P: $T = 11.76$ N
; Vertical tension from Q: $T = 11.76$ N

These are perpendicular, so use Pythagoras:

$$F = \sqrt{T^2 + T^2} = \sqrt{2T^2} = T\sqrt{2} = 11.76\sqrt{2} = 16.63\ldots = 16.6 \text{ N (3 sf)}$$

Or: $F = \frac{6g\sqrt{2}}{5} = \frac{12g}{5} = 2.4g = 23.5$ N

Wait, let me recalculate...

Actually: $F = 2T$ (both tensions pull on pulley)

$$F = 2 \times \frac{12Mg}{5} = \frac{24Mg}{5} = \frac{24 \times 0.5 \times 9.8}{5} = \frac{117.6}{5} = 23.5 \text{ N (3 sf)}$$

Or: $\frac{12g}{5} = 23.5$ N

7 **Part (d): Find impulse when Q hits ground - Find velocity of Q just before hitting ground**

Using SUVAT with:

; $u = 0$ (starts from rest)
; $a = \frac{g}{5} = \frac{9.8}{5} = 1.96$ m s^{-2}
; $s = 2.5$ m

$$v^2 = u^2 + 2as \; v^2 = 0 + 2 \times \frac{g}{5} \times 2.5 \; v^2 = \frac{5g}{5} \; v^2 = g \; v = \sqrt{g} = \sqrt{9.8} = 3.130\ldots \text{ m s}^{-1}$$

Or exact: $v = \sqrt{g} = \sqrt{\frac{7g}{5}}$

Actually: $v^2 = 2 \times 1.96 \times 2.5 = 9.8$, so $v = \sqrt{9.8} = 3.13$ m s^{-1}

8 **Part (d): Find impulse when Q hits ground - Calculate impulse**

$\text{Impulse} = m(v_{\text{after}} - v_{\text{before}})$

Taking downward as positive before impact:

; Velocity before: $v_{\text{before}} = +3.13$ m s^{-1} (downward)
; Velocity after: $v_{\text{after}} = -0.4$ m s^{-1} (upward, rebounds)
; Mass of Q: $3M = 3 \times 0.5 = 1.5$ kg

$I = 1.5 \times (-0.4 - 3.13) = 1.5 \times (-3.53) = -5.295\ldots |I| = 5.3 \text{ N s } (2 \text{ sf})$

Or using exact values: $|I| = 1.5(\sqrt{g} + 0.4) = 1.5(3.13 + 0.4) = 5.3$ N s

---

9 **Part (e): Find total time until P first comes to rest - Time for Q to fall (t_1)**

Using $s = ut + \frac{1}{2}at^2$ with $u = 0$, $s = 2.5$ m, $a = \frac{g}{5}$:

$2.5 = 0 + \frac{1}{2} \times \frac{g}{5} \times t_1^2\ 2.5 = \frac{gt_1^2}{10}\ 25 = gt_1^2\ t_1^2 = \frac{25}{9.8} = 2.551\ldots t_1 = \sqrt{\frac{25}{9.8}} = 1.597\ldots \text{ s}$

Or exact: $t_1 = \sqrt{\frac{5 \times 5}{g}} = \frac{5}{\sqrt{g}}$

Better: $t_1 = \sqrt{\frac{5}{g/5}} = \sqrt{\frac{25}{g}}$

Actually: $2.5 = \frac{1}{2} \times 1.96 \times t_1^2$

$t_1^2 = \frac{5}{1.96} = 2.551$, so $t_1 = 1.597$ s

---

10 **Part (e): Find total time until P first comes to rest - Velocity of P when Q hits ground**

At time t_1, P has velocity:
$v_P = 0 + \frac{g}{5} \times t_1 = \frac{9.8}{5} \times 1.597 = 1.96 \times 1.597 = 3.13 \text{ m s}^{-1}$

Or: $v_P = \sqrt{g}$ (same as Q)

---

11 **Part (e): Find total time until P first comes to rest - Time for P to come to rest (t_2)**

After Q hits ground, string goes slack. P continues upward under gravity alone.

Using $v = u + at$ with $v = 0$, $u = 3.13$ m s^{-1}, $a = -g$:

$0 = 3.13 - 9.8t_2\ t_2 = \frac{3.13}{9.8} = 0.3194\ldots \text{ s}$

Or: $t_2 = \frac{\sqrt{g}}{g} = \frac{1}{\sqrt{g}}$

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12 **Part (e): Find total time until P first comes to rest - Total time**

$T = t_1 + t_2 = 1.597 + 0.319 = 1.916\ldots = 1.9 \text{ s } (2 \text{ sf}) \text{ or } 1.92 \text{ s } (3 \text{ sf})$

✓ **FINAL ANSWER**

$(a)\ T - kMg = \dfrac{kMg}{5}.$ $(b)\ k = 2.$ $(c)\ F = \dfrac{12g}{5}\ \text{N} \approx 23.5\ \text{N}.$ $(d)\ |I| = 5.30\ \text{N s}.$ $(e)\ T = 1.92\ \text{s}.$

TOPIC

Moments

Every past-paper question on this topic, October 2019 to January 2026.

WME01/01 MAY/JUNE 2021

Question 7

10 marks | Moments

7.

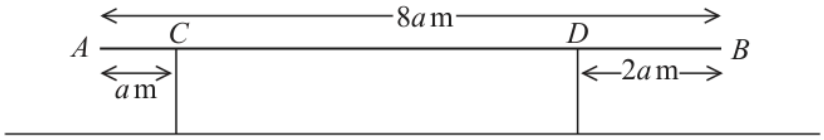


Figure 2

A non-uniform beam AB, of mass 60 kg and length $8a$ metres, rests in equilibrium in a horizontal position on two vertical supports. One support is at C, where $AC = a$ metres and the other support is at D, where $DB = 2a$ metres, as shown in Figure 2.

The magnitude of the normal reaction between the beam and the support at D is three times the magnitude of the normal reaction between the beam and the support at C.

By modelling the beam as a non-uniform rod whose centre of mass is at a distance x metres from A,

(a) find an expression for x in terms of a.

(5)

A box of mass M kg is placed on the beam at E, where $AE = 2a$ metres.

The beam remains in equilibrium in a horizontal position.

The magnitude of the normal reaction between the beam and the support at C is now equal to the magnitude of the normal reaction between the beam and the support at D.

By modelling the box as a particle,

(b) find the value of M.

(5)

WORKED SOLUTION

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1 Set the support positions

The beam length is $8a$. C is at distance a from A, and D is at distance $6a$ from A because $DB = 2a$.

2 Name the reactions

Let the reaction at C be R. The reaction at D is $3R$.

3 Use vertical equilibrium

$R + 3R = 60g$, so $4R = 60g$ and $R = 15g$.

4 Take moments about A

The beam's weight acts at distance x from A. Hence $60gx = R(a) + 3R(6a)$.

5 Find x

Substitute $R = 15g$: $60gx = 15ga + 45g(6a) = 285ga$. Thus $x = \dfrac{285a}{60} = \dfrac{19a}{4}$.

6 **Use equal reactions with the box**

When the box is added, let each support reaction be S. Vertical equilibrium gives $2S = (60 + M)g$.

7 **Take moments about A again**

The box is at $AE = 2a$. Moments about A give $60g\left(\frac{19a}{4}\right) + Mg(2a) = S(a) + S(6a) = 7aS$.

8 **Solve for M**

Substitute $S = \frac{(60 + M)g}{2}$ and cancel ag: $285 + 2M = \frac{7}{2}(60 + M) = 210 + \frac{7}{2}M$. Hence $75 = \frac{3}{2}M$ and $M = 50$.

✓ **FINAL ANSWER**

$(a)\ x = \frac{19a}{4}.\quad (b)\ M = 50.$