Elite Mathematics
Edexcel International A Level

IAL MECHANICS 1 · WME01

WME01/01 October 2025 Worked Solutions

October 2025 paper rebuilt with the worked solution after each question.

7	2019–2026	Single-paper solutions
paper questions	18 exam sessions	checked by Dr Eslam



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PAST PAPER

WME01/01 October 2025

October 2025

7	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

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Question 1

7 marks | Moments

1.

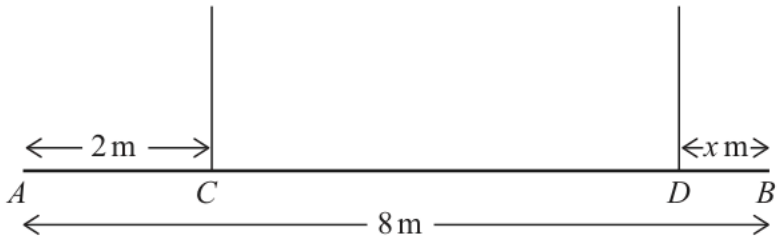


Figure 1

Figure 1 shows a sketch of a beam AB, with weight 240 N and length 8 m.

The beam is held in equilibrium in a horizontal position by two vertical ropes. The ropes are attached to the beam at the points C and D, where $AC = 2$ m and $DB = x$ metres.

The beam is modelled as a uniform rod and the ropes are modelled as light inextensible strings.

The tension in the rope at D is 90 N.

(a) Show that $x = \frac{2}{3}$

(3)

The rope at C will break if its tension exceeds 183 N. The rope at D cannot break. A package of weight W newtons is now attached to the beam at A. The beam remains horizontal and in equilibrium.

The package is modelled as a particle.

It is given that the rope at C does not break.

(b) Find the greatest possible value of W.

(4)

WORKED SOLUTION

Question 1 · WME01/01 October 2025

1 **Part (a): Show that x = 2/3 - Identify distances from C**

From the diagram:

; Distance AC = 2 m

; Distance CD = 8 - 2 - x = (6-x) m

; Distance from A to center = 4 m, so distance from C to center = 4 - 2 = 2 m

2 **Part (a): Show that x = 2/3 - Take moments about C**

$\sum M_C = 0$

Clockwise moments = Anticlockwise moments

$90 \times (6 - x) = 240 \times 2\ 90(6 - x) = 480\ 540 - 90x = 480\ 90x = 60\ x = \frac{60}{90} = \frac{2}{3}$

---

3 **Part (b): Find the greatest possible value of W - Apply vertical equilibrium**

$\sum F_{\text{vertical}} = 0$

$T_C + T_D = 240 + W\ 183 + T_D = 240 + W\ T_D = 57 + W$

---

4 **Part (b): Find the greatest possible value of W - Take moments about C**

$\sum M_C = 0$

Clockwise moments = Anticlockwise moments

Distance CD = $6 - \frac{2}{3} = \frac{16}{3}$ m

$T_D \times \frac{16}{3} + W \times 2 = 240 \times 2\ (57 + W) \times \frac{16}{3} + 2W = 480\ \frac{16(57+W)}{3} + 2W = 480$
$\frac{912+16W}{3} + 2W = 480$

Multiply through by 3:
$912 + 16W + 6W = 1440\ 22W = 528\ W = 24$

✓ **FINAL ANSWER**

$(a)\ x = \dfrac{2}{3}.\quad (b)\ W = 24\ \text{N}.$

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Question 2

8 marks | Momentum, Impulse & Collisions

2.

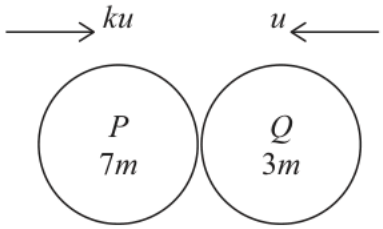


Figure 2

Particle P of mass $7m$ and particle Q of mass $3m$ are moving in opposite directions along the same straight line on a smooth horizontal surface. The particles collide directly.

Immediately **before** the collision, the speed of P is ku and the speed of Q is u, as shown in Figure 2.

Immediately **after** the collision, the speed of P is w and the speed of Q is $2w$.

The direction of motion of Q is reversed by the collision.

The impulse received by Q in the collision has magnitude $\frac{7}{2}mu$.

(a) Find w in terms of u.

(3)

(b) Find the two possible values of k.

(5)

WORKED SOLUTION

Question 2 · WME01/01 October 2025

1 **Part (a): Find w in terms of u - Apply impulse-momentum theorem to Q**

$\text{Impulse} = \text{Final momentum} - \text{Initial momentum}$

For particle Q (mass = 3m):

; Initial velocity: $-u$ (moving left, negative)
; Final velocity: $+2w$ (moving right, positive)

$\text{Impulse} = 3m(2w) - 3m(-u)$ $\frac{7mu}{2} = 6mw + 3mu$ $\frac{7mu}{2} = 3m(2w + u)$ $\frac{7u}{2} = 6w + 3u$
$6w = \frac{7u}{2} - 3u$ $6w = \frac{7u-6u}{2}$ $6w = \frac{u}{2}$ $w = \frac{u}{12}$

2 **Part (b): Find the two possible values of k - Apply conservation of momentum**

$\text{Total momentum before} = \text{Total momentum after}$

Before collision:

$\text{P momentum} = 7m \times ku = 7mku \text{ (right)}$ $\text{Q momentum} = 3m \times (-u) = -3mu \text{ (left)}$

$\text{Total} = 7mku - 3mu$

After collision:

$\text{P momentum} = 7m \times w = 7mw \text{ (right)}$ $\text{Q momentum} = 3m \times 2w = 6mw \text{ (right)}$

$\text{Total} = 7mw + 6mw = 13mw$

---

3 **Part (b): Find the two possible values of k - Set up equation**

$7mku - 3mu = 13mw$ $7ku - 3u = 13w$

---

4 **Part (b): Find the two possible values of k - Substitute $w = \frac{u}{12}$**

$7ku - 3u = 13 \times \frac{u}{12}$ $7ku - 3u = \frac{13u}{12}$ $7ku = \frac{13u}{12} + 3u$ $7ku = \frac{13u+36u}{12}$ $7ku = \frac{49u}{12}$ $7k = \frac{49}{12}$

$k = \frac{49}{84} = \frac{7}{12}$

---

5 **Part (b): Find the two possible values of k - Consider both directions for P after collision**

P could move either right (velocity = $+w$) or left (velocity = $-w$)

Case 1: P moves right with velocity $+w$

Already solved: $k = \frac{7}{12}$

Case 2: P moves left with velocity $-w$

Total momentum after = $-7mw + 6mw = -mw$

$7mku - 3mu = -mw$ $7ku - 3u = -w$ $7ku - 3u = -\frac{u}{12}$ $7ku = 3u - \frac{u}{12}$ $7ku = \frac{36u-u}{12}$

$7ku = \frac{35u}{12}$ $k = \frac{35}{84} = \frac{5}{12}$

✓ **FINAL ANSWER**

$(a)\ w = \dfrac{u}{12}.$ $(b)\ k = \dfrac{7}{12}$ or $k = \dfrac{5}{12}.$

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Question 3

14 marks | Forces

3. [*In this question* **i** *and* **j** *are horizontal perpendicular unit vectors.*]

A particle P of mass 2 kg moves on a smooth horizontal surface under the action of two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, where $\mathbf{F}_1 = (-2\mathbf{i} + 3\mathbf{j})$ N and $\mathbf{F}_2 = (4\mathbf{i} + 2\mathbf{j})$ N.

(a) Find the acceleration of P.

(3)

At time $t = 0$, the velocity of P is $(3\mathbf{i} - 4\mathbf{j})\,\text{m s}^{-1}$

(b) Find the speed of P when $t = 3$ seconds.

(4)

An additional force, $\mathbf{F}_3 = (b\mathbf{i} + c\mathbf{j})$ N, is applied to P.

The resultant of $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ is equal to $\lambda(\mathbf{i} + \mathbf{j})$ N, where λ is a constant.

(c) Show that $b - c = 3$

(3)

The resultant of $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ has magnitude $10\sqrt{2}$ N.

(d) Find the two possible $\mathbf{F}_3$ forces.

(4)

WORKED SOLUTION

Question 3 · WME01/01 October 2025

1 **Part (a): Find the acceleration of P - Find resultant force**

$\mathbf{F}_{\text{resultant}} = \mathbf{F}_1 + \mathbf{F}_2$

$\mathbf{F}_{\text{resultant}} = (-2\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} + 2\mathbf{j}) = (-2 + 4)\mathbf{i} + (3 + 2)\mathbf{j} = 2\mathbf{i} + 5\mathbf{j}\ \text{N}$

2 **Part (a): Find the acceleration of P - Apply Newton's Second Law**

$\mathbf{F} = m\mathbf{a}$

$2\mathbf{i} + 5\mathbf{j} = 2 \times \mathbf{a}\ \mathbf{a} = \frac{2\mathbf{i}+5\mathbf{j}}{2}\ \mathbf{a} = \mathbf{i} + 2.5\mathbf{j}\ \text{m s}^{-2}$

Or: $\mathbf{a} = \mathbf{i} + \frac{5}{2}\mathbf{j}\ \text{m s}^{-2}$

3 **Part (b): Find the speed of P when t = 3 seconds - Find velocity at t = 3**

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

Given:

; $\mathbf{u} = 3\mathbf{i} - 4\mathbf{j}$ m s^{-1}
; $\mathbf{a} = \mathbf{i} + 2.5\mathbf{j}$ m s^{-2}
; $t = 3$ seconds

$\mathbf{v} = (3\mathbf{i} - 4\mathbf{j}) + (\mathbf{i} + 2.5\mathbf{j}) \times 3 = 3\mathbf{i} - 4\mathbf{j} + 3\mathbf{i} + 7.5\mathbf{j} = 6\mathbf{i} + 3.5\mathbf{j}\ \text{m s}^{-1}$

---

4 **Part (b): Find the speed of P when t = 3 seconds - Find the speed (magnitude of velocity)**

$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$

$\text{Speed} = \sqrt{6^2 + 3.5^2} = \sqrt{36 + 12.25} = \sqrt{48.25} = 6.946... = 6.95\ \text{m s}^{-1}\ (3\ \text{s.f.})$

---

5 **Part (c): Show that b - c = 3 - Find resultant of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$**

$\mathbf{F}_{\text{resultant}} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = (-2\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} + 2\mathbf{j}) + (b\mathbf{i} + c\mathbf{j})$
$= (-2 + 4 + b)\mathbf{i} + (3 + 2 + c)\mathbf{j} = (2 + b)\mathbf{i} + (5 + c)\mathbf{j}$

---

6 **Part (c): Show that b - c = 3 - Apply the condition**

Given that resultant = $\lambda(\mathbf{i} + \mathbf{j})$, we can write:

$(2 + b)\mathbf{i} + (5 + c)\mathbf{j} = \lambda\mathbf{i} + \lambda\mathbf{j}$

---

7 **Part (c): Show that b - c = 3 - Compare coefficients**

For this to be in the form $\lambda(\mathbf{i} + \mathbf{j})$, the coefficients must be equal:

$2 + b = \lambda\ 5 + c = \lambda$

Therefore:
$2 + b = 5 + c\ b = 5 + c - 2\ b = 3 + c\ b - c = 3$

---

8 **Part (d): Find the two possible $\mathbf{F}_3$ forces - Use the magnitude condition**

Resultant = $(2+b)\mathbf{i}+(5+c)\mathbf{j}$

Since resultant = $\lambda(\mathbf{i}+\mathbf{j})$, we have $2+b=5+c=\lambda$

$|\mathbf{F}_{\text{resultant}}|=\lambda\sqrt{1^2+1^2}=\lambda\sqrt{2}$

Given: $|\mathbf{F}_{\text{resultant}}|=10\sqrt{2}$

$\lambda\sqrt{2}=10\sqrt{2}\,\lambda=10 \quad \text{or} \quad \lambda=-10$

(The negative value comes from considering the opposite direction)

---

9 **Part (d): Find the two possible $\mathbf{F}_3$ forces - Find b and c for each value of λ**

Case 1: $\lambda=10$

$2+b=10 \quad \Rightarrow \quad b=8\,5+c=10 \quad \Rightarrow \quad c=5$

Check: $b-c=8-5=3$ ✓

Therefore: $\mathbf{F}_3=8\mathbf{i}+5\mathbf{j}$ N

Case 2: $\lambda=-10$

$2+b=-10 \quad \Rightarrow \quad b=-12\,5+c=-10 \quad \Rightarrow \quad c=-15$

Check: $b-c=-12-(-15)=3$ ✓

Therefore: $\mathbf{F}_3=-12\mathbf{i}-15\mathbf{j}$ N

✓ **FINAL ANSWER**

$(a)\ \mathbf{a}=\mathbf{i}+2.5\mathbf{j}\ \mathrm{m\,s^{-2}}.\quad (b)\ \text{speed}=6.95\ \mathrm{m\,s^{-1}}.\quad (c)\ b-c=3.\quad (d)\ \mathbf{F}_3=(8\mathbf{i}+5\mathbf{j})\ \text{N or}\ (-12\mathbf{i}-15\mathbf{j})\ \text{N}$

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Question 4

9 marks | Constant Acceleration in 2D

4. The point A is 10 m above horizontal ground.
At time $t = 0$, a particle P is projected vertically upwards with speed $5\,\text{m s}^{-1}$ from A.
Particle P moves freely under gravity.

(a) Find the greatest height above A reached by P.

(3)

The point B is on the ground, vertically below A.
At time $t = 1$ second, a particle Q is projected vertically upwards with speed $7\,\text{m s}^{-1}$ from B.
Particle Q moves freely under gravity.

Particles P and Q collide at time $t = T$ seconds.

(b) Find the value of T.

(4)

(c) Find the speed of P at the instant immediately before the particles collide.

(2)

WORKED SOLUTION

Question 4 · WME01/01 October 2025

1 **Part (a): Find the greatest height above A reached by P - Identify the SUVAT variables**

At maximum height, velocity = 0

; $u = 5$ m s^{-1} (initial velocity)
; $v = 0$ m s^{-1} (at maximum height)
; $a = -9.8$ m s^{-2} (gravity, downward)
; $s = ?$ (displacement above A)

2 **Part (a): Find the greatest height above A reached by P - Apply $v^2 = u^2 + 2as$**

$v^2 = u^2 + 2as$

$0^2 = 5^2 + 2(-9.8)s \; 0 = 25 - 19.6s \; 19.6s = 25 \; s = \frac{25}{19.6} \; s = 1.276\ldots \; s = 1.28 \text{ m (3 s.f.)}$

3 **Part (b): Find the value of T - Position of P at time t**

Taking upward as positive, with origin at ground level:

$$s = s_0 + ut + \tfrac{1}{2}at^2$$

For P (starting at A, 10 m above ground):

$s_P = 10 + 5t + \tfrac{1}{2}(-9.8)t^2$ $s_P = 10 + 5t - 4.9t^2$

---

4 **Part (b): Find the value of T - Position of Q at time t**

Q starts at $t = 1$, so Q has been moving for $(t - 1)$ seconds:

For Q (starting at B, ground level):

$s_Q = 0 + 7(t-1) + \tfrac{1}{2}(-9.8)(t-1)^2$ $s_Q = 7(t-1) - 4.9(t-1)^2$

$s_Q = 7t - 7 - 4.9(t^2 - 2t + 1)$ $s_Q = 7t - 7 - 4.9t^2 + 9.8t - 4.9$ $s_Q = -4.9t^2 + 16.8t - 11.9$

---

5 **Part (b): Find the value of T - Set $s_P = s_Q$ at collision (when $t = T$)**

$10 + 5T - 4.9T^2 = -4.9T^2 + 16.8T - 11.9$ $10 + 5T = 16.8T - 11.9$ $10 + 11.9 = 16.8T - 5T$

$21.9 = 11.8T$ $T = \tfrac{21.9}{11.8}$ $T = 1.855\ldots$ $T = 1.86\ \text{s}\ (3\ \text{s.f.})$

---

6 **Part (c): Find the speed of P immediately before collision - Find velocity of P at time $T = 1.86$ s**

$v = u + at$

For particle P:

; $u = 5\ \text{m s}^{-1}$

; $a = -9.8\ \text{m s}^{-2}$

; $t = 1.86\ \text{s}$

$$v_P = 5 + (-9.8)(1.86) = 5 - 18.228 = -13.228\ldots = -13.2\ \text{m s}^{-1}$$

The negative sign indicates P is moving downward.

---

7 **Part (c): Find the speed of P immediately before collision - Find speed**

Speed = $|v|$

$$\text{Speed} = |-13.2| = 13.2\ \text{m s}^{-1}$$

✓ **FINAL ANSWER**

(a) maximum height above A $= 1.28$ m. (b) $T = 1.86$ s. (c) speed of P $= 13.2\ \text{m s}^{-1}$.

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Question 5

13 marks | Resolving Forces, Inclined Planes

5.

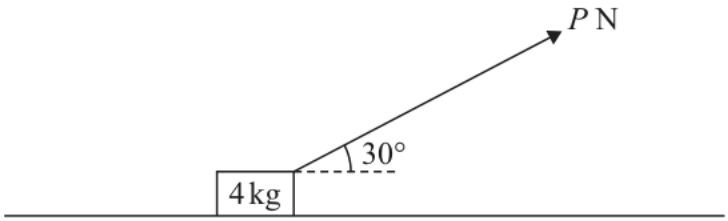


Figure 3

A box of mass 4 kg is placed on a rough horizontal surface.

A force of magnitude P newtons, acting at 30° to the horizontal, is applied to the box, as shown in Figure 3.

The coefficient of friction between the box and the surface is $\frac{2}{3}$

The box is modelled as a particle.

(a) Find the value of P when the box is on the point of sliding along the surface.

(6)

The value of P is now increased to 25 and the box moves along the surface.

Find

(b) the acceleration of the box,

(5)

(c) the speed of the box when it has moved 1.5 m.

(2)

WORKED SOLUTION

Question 5 · WME01/01 October 2025

1 **Part (a): Find P when box is on point of sliding - Resolve vertically**

$\uparrow: \sum F_{\text{vertical}} = 0$

$R + P\sin 30^\circ = mg \; R + 0.5P = 4 \times 9.8 \; R = 39.2 - 0.5P \quad ...(1)$

2 **Part (a): Find P when box is on point of sliding - Apply friction law**

At limiting equilibrium (on point of sliding):

$F = \mu R$

$F = \frac{2}{3} \times R \; F = \frac{2}{3}(39.2 - 0.5P) \; F = 26.133... - \frac{P}{3} \quad ...(2)$

3 **Part (a): Find P when box is on point of sliding - Resolve horizontally**

Box on point of sliding means $a = 0$:

$\rightarrow: \sum F_{\text{horizontal}} = 0$

$P\cos 30^\circ = F\ P \times \frac{\sqrt{3}}{2} = 26.133... - \frac{P}{3}\ 0.866P = 26.133 - 0.333P$
$0.866P + 0.333P = 26.133\ 1.199P = 26.133\ P = 21.8\ \text{N}\ (3\ \text{s.f.})$

---

4 **Part (b): Find acceleration when P = 25 N - Find normal reaction when P = 25 N**

From equation (1) in part (a):

$R = 39.2 - 0.5P\ R = 39.2 - 0.5(25)\ R = 39.2 - 12.5\ R = 26.7\ \text{N}$

---

5 **Part (b): Find acceleration when P = 25 N - Find friction force**

Box is now moving, so:

$F = \mu R\ F = \frac{2}{3} \times 26.7\ F = 17.8\ \text{N}$

---

6 **Part (b): Find acceleration when P = 25 N - Resolve horizontally and apply F = ma**

$\rightarrow: \sum F = ma$

$P\cos 30^\circ - F = ma\ 25 \times 0.866 - 17.8 = 4a\ 21.65 - 17.8 = 4a\ 3.85 = 4a\ a = 0.9625$
$a = 0.96\ \text{m s}^{-2}\ (2\ \text{s.f.})$

---

7 **Part (c): Find speed when box has moved 1.5 m - Identify SUVAT variables**

$u = 0$ m s^{-1} (starts from rest)
; $a = 0.9625$ m s^{-2} (from part b)
; $s = 1.5$ m
; $v = ?$

---

8 **Part (c): Find speed when box has moved 1.5 m - Apply $v^2 = u^2 + 2as$**

$v^2 = u^2 + 2as$

$v^2 = 0^2 + 2(0.9625)(1.5)\ v^2 = 2.8875\ v = \sqrt{2.8875}\ v = 1.699...\ v = 1.7\ \text{m s}^{-1}\ (2\ \text{s.f.})$

✓ **FINAL ANSWER**

$(a)\ P = 21.8\ \text{N}.\quad (b)\ a = 0.96\ \text{m s}^{-2}.\quad (c)\ v = 1.7\ \text{m s}^{-1}.$

Question 6

10 marks | Kinematics Graphs

6.

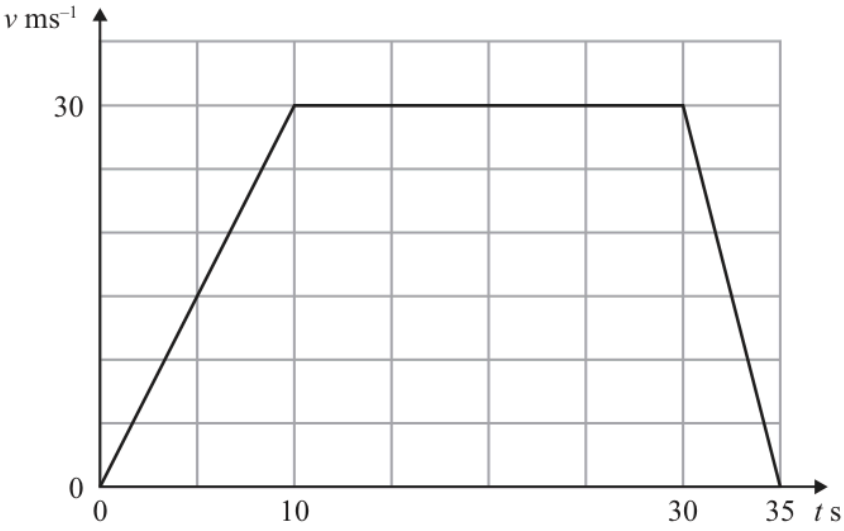


Figure 4

The point O lies on a straight horizontal road.
At time $t = 0$, a car leaves O and travels along the road.

The velocity-time graph in Figure 4 shows the velocity, $v\,\text{m s}^{-1}$, of the car at time t seconds for the first 35 seconds of its journey.

(a) Find

(i) the acceleration of the car for the period $0 \leqslant t \leqslant 10$

(ii) the deceleration of the car for the period $30 \leqslant t \leqslant 35$

(2)

(b) Sketch an acceleration-time graph for the car for the period $0 \leqslant t \leqslant 35$

(2)

(c) Find the distance travelled by the car for the period $0 \leqslant t \leqslant 35$

(2)

When $t = 5$, a motorcycle starts from rest at O.
The motorcycle travels along the same road as the car and in the same direction.

For the period $5 \leqslant t \leqslant 20$, the acceleration of the motorcycle is $A\,\text{m s}^{-2}$, where A is a positive constant.

The motorcycle catches up with the car when $t = 20$

(d) Find the value of A.

(4)

1 Part (a): Find accelerations

(i) Acceleration for $0 \leq t \leq 10$

$$\text{Acceleration} = \frac{\Delta v}{\Delta t} = \text{gradient of v-t graph}$$

$$a = \frac{30-0}{10-0} \; a = 3 \text{ m s}^{-2}$$

(ii) Deceleration for $30 \leq t \leq 35$

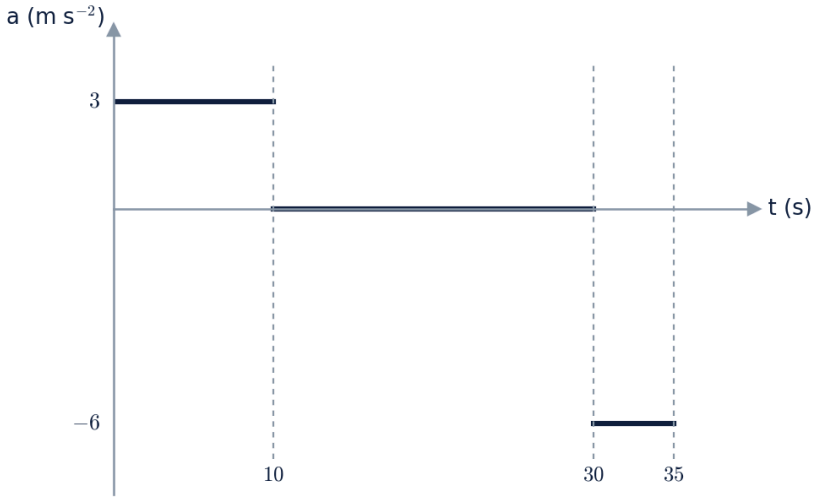
$$a = \frac{0-30}{35-30} \; a = \frac{-30}{5} \; a = -6 \text{ m s}^{-2}$$

Deceleration = $|-6| = 6$ m s^{-2}

2 Part (b): Sketch acceleration-time graph

Analysis of each section:

; $0 \leq t \leq 10$: Constant acceleration = 3 m s^{-2}
; $10 < t \leq 30$: Constant velocity, so acceleration = 0 m s^{-2}
; $30 < t \leq 35$: Constant deceleration = -6 m s^{-2}



3 **Part (c): Find distance travelled for $0 \leq t \leq 35$ - Calculate area under v-t graph**

$\text{Distance} = \text{Area under v-t graph}$

The shape is a trapezium. We can split it into regions:

Method 1: Trapezium formula

$\text{Area} = \frac{1}{2}(a + b)h$

Where the parallel sides are the time intervals and height is velocity.

Better to calculate as sum of shapes:

Method 2: Sum of triangle + rectangle + triangle

$\text{Area}_1 = \frac{1}{2} \times 10 \times 30 = 150 \text{ m}$ $\text{Area}_2 = 20 \times 30 = 600 \text{ m}$ $\text{Area}_3 = \frac{1}{2} \times 5 \times 30 = 75 \text{ m}$

$\text{Total distance} = 150 + 600 + 75 = 825 \text{ m}$

---

4 **Part (d): Find value of A - Distance travelled by car (0 to 20 seconds)**

Area under v-t graph from $t = 0$ to $t = 20$:

$\text{Distance}_{\text{car}} = \text{Triangle} + \text{Rectangle} = \frac{1}{2}(10)(30) + (10)(30) = 150 + 300 = 450 \text{ m}$

---

5 **Part (d): Find value of A - Distance travelled by motorcycle (5 to 20 seconds)**

Motorcycle starts at $t = 5$ with constant acceleration A.

Time of travel: $20 - 5 = 15$ seconds

$s = ut + \frac{1}{2}at^2$

$\text{Distance}_{\text{motorcycle}} = 0 + \frac{1}{2}A(15)^2 = \frac{225A}{2} = 112.5A \text{ m}$

---

6 **Part (d): Find value of A - Set distances equal**

Motorcycle catches up with car when $t = 20$:

$112.5A = 450$ $A = \frac{450}{112.5}$ $A = 4 \text{ m s}^{-2}$

FINAL ANSWER

$(a)(i)\ a = 3\ \text{m s}^{-2},\ (ii)$ deceleration $= 6\ \text{m s}^{-2}$. (b) correct acceleration-time graph with levels $3, 0, -6$. (c) distance $= 825$ m. $(d)\ A = 4\ \text{m s}^{-2}$

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Question 7

14 marks | Resolving Forces, Inclined Planes

7.

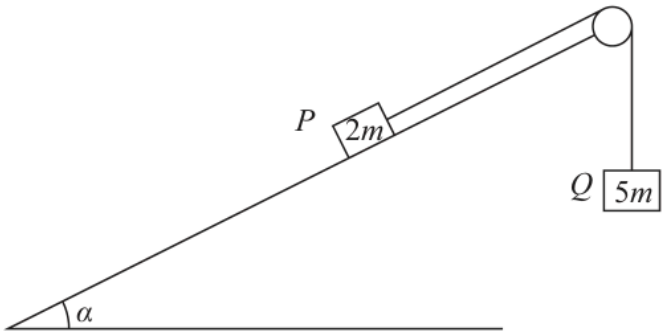


Figure 5

A block P of mass $2m$ is held at rest on a fixed rough plane.

The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{5}{12}$

One end of a light inextensible string is attached to P.
The string is parallel to a line of greatest slope of the plane and passes over a smooth light pulley which is fixed at the top of the plane.
The other end of the string is attached to a block Q of mass $5m$.
Block Q hangs vertically below the pulley, as shown in Figure 5.

The system is released from rest with the string taut and block P moves up the plane.

Immediately after the system is released, the tension in the string is T and the acceleration of the blocks is a.

The blocks are modelled as particles and air resistance is ignored.

(a) Write down an equation of motion for Q.

(2)

The coefficient of friction between P and the plane is $\frac{1}{8}$

(b) Find T in terms of m and g.

(7)

(c) State how the solution to part (b) uses the fact that the string is inextensible.

(1)

The magnitude of the force exerted on the pulley by the string is kmg.

(d) Find the value of k to 3 significant figures.

(4)

1 **Part (a): Write equation of motion for Q**

Forces on Q (mass = 5m):

Taking downward as positive for Q:

$\text{For Q: } \sum F = ma$

$5mg - T = 5ma$

2 **Part (b): Find T in terms of m and g - Resolve perpendicular to plane**

$\perp \text{ to plane: } \sum F = 0$

$R = 2mg\cos\alpha \; R = 2mg \times \frac{12}{13} \; R = \frac{24mg}{13}$

3 **Part (b): Find T in terms of m and g - Find friction force**

P is moving UP the plane, so friction acts DOWN the plane:

$F = \mu R = \frac{1}{8} \times \frac{24mg}{13} = \frac{3mg}{13}$

4 **Part (b): Find T in terms of m and g - Resolve parallel to plane**

Taking up the plane as positive:

$\parallel \text{ to plane: } \sum F = ma$

$T - 2mg\sin\alpha - F = 2ma \; T - 2mg \times \frac{5}{13} - \frac{3mg}{13} = 2ma \; T - \frac{10mg}{13} - \frac{3mg}{13} = 2ma$
$T - \frac{13mg}{13} = 2ma \; T - mg = 2ma \quad ...(1)$

5 **Part (b): Find T in terms of m and g - Combine with equation for Q**

From part (a): $5mg - T = 5ma$ ...(2)

Add equations (1) and (2):

$(T - mg) + (5mg - T) = 2ma + 5ma \; 4mg = 7ma \; a = \frac{4g}{7}$

6 **Part (b): Find T in terms of m and g - Substitute back to find T**

From equation (1):

$T = mg + 2ma = mg + 2m \times \frac{4g}{7} = mg + \frac{8mg}{7} = \frac{7mg+8mg}{7} = \frac{15mg}{7}$

7 **Part (c): State how solution uses fact that string is inextensible**

An inextensible string means the string cannot stretch.

This has the following consequence:

For an inextensible string over a pulley:

The magnitude of acceleration of both connected particles must be equal.

In this problem: $|a_P| = |a_Q| = a$

How this was used in part (b):

When we wrote the equations of motion:

; For P: $T - mg = 2ma$
; For Q: $5mg - T = 5ma$

We used the same acceleration a for both particles because the string is inextensible.

If the string could stretch, the two particles could have different accelerations.

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8 **Part (d): Find value of k (force on pulley = kmg) - Identify the angle between tensions**

The string from Q is vertical (downward).
The string from P is along the plane at angle α to horizontal.

Angle between the two strings = $90° + \alpha$

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9 **Part (d): Find value of k (force on pulley = kmg) - Use vector components**

Let's take horizontal (right) and vertical (up) as positive directions.

Tension from Q:

; Horizontal component: 0
; Vertical component: $-T = -\frac{15mg}{7}$ (downward)

Tension from P along plane:
The plane makes angle α with horizontal, so:

; Horizontal component: $-T\cos\alpha = -\frac{15mg}{7} \times \frac{12}{13} = -\frac{180mg}{91}$
; Vertical component: $T\sin\alpha = \frac{15mg}{7} \times \frac{5}{13} = \frac{75mg}{91}$

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10 **Part (d): Find value of k (force on pulley = kmg) - Find resultant**

$R_x = 0 - \frac{180mg}{91} = -\frac{180mg}{91}$ $R_y = -\frac{15mg}{7} + \frac{75mg}{91}$
Convert $\frac{15mg}{7}$ to denominator 91: $\frac{15mg}{7} = \frac{195mg}{91}$
$R_y = -\frac{195mg}{91} + \frac{75mg}{91} = -\frac{120mg}{91}$

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11 **Part (d): Find value of k (force on pulley = kmg) - Find magnitude**

$|R| = \sqrt{R_x^2 + R_y^2} = \sqrt{\left(\frac{180mg}{91}\right)^2 + \left(\frac{120mg}{91}\right)^2} = \frac{mg}{91}\sqrt{180^2 + 120^2} = \frac{mg}{91}\sqrt{32400 + 14400}$
$= \frac{mg}{91}\sqrt{46800} = \frac{mg}{91} \times 216.333\ldots = 2.377\ldots mg = 2.38mg$ (3 s.f.)
Therefore: $k = 2.38$

✓ **FINAL ANSWER**

$(a)\ 5mg - T = 5ma.$ $(b)\ T = \frac{15mg}{7}.$ (c) both particles have the same magnitude of acceleration. $(d)\ k = 2.38$

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