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Elite Mathematics
Edexcel International A Level

IAL MECHANICS 1 · WME01

WME01/01 May/June 2024 Worked Solutions

May/June 2024 paper rebuilt with the worked solution after each question.

8	2019–2026	Single-paper solutions
paper questions	18 exam sessions	checked by Dr Eslam



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PAST PAPER

WME01/01 May/June 2024

May/June 2024

8	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WME01/01 MAY/JUNE 2024

Question 1

5 marks | Momentum, Impulse & Collisions

1. Two particles, A and B, have masses m and $3m$ respectively. The particles are connected by a light inextensible string. Initially A and B are at rest on a smooth horizontal plane with the string slack.

Particle A is then projected along the plane away from B with speed U.

Given that the common speed of the particles immediately after the string becomes taut is S

(a) find S in terms of U.

(2)

(b) Find, in terms of m and U, the magnitude of the impulse exerted on A immediately after the string becomes taut.

(3)

WORKED SOLUTION

Question 1 · WME01/01 May/June 2024

1 Use conservation of momentum

When the string becomes taut, A and B move with the same speed S. Before this, only A is moving. Hence $mU = (m + 3m)S = 4mS$.

2 Find the common speed

From $mU = 4mS$, cancel m to get $S = \dfrac{U}{4}$.

3 Use impulse on A

The impulse on A equals the magnitude of its change in momentum: $I = m\left|\dfrac{U}{4} - U\right|$.

4 Simplify the impulse

$$I = m\left(\frac{3U}{4}\right) = \frac{3mU}{4}.$$

FINAL ANSWER

$(a)\ S = \dfrac{U}{4}.\quad (b)\ I = \dfrac{3mU}{4}.$

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Question 2

4 marks | Forces

2. Two forces, **P** and **Q**, act on a particle.

- **P** has magnitude 10 N and acts due west
- **Q** has magnitude 8 N and acts on a bearing of 330°

Given that $\mathbf{F} = \mathbf{P} + \mathbf{Q}$, find the magnitude of **F**.

(4)

WORKED SOLUTION

Question 2 · WME01/01 May/June 2024

1 Resolve the two forces

Take east as $\mathbf{i}$ and north as $\mathbf{j}$. The force due west is $-10\mathbf{i}$. A bearing of $330°$ is $30°$ west of north, so the 8 N force has components $-8\sin 30°\mathbf{i} + 8\cos 30°\mathbf{j}$.

2 Add the components

$\mathbf{F} = (-10 - 4)\mathbf{i} + 4\sqrt{3}\mathbf{j} = -14\mathbf{i} + 4\sqrt{3}\mathbf{j}$.

3 Find the magnitude

$|\mathbf{F}| = \sqrt{(-14)^2 + (4\sqrt{3})^2} = \sqrt{196 + 48} = \sqrt{244} = 2\sqrt{61}\ \text{N}$.

4 Give a decimal value

$2\sqrt{61} = 15.620\ldots$, so the magnitude is about $16\ \text{N}$ to 2 significant figures.

FINAL ANSWER

$|\mathbf{F}| = 2\sqrt{61}\ \text{N} \approx 15.6\ \text{N}$.

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Question 3

6 marks | Newton's Second Law

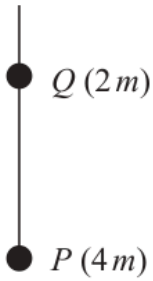


Figure 1

Two particles, P and Q, have masses $4m$ and $2m$ respectively. The particles are connected by a light inextensible string. A second light inextensible string has one end attached to Q. Both strings are taut and vertical, as shown in Figure 1.

The particles are **accelerating** vertically **downwards**.

Given that the tension in the string connecting the two particles is $3mg$, find, in terms of m and g, the tension in the upper string.

(6)

WORKED SOLUTION

Question 3 · WME01/01 May/June 2024

1 Use the lower particle to find acceleration

For P, take downward as positive. Its weight is $4mg$ and the tension in the string between P and Q is $3mg$ upward, so $4mg - 3mg = 4ma$.

2 Find the acceleration

The equation gives $mg = 4ma$, hence $a = \frac{g}{4}$ downward.

3 Write the equation for Q

For Q, downward forces are its weight $2mg$ and the lower string tension $3mg$. The upper string tension T acts upward. Taking downward as positive: $2mg + 3mg - T = 2ma$.

4 Substitute acceleration

$5mg - T = 2m\left(\frac{g}{4}\right) = \frac{mg}{2}$.

5 Find the upper tension

$T = 5mg - \frac{mg}{2} = \frac{9mg}{2}$.

FINAL ANSWER

$$T = \frac{9mg}{2} = 4.5mg.$$

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Question 4

6 marks | Moments

4.

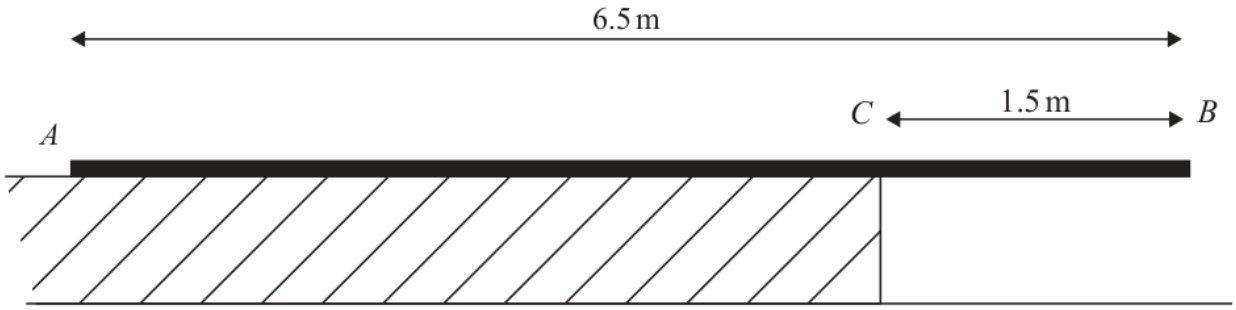


Figure 2

A non-uniform rod AB has length 6.5 m and mass 1.2 kg. The centre of mass of the rod is 3 m from A. The rod rests on a horizontal step and overhangs the end of the step C by 1.5 m, as shown in Figure 2.

The rod is perpendicular to the edge of the step.

A particle of mass 4 kg is placed on the rod at B and another particle, whose mass is M kg, is placed on the rod at D, where $AD = 0.5$ m.

The rod remains in equilibrium in a horizontal position.

(a) Find the smallest possible value of M.

(3)

The particle at B **and** the particle at D are now **removed**.

A new particle is placed on the rod at the point E, where $EB = 0.9$ m.

The rod remains in equilibrium in a horizontal position but is on the point of tilting about C.

(b) Find the magnitude of the force acting on the rod at C.

(3)

WORKED SOLUTION

Question 4 · WME01/01 May/June 2024

1 **Set the positions relative to C**

The rod is 6.5 m long and overhangs C by 1.5 m, so C is 5.0 m from A. The rod centre of mass is 3.0 m from A, so it is 2.0 m to the left of C. Point D is 0.5 m from A, so it is 4.5 m to the left of C.

2 **Use the limiting balance for smallest M**

For the smallest M, the rod is just prevented from tilting clockwise about C. Taking moments about C: $Mg(4.5) + 1.2g(2) = 4g(1.5)$.

3 **Find M**

$4.5M + 2.4 = 6$, so $4.5M = 3.6$ and $M = 0.8$.

4 **Place the new particle E**

Now the old particles are removed. Since $EB = 0.9$ m and $AB = 6.5$ m, E is 5.6 m from A, so E is 0.6 m to the right of C.

5 **Find the new particle mass from moments**

At tilting about C, moments about C give $Xg(0.6) = 1.2g(2)$, so $X = 4$.

6 **Find the force at C**

The upward force at C balances the rod and the new particle: $R_C = 1.2g + 4g = 5.2g\ \text{N}$, which is about $51\ \text{N}$.

✓ **FINAL ANSWER**

$(a)\ M = 0.8.\quad (b)\ R_C = 5.2g\ \text{N} \approx 51\ \text{N}.$

WME01/01 MAY/JUNE 2024

Question 5

14 marks | Constant Acceleration in 1D

5. A parachute is used to deliver a box of supplies. The parachute is attached to the box.

- the parachute and box are dropped from rest from a helicopter that is hovering at a height of 520 m above the ground
- the parachute and box fall vertically and freely under gravity for 5 seconds, then the parachute opens
- from the instant the parachute opens, it provides a resistance to motion of magnitude 3200 N
- the parachute and box continue to fall vertically downwards after the parachute opens
- the parachute and box are modelled throughout the motion as a particle P of mass 250 kg

(a) Find the distance fallen by P in the first 5 seconds.

(2)

(b) Find the speed with which P lands on the ground.

(7)

(c) Find the total time from the instant when P is dropped from the helicopter to the instant when P lands on the ground.

(3)

(d) Sketch a speed-time graph for the motion of P from the instant when P is dropped from the helicopter to the instant when P lands on the ground.

(2)

WORKED SOLUTION

Question 5 · WME01/01 May/June 2024

1 Find the first distance

For the first 5 seconds, P falls freely from rest. Using $s = ut + \frac{1}{2}at^2$, $s = 0 + \frac{1}{2}(9.8)(5^2) = 122.5\ \text{m}$.

2 Find the speed when the parachute opens

After 5 seconds of free fall, $v = 0 + 9.8(5) = 49\ \text{m s}^{-1}$ downward.

3 Find the new acceleration

After the parachute opens, weight is $250g = 2450$ N downward and resistance is 3200 N upward.

Taking downward as positive, the resultant is $2450 - 3200 = -750$ N, so $a = \frac{-750}{250} = -3\ \text{m s}^{-2}$.

4 **Use the remaining distance**

The remaining distance is $520 - 122.5 = 397.5$ m. Using $v^2 = u^2 + 2as$: $v^2 = 49^2 + 2(-3)(397.5) = 16$.

5 **Find the landing speed**

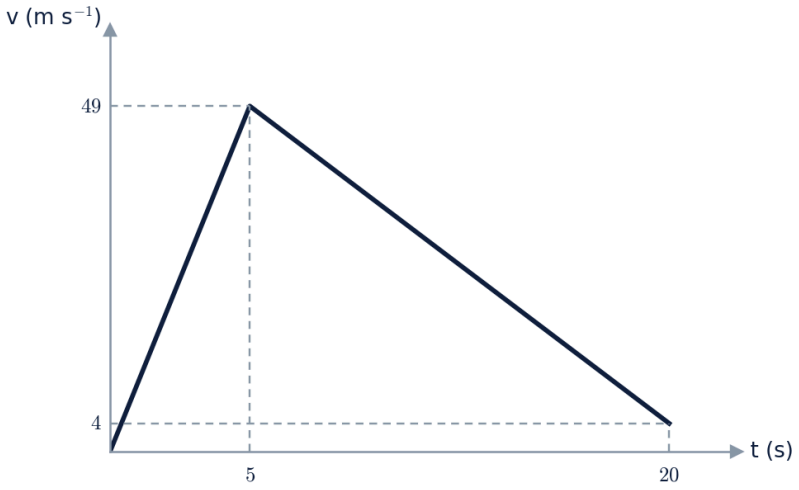
$v = 4\ \text{m s}^{-1}$, so P lands with speed $4\ \text{m s}^{-1}$.

6 **Find the time after the parachute opens**

Using $v = u + at$, $4 = 49 - 3t$, so $t = 15$ s after the parachute opens.

7 **Find the total time and describe the graph**

The total time is $5 + 15 = 20$ s. The speed-time graph rises linearly from 0 to 49 over the first 5 s, then falls linearly to 4 at $t = 20$; it does not meet the time-axis.



FINAL ANSWER

$(a)\ 122.5\ \text{m}$ or $123\ \text{m}$. $(b)\ 4\ \text{m s}^{-1}$. $(c)\ 20\ \text{s}$. (d) Speed-time graph: straight line from $(0, 0)$ to $(5, 49)$ then straight line down to $(20, 4)$.

WME01/01 MAY/JUNE 2024

Question 6

12 marks | Resolving Forces, Inclined Planes

6.

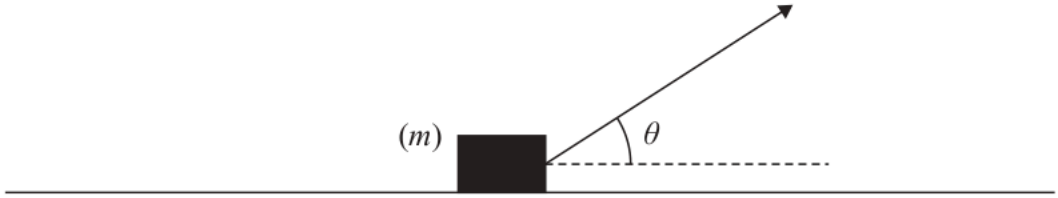


Figure 3

A box of mass m lies on a rough horizontal plane. The box is pulled along the plane in a straight line at **constant speed** by a light rope. The rope is inclined at an angle θ to the plane, as shown in Figure 3.

The coefficient of friction between the box and the plane is $\frac{1}{3}$

The box is modelled as a particle.

Given that $\tan\theta = \frac{3}{4}$

(a) find, in terms of m and g, the tension in the rope.

(7)

The rope is now removed and the box is placed at rest on the plane.
The box is then projected horizontally along the plane with speed u.

The box is again modelled as a particle.

When the box has moved a distance d along the plane, the speed of the box is $\frac{1}{2}u$.

(b) Find d in terms of u and g.

(5)

WORKED SOLUTION

Question 6 · WME01/01 May/June 2024

1 Use the trig ratios

Given $\tan\theta = \frac{3}{4}$, use $\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$.

2 Resolve vertically

The box moves at constant speed, so it is in equilibrium. Vertically, $R + T\sin\theta = mg$, hence $R = mg - T\sin\theta$.

3 Resolve horizontally

Horizontally, the tension component balances friction: $T\cos\theta = F$.

4 **Use friction**

With $\mu = \frac{1}{3}$, $F = \frac{1}{3}R$. Therefore $T\cos\theta = \frac{1}{3}(mg - T\sin\theta)$.

5 **Find T**

Substitute $\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$: $\frac{4}{5}T = \frac{1}{3}\left(mg - \frac{3}{5}T\right)$. This gives $\frac{4}{5}T = \frac{mg}{3} - \frac{1}{5}T$, so $T = \frac{mg}{3}$.

6 **Find the deceleration when the rope is removed**

On the horizontal plane without the rope, $R = mg$ and friction is $\frac{1}{3}mg$. Hence the acceleration is $-\frac{g}{3}$.

7 **Use SUVAT to find d**

When the speed has fallen from u to $\frac{1}{2}u$, use $v^2 = u^2 + 2as$: $\left(\frac{u}{2}\right)^2 = u^2 + 2\left(-\frac{g}{3}\right)d$.

8 **Solve for d**

$\frac{u^2}{4} = u^2 - \frac{2gd}{3}$, so $\frac{2gd}{3} = \frac{3u^2}{4}$ and $d = \frac{9u^2}{8g}$.

FINAL ANSWER

$(a)\ T = \frac{mg}{3}$. $\quad (b)\ d = \frac{9u^2}{8g}$.

WME01/01 MAY/JUNE 2024

Question 7

13 marks | Working with Vectors

7. [*In this question, the horizontal unit vectors* **i** *and* **j** *are directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two speedboats, A and B, are each moving with constant velocity.

- the velocity of A is $40\,\text{km h}^{-1}$ due east
- the velocity of B is $20\,\text{km h}^{-1}$ on a bearing of angle α $(0° < \alpha < 90°)$, where $\tan\alpha = \frac{4}{3}$

The boats are modelled as particles.

(a) Find, in terms of **i** and **j**, the velocity of B in km h^{-1}

(2)

At noon

- the position vector of A is $20\mathbf{j}\,\text{km}$
- the position vector of B is $(10\mathbf{i} + 5\mathbf{j})\,\text{km}$

At time t hours after noon

- the position vector of A is $\mathbf{r}\,\text{km}$, where $\mathbf{r} = 20\mathbf{j} + 40t\mathbf{i}$
- the position vector of B is $\mathbf{s}\,\text{km}$

(b) Find an expression for $\mathbf{s}$ in terms of t, **i** and **j**.

(2)

(c) Show that at time t hours after noon,

$$\overrightarrow{AB} = [(10 - 24t)\mathbf{i} + (12t - 15)\mathbf{j}]\,\text{km}$$

(2)

(d) Show that the boats will never collide.

(3)

(e) Find the distance between the boats when the bearing of B from A is 225°

(4)

WORKED SOLUTION

Question 7 · WME01/01 May/June 2024

1 **Resolve the velocity of B**

For a bearing angle α with $\tan\alpha = \frac{4}{3}$, use $\sin\alpha = \frac{4}{5}$ and $\cos\alpha = \frac{3}{5}$. Therefore $\mathbf{v}_B = 20\sin\alpha\mathbf{i} + 20\cos\alpha\mathbf{j} = 16\mathbf{i} + 12\mathbf{j}$.

2 **Write the position of B**

At noon, B is at $10\mathbf{i} + 5\mathbf{j}$. With constant velocity $16\mathbf{i} + 12\mathbf{j}$, $\mathbf{s} = (10 + 16t)\mathbf{i} + (5 + 12t)\mathbf{j}$.

3 **Find AB vector**

A has position vector $\mathbf{r} = 40t\mathbf{i} + 20\mathbf{j}$. Hence $\overrightarrow{AB} = \mathbf{s} - \mathbf{r} = (10 + 16t - 40t)\mathbf{i} + (5 + 12t - 20)\mathbf{j}$.

4 **Show the printed expression**

This simplifies to $\overrightarrow{AB} = (10 - 24t)\mathbf{i} + (12t - 15)\mathbf{j}$ km.

5 **Show the boats do not collide**

For collision, both components must be zero. From $10 - 24t = 0$, $t = \frac{5}{12}$. From $12t - 15 = 0$, $t = \frac{5}{4}$. These times are different, so the boats never collide.

6 **Use the bearing 225 degrees condition**

A bearing of $225°$ means B is south-west of A, so the i and j components of $\overrightarrow{AB}$ are equal and negative. Set $10 - 24t = 12t - 15$.

7 **Find the distance**

The equation gives $t = \frac{25}{36}$. Then each component is $-\frac{20}{3}$, so the distance is $\sqrt{\left(\frac{20}{3}\right)^2 + \left(\frac{20}{3}\right)^2} = \frac{20\sqrt{2}}{3}\ \text{km}$.

✓ **FINAL ANSWER**

$(a)\ \mathbf{v}_B = 16\mathbf{i} + 12\mathbf{j}\ \text{km h}^{-1}$. $(b)\ \mathbf{s} = (10 + 16t)\mathbf{i} + (5 + 12t)\mathbf{j}$. $(c)\ \overrightarrow{AB} = (10 - 24t)\mathbf{i} + (12t - 15)\mathbf{j}$. (d) no collision. $(e)\ \frac{20\sqrt{2}}{3}\ \text{km} \approx 9.4\ \text{km}$.

Question 8

15 marks | Resolving Forces, Inclined Planes

8.

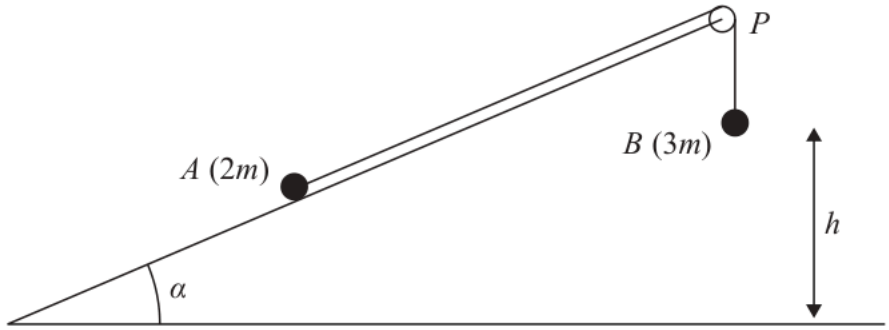


Figure 4

One end of a light inextensible string is attached to a particle A of mass $2m$. The other end of the string is attached to a particle B of mass $3m$. Particle A is held at rest on a rough plane which is inclined to horizontal ground at an angle α, where $\tan\alpha = \frac{5}{12}$

The string passes over a small smooth pulley P which is fixed at the top of the plane. Particle B hangs vertically below P with the string taut, at a height h above the ground, as shown in Figure 4.

The part of the string between A and P lies along a line of greatest slope of the plane. The two particles, the string and the pulley all lie in the same vertical plane.

The coefficient of friction between A and the plane is $\frac{11}{36}$

The particle A is released from rest and begins to move up the plane.

(a) Show that the frictional force acting on A as it moves up the plane is $\frac{22mg}{39}$

(3)

(b) Write down an equation of motion for B.

(2)

(c) Show that the acceleration of A immediately after its release is $\frac{1}{3}g$

(4)

In the subsequent motion, A comes to rest before it reaches the pulley.

(d) Find, in terms of h, the total distance travelled by A from when it was released from rest to when it first comes to rest again.

(6)

1 Use the trig ratios

Here $\tan\alpha = \frac{5}{12}$, so $\sin\alpha = \frac{5}{13}$ and $\cos\alpha = \frac{12}{13}$.

2 Show the frictional force on A

For A, $R = 2mg\cos\alpha = 2mg \cdot \frac{12}{13} = \frac{24mg}{13}$. Since $\mu = \frac{11}{36}$, $F = \mu R = \frac{11}{36} \cdot \frac{24mg}{13} = \frac{22mg}{39}$.

3 Write the equation for B

B moves down as A moves up. Taking downward as positive for B: $3mg - T = 3ma$.

4 Write the equation for A

Taking up the plane as positive for A: $T - 2mg\sin\alpha - F = 2ma$. Substituting $\sin\alpha = \frac{5}{13}$ and $F = \frac{22mg}{39}$ gives $T - \frac{10mg}{13} - \frac{22mg}{39} = 2ma$.

5 Show the acceleration

From B, $T = 3mg - 3ma$. Substitute into A equation: $3mg - 3ma - \frac{10mg}{13} - \frac{22mg}{39} = 2ma$. The constant part is $\frac{5mg}{3}$, so $\frac{5mg}{3} = 5ma$ and $a = \frac{g}{3}$.

6 Find the speed when B reaches the ground

While B descends distance h, A moves distance h up the plane from rest with acceleration $g/3$. Thus $v^2 = 2\left(\frac{g}{3}\right)h = \frac{2gh}{3}$.

7 Find the later deceleration of A

After B reaches the ground, the string no longer pulls A. The forces down the plane on A are $2mg\sin\alpha + F = \frac{10mg}{13} + \frac{22mg}{39} = \frac{4mg}{3}$. Since A has mass $2m$, its deceleration is $\frac{2g}{3}$.

8 Find the extra distance and total distance

Let the extra distance be d. Using $0 = v^2 - 2\left(\frac{2g}{3}\right)d$ with $v^2 = \frac{2gh}{3}$ gives $d = \frac{h}{2}$. Total distance travelled by A is $h + \frac{h}{2} = \frac{3h}{2}$.

✓ FINAL ANSWER

$(a)\ F = \frac{22}{39}mg.$ $(b)\ 3mg - T = 3ma.$ $(c)\ a = \frac{g}{3}.$ $(d)\ \text{total distance} = \frac{3h}{2}.$