Elite Mathematics
Edexcel International A Level

IAL MECHANICS 1 · WME01

WME01/01 October 2024 Worked Solutions

October 2024 paper rebuilt with the worked solution after each question.

7	2019–2026	Single-paper solutions
paper questions	18 exam sessions	checked by Dr Eslam



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PAST PAPER

WME01/01 October 2024

October 2024

7	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

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Question 1

6 marks | Momentum, Impulse & Collisions

1. Particle A has mass $4m$ and particle B has mass $3m$.

The particles are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.

Immediately **before** the collision, the speed of A is $2x$ and the speed of B is x.

Immediately **after** the collision, the speed of A is y and the speed of B is $5y$.

The direction of motion of each particle is reversed as a result of the collision.

(a) Show that $y = \frac{5}{11}x$.

(3)

(b) Find, in terms of m and x, the magnitude of the impulse received by A in the collision.

(3)

WORKED SOLUTION

Question 1 · WME01/01 October 2024

1 Choose a sign convention

Take the initial direction of A as positive. Before collision, A has velocity $2x$ and B has velocity $-x$. After collision, both reverse direction, so A has velocity $-y$ and B has velocity $5y$.

2 Apply conservation of momentum

$4m(2x) + 3m(-x) = 4m(-y) + 3m(5y)$. The left side is $5mx$ and the right side is $11my$.

3 Show the ratio

$5mx = 11my$, so $\frac{y}{x} = \frac{5}{11}$.

4 Use change in momentum for A

The impulse received by A is the change in momentum of A: $I = 4m|-y-2x| = 4m(y+2x)$.

5 Substitute y

Using $y = \frac{5x}{11}$, $I = 4m\left(\frac{5x}{11} + 2x\right) = 4m\left(\frac{27x}{11}\right) = \frac{108}{11}mx$.

FINAL ANSWER

$(a)\ \frac{y}{x} = \frac{5}{11}$. $(b)\ I = \frac{108}{11}mx$.

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Question 2

9 marks | Moments

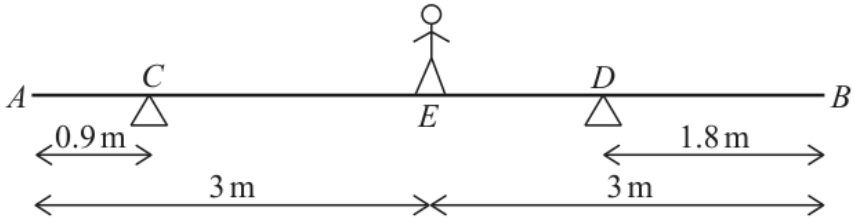


Figure 1

A non-uniform beam AB has length 6 m and mass 50 kg. The beam rests horizontally on two supports at C and D, where $AC = 0.9$ m and $DB = 1.8$ m.

A child of mass 25 kg stands on the beam at E, where $AE = EB = 3$ m, as shown in Figure 1.

The beam is in equilibrium.

The magnitude of the normal reaction between the beam and the support at C is R_C newtons.

The magnitude of the normal reaction between the beam and the support at D is R_D newtons.

The beam is modelled as a rod and the child is modelled as a particle.

The centre of mass of the beam is between C and D and is a distance x metres from D.

Given that $2R_D = 3R_C$

(a) show that $x = 1.38$

(6)

The child remains at E and a block of mass M kg is placed on the beam at B.

The block is modelled as a particle.

Given that the beam is on the point of tilting,

(b) find the value of M.

(3)

WORKED SOLUTION

Question 2 · WME01/01 October 2024

1 **Find the reactions from vertical equilibrium**

Vertically, $R_C + R_D = 50g + 25g = 75g$. Given $2R_D = 3R_C$, we have $R_D = 1.5R_C$. Therefore $2.5R_C = 75g$, so $R_C = 30g$ and $R_D = 45g$.

2 **Set the distances from D**

D is $6 - 1.8 = 4.2$ m from A. The support C is $4.2 - 0.9 = 3.3$ m to the left of D, the child at E is $4.2 - 3 = 1.2$ m to the left of D, and the centre of mass is x m to the left of D.

3 **Take moments about D**

Balancing moments about D gives $50gx + 25g(1.2) = R_C(3.3)$.

4 **Show x is 1.38**

Substitute $R_C = 30g$: $50x + 30 = 99$, so $50x = 69$ and $x = \dfrac{69}{50} = 1.38$.

5 **Use the tilting condition with the block**

When the block at B is on the point of making the beam tilt, the beam is about to pivot about D and the reaction at C is zero.

6 **Take moments about D for the block**

The child and the beam act to the left of D, and the block at B acts 1.8 m to the right of D. Hence $25g(1.2) + 50g(1.38) = Mg(1.8)$.

7 **Find M**

$30 + 69 = 1.8M$, so $M = \dfrac{99}{1.8} = 55$.

✓ **FINAL ANSWER**

$(a)\ x = 1.38.\quad (b)\ M = 55.$

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Question 3

11 marks | Working with Vectors

3. [*In this question* **i** *and* **j** *are horizontal unit vectors and position vectors are given relative to a fixed origin.*]

A ship A is moving with constant velocity.

At 1 pm, the position vector of A is $(25\mathbf{i} + 10\mathbf{j})$ km.

At 3 pm, the position vector of A is $(55\mathbf{i} + 34\mathbf{j})$ km.

At time t hours after 1 pm, the position vector of A is $\mathbf{r}_A$ km.

(a) Show that $\mathbf{r}_A = (25 + 15t)\mathbf{i} + (10 + 12t)\mathbf{j}$

(4)

The speed of A is V m s^{-1}

(b) Find the value of V.

(2)

A ship B is moving with constant velocity $(20\mathbf{i} - 6\mathbf{j})$ km h^{-1}

At 1 pm, the position vector of B is $(35\mathbf{i} + 51\mathbf{j})$ km.

At 2:30 pm, B passes through the point P.

(c) Show that A also passes through P.

(5)

WORKED SOLUTION

Question 3 · WME01/01 October 2024

1 Find the velocity of A

From 1 pm to 3 pm is 2 hours. The displacement of A is $(55\mathbf{i} + 34\mathbf{j}) - (25\mathbf{i} + 10\mathbf{j}) = 30\mathbf{i} + 24\mathbf{j}$ km, so $\mathbf{v}_A = 15\mathbf{i} + 12\mathbf{j}\ \text{km h}^{-1}$.

2 Write the position vector of A

At time t hours after 1 pm, $\mathbf{r}_A = (25\mathbf{i} + 10\mathbf{j}) + t(15\mathbf{i} + 12\mathbf{j}) = (25 + 15t)\mathbf{i} + (10 + 12t)\mathbf{j}$.

3 Convert the speed to metres per second

The speed of A is $\sqrt{15^2 + 12^2} = \sqrt{369}\ \text{km h}^{-1}$. Converting to m s^{-1},

$V = \sqrt{369} \times \dfrac{1000}{3600} = \dfrac{5\sqrt{41}}{6}\ \text{m s}^{-1}$.

4 Find the point P reached by B

At 2:30 pm, $t = 1.5$ hours after 1 pm. For B, $\mathbf{r}_B = (35\mathbf{i} + 51\mathbf{j}) + 1.5(20\mathbf{i} - 6\mathbf{j}) = 65\mathbf{i} + 42\mathbf{j}$.

5 **Check whether A reaches P**

Set $\mathbf{r}_A = 65\mathbf{i} + 42\mathbf{j}$. From the i-component, $25 + 15t = 65$, so $t = \dfrac{8}{3}$.

6 **Confirm with the j-component**

When $t = \dfrac{8}{3}$, $10 + 12t = 10 + 32 = 42$. Both components match, so A also passes through P.

✓ **FINAL ANSWER**

$(a)\ \mathbf{r}_A = (25 + 15t)\mathbf{i} + (10 + 12t)\mathbf{j}.$ $(b)\ V = \dfrac{5\sqrt{41}}{6}\ \mathrm{m\,s^{-1}} \approx 5.3\ \mathrm{m\,s^{-1}}.$ (c) A also passes through P.

Question 4

13 marks | Kinematics Graphs

4. The points A and B lie on the same straight horizontal road.

Figure 2, on page 11, shows the speed-time graph of a cyclist P, for his journey from A to B.
At time $t = 0$, P starts from rest at A and accelerates uniformly for 9 seconds until his speed is $V\,\text{m s}^{-1}$

He then travels at constant speed $V\,\text{m s}^{-1}$

When $t = 42$, cyclist P passes B.

Given that the distance AB is 120 m,

(a) show that $V = 3.2$

(3)

(b) Find the acceleration of cyclist P between $t = 0$ and $t = 9$

(2)

Cyclist P continues to cycle along the road in the same direction at the same constant speed, $V\,\text{m s}^{-1}$

When $t = 6$, a second cyclist Q sets off from A and travels in the same direction as P along the same road. She accelerates for T seconds until her speed is $3.6\,\text{m s}^{-1}$

She then travels at constant speed $3.6\,\text{m s}^{-1}$

Cyclist Q catches up with P when $t = 54$

(c) On Figure 2, on page 11, sketch a speed-time graph showing the journeys of **both** cyclists, for the interval $0 \leqslant t \leqslant 54$

(3)

(d) Find the value of T

(5)

Question 4 continued

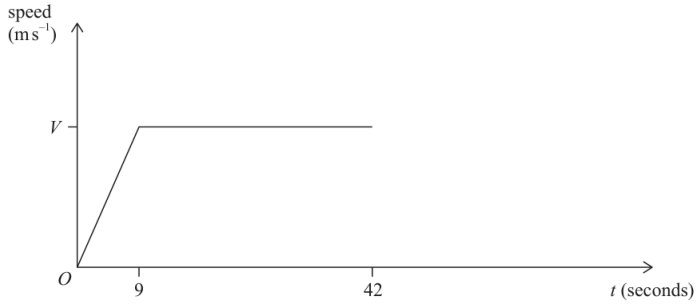
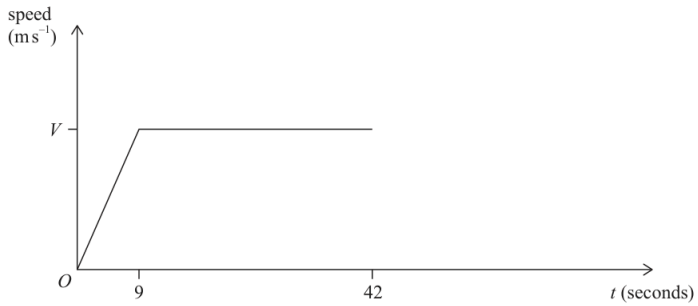


Figure 2

A copy of Figure 2 is on page 13 if you need to redraw your answer to part (c).

Question 4 continued

Only use this copy of Figure 2 if you need to redraw your answer to part (c).



Copy of Figure 2

1 **Use the area under P graph**

For cyclist P, the distance from A to B is the area under the speed-time graph from 0 to 42. This is $\frac{1}{2}(9)V + 33V = 120$.

2 **Show V is 3.2**

$4.5V + 33V = 37.5V = 120$, so $V = 3.2\ \mathrm{m\,s^{-1}}$.

3 **Find P acceleration**

Between $t = 0$ and $t = 9$, P accelerates from 0 to 3.2. Hence $a = \frac{3.2}{9} = \frac{16}{45} = 0.356\ldots\ \mathrm{m\,s^{-2}}$.

4 **Describe Q graph**

Cyclist Q starts at $t = 6$ from rest, accelerates in a straight line for T seconds to speed $3.6\ \mathrm{m\,s^{-1}}$, then continues horizontally at $3.6\ \mathrm{m\,s^{-1}}$ until $t = 54$.

5 **Find P distance by t equals 54**

By $t = 54$, P has travelled $\frac{1}{2}(9)(3.2) + 45(3.2) = 14.4 + 144 = 158.4\ \mathrm{m}$.

6 **Form Q distance in terms of T**

Q travels for 48 seconds after starting at $t = 6$. Her distance is $\frac{1}{2}T(3.6) + (48 - T)(3.6) = 172.8 - 1.8T$.

7 **Solve for T**

Q catches P at $t = 54$, so $172.8 - 1.8T = 158.4$. Thus $1.8T = 14.4$ and $T = 8$ s.

FINAL ANSWER

$(a)\ V = 3.2. \quad (b)\ a = \frac{16}{45}\ \mathrm{m\,s^{-2}} \approx 0.36\ \mathrm{m\,s^{-2}}. \quad (d)\ T = 8\ \mathrm{s}.$

Question 5

13 marks | Newton's Second Law

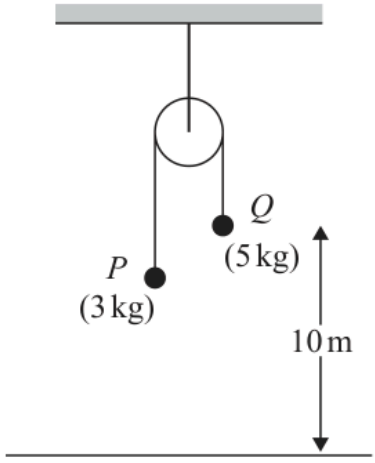


Diagram not to scale

Figure 3

Two particles, P and Q, have masses 3 kg and 5 kg respectively. The particles are connected by a light inextensible string which passes over a small smooth fixed pulley.

The particles are released from rest with the string taut and the hanging parts of the string vertical, as shown in Figure 3.

Immediately after the particles are released from rest, P moves upwards with acceleration $a\,\text{m s}^{-2}$ and the tension in the string is T newtons.

(a) Write down an equation of motion for P.

(2)

(b) Find the value of T.

(4)

The total force acting on the pulley due to the string has magnitude F newtons.

(c) Find the value of F.

(2)

Initially, Q is 10 m above horizontal ground and P is more than 2 m below the pulley.

At the instant when Q has descended a distance of 2 m, the string breaks and Q falls to the ground.

(d) Find the speed of Q at the instant it hits the ground.

(5)

1 Write the equation for P

P has mass 3 kg and moves upwards, so taking upwards as positive gives $T - 3g = 3a$.

2 Write the equation for Q

Q has mass 5 kg and moves downwards, so taking downwards as positive for Q gives $5g - T = 5a$.

3 Find acceleration and tension

Adding the two equations gives $2g = 8a$, so $a = \dfrac{g}{4}$. Substitute into $T - 3g = 3a$:

$T = 3g + 3\left(\dfrac{g}{4}\right) = \dfrac{15g}{4} = 36.8\ \text{N}$.

4 Find the force on the pulley

The pulley is pulled by two vertical tensions, each of magnitude T. Hence $F = 2T = \dfrac{15g}{2} = 73.5\ \text{N}$.

5 Find Q speed when the string breaks

Before the string breaks, Q descends 2 m from rest with acceleration $g/4$. Thus $v^2 = 2\left(\dfrac{g}{4}\right)(2) = g$, so $v = \sqrt{g}$.

6 Find the remaining fall

After the string breaks, Q is still 8 m above the ground and accelerates freely under gravity. Let the impact speed be w. Then $w^2 = v^2 + 2g(8) = g + 16g = 17g$.

7 Calculate the impact speed

$w = \sqrt{17g} = \sqrt{166.6} = 12.9\ \text{m s}^{-1}$ to 3 significant figures.

FINAL ANSWER

$(a)\ T - 3g = 3a.$ $(b)\ T = 36.8\ \text{N}.$ $(c)\ F = 73.5\ \text{N}.$ $(d)\ \text{speed} = 12.9\ \text{m s}^{-1}.$

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Question 6

12 marks | Resolving Forces, Inclined Planes

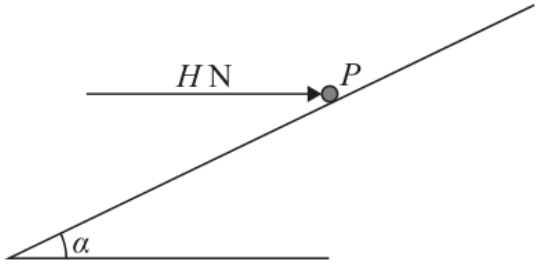


Figure 4

A particle P of mass 5 kg lies on the surface of a rough plane.

The plane is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$

The particle is held in equilibrium by a horizontal force of magnitude H newtons, as shown in Figure 4.

The horizontal force acts in a vertical plane containing a line of greatest slope of the inclined plane.

The coefficient of friction between the particle and the plane is $\frac{1}{4}$

(a) Find the smallest possible value of H.

(6)

The horizontal force is now removed, and P starts to slide down the slope.

In the first T seconds after P is released from rest, P slides 1.5 m down the slope.

(b) Find the value of T.

(6)

WORKED SOLUTION

Question 6 · WME01/01 October 2024

1 Use the triangle for alpha

From $\tan \alpha = \frac{3}{4}$, use $\sin \alpha = \frac{3}{5}$ and $\cos \alpha = \frac{4}{5}$.

2 Resolve perpendicular to the plane

The horizontal force has a component into the plane, so $R = 5g\cos\alpha + H\sin\alpha = 4g + \frac{3}{5}H$.

3 **Resolve parallel for the smallest H**

For the smallest H, the particle is on the point of sliding down, so friction acts up the plane. Parallel to the plane, $H\cos\alpha + F = 5g\sin\alpha$.

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4 **Use friction to find H**

With $F = \frac{1}{4}R$, the parallel equation becomes $\frac{4}{5}H + \frac{1}{4}\left(4g + \frac{3}{5}H\right) = 3g$. Hence $0.95H + g = 3g$, so $H = \frac{40g}{19} = 20.6\ \text{N}$.

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5 **Find the acceleration after H is removed**

When H is removed, $R = 5g\cos\alpha = 4g$ and friction is $\frac{1}{4}R = g$. The component of weight down the plane is $5g\sin\alpha = 3g$, so the resultant down the plane is $2g$.

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6 **Apply Newtons second law**

$2g = 5a$, so $a = \frac{2g}{5} = 3.92\ \text{m s}^{-2}$ down the plane.

---

7 **Find T from the displacement**

From rest, the particle moves 1.5 m with acceleration $2g/5$. Using $s = \frac{1}{2}at^2$: $1.5 = \frac{1}{2}\left(\frac{2g}{5}\right)T^2 = \frac{g}{5}T^2$. Thus $T = \sqrt{\frac{7.5}{9.8}} = 0.875\ \text{s}$.

✓ **FINAL ANSWER**

$(a)\ H = \frac{40g}{19}\ \text{N} \approx 20.6\ \text{N}.\quad (b)\ T = 0.875\ \text{s}.$

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Question 7

11 marks | Constant Acceleration in 1D

7 At time $t = 0$, a small ball A is projected vertically upwards with speed $8\,\text{m s}^{-1}$ from a fixed point on horizontal ground.
The ball hits the ground again for the first time at time $t = T_1$ seconds.

Ball A is modelled as a particle moving freely under gravity.

(a) Show that $T_1 = 1.63$ to 3 significant figures.

(2)

After the first impact with the ground, A rebounds to a height of 2 m above the ground.

Given that the mass of A is 0.1 kg,

(b) find the magnitude of the impulse received by A as a result of its first impact with the ground.

(5)

At time $t = 1$ second, another small ball B is projected vertically upwards from another point on the ground with speed $5\,\text{m s}^{-1}$

Ball B is modelled as a particle moving freely under gravity.

At time $t = T_2$ seconds ($T_2 > 1$), A and B are at the same height above the ground for the first time.

(c) Find the value of T_2

(4)

WORKED SOLUTION

Question 7 · WME01/01 October 2024

1 Find the first flight time

For ball A, take upward as positive. Returning to the ground gives $0 = 8T_1 - 4.9T_1^2$. The non-zero solution is $T_1 = \dfrac{8}{4.9} = 1.63\ \text{s}$ to 3 significant figures.

2 Find the rebound speed

After impact, A rebounds to a maximum height of 2 m. At the top, $v = 0$, so $0 = u^2 - 2g(2)$. Hence $u = \sqrt{4g} = 6.261\ \text{m s}^{-1}$.

3 Use impulse as change in momentum

Just before the first impact, A has velocity $-8\ \text{m s}^{-1}$. Just after impact, it has velocity $+6.261\ \text{m s}^{-1}$. With mass 0.1 kg, $I = 0.1[6.261 - (-8)]$.

4 **Calculate the impulse**

$I = 1.426\ldots\ \mathrm{N\,s}$, so the impulse is $1.43\ \mathrm{N\,s}$ to 3 significant figures.

5 **Write the height of A before first impact**

For the first meeting with B, A is still in its first flight. Its height at time T_2 is $h_A = 8T_2 - 4.9T_2^2$.

6 **Write the height of B**

B starts at time $t = 1$, so at time T_2 it has been moving for $T_2 - 1$ seconds. Its height is $h_B = 5(T_2 - 1) - 4.9(T_2 - 1)^2$.

7 **Set the heights equal**

$8T_2 - 4.9T_2^2 = 5(T_2 - 1) - 4.9(T_2 - 1)^2$. Expanding and simplifying gives $6.8T_2 = 9.9$.

8 **Find T2**

$T_2 = \dfrac{9.9}{6.8} = 1.456\ldots$, so $T_2 = 1.46\ \mathrm{s}$ to 3 significant figures.

✓ **FINAL ANSWER**

$(a)\ T_1 = 1.63\ \mathrm{s}.\quad (b)\ I = 1.43\ \mathrm{N\,s}.\quad (c)\ T_2 = 1.46\ \mathrm{s}.$