Elite Mathematics
Edexcel International A Level

IAL MECHANICS 1 · WME01

WME01/01 January 2024 Worked Solutions

January 2024 paper rebuilt with the worked solution after each question.

8	2019–2026	Single-paper solutions
paper questions	18 exam sessions	checked by Dr Eslam



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PAST PAPER

WME01/01 January 2024

January 2024

8	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WME01/01 JANUARY 2024

Question 1

6 marks | Forces

1.

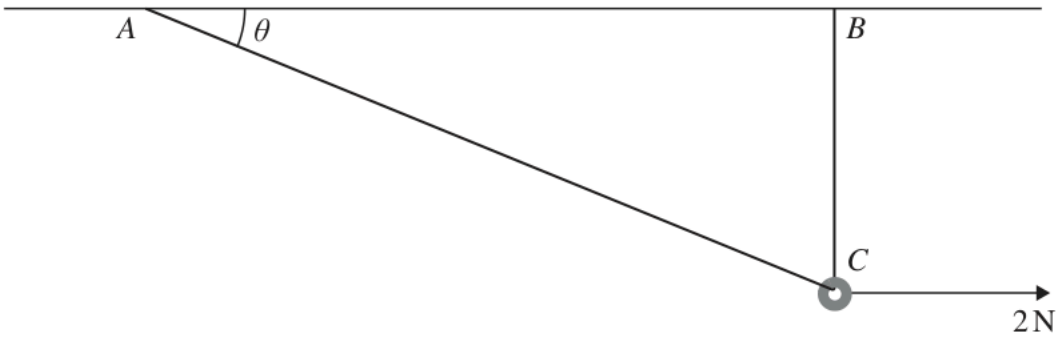


Figure 1

Figure 1 shows a small smooth ring **threaded** onto a light inextensible string.

One end of the string is attached to a fixed point A on a horizontal ceiling and the other end of the string is attached to a fixed point B on the ceiling.

A horizontal force of magnitude 2 N acts on the ring so that the ring rests in equilibrium at a point C, vertically below B, with the string taut.

The line of action of the horizontal force and the string both lie in the same vertical plane.

The angle that the string makes with the ceiling at A is θ, where $\tan\theta = \frac{3}{4}$

The tension in the string is T newtons. The mass of the ring is M kg.

(a) Find the value of T

(3)

(b) Find the value of M

(3)

WORKED SOLUTION

Question 1 · WME01/01 January 2024

1 Use the trig ratios

Given $\tan\theta = \frac{3}{4}$, use $\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$.

2 Resolve horizontally

The horizontal force of 2 N is balanced by the horizontal component of the tension in the sloping part of the string: $T\cos\theta = 2$.

3 **Find the tension**

$T \cdot \dfrac{4}{5} = 2$, so $T = 2.5\ \text{N}$.

4 **Resolve vertically**

The ring is smooth, so the tension is the same in both parts of the string. Upward components are T from the vertical part and $T\sin\theta$ from the sloping part. Therefore $T + T\sin\theta = Mg$.

5 **Find the mass**

$Mg = 2.5 + 2.5 \cdot \dfrac{3}{5} = 4.0$, so $M = \dfrac{4.0}{9.8} = 0.408\ \text{kg}$.

✓ **FINAL ANSWER**

$(a)\ T = 2.5\ \text{N}$. $(b)\ M = 0.408\ \text{kg}$ approximately.

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Question 2

6 marks | Momentum, Impulse & Collisions

2.

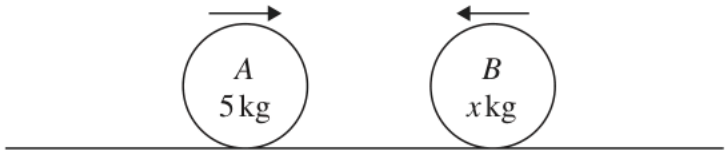


Figure 2

Figure 2 shows two particles, A and B, moving in opposite directions on a smooth horizontal surface. Particle A has mass 5 kg and particle B has mass x kg.

The particles collide directly.

Immediately before the collision, the speed of A is $3\,\text{m s}^{-1}$ and the speed of B is $x\,\text{m s}^{-1}$

Immediately after the collision, the speed of A is $1\,\text{m s}^{-1}$ and its direction of motion is unchanged.

Immediately after the collision, the speed of B is $1.5\,\text{m s}^{-1}$

(a) Find the value of x.

(3)

(b) Find the magnitude of the impulse exerted on A by B in the collision.

(3)

WORKED SOLUTION

Question 2 · WME01/01 January 2024

1 Choose a sign convention

Take the initial direction of A as positive. Before collision, A has velocity 3 and B has velocity $-x$.

2 Set up conservation of momentum

After collision, A still moves in the same direction with velocity 1. B has speed 1.5 in the positive direction for the valid solution. Hence $5(3) + x(-x) = 5(1) + 1.5x$.

3 Solve for x

$15 - x^2 = 5 + 1.5x$, so $x^2 + 1.5x - 10 = 0$. This gives $x = 2.5$ or $x = -4$. Since mass is positive, $x = 2.5$.

4 Use change in momentum of A

The impulse exerted on A equals the change in momentum of A: $I = 5(1 - 3) = -10\,\text{N s}$ using the chosen positive direction.

5 **Give the magnitude**

The magnitude of the impulse is $10\ \mathrm{N\,s}$.

✓ **FINAL ANSWER**

$(a)\ x = 2.5. \quad (b)\ I = 10\ \mathrm{N\,s}.$

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Question 3

10 marks | Constant Acceleration in 1D

3. A van travels with constant acceleration along a straight horizontal road.

The van passes a point A with speed $u\,\mathrm{m\,s^{-1}}$ and 20 seconds later passes a point B with speed $28\,\mathrm{m\,s^{-1}}$

The distance AB is 400 m.

(a) Show that $u = 12$

(2)

(b) Find the time taken for the van to travel from A to the midpoint of AB.

(5)

The van has mass 1200 kg.

During its motion the van experiences a constant resistive force of magnitude 260 N

(c) Find the magnitude of the driving force exerted by the engine of the van as it travels from A to B.

(3)

WORKED SOLUTION

Question 3 · WME01/01 January 2024

1 Use average speed from A to B

With constant acceleration, distance equals average speed times time. So $400 = \dfrac{u + 28}{2} \cdot 20$.

2 Show u is 12

$400 = 10(u + 28)$, so $u + 28 = 40$ and $u = 12$.

3 Find the acceleration

Use $v = u + at$: $28 = 12 + 20a$, hence $a = 0.8\ \mathrm{m\,s^{-2}}$.

4 Use the midpoint distance

The midpoint of AB is 200 m from A. Use $s = ut + \dfrac{1}{2}at^2$: $200 = 12t + \dfrac{1}{2}(0.8)t^2$.

5 Solve for the time to the midpoint

$200 = 12t + 0.4t^2$. The positive solution is $t = 5\sqrt{29} - 15 = 11.9258\ldots$ s.

6 **Use Newtons second law for the van**

Let the driving force be D. The resultant forward force is $D - 260$. Since $ma = 1200(0.8) = 960$, $D - 260 = 960$.

7 **Find the driving force**

$D = 1220\ \text{N}$.

FINAL ANSWER

$(a)\ u = 12.\quad (b)\ t = 5\sqrt{29} - 15 \approx 11.9\ \text{s}.\quad (c)\ \text{driving force} = 1220\ \text{N}.$

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Question 4

8 marks | Forces

4.

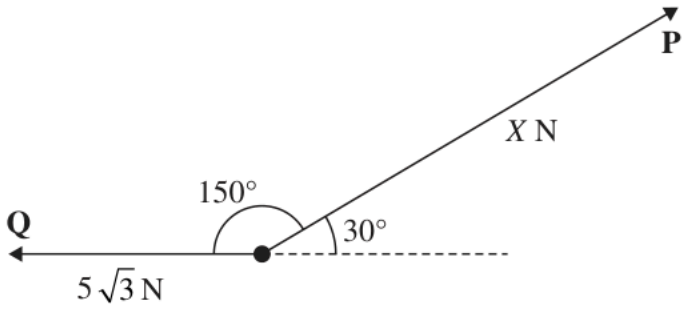


Figure 3

Figure 3 shows two horizontal forces **P** and **Q** acting on a particle.

The angle between the direction of **P** and the direction of **Q** is 150°

Force **P** has magnitude X newtons.

Force **Q** has magnitude $5\sqrt{3}$ N.

The resultant of **P** and **Q** has magnitude $\sqrt{129}$ N.

Find

(i) the value of X.

(ii) the angle between **Q** and the resultant, giving your answer to the nearest degree.

(8)

WORKED SOLUTION

Question 4 · WME01/01 January 2024

1 Use the included angle in the force triangle

The two given forces have angle $150°$ between them, so the triangle used for adding them has the supplementary angle $30°$ between the side of length X and the side of length $5\sqrt{3}$.

2 Use the cosine rule to find X

The resultant has magnitude $\sqrt{129}$. Therefore $129 = X^2 + (5\sqrt{3})^2 - 2X(5\sqrt{3})\cos 30°$.

3 Solve for X

Since $(5\sqrt{3})^2 = 75$ and $2(5\sqrt{3})\cos 30° = 15$, the equation becomes $129 = X^2 + 75 - 15X$. Thus $X^2 - 15X - 54 = 0$, giving $X = 18$.

4 Find the angle with Q

Let β be the angle between Q and the resultant. Using the cosine rule in the same triangle,

$$\frac{18}{\sin\beta} = \frac{\sqrt{129}}{\sin 30^\circ}.$$

5 Choose the correct angle

This gives an obtuse angle for β. Equivalently, $\cos\beta = \dfrac{(5\sqrt{3})^2 + 129 - 18^2}{2(5\sqrt{3})\sqrt{129}}$, so $\beta = 127.6\ldots^\circ$.

6 Round the angle

To the nearest degree, the angle between Q and the resultant is 128°.

FINAL ANSWER

$(i)\ X = 18\ \text{N}.\quad (ii)\ \text{angle} = 128^\circ\ \text{to the nearest degree}.$

Question 5

10 marks | Moments

5.

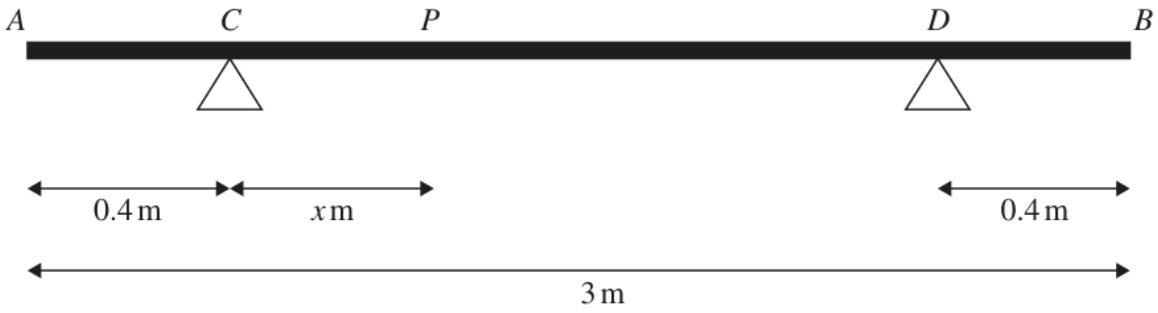


Figure 4

A beam AB has mass 30 kg and length 3 m.

The beam rests on supports at C and D where $AC = 0.4$ m and $DB = 0.4$ m, as shown in Figure 4.

A person of mass 55 kg stands on the beam between C and D.

The person is modelled as a particle at the point P, where $CP = x$ metres and $0 < x < 2.2$

The beam is modelled as a uniform rod resting in equilibrium in a horizontal position.

Using the model,

(a) show that the magnitude of the reaction at C is $(686 - 245x)$ N.

(3)

The magnitude of the reaction at C is **four** times the magnitude of the reaction at D.

Using the model,

(b) find the value of x

(4)

The person steps off the beam and places a package of mass M kg at A.

The package is modelled as a particle at the point A.

The beam is now on the point of tilting about C.

Using the model,

(c) find the value of M

(3)

1 Set the positions on the beam

$AC = 0.4$ m and $DB = 0.4$ m, so $CD = 2.2$ m. The beam is uniform, so its weight $30g$ acts at the midpoint, which is 1.1 m from D. The person is $2.2 - x$ m from D.

2 Take moments about D

Taking moments about D removes R_D: $R_C(2.2) = 55g(2.2 - x) + 30g(1.1)$.

3 Show the expression for RC

Using $g = 9.8$, $R_C = \dfrac{539(2.2 - x) + 294(1.1)}{2.2} = 686 - 245x$.

4 Use the reaction ratio

Given $R_C = 4R_D$ and vertical equilibrium gives $R_C + R_D = (55 + 30)g = 833$. Hence $5R_D = 833$ and $R_C = 666.4$.

5 Find x

$686 - 245x = 666.4$, so $245x = 19.6$ and $x = 0.08$.

6 Use the tilting condition with the package

When the package is at A and the beam is on the point of tilting about C, the reaction at D is zero.

7 Find M

Take moments about C: $Mg(0.4) = 30g(1.1)$. Thus $M = \dfrac{33}{0.4} = 82.5$.

FINAL ANSWER

$(a)\ R_C = 686 - 245x.$ $(b)\ x = 0.08.$ $(c)\ M = 82.5.$

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Question 6

12 marks | Constant Acceleration in 1D

6. A particle is projected vertically upwards from a point A with speed $24\,\text{m s}^{-1}$

The point A is 2.5 m vertically above the point B.

Point B lies on horizontal ground.

The particle moves freely under gravity until it hits the ground at B with speed $V\,\text{m s}^{-1}$

After hitting the ground the particle does not rebound.

(a) Find the value of V.

(3)

(b) Find the time taken for the particle to reach B.

(3)

The point C is 10 m vertically above A.

(c) Find the length of time for which the particle is above C.

(4)

(d) Sketch a speed-time graph for the motion of the particle from projection to the instant that it reaches B. (No further calculations are required.)

(2)

WORKED SOLUTION

Question 6 · WME01/01 January 2024

1 Find the impact speed

Take upward as positive from A. At B, the displacement is -2.5 m, with $u = 24$ and $a = -9.8$. Use $v^2 = u^2 + 2as$: $V^2 = 24^2 + 2(-9.8)(-2.5) = 625$.

2 State V

$V = 25\ \text{m s}^{-1}$.

3 Find the time to reach B

The velocity at B is $-25\ \text{m s}^{-1}$. Using $v = u + at$, $-25 = 24 - 9.8t$, so $t = 5$ s.

4 Set up the height C equation

C is 10 m above A. The particle is above C between the two times when $s = 10$: $10 = 24t - 4.9t^2$.

5 **Solve for the two crossing times**

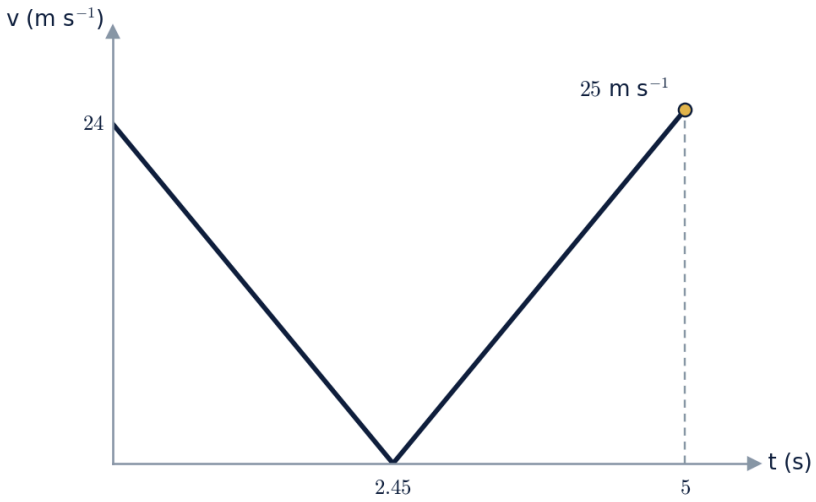
$4.9t^2 - 24t + 10 = 0$, giving $t = 0.459\ldots$ and $t = 4.438\ldots$.

6 **Find the time above C**

The length of time above C is $4.438\ldots - 0.459\ldots = 3.98\ldots\ \text{s}$, about $4.0\ \text{s}$.

7 **Describe the speed-time graph**

The graph starts at speed 24, decreases linearly to 0 at the highest point, then increases linearly to speed 25 at $t = 5$. It is a V-shaped speed-time graph.



FINAL ANSWER

$(a)\ V = 25\ \text{m s}^{-1}$. $(b)\ t = 5\ \text{s}$. $(c)\ 4.0\ \text{s}$ approximately. (d) Speed-time graph is V-shaped, from speed 24 to 0, then to speed 25 at t=5.

WME01/01 JANUARY 2024

Question 7

11 marks | Working with Vectors

7. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

At midnight, a ship S is at the point with position vector $(19\mathbf{i} + 22\mathbf{j})$ km

The ship travels with constant velocity $(12\mathbf{i} - 16\mathbf{j})\,\text{km h}^{-1}$

(a) Find the speed of S.

(2)

At time t hours after midnight, the position vector of S is $\mathbf{s}$ km.

(b) Find an expression for $\mathbf{s}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and t.

(2)

A lighthouse stands on a small rocky island. The lighthouse is modelled as being at the point with position vector $(26\mathbf{i} + 15\mathbf{j})$ km.

It is not safe for ships to be within 1.3 km of the lighthouse.

(c) (i) Find the value of t when S is closest to the lighthouse.

(ii) Hence determine whether it is safe for S to continue its course.

(7)

WORKED SOLUTION

Question 7 · WME01/01 January 2024

1 Find the speed

The velocity is $12\mathbf{i} - 16\mathbf{j}$, so the speed is $\sqrt{12^2 + (-16)^2} = 20\ \text{km h}^{-1}$.

2 Write the ship position vector

At time t hours after midnight, $\mathbf{s} = (19\mathbf{i} + 22\mathbf{j}) + t(12\mathbf{i} - 16\mathbf{j}) = (19 + 12t)\mathbf{i} + (22 - 16t)\mathbf{j}$.

3 Form the vector from lighthouse to ship

The lighthouse is at $26\mathbf{i} + 15\mathbf{j}$. Therefore $\overrightarrow{LS} = \mathbf{s} - \mathbf{l} = (12t - 7)\mathbf{i} + (7 - 16t)\mathbf{j}$.

4 Use distance squared

The square of the distance is $d^2 = (12t - 7)^2 + (7 - 16t)^2 = 400t^2 - 392t + 98$.

5 Find the time of closest approach

This quadratic is minimum when $t = \dfrac{392}{2 \cdot 400} = 0.49$.

6 **Find the minimum distance**

At $t = 0.49$, $d^2 = 1.96$, so $d = 1.4\ \text{km}$.

7 **Decide if the course is safe**

The unsafe distance is 1.3 km. Since $1.4 > 1.3$, it is safe for the ship to continue its course.

✓ **FINAL ANSWER**

$(a)\ 20\ \text{km h}^{-1}$. $(b)\ \mathbf{s} = (19 + 12t)\mathbf{i} + (22 - 16t)\mathbf{j}$. $(c)(i)\ t = 0.49$. $(c)(ii)\ \text{minimum distance} = 1.4\ \text{km, so it is safe}$.

WME01/01 JANUARY 2024

Question 8

12 marks | Resolving Forces, Inclined Planes

8.

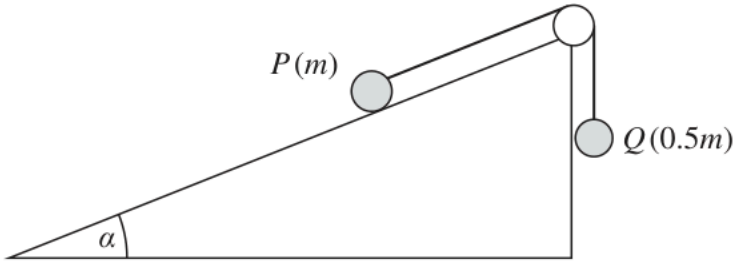


Figure 5

A fixed rough plane is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{5}{12}$

A small smooth pulley is fixed at the top of the plane.

One end of a light inextensible string is attached to a particle P which is at rest on the plane. The string passes over the pulley and the other end of the string is attached to a particle Q which hangs vertically below the pulley, as shown in Figure 5.

Particle P has mass m and particle Q has mass $0.5m$

The string from P to the pulley lies along a line of greatest slope of the plane.

The coefficient of friction between P and the plane is μ.

The system is in **limiting equilibrium** with the string taut and P is on the point of slipping **up** the plane.

(a) Find the value of μ.

(8)

The string breaks and P begins to move down the plane.

When particle P has travelled a distance of 0.8 m down the plane, the speed of P is $V\,\text{m s}^{-1}$

(b) Find the value of V.

(4)

WORKED SOLUTION

Question 8 · WME01/01 January 2024

1 **Use the trig ratios**

Given $\tan\alpha = \frac{5}{12}$, use $\sin\alpha = \frac{5}{13}$ and $\cos\alpha = \frac{12}{13}$.

2 **Find the tension**

Q has mass $0.5m$ and the system is in equilibrium, so the tension is $T = 0.5mg$.

3 **Resolve P parallel to the plane**

P is on the point of slipping up the plane, so friction acts down the plane. Along the plane, $T = mg\sin\alpha + F$.

4 **Find the friction force**

$F = T - mg\sin\alpha = \dfrac{1}{2}mg - \dfrac{5}{13}mg = \dfrac{3}{26}mg$.

5 **Find the normal reaction and mu**

$R = mg\cos\alpha = \dfrac{12}{13}mg$. Since $F = \mu R$, $\mu = \dfrac{3mg/26}{12mg/13} = \dfrac{1}{8}$.

6 **Find acceleration after the string breaks**

P moves down the plane, so friction acts up the plane. The resultant down the plane is $mg\sin\alpha - F = \dfrac{5}{13}mg - \dfrac{1}{8}\cdot\dfrac{12}{13}mg = \dfrac{7}{26}mg$.

7 **Apply Newtons second law**

As P has mass m, $a = \dfrac{7g}{26}$ down the plane.

8 **Find V**

From rest over 0.8 m, $V^2 = 2\left(\dfrac{7g}{26}\right)(0.8)$. With $g = 9.8$, $V = 2.05\ \mathrm{m\,s^{-1}}$ to 3 significant figures.

✓ **FINAL ANSWER**

$(a)\ \mu = \dfrac{1}{8}$. $\quad (b)\ V = 2.05\ \mathrm{m\,s^{-1}}$ approximately.