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Elite Mathematics
Edexcel International A Level

IAL MECHANICS 1 · WME01

WME01/01 May/June 2025 Worked Solutions

May/June 2025 paper rebuilt with the worked solution after each question.

8	2019–2026	Single-paper solutions
paper questions	18 exam sessions	checked by Dr Eslam



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PAST PAPER

WME01/01 May/June 2025

May/June 2025

8	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WME01/01 MAY/JUNE 2025

Question 1

6 marks | Momentum, Impulse & Collisions

1.

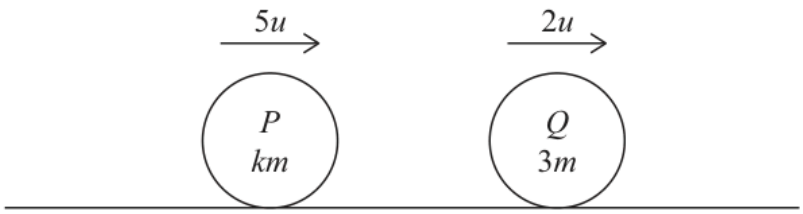


Figure 1

Figure 1 shows two particles, P and Q, moving in the same direction along the same straight line on a smooth horizontal surface.
Particle P has mass km and particle Q has mass $3m$
The particles collide directly.

Immediately before the collision, the speed of P is $5u$ and the speed of Q is $2u$

Immediately after the collision, the speed of P is $2u$ and its direction of motion is unchanged.

Immediately after the collision, the speed of Q is v

The impulse exerted on Q by P in the collision has magnitude $4.5mu$

(a) Find v in terms of u only.

(3)

(b) Find the value of k

(3)

WORKED SOLUTION

Question 1 · WME01/01 May/June 2025

1 **Part (a): Find v in terms of u - Apply impulse-momentum theorem to Q**

$\text{Impulse} = \text{Final momentum} - \text{Initial momentum}$

For particle Q:

; Mass = $3m$
; Initial velocity = $2u$
; Final velocity = v
; Impulse received = $4.5mu$

2 **Part (a): Find v in terms of u - Set up equation**

$4.5mu = 3m \times v - 3m \times 2u \; 4.5mu = 3mv - 6mu \; 4.5u = 3v - 6u \; 4.5u + 6u = 3v \; 10.5u = 3v \; v = \frac{10.5u}{3} \; v = 3.5u$

3 **Part (b): Find the value of k - Apply conservation of momentum**

$\text{Total momentum before} = \text{Total momentum after}$

Before collision:

$\text{P momentum} = km \times 5u = 5kmu$ $\text{Q momentum} = 3m \times 2u = 6mu$ $\text{Total} = 5kmu + 6mu$

After collision:

$\text{P momentum} = km \times 2u = 2kmu$ $\text{Q momentum} = 3m \times 3.5u = 10.5mu$ $\text{Total} = 2kmu + 10.5mu$

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4 **Part (b): Find the value of k - Set up equation**

$5kmu + 6mu = 2kmu + 10.5mu$ $5km + 6m = 2km + 10.5m$ $5k + 6 = 2k + 10.5$ $5k - 2k = 10.5 - 6$ $3k = 4.5$ $k = \frac{4.5}{3}$ $k = 1.5$

✓ **FINAL ANSWER**

$(a)\ v = 3.5u = \dfrac{7u}{2}.$ $(b)\ k = 1.5 = \dfrac{3}{2}.$

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Question 2

7 marks | Working with Vectors

2. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively. Position vectors are given relative to a fixed origin.*]

Two particles, A and B, are moving on a smooth horizontal surface.
Each particle is moving with constant velocity.

At time t seconds, the position vector of A is given by $\mathbf{r}$ metres.

- When $t = 2$, $\mathbf{r} = (-5\mathbf{i} + 16\mathbf{j})$
- When $t = 5$, $\mathbf{r} = (10\mathbf{i} + 4\mathbf{j})$

(a) Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the velocity of A.

(2)

(b) Find an expression for $\mathbf{r}$ at time t seconds.
Give your answer in the form $p\mathbf{i} + q\mathbf{j}$, where p and q are functions of t

(2)

At time t seconds, the position vector of B is given by $\mathbf{s}$ metres where

$$\mathbf{s} = -5\mathbf{i} + 7\mathbf{j} + t(2\mathbf{i} - 3\mathbf{j})$$

(c) Find, to the nearest degree, the bearing of B from A when $t = 5$

(3)

WORKED SOLUTION

Question 2 · WME01/01 May/June 2025

1 **Part (a): Find velocity of A - Find change in position**

Given positions for particle A:

; At $t = 2$: $\mathbf{r} = -5\mathbf{i} + 16\mathbf{j}$
; At $t = 5$: $\mathbf{r} = 10\mathbf{i} + 4\mathbf{j}$

$\mathbf{v} = \frac{\Delta \mathbf{r}}{\Delta t}$

$\mathbf{v}_A = \frac{(10\mathbf{i}+4\mathbf{j})-(-5\mathbf{i}+16\mathbf{j})}{5-2} = \frac{(10+5)\mathbf{i}+(4-16)\mathbf{j}}{3} = \frac{15\mathbf{i}-12\mathbf{j}}{3} = 5\mathbf{i} - 4\mathbf{j}\ \text{m s}^{-1}$

2 **Part (b): Find expression for r at time t - Use position-velocity relationship**

$\mathbf{r}(t) = \mathbf{r}(t_0) + \mathbf{v}(t - t_0)$

Using $t_0 = 2$ with $\mathbf{r}(2) = -5\mathbf{i} + 16\mathbf{j}$:

$\mathbf{r}(t) = (-5\mathbf{i} + 16\mathbf{j}) + (5\mathbf{i} - 4\mathbf{j})(t - 2) = -5\mathbf{i} + 16\mathbf{j} + 5(t-2)\mathbf{i} - 4(t-2)\mathbf{j} = -5\mathbf{i} + 16\mathbf{j} + (5t - 10)\mathbf{i} + (-4t + 8)\mathbf{j}$
$= (-5 + 5t - 10)\mathbf{i} + (16 - 4t + 8)\mathbf{j} = (5t - 15)\mathbf{i} + (24 - 4t)\mathbf{j}$

3 **Part (c): Find bearing of B from A when t = 5 - Find positions at t = 5**

For A at $t = 5$: $\mathbf{r}_A = 10\mathbf{i} + 4\mathbf{j}$ (given)

For B at $t = 5$:

$\mathbf{s}_B = -5\mathbf{i} + 7\mathbf{j} + 5(2\mathbf{i} - 3\mathbf{j}) = -5\mathbf{i} + 7\mathbf{j} + 10\mathbf{i} - 15\mathbf{j} = 5\mathbf{i} - 8\mathbf{j}$

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4 **Part (c): Find bearing of B from A when t = 5 - Find vector from A to B**

$\overrightarrow{AB} = \mathbf{s}_B - \mathbf{r}_A = (5\mathbf{i} - 8\mathbf{j}) - (10\mathbf{i} + 4\mathbf{j}) = -5\mathbf{i} - 12\mathbf{j}$

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5 **Part (c): Find bearing of B from A when t = 5 - Find angle and bearing**

Angle from South (negative j direction):

$\tan\alpha = \frac{5}{12}$ $\alpha = \tan^{-1}\left(\frac{5}{12}\right)$ $\alpha = 22.619...^\circ$

Bearing = $180^\circ + \alpha = 180^\circ + 22.619...^\circ = 202.619...^\circ$

FINAL ANSWER

$(a)\ \mathbf{v}_A = 5\mathbf{i} - 4\mathbf{j}\ \text{m s}^{-1}.$ $(b)\ \mathbf{r} = (5t - 15)\mathbf{i} + (24 - 4t)\mathbf{j}.$ $(c)\ \text{bearing} = 203°.$

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Question 3

8 marks | Constant Acceleration in 1D

3. A particle is projected vertically upwards with speed $U\,\text{m s}^{-1}$ from a point A.

The point A is 12 m vertically above the point B.

Point B is on horizontal ground.

The particle moves freely under gravity until it hits the ground at B.

The time taken for the particle to travel from A to B is 4 seconds.

(a) Find the value of U.

(3)

(b) Find the speed of the particle as it hits the ground at B.

(3)

(c) Sketch a speed-time graph for the motion of the particle from the instant it leaves A to the instant it reaches B. (No further calculations are required.)

(2)

WORKED SOLUTION

Question 3 · WME01/01 May/June 2025

1 **Part (a): Find the value of U - Identify SUVAT variables**

Taking upward as positive:

; $u = U$ m s^{-1} (unknown, upward)
; $s = -12$ m (displacement from A to B, downward)
; $t = 4$ seconds
; $a = -9.8$ m s^{-2} (gravity, downward)

2 **Part (a): Find the value of U - Apply $s = ut + \frac{1}{2}at^2$**

$s = ut + \frac{1}{2}at^2$

$-12 = U(4) + \frac{1}{2}(-9.8)(4)^2$ $-12 = 4U + \frac{1}{2}(-9.8)(16)$ $-12 = 4U - 78.4$ $4U = -12 + 78.4$ $4U = 66.4$ $U = 16.6\ \text{m s}^{-1}$

3 **Part (b): Find speed as particle hits ground - Use SUVAT to find velocity at B**

$v^2 = u^2 + 2as$

$v^2 = (16.6)^2 + 2(-9.8)(-12)$ $v^2 = 275.56 + 235.2$ $v^2 = 510.76$ $v = \pm 22.6$

Since particle is moving downward at B, $v = -22.6$ m s^{-1} (negative)

4 **Part (b): Find speed as particle hits ground - Find speed**

Speed $= |v| = 22.6\ \text{m s}^{-1}$, or about $23\ \text{m s}^{-1}$ to 2 significant figures.

5 **Part (c): Sketch speed-time graph - Analyze the motion**

The particle:

; Starts at A with speed $U = 16.6$ m s^{-1} upward
; Decelerates (speed decreases) as it goes up
; Reaches maximum height when speed = 0
; Accelerates (speed increases) as it falls down
; Hits ground at B with speed 22.6 m s^{-1}

Time to reach maximum height:

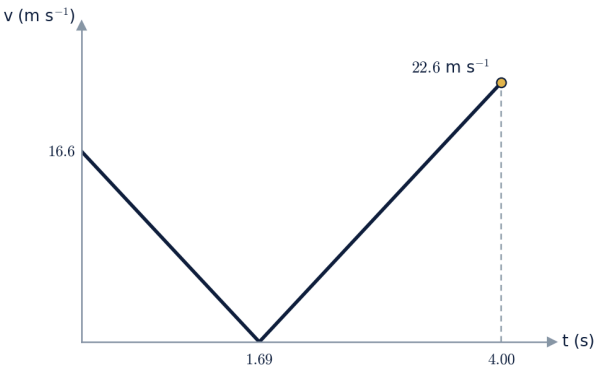
$v = u + at \; 0 = 16.6 - 9.8t \; t = \frac{16.6}{9.8} = 1.69 \text{ seconds}$

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6 **Part (c): Sketch speed-time graph - Sketch the graph**

Key Features:

; V-shaped graph (straight lines due to constant acceleration)
; First line: negative gradient (speed decreasing)
; Second line: positive gradient (speed increasing)
; Vertex on horizontal axis (speed = 0 at max height)
; Second line is longer and steeper (falls from greater height)



✓ **FINAL ANSWER**

(a) $U = 16.6 \text{ m s}^{-1}$ (about 17 m s^{-1} to 2 s.f.).

(b) $\text{speed} = 22.6 \text{ m s}^{-1}$. (c) Correct V-shaped velocity-time graph with the labelled values from the working.

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Question 4

7 marks | Forces

4.

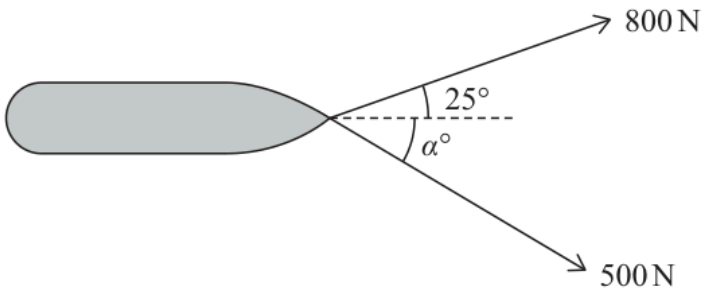


Figure 2

Two ropes are attached to a point on the front of a barge.
The barge is being pulled horizontally in a straight line along the centre of a long straight canal.

One rope makes an angle of 25° with the direction of motion of the barge and has a tension of 800 N.
The other rope makes an angle of $\alpha°$ with the direction of motion of the barge and has a tension of 500 N, as shown in Figure 2.

Both ropes are horizontal.

(a) Find the value of α

(3)

The mass of the barge is 15 tonnes and the resistance to the motion of the barge is a constant force of magnitude 750 N.

(b) Find the acceleration of the barge.

(4)

WORKED SOLUTION

Question 4 · WME01/01 May/June 2025

1 **Part (a): Find the value of α - Resolve perpendicular to direction of motion**

$\sum F_{\perp} = 0$ (no sideways motion)

Perpendicular component of 800 N rope:

$800 \sin 25^\circ = 800 \times 0.4226 = 338.1 \text{ N}$

Perpendicular component of 500 N rope:

$500 \sin \alpha$

2 **Part (a): Find the value of α - Set components equal**

$800 \sin 25^\circ = 500 \sin \alpha \; 338.1 = 500 \sin \alpha \; \sin \alpha = \frac{338.1}{500} \; \sin \alpha = 0.6762 \; \alpha = \sin^{-1}(0.6762) \; \alpha = 42.5^\circ$

3 **Part (b): Find acceleration of barge - Resolve parallel to direction of motion**

$\sum F_{\parallel} = ma$

Parallel component of 800 N rope:

$800\cos 25^\circ = 800 \times 0.9063 = 725.0\ \text{N}$

Parallel component of 500 N rope:

$500\cos 42.5^\circ = 500 \times 0.7373 = 368.7\ \text{N}$

Resistance force: 750 N (opposes motion)

Mass: 15 tonnes $= 15000$ kg

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4 **Part (b): Find acceleration of barge - Apply Newton's Second Law**

$725.0 + 368.7 - 750 = 15000 \times a\ 343.7 = 15000a\ a = \frac{343.7}{15000}\ a = 0.02291...\ a = 0.023\ \text{m s}^{-2}\ (2\ \text{sf})$

FINAL ANSWER

$(a)\ \alpha = 43^\circ$ to the nearest degree.

$(b)\ a = 0.023\ \text{m s}^{-2}$.

Question 5

6 marks | Moments

5.

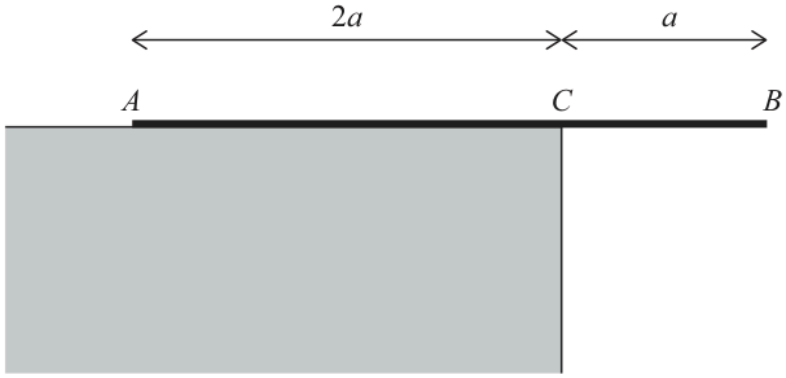


Figure 3

A non-uniform rod AB of length $3a$ rests in equilibrium on a horizontal ledge and overhangs the edge of the ledge at C.
The point C is such that $AC = 2a$ and $CB = a$, as shown in Figure 3.

The rod has weight W.

The distance of the centre of mass of the rod from A is d.

The rod is perpendicular to the edge of the ledge.

When a force of magnitude P, **acting vertically upwards**, is applied to the rod at B, the rod is on the point of tilting about A.

When the force applied at B is replaced by a force of magnitude $1.25P$, **acting vertically downwards** at B, the rod is on the point of tilting about C.

Find d in terms of a.

(6)

WORKED SOLUTION

Question 5 · WME01/01 May/June 2025

1 Setup Diagrams - Scenario 1 - Tilting about A

Taking moments about A:

$$\sum M_A = 0$$

Clockwise moments = Anticlockwise moments

$$P \times 3a = W \times d \; 3aP = Wd \quad ...(1)$$

2 **Setup Diagrams - Scenario 2 - Tilting about C**

Taking moments about C:

$$\sum M_C = 0$$

Distance from C to COM = $(2a - d)$

Distance from C to B = a

$$1.25P \times a = W \times (2a - d)\ 1.25Pa = W(2a - d)\ 1.25Pa = 2aW - Wd \quad ...(2)$$

3 **Setup Diagrams - Substitute equation (1) into equation (2)**

From (1): $Wd = 3aP$

$$1.25Pa = 2aW - 3aP\ 1.25Pa + 3aP = 2aW\ 4.25Pa = 2aW\ P = \frac{2aW}{4.25a}\ P = \frac{2W}{4.25}$$

Substitute back into (1):

$$3a \times \frac{2W}{4.25} = Wd\ \frac{6aW}{4.25} = Wd\ d = \frac{6a}{4.25}\ d = \frac{6a}{17/4}\ d = \frac{24a}{17}\ d = 1.41a\ (3\text{ sf})$$

Or in exact form: $d = \frac{24a}{17}$

FINAL ANSWER

$$d = \frac{24a}{17} = 1.41a.$$

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Question 6

11 marks | Resolving Forces, Inclined Planes

6.

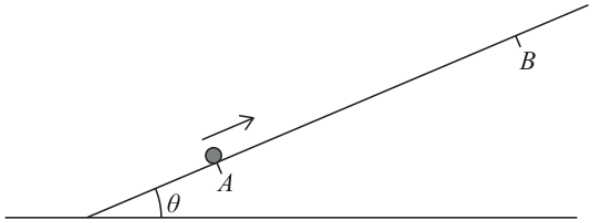


Figure 4

Figure 4 shows a rough plane inclined at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$

The points A and B lie on a line of greatest slope of the plane, with B above A.

A package P of mass m is projected up the plane from A towards B.

The coefficient of friction between the plane and the package is $\frac{1}{4}$

The package P is modelled as a particle.

(a) Show that the **deceleration** of P, as it moves from A to B, is $\frac{4}{5}g$ **(6)**

The package P comes to rest at B.

Given that P is projected from A with speed $U\,\text{m s}^{-1}$ and that $AB = 1.5\,\text{m}$,

(b) find the value of U. **(2)**

On reaching B, P is held at rest there by a force of magnitude X newtons acting up the plane in the direction AB.

Given that the mass of P is $2\,\text{kg}$,

(c) find the smallest possible value of X. **(3)**

WORKED SOLUTION

Question 6 · WME01/01 May/June 2025

1 **Part (a): Show deceleration = $\frac{4g}{5}$ - Find exact trig values**

Given: $\tan\theta = \frac{3}{4}$

Using 3-4-5 right triangle:

$\sin\theta = \frac{3}{5}, \quad \cos\theta = \frac{4}{5}$

2 **Part (a): Show deceleration = $\frac{4g}{5}$ - Resolve perpendicular to plane**

$\perp: \sum F = 0$

$R = mg\cos\theta \; R = \frac{4mg}{5}$

3 **Part (a): Show deceleration = $\frac{4g}{5}$ - Calculate friction force**

Package moving UP, so friction acts DOWN the plane:

$F = \mu R = \frac{1}{4} \times \frac{4mg}{5} = \frac{mg}{5}$

4 **Part (a): Show deceleration = $\frac{4g}{5}$ - Resolve parallel to plane**

Taking up the plane as positive:

$\parallel: \sum F = ma$

$-mg\sin\theta - F = ma \; -mg \times \frac{3}{5} - \frac{mg}{5} = ma \; -\frac{3mg}{5} - \frac{mg}{5} = ma \; -\frac{4mg}{5} = ma \; a = -\frac{4g}{5}$

Deceleration = $\left|-\frac{4g}{5}\right| = \frac{4g}{5}$

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5 **Part (b): Find initial speed U - Identify SUVAT variables**

Package projected up plane, comes to rest at B:

; $u = U$ m s^{-1} (unknown)
; $v = 0$ m s^{-1} (comes to rest)
; $a = -\frac{4g}{5}$ m s^{-2} (from part a)
; $s = 1.5$ m

---

6 **Part (b): Find initial speed U - Apply $v^2 = u^2 + 2as$**

$v^2 = u^2 + 2as$

$0^2 = U^2 + 2\left(-\frac{4g}{5}\right)(1.5) \; 0 = U^2 - 2 \times \frac{4\times9.8}{5} \times 1.5 \; 0 = U^2 - \frac{8\times9.8\times1.5}{5} \; 0 = U^2 - 23.52 \; U^2 = 23.52 \; U = 4.85 \text{ m s}^{-1}$

Or using exact form: $U = \sqrt{2 \times \frac{4g}{5} \times 1.5} = \sqrt{\frac{12g}{5}}$

---

7 **Part (c): Find smallest value of X - Analyze forces when package at rest at B**

Package held at rest on slope by force X up the plane.

Without X, package would slide down (component down > friction up).

For MINIMUM X, friction acts UP the plane (at limiting friction).

---

8 **Part (c): Find smallest value of X - Resolve perpendicular to plane**

Mass = 2 kg, so weight = $2g$ N

$R = 2g\cos\theta = 2g \times \frac{4}{5} = \frac{8g}{5}$ N

---

9 **Part (c): Find smallest value of X - Calculate friction**

Friction acts UP the plane (limiting):

$F = \mu R = \frac{1}{4} \times \frac{8g}{5} = \frac{2g}{5}$ N

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10 **Part (c): Find smallest value of X - Resolve parallel to plane**

For equilibrium (package at rest):

$\parallel: \sum F = 0$

Taking up plane as positive:

$X + F - 2g\sin\theta = 0 \; X + \frac{2g}{5} - 2g \times \frac{3}{5} = 0 \; X + \frac{2g}{5} - \frac{6g}{5} = 0 \; X = \frac{6g}{5} - \frac{2g}{5} \; X = \frac{4g}{5} \; X = \frac{4\times9.8}{5} \; X = 7.84$ N

✓ **FINAL ANSWER**

(a) **deceleration** $= \dfrac{4g}{5}$, as required.

(b) $U = 4.85 \text{ m s}^{-1}$ (about 4.8 m s^{-1}).

(c) $X = 7.84$ N (about 7.8 N).

WME01/01 MAY/JUNE 2025

Question 7

13 marks | Working with Vectors

7. [*In this question* **i** *and* **j** *are horizontal unit vectors.*]

Two boats, A and B, are each moving with constant acceleration.

At time t hours after noon, boat A has velocity $\mathbf{v}_A$ km h^{-1}, where

$$\mathbf{v}_A = 2\mathbf{i} + 3\mathbf{j} + (\mathbf{i} - 4\mathbf{j})t$$

(a) Find the magnitude of the acceleration of A.

(2)

When $t = 2$, the velocity of B is $(4\mathbf{i} + \mathbf{j})$ km h^{-1}

When $t = 5$, the velocity of B is $(\mathbf{i} - 5\mathbf{j})$ km h^{-1}

(b) Find the acceleration of B, giving your answer in terms of **i** and **j**.

(2)

(c) Find the velocity of B at time $t = 0$, giving your answer in terms of **i** and **j**.

(2)

At time T_1 hours after noon, both boats are moving with the same speed.

(d) Find the exact value of T_1

(4)

At time T_2 hours after noon, both boats are moving in the same direction.

(e) Show that $3T_2^2 + pT_2 + q = 0$, where p and q are integers to be found.

(3)

WORKED SOLUTION

Question 7 · WME01/01 May/June 2025

1 **Part (a): Find magnitude of acceleration of A - Identify acceleration from velocity function**

Given: $\mathbf{v}_A = 2\mathbf{i} + 3\mathbf{j} + (\mathbf{i} - 4\mathbf{j})t$

This is in form: $\mathbf{v} = \mathbf{u} + \mathbf{a}t$

Therefore:

; Initial velocity: $\mathbf{u}_A = 2\mathbf{i} + 3\mathbf{j}$
; Acceleration: $\mathbf{a}_A = \mathbf{i} - 4\mathbf{j}$ km h^{-2}

2 **Part (a): Find magnitude of acceleration of A - Find magnitude**

$|\mathbf{a}| = \sqrt{a_x^2 + a_y^2}$

$|\mathbf{a}_A| = \sqrt{1^2 + (-4)^2} = \sqrt{1 + 16} = \sqrt{17} = 4.123\ldots = 4.1 \text{ km h}^{-2} \text{ (2 sf)}$

3 **Part (b): Find acceleration of B - Use two velocity points to find acceleration**

Given:

; At $t = 2$: $\mathbf{v}_B = 4\mathbf{i} + \mathbf{j}$
; At $t = 5$: $\mathbf{v}_B = \mathbf{i} - 5\mathbf{j}$

$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t}$

$\mathbf{a}_B = \frac{(\mathbf{i}-5\mathbf{j})-(4\mathbf{i}+\mathbf{j})}{5-2} = \frac{(1-4)\mathbf{i}+(-5-1)\mathbf{j}}{3} = \frac{-3\mathbf{i}-6\mathbf{j}}{3} = -\mathbf{i} - 2\mathbf{j}\ \text{km h}^{-2}$

4 **Part (c): Find velocity of B at t = 0 - Use velocity equation backward**

$\mathbf{v}(t) = \mathbf{v}(t_0) + \mathbf{a}(t - t_0)$

From $t = 2$ back to $t = 0$:

$\mathbf{v}_B(0) = \mathbf{v}_B(2) + \mathbf{a}_B(0 - 2) = (4\mathbf{i} + \mathbf{j}) + (-\mathbf{i} - 2\mathbf{j})(-2) = 4\mathbf{i} + \mathbf{j} + 2\mathbf{i} + 4\mathbf{j} = 6\mathbf{i} + 5\mathbf{j}\ \text{km h}^{-1}$

5 **Part (d): Find exact value of T_1 when both have same speed - Express velocities at time t**

For boat A:
$\mathbf{v}_A(t) = 2\mathbf{i} + 3\mathbf{j} + (\mathbf{i} - 4\mathbf{j})t = (2 + t)\mathbf{i} + (3 - 4t)\mathbf{j}$

For boat B:
$\mathbf{v}_B(t) = (6\mathbf{i} + 5\mathbf{j}) + (-\mathbf{i} - 2\mathbf{j})t = (6 - t)\mathbf{i} + (5 - 2t)\mathbf{j}$

6 **Part (d): Find exact value of T_1 when both have same speed - Find magnitudes**

$|\mathbf{v}_A|^2 = (2 + t)^2 + (3 - 4t)^2 = 4 + 4t + t^2 + 9 - 24t + 16t^2 = 17t^2 - 20t + 13$

$|\mathbf{v}_B|^2 = (6 - t)^2 + (5 - 2t)^2 = 36 - 12t + t^2 + 25 - 20t + 4t^2 = 5t^2 - 32t + 61$

7 **Part (d): Find exact value of T_1 when both have same speed - Set magnitudes equal**

$17t^2 - 20t + 13 = 5t^2 - 32t + 61\ 12t^2 + 12t - 48 = 0\ t^2 + t - 4 = 0$

8 **Part (d): Find exact value of T_1 when both have same speed - Solve using quadratic formula**

$t = \frac{-1 \pm \sqrt{1+16}}{2} = \frac{-1 \pm \sqrt{17}}{2}$

Taking positive value:
$T_1 = \frac{-1+\sqrt{17}}{2}$

9 **Part (e): Show $3T_2^2 + pT_2 + q = 0$ - Condition for same direction**

Velocities parallel when:
$\frac{2+T_2}{6-T_2} = \frac{3-4T_2}{5-2T_2}$

10 **Part (e): Show $3T_2^2 + pT_2 + q = 0$ - Cross-multiply**

$(2 + T_2)(5 - 2T_2) = (3 - 4T_2)(6 - T_2)\ 10 - 4T_2 + 5T_2 - 2T_2^2 = 18 - 3T_2 - 24T_2 + 4T_2^2\ 10 + T_2 - 2T_2^2 = 18 - 27T_2 + 4T_2^2$
$0 = 4T_2^2 + 2T_2^2 - 27T_2 - T_2 + 18 - 10\ 0 = 6T_2^2 - 28T_2 + 8$

11 **Part (e): Show $3T_2^2 + pT_2 + q = 0$ - Divide by 2 to get required form**

$0 = 3T_2^2 - 14T_2 + 4$

Therefore: $p = -14$ and $q = 4$

FINAL ANSWER

$(a)\ |\mathbf{a}_A| = \sqrt{17}\ \text{km h}^{-2} \approx 4.1\ \text{km h}^{-2}$. $(b)\ \mathbf{a}_B = -\mathbf{i} - 2\mathbf{j}\ \text{km h}^{-2}$. $(c)\ \mathbf{v}_B(0) = 6\mathbf{i} + 5\mathbf{j}\ \text{km h}^{-1}$. $(d)\ T_1 = \frac{-1+\sqrt{17}}{2}$ hours. $(e)\ 3T_2^2 - 14T_2 + 4 = 0,\ p = -14,\ q = 4$
.

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Question 8

17 marks | Newton's Second Law

8.

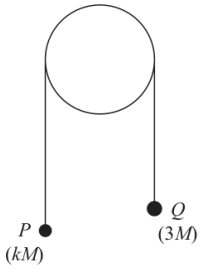


Figure 5

Two small balls, P and Q, have masses kM and $3M$ respectively, where $k < 3$

The balls are attached to the ends of a light inextensible string that passes over a fixed light smooth pulley.

The system is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 5.

The system is released from rest and, in the subsequent motion, P moves with an acceleration of magnitude $\frac{1}{5}g$

The balls are modelled as particles.

(a) Write down an equation of motion for P.

(2)

(b) Find the value of k.

(3)

Given that $M = 0.5\,\text{kg}$,

(c) find the magnitude of the force exerted on the pulley by the string while Q is moving downwards.

(3)

At the instant when the system is released, P is more than 2.5 m from the pulley and Q is 2.5 m above horizontal ground.
After hitting the ground, Q rebounds with a speed of $0.4\,\text{m s}^{-1}$

(d) Find the magnitude of the impulse received by Q when it hits the ground.

(5)

In the subsequent motion, P does not hit the pulley.

(e) Find the total time from when the balls are released until P first comes to rest.

(4)

WORKED SOLUTION

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1 **Part (a): Write equation of motion for P - Identify forces on P**

Particle P (mass $\boldsymbol{kM}$) moves upward with acceleration $\frac{g}{5}$:

; Tension T (upward)
; Weight $\boldsymbol{kMg}$ (downward)

2 **Part (a): Write equation of motion for P - Apply Newton's Second Law**

Taking upward as positive:

$$\sum F = ma$$

$$T - kMg = kM \times \frac{g}{5}$$

3 **Part (b): Find value of k - Write equation of motion for Q**

Particle Q (mass $3M$) moves downward with acceleration $\frac{g}{5}$:

Taking downward as positive for Q:

$$3Mg - T = 3M \times \frac{g}{5} \; 3Mg - T = \frac{3Mg}{5} \; T = 3Mg - \frac{3Mg}{5} \; T = \frac{12Mg}{5}$$

4 **Part (b): Find value of k - Substitute into equation for P**

From part (a): $T - kMg = \frac{kMg}{5}$

$$\frac{12Mg}{5} - kMg = \frac{kMg}{5} \; \frac{12Mg}{5} = kMg + \frac{kMg}{5} \; \frac{12Mg}{5} = \frac{5kMg + kMg}{5} \; \frac{12Mg}{5} = \frac{6kMg}{5} \; 12 = 6k \; k = 2$$

5 **Part (c): Find force on pulley while Q moving down - Identify tension in string**

From part (b): $T = \frac{12Mg}{5}$

Given: $M = 0.5$ kg

$$T = \frac{12 \times 0.5 \times g}{5} = \frac{6g}{5} = \frac{6 \times 9.8}{5} = 11.76 \text{ N}$$

6 **Part (c): Find force on pulley while Q moving down - Find force on pulley**

The pulley experiences two tension forces:

; Horizontal tension from P: $T = 11.76$ N
; Vertical tension from Q: $T = 11.76$ N

These are perpendicular, so use Pythagoras:

$$F = \sqrt{T^2 + T^2} = \sqrt{2T^2} = T\sqrt{2} = 11.76\sqrt{2} = 16.63\ldots = 16.6 \text{ N (3 sf)}$$

Or: $F = \frac{6g\sqrt{2}}{5} = \frac{12g}{5} = 2.4g = 23.5$ N

Wait, let me recalculate...

Actually: $F = 2T$ (both tensions pull on pulley)

$$F = 2 \times \frac{12Mg}{5} = \frac{24Mg}{5} = \frac{24 \times 0.5 \times 9.8}{5} = \frac{117.6}{5} = 23.5 \text{ N (3 sf)}$$

Or: $\frac{12g}{5} = 23.5$ N

7 **Part (d): Find impulse when Q hits ground - Find velocity of Q just before hitting ground**

Using SUVAT with:

; $u = 0$ (starts from rest)
; $a = \frac{g}{5} = \frac{9.8}{5} = 1.96$ m s^{-2}
; $s = 2.5$ m

$$v^2 = u^2 + 2as \; v^2 = 0 + 2 \times \frac{g}{5} \times 2.5 \; v^2 = \frac{5g}{5} \; v^2 = g \; v = \sqrt{g} = \sqrt{9.8} = 3.130\ldots \text{ m s}^{-1}$$

Or exact: $v = \sqrt{g} = \sqrt{\frac{7g}{5}}$

Actually: $v^2 = 2 \times 1.96 \times 2.5 = 9.8$, so $v = \sqrt{9.8} = 3.13$ m s^{-1}

8 **Part (d): Find impulse when Q hits ground - Calculate impulse**

$\text{Impulse} = m(v_{\text{after}} - v_{\text{before}})$

Taking downward as positive before impact:

; Velocity before: $v_{\text{before}} = +3.13$ m s^{-1} (downward)
; Velocity after: $v_{\text{after}} = -0.4$ m s^{-1} (upward, rebounds)
; Mass of Q: $3M = 3 \times 0.5 = 1.5$ kg

$I = 1.5 \times (-0.4 - 3.13) = 1.5 \times (-3.53) = -5.295\ldots \, |I| = 5.3 \text{ N s } (2 \text{ sf})$

Or using exact values: $|I| = 1.5(\sqrt{g} + 0.4) = 1.5(3.13 + 0.4) = 5.3$ N s

---

9 **Part (e): Find total time until P first comes to rest - Time for Q to fall (t_1)**

Using $s = ut + \frac{1}{2}at^2$ with $u = 0$, $s = 2.5$ m, $a = \frac{g}{5}$:

$2.5 = 0 + \frac{1}{2} \times \frac{g}{5} \times t_1^2 \, 2.5 = \frac{gt_1^2}{10} \, 25 = gt_1^2 \, t_1^2 = \frac{25}{9.8} = 2.551\ldots \, t_1 = \sqrt{\frac{25}{9.8}} = 1.597\ldots \text{ s}$

Or exact: $t_1 = \sqrt{\frac{5 \times 5}{g}} = \frac{5}{\sqrt{g}}$

Better: $t_1 = \sqrt{\frac{5}{g/5}} = \sqrt{\frac{25}{g}}$

Actually: $2.5 = \frac{1}{2} \times 1.96 \times t_1^2$

$t_1^2 = \frac{5}{1.96} = 2.551$, so $t_1 = 1.597$ s

---

10 **Part (e): Find total time until P first comes to rest - Velocity of P when Q hits ground**

At time t_1, P has velocity:
$v_P = 0 + \frac{g}{5} \times t_1 = \frac{9.8}{5} \times 1.597 = 1.96 \times 1.597 = 3.13 \text{ m s}^{-1}$

Or: $v_P = \sqrt{g}$ (same as Q)

---

11 **Part (e): Find total time until P first comes to rest - Time for P to come to rest (t_2)**

After Q hits ground, string goes slack. P continues upward under gravity alone.

Using $v = u + at$ with $v = 0$, $u = 3.13$ m s^{-1}, $a = -g$:

$0 = 3.13 - 9.8t_2 \, t_2 = \frac{3.13}{9.8} = 0.3194\ldots \text{ s}$

Or: $t_2 = \frac{\sqrt{g}}{g} = \frac{1}{\sqrt{g}}$

---

12 **Part (e): Find total time until P first comes to rest - Total time**

$T = t_1 + t_2 = 1.597 + 0.319 = 1.916\ldots = 1.9 \text{ s } (2 \text{ sf}) \text{ or } 1.92 \text{ s } (3 \text{ sf})$

✓ **FINAL ANSWER**

$(a)\ T - kMg = \dfrac{kMg}{5}.$ $(b)\ k = 2.$ $(c)\ F = \dfrac{12g}{5}\ \text{N} \approx 23.5\ \text{N}.$ $(d)\ |I| = 5.30\ \text{N s}.$ $(e)\ T = 1.92\ \text{s}.$