Elite Mathematics
Edexcel International A Level

IAL MECHANICS 1 · WME01

WME01/01 October 2023 Worked Solutions

October 2023 paper rebuilt with the worked solution after each question.

7	2019–2026	Single-paper solutions
paper questions	18 exam sessions	checked by Dr Eslam



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PAST PAPER

WME01/01 October 2023

October 2023

7	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

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Question 1

5 marks | Moments

1.

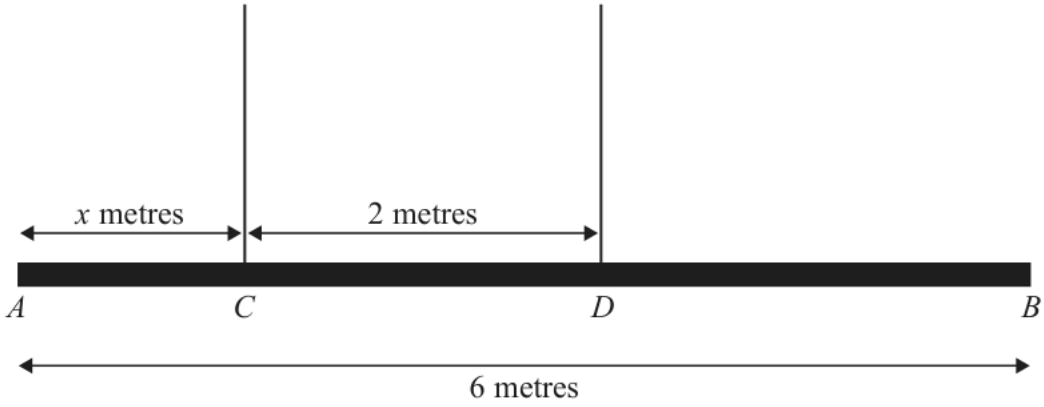


Figure 1

Figure 1 shows a beam AB with weight 24 N and length 6 m.

The beam is suspended by two light vertical ropes. The ropes are attached to the points C and D on the beam where $AC = x$ metres and $CD = 2$ m.

The tension in the rope attached to the beam at C is double the tension in the rope attached to the beam at D.

The beam is modelled as a uniform rod, resting horizontally in equilibrium.

Find

(i) the tension in the rope attached to the beam at D.

(ii) the value of x.

(5)

WORKED SOLUTION

Question 1 · WME01/01 October 2023

1 Name the tensions

Let the tension at D be T. The tension at C is double this, so it is $2T$.

2 Use vertical equilibrium

The beam is in equilibrium, so the upward forces equal the weight: $2T + T = 24$. Hence $3T = 24$ and $T = 8\ \text{N}$.

3 Set the moment distances

The beam is uniform, so its weight acts at the midpoint, 3 m from A. Also $AC = x$ and $AD = x + 2$.

4 **Take moments about A**

Clockwise and anticlockwise moments balance: $(2T)x + T(x+2) = 24 \cdot 3$.

5 **Solve for x**

Substitute $T = 8$: $16x + 8(x+2) = 72$. Thus $24x + 16 = 72$, so $24x = 56$ and $x = \dfrac{7}{3}$ m.

✓ **FINAL ANSWER**

$(i)\ T_D = 8\ \text{N}. \quad (ii)\ x = \dfrac{7}{3}\ \text{m} \approx 2.33\ \text{m}.$

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Question 2

10 marks | Kinematics Graphs

2.

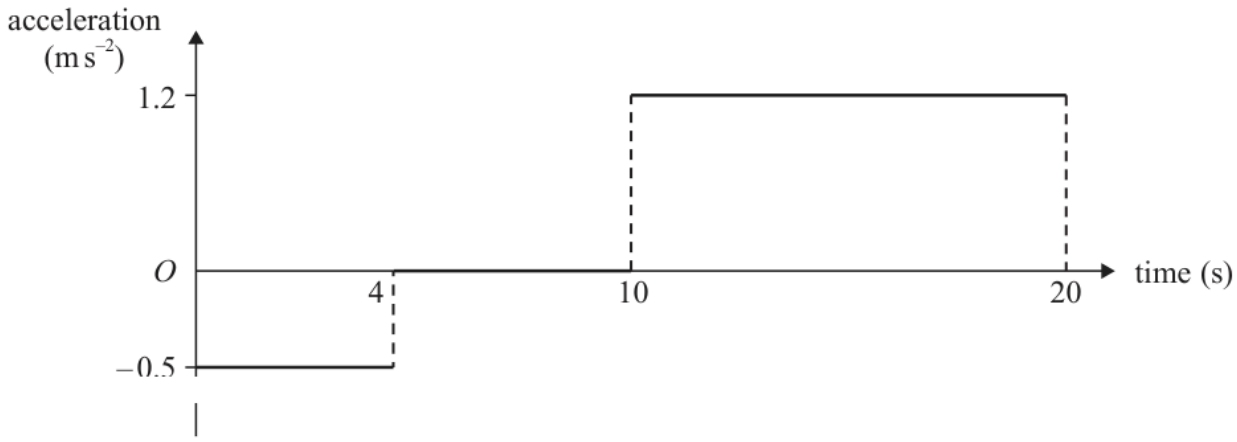


Figure 2

Two fixed points, A and B, are on a straight horizontal road.

The **acceleration-time** graph in Figure 2 represents the motion of a car travelling along the road as it moves from A to B.

At time $t = 0$, the car passes through A with speed $8\,\text{m s}^{-1}$

At time $t = 20\,\text{s}$, the car passes through B with speed $v\,\text{m s}^{-1}$

(a) Show that $v = 18$

(3)

(b) Sketch a speed-time graph for the motion of the car from A to B.

(3)

(c) Find the distance AB.

(4)

WORKED SOLUTION

Question 2 · WME01/01 October 2023

1 Find the speed at t = 4

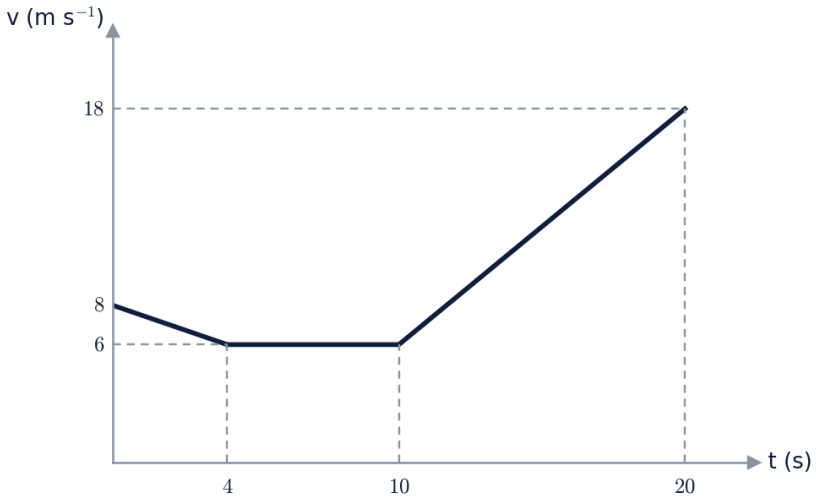
From 0 to 4 s the acceleration is $-0.5\ \text{m s}^{-2}$. Therefore $w = 8 + (-0.5)(4) = 6\ \text{m s}^{-1}$.

2 Find the final speed

From 10 to 20 s the acceleration is $1.2\ \text{m s}^{-2}$, while the speed at $t = 10$ is still 6. Hence $v = 6 + 1.2(10) = 18\ \text{m s}^{-1}$.

3 **Describe the speed-time graph**

The graph starts at $(0, 8)$, falls linearly to $(4, 6)$, stays horizontal to $(10, 6)$, then rises linearly to $(20, 18)$.



4 **Use area under the speed-time graph**

The distance is the area under the graph. From 0 to 4: $\frac{8+6}{2} \cdot 4 = 28$. From 4 to 10: $6 \cdot 6 = 36$. From 10 to 20: $\frac{6+18}{2} \cdot 10 = 120$.

5 **Add the distances**

$AB = 28 + 36 + 120 = 184\ \text{m}$.

✓ **FINAL ANSWER**

(a) $v = 18\ \text{m s}^{-1}$.

(b) Speed-time graph through $(0, 8), (4, 6), (10, 6), (20, 18)$.

(c) $AB = 184\ \text{m}$.

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Question 3

10 marks | Momentum, Impulse & Collisions

3. A hammer is used to hit a tent peg into soft ground.

The hammer has mass 1.8 kg and the tent peg has mass 0.2 kg.

The hammer and tent peg are both modelled as particles and the impact is modelled as a direct collision.

Immediately before the impact, the tent peg is stationary and the hammer is moving vertically downwards with speed $10\,\mathrm{m\,s^{-1}}$

Immediately after the impact, the hammer and tent peg move together, vertically downwards, with the **same** speed $v\,\mathrm{m\,s^{-1}}$

(a) Find the value of v

(2)

(b) Find the magnitude of the impulse exerted on the tent peg by the hammer, stating the units of your answer.

(3)

The ground exerts a constant vertical resistive force of magnitude R newtons, bringing the hammer and tent peg to rest after they travel a distance of 12 cm.

(c) Find the value of R.

(5)

WORKED SOLUTION

Question 3 · WME01/01 October 2023

1 **Use conservation of momentum**

Take downward as positive during the impact. Before impact, the hammer has momentum $1.8(10)$ and the peg has momentum 0. After impact, the combined mass is $1.8 + 0.2 = 2.0$ kg.

2 **Find the common speed**

$1.8(10) = 2.0v$, so $18 = 2v$ and $v = 9\ \mathrm{m\,s^{-1}}$.

3 **Use impulse on the peg**

The peg changes from rest to speed $9\ \mathrm{m\,s^{-1}}$ downward. The impulse magnitude is $0.2(9 - 0) = 1.8\ \mathrm{N\,s}$.

4 **Find the deceleration in the ground**

The hammer and peg move together from speed 9 to rest over 0.12 m. With downward positive, $0 = 9^2 + 2a(0.12)$, so $a = -337.5\ \mathrm{m\,s^{-2}}$.

5 **Apply Newtons second law**

For the combined mass 2.0 kg, forces are weight $2g$ downward and resistance R upward. With downward positive, $2g - R = 2a$.

6 **Find R**

$19.6 - R = 2(-337.5) = -675$, so $R = 694.6\ \text{N}$, which is about $695\ \text{N}$.

FINAL ANSWER

$(a)\ v = 9\ \text{m s}^{-1}$.

$(b)\ I = 1.8\ \text{N s}$.

$(c)\ R \approx 695\ \text{N}$.

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Question 4

10 marks | Working with Vectors

4. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively.*]

A particle P moves with constant acceleration $(-\lambda\mathbf{i} + 2\lambda\mathbf{j})\,\text{m s}^{-2}$, where λ is a positive constant.

At time $t = 0$, the velocity of P is $(5\mathbf{i} - 8\mathbf{j})\,\text{m s}^{-1}$

(a) Find the velocity of P when $t = 5\,\text{s}$, giving your answer in terms of **i**, **j** and λ.

(2)

The speed of P when $t = 5\,\text{s}$ is $13\,\text{m s}^{-1}$

(b) Show that

$$25\lambda^2 - 42\lambda - 16 = 0$$

(3)

(c) Find the direction of motion of P when $t = 4\,\text{s}$, giving your answer as a bearing to the nearest degree.

(5)

WORKED SOLUTION

Question 4 · WME01/01 October 2023

1 Use v = u + at

The initial velocity is $(5\mathbf{i} - 8\mathbf{j})$ and the acceleration is $(-\lambda\mathbf{i} + 2\lambda\mathbf{j})$. At $t = 5$, $\mathbf{v} = (5\mathbf{i} - 8\mathbf{j}) + 5(-\lambda\mathbf{i} + 2\lambda\mathbf{j})$.

2 Write the velocity at t = 5

So $\mathbf{v} = (5 - 5\lambda)\mathbf{i} + (-8 + 10\lambda)\mathbf{j}$.

3 Use the speed at t = 5

The speed is 13, so $(5 - 5\lambda)^2 + (-8 + 10\lambda)^2 = 13^2$.

4 Simplify to the printed equation

Expanding gives $25 - 50\lambda + 25\lambda^2 + 64 - 160\lambda + 100\lambda^2 = 169$. Therefore $125\lambda^2 - 210\lambda - 80 = 0$, and dividing by 5 gives $25\lambda^2 - 42\lambda - 16 = 0$.

5 Find lambda

Solving $25\lambda^2 - 42\lambda - 16 = 0$ gives $\lambda = 2$ or $\lambda = -\frac{8}{25}$. Since λ is positive, $\lambda = 2$.

6 **Find the velocity at t = 4**

$\mathbf{v}(4) = (5\mathbf{i} - 8\mathbf{j}) + 4(-2\mathbf{i} + 4\mathbf{j}) = -3\mathbf{i} + 8\mathbf{j}$.

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7 **Convert to a bearing**

The motion is north-west. The angle west of north satisfies $\tan\alpha = \dfrac{3}{8}$, so $\alpha = 20.6°$. The bearing is $360° - 20.6° = 339.4°$, so $339°$ to the nearest degree.

✓ **FINAL ANSWER**

$(a)\ \mathbf{v} = (5 - 5\lambda)\mathbf{i} + (-8 + 10\lambda)\mathbf{j}$.

$(b)\ 25\lambda^2 - 42\lambda - 16 = 0$.

$(c)\ \text{bearing} = 339°$.

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Question 5

12 marks | Forces

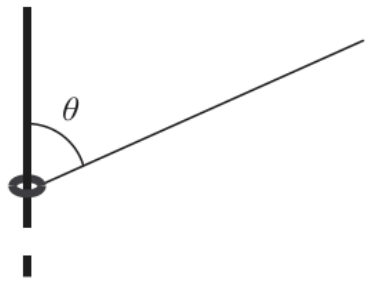


Figure 3

A small ring of mass 0.2 kg is attached to one end of a light inextensible string.

The ring is **threaded** onto a fixed rough vertical rod.

The string is taut and makes an angle θ with the rod, as shown in Figure 3, where $\tan\theta = \frac{12}{5}$

Given that the ring is in equilibrium and that the tension in the string is 10 N,

(a) find the magnitude of the frictional force acting on the ring,

(3)

(b) state the direction of the frictional force acting on the ring.

(1)

The coefficient of friction between the ring and the rod is $\frac{1}{4}$

Given that the ring is in equilibrium, and that the tension in the string, T newtons, can now vary,

(c) (i) find the minimum value of T

(ii) find the maximum value of T

(8)

WORKED SOLUTION

Question 5 · WME01/01 October 2023

1 **Use the trig ratios**

The string makes angle θ with the vertical rod and $\tan\theta = \frac{12}{5}$. Hence $\sin\theta = \frac{12}{13}$ and $\cos\theta = \frac{5}{13}$.

2 **Find the friction for T = 10**

The upward component of the tension is $10\cos\theta = 10 \cdot \frac{5}{13} = \frac{50}{13}$ N. The weight is $0.2g = 1.96$ N.

3 **Balance vertical forces**

Since the upward component of tension is larger than the weight, friction must act downwards. Its magnitude is $\frac{50}{13} - 1.96 = 1.89\ldots$ N, so $F = 1.9$ N.

4 **Write the normal reaction**

For variable tension T, the normal reaction from the rod is produced by the horizontal component of tension: $R = T\sin\theta = \frac{12T}{13}$.

5 **Use limiting friction**

Since $\mu = \frac{1}{4}$, limiting friction is $F = \mu R = \frac{1}{4} \cdot \frac{12T}{13} = \frac{3T}{13}$.

6 **Find the minimum tension**

For the minimum tension, the ring would tend to slide down, so friction acts upwards. Vertical equilibrium gives $T\cos\theta + F = 0.2g$, hence $\frac{5T}{13} + \frac{3T}{13} = 1.96$. Thus $T = 3.185\ldots$ N, so $T_{\min} = 3.2$ N.

7 **Find the maximum tension**

For the maximum tension, the ring would tend to move up, so friction acts downwards. Vertical equilibrium gives $T\cos\theta = 0.2g + F$, hence $\frac{5T}{13} = 1.96 + \frac{3T}{13}$. Thus $T = 12.74\ldots$ N, so $T_{\max} = 13$ N.

✓ **FINAL ANSWER**

(a) $F = 1.9$ N.

(b) Friction acts downwards.

$(c)(i)$ $T_{\min} = 3.2$ N. $(c)(ii)$ $T_{\max} = 13$ N.

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Question 6

15 marks | Working with Vectors

6. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

At 12:00, a ship P sets sail from a harbour with position vector $(15\mathbf{i} + 36\mathbf{j})$ km.
At 12:30, P is at the point with position vector $(20\mathbf{i} + 34\mathbf{j})$ km.

Given that P moves with constant velocity,

(a) show that the velocity of P is $(10\mathbf{i} - 4\mathbf{j})\,\text{km h}^{-1}$

(2)

At time t hours after 12:00, the position vector of P is $\mathbf{p}$ km.

(b) Find an expression for $\mathbf{p}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and t.

(2)

A second ship Q is also travelling at a constant velocity.

At time t hours after 12:00, the position vector of Q is given by $\mathbf{q}$ km, where

$$\mathbf{q} = (42 - 8t)\mathbf{i} + (9 + 14t)\mathbf{j}$$

Ships P and Q are modelled as particles.

If both ships maintained their course,

(c) (i) verify that they would collide at 13:30

(ii) find the position vector of the point at which the collision would occur.

(4)

At 12:30 Q changes speed and direction to avoid the collision.

Ship Q now travels due north with a constant speed of $15\,\text{km h}^{-1}$

Ship P maintains the same constant velocity throughout.

(d) Find the exact distance between P and Q at 14:30

(7)

WORKED SOLUTION

Question 6 · WME01/01 October 2023

1 **Find the velocity of P**

From 12:00 to 12:30, the displacement of P is $(20\mathbf{i} + 34\mathbf{j}) - (15\mathbf{i} + 36\mathbf{j}) = 5\mathbf{i} - 2\mathbf{j}$ km. The time is 0.5 hours, so $\mathbf{v}_P = \dfrac{5\mathbf{i} - 2\mathbf{j}}{0.5} = 10\mathbf{i} - 4\mathbf{j}$.

2 **Write the position of P**

At t hours after 12:00, $\mathbf{p} = (15\mathbf{i} + 36\mathbf{j}) + t(10\mathbf{i} - 4\mathbf{j}) = (15 + 10t)\mathbf{i} + (36 - 4t)\mathbf{j}$.

3 **Verify the collision time**

At 13:30, $t = 1.5$. Then $\mathbf{p} = (15 + 15)\mathbf{i} + (36 - 6)\mathbf{j} = 30\mathbf{i} + 30\mathbf{j}$. Also $\mathbf{q} = (42 - 8(1.5))\mathbf{i} + (9 + 14(1.5))\mathbf{j} = 30\mathbf{i} + 30\mathbf{j}$.

4 **State the collision position**

Since both position vectors are the same, the collision point has position vector $30\mathbf{i} + 30\mathbf{j}$ km.

5 **Find P at 14:30**

At 14:30, $t = 2.5$, so $\mathbf{p} = (15 + 25)\mathbf{i} + (36 - 10)\mathbf{j} = 40\mathbf{i} + 26\mathbf{j}$.

6 **Find Q after changing course**

At 12:30, $t = 0.5$, so $\mathbf{q} = 38\mathbf{i} + 16\mathbf{j}$. From 12:30 to 14:30 is 2 hours due north at $15\ \text{km}\,\text{h}^{-1}$, so the extra displacement is $30\mathbf{j}$. Thus Q is at $38\mathbf{i} + 46\mathbf{j}$.

7 **Find the distance**

The separation vector is $(40 - 38)\mathbf{i} + (26 - 46)\mathbf{j} = 2\mathbf{i} - 20\mathbf{j}$. The distance is $\sqrt{2^2 + 20^2} = \sqrt{404} = 2\sqrt{101}\ \text{km}$.

✓ **FINAL ANSWER**

$(a)\ \mathbf{v}_P = 10\mathbf{i} - 4\mathbf{j}\ \text{km}\,\text{h}^{-1}$. $(b)\ \mathbf{p} = (15 + 10t)\mathbf{i} + (36 - 4t)\mathbf{j}$. $(c)(ii)\ 30\mathbf{i} + 30\mathbf{j}$. $(d)\ 2\sqrt{101}\ \text{km}$.

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Question 7

13 marks | Resolving Forces, Inclined Planes

7.

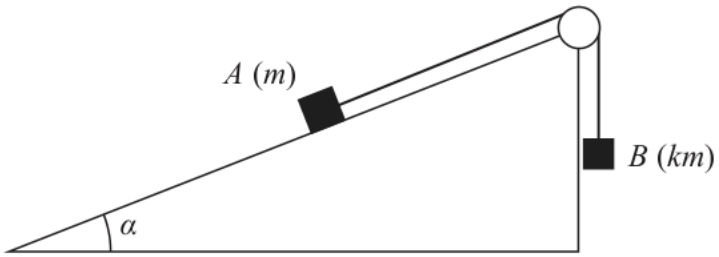


Figure 4

Figure 4 shows a block A of mass m held at rest on a rough plane.
The plane is inclined at an angle α to the horizontal and the coefficient of friction between the block and the plane is μ.

One end of a light inextensible string is now attached to A. The string passes over a small smooth pulley which is fixed at the top of the plane.
The other end of the string is attached to a block B of mass km.
Block B hangs vertically below the pulley, with the string taut.

The string from A to the pulley lies along a line of greatest slope of the plane.

Both A and B are modelled as particles.

When the system is released from rest, A moves up the plane and the tension in the string is $\frac{4mg}{3}$

Given that $\mu = \frac{1}{3}$ and $\tan\alpha = \frac{7}{24}$

(a) (i) find the magnitude of the acceleration of A, giving your answer in terms of g,

(ii) find the value of k.

(9)

(b) Find the magnitude of the resultant force exerted on the pulley by the string, giving your answer in terms of m and g.

(4)

WORKED SOLUTION

Question 7 · WME01/01 October 2023

1 Use the trig ratios

Given $\tan\alpha = \frac{7}{24}$, use $\sin\alpha = \frac{7}{25}$ and $\cos\alpha = \frac{24}{25}$.

2 **Find friction on A**

For block A, $R = mg\cos\alpha = \frac{24mg}{25}$. Since $\mu = \frac{1}{3}$, friction is $F = \frac{1}{3}R = \frac{8mg}{25}$.

3 **Apply Newtons second law to A**

A moves up the plane, so friction and the component of weight act down the plane. Taking up the plane as positive: $T - mg\sin\alpha - F = ma$.

4 **Find the acceleration**

Substitute $T = \frac{4}{3}mg$, $mg\sin\alpha = \frac{7mg}{25}$ and $F = \frac{8mg}{25}$:
$ma = \frac{4}{3}mg - \frac{15mg}{25} = \left(\frac{4}{3} - \frac{3}{5}\right)mg = \frac{11mg}{15}$. Hence $a = \frac{11g}{15}$.

5 **Apply Newtons second law to B**

Block B moves downwards with the same acceleration. Taking downward as positive:
$kmg - T = kma$.

6 **Find k**

Substitute $T = \frac{4}{3}mg$ and $a = \frac{11g}{15}$: $kmg - \frac{4}{3}mg = km\frac{11g}{15}$. Divide by mg to get $k - \frac{4}{3} = \frac{11k}{15}$, so $15k - 20 = 11k$ and $k = 5$.

7 **Resolve the force on the pulley**

The pulley is pulled by two tensions of magnitude T. One is vertical and the other is along the plane. The horizontal component of the resultant is $T\cos\alpha$ and the vertical component is $T + T\sin\alpha$.

8 **Find the resultant**

The magnitude is $T\sqrt{\cos^2\alpha + (1+\sin\alpha)^2}$. With $\sin\alpha = \frac{7}{25}$ and $\cos\alpha = \frac{24}{25}$, this is $T\sqrt{\frac{576}{625} + \frac{1024}{625}} = \frac{8T}{5}$. Since $T = \frac{4}{3}mg$, the resultant is $\frac{32}{15}mg$.

FINAL ANSWER

$(a)(i)\ a = \frac{11g}{15}.\quad (a)(ii)\ k = 5.\quad (b)\ \text{resultant} = \frac{32}{15}mg.$