Elite Mathematics
Edexcel International A Level

IAL MECHANICS 1 · WME01

WME01/01 May/June 2023 Worked Solutions

May/June 2023 paper rebuilt with the worked solution after each question.

8	2019–2026	Single-paper solutions
paper questions	18 exam sessions	checked by Dr Eslam



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PAST PAPER

WME01/01 May/June 2023

May/June 2023

8	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WME01/01 MAY/JUNE 2023

Question 1

7 marks | Momentum, Impulse & Collisions

1. A particle A has mass 4 kg and a particle B has mass 2 kg.

The particles move towards each other in opposite directions along the same straight line on a smooth horizontal table and collide directly.

Immediately before the collision, the speed of A is $2u\,\text{m s}^{-1}$ and the speed of B is $3u\,\text{m s}^{-1}$

Immediately after the collision, the speed of B is $2u\,\text{m s}^{-1}$

The direction of motion of B is reversed by the collision.

(a) Find, in terms of u, the speed of A immediately after the collision.

(3)

(b) State the direction of motion of A immediately after the collision.

(1)

(c) Find, in terms of u, the magnitude of the impulse received by B in the collision. State the units of your answer.

(3)

WORKED SOLUTION

Question 1 · WME01/01 May/June 2023

1 Choose a positive direction

Take the original direction of A as positive. Before the collision, A has velocity $2u$ and B has velocity $-3u$. After the collision, B has velocity $2u$ because its direction is reversed.

2 Use conservation of momentum

Let the velocity of A after the collision be v. Then $4(2u) + 2(-3u) = 4v + 2(2u)$.

3 Solve for the velocity of A

$8u - 6u = 4v + 4u$, so $2u = 4v + 4u$. Hence $4v = -2u$ and $v = -\frac{u}{2}$.

4 State the speed and direction

The speed of A is the magnitude of its velocity, so it is $\frac{u}{2}$. The negative sign shows that A has reversed direction.

5 Find the impulse on B

Impulse equals change in momentum. For B, $I = 2\left(2u - (-3u)\right) = 2(5u) = 10u$.

6 **Attach the units**

The impulse is $10u\ \mathrm{N\,s}$, equivalently $10u\ \mathrm{kg\,m\,s^{-1}}$.

✓ **FINAL ANSWER**

(a) speed of A $= \dfrac{u}{2}$. (b) A reverses direction. (c) $I = 10u\ \mathrm{N\,s}$.

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Question 2

10 marks | Working with Vectors

[*In this question* **i** *and* **j** *are horizontal perpendicular unit vectors.*]

2. A particle P rests in equilibrium on a smooth horizontal plane.

A system of **three** forces, $\mathbf{F}_1$ N, $\mathbf{F}_2$ N and $\mathbf{F}_3$ N where

$$\mathbf{F}_1 = (3c\mathbf{i} + 4c\mathbf{j})$$
$$\mathbf{F}_2 = (-14\mathbf{i} + 7\mathbf{j})$$

is applied to P.

Given that P remains in equilibrium,

(a) find $\mathbf{F}_3$ in terms of c, **i** and **j**.

(2)

The force $\mathbf{F}_3$ is **removed** from the system.

Given that $c = 2$

(b) find the size of the angle between the direction of **i** and the direction of the resultant force acting on P.

(4)

The mass of P is m kg.

Given that the magnitude of the acceleration of P is $8.5\,\text{m s}^{-2}$

(c) find the value of m.

(4)

WORKED SOLUTION

Question 2 · WME01/01 May/June 2023

1 **Use equilibrium**

For equilibrium, $\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} = \mathbf{0}$. Hence $\mathbf{F_3} = -(\mathbf{F_1} + \mathbf{F_2})$.

2 **Find F3**

$\mathbf{F_1} + \mathbf{F_2} = (3c - 14)\mathbf{i} + (4c + 7)\mathbf{j}$, so $\mathbf{F_3} = (14 - 3c)\mathbf{i} + (-7 - 4c)\mathbf{j}$.

3 **Find the resultant when c = 2**

When $c = 2$, $\mathbf{F_1} = 6\mathbf{i} + 8\mathbf{j}$ and $\mathbf{F_2} = -14\mathbf{i} + 7\mathbf{j}$. Therefore the resultant is $-8\mathbf{i} + 15\mathbf{j}$.

4 **Find the angle**

The resultant is in the second quadrant. Its angle from the positive $\mathbf{i}$ direction is $180° - \tan^{-1}\left(\frac{15}{8}\right) = 118.1°$, so the angle is about $118°$.

5 **Find the magnitude of the resultant**

$|\mathbf{R}| = \sqrt{(-8)^2 + 15^2} = 17\ \text{N}$.

6 **Use F = ma**

The acceleration magnitude is $8.5\ \text{m s}^{-2}$, so $17 = 8.5m$. Hence $m = 2$.

✓ **FINAL ANSWER**

$(a)\ \mathbf{F}_3 = (14 - 3c)\mathbf{i} + (-7 - 4c)\mathbf{j}$. $(b)\ 118°$ approximately. $(c)\ m = 2$.

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Question 3

8 marks | Constant Acceleration in 1D

3. Two students observe a book of mass 0.2 kg fall vertically from rest from a shelf that is 1.5 m above the floor.

Student A suggests that the book is modelled as a particle falling freely under gravity.

(a) Use student A's model to find the time taken for the book to reach the floor.

(3)

Student B suggests an improved model where the book is modelled as a particle experiencing a constant resistance to motion of magnitude R newtons.

Given that the time taken for the book to reach the floor is 0.6 seconds,

(b) use student B's model to find the value of R

(5)

WORKED SOLUTION

Question 3 · WME01/01 May/June 2023

1 Use free fall for student A

From rest, with downward displacement 1.5 m and acceleration g, use $s = ut + \frac{1}{2}at^2$.

2 Find the time

$1.5 = 0 + \frac{1}{2}(9.8)t^2$, so $t^2 = \frac{3}{9.8}$ and $t = 0.553\ldots$ s.

3 Find the acceleration in student B's model

Now the time is 0.6 s. Again starting from rest, $1.5 = \frac{1}{2}a(0.6)^2 = 0.18a$, so $a = 8.333\ldots\ \text{m s}^{-2}$ downward.

4 Apply Newtons second law

Taking downward as positive, weight is $0.2g$ and resistance R acts upward. Therefore $0.2g - R = 0.2a$.

5 Find R

$1.96 - R = 0.2(8.333\ldots) = 1.666\ldots$, so $R = 0.293\ldots$ N.

FINAL ANSWER

$(a)\ t = 0.553\ \text{s}$ approximately. $\quad (b)\ R = 0.293\ \text{N}$.

Question 4

12 marks | Moments

4.

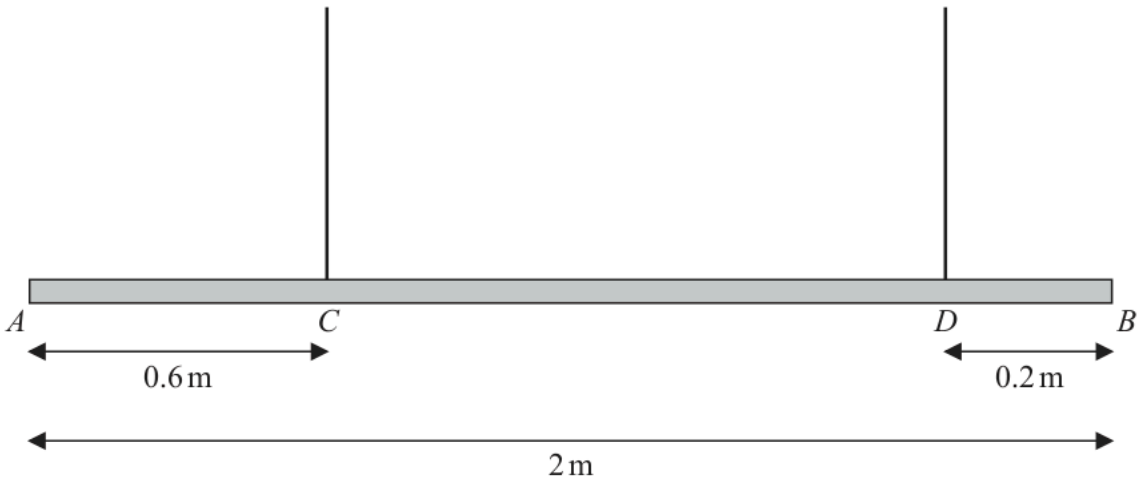


Figure 1

Figure 1 shows a beam AB, of mass m kg and length 2 m, suspended by two light vertical ropes.
The ropes are attached to the points C and D on the beam, where $AC = 0.6$ m and $DB = 0.2$ m
The beam is in equilibrium in a horizontal position.

A particle of mass pm kg is attached to the beam at A and the beam remains in equilibrium in a horizontal position.

The beam is modelled as a uniform rod.

(a) Given that the tension in the rope attached at C is four times the tension in the rope attached at D, use the model to find the exact value of p.

(7)

The particle of mass pm kg at A is removed and replaced by a particle of mass qm kg at A.
The beam remains in equilibrium in a horizontal position but is now on the point of tilting.

(b) Using the model, find the exact value of q

(4)

(c) State how you have used the modelling assumption that the beam is uniform.

(1)

1 Set the geometry

The beam is 2 m long. Since $AC = 0.6$ m and $DB = 0.2$ m, point D is 1.8 m from A. The uniform beam has weight mg acting 1.0 m from A.

2 Name the tensions

Let the tension at D be T. The tension at C is four times this, so it is $4T$.

3 Use vertical equilibrium

$4T + T = pmg + mg$, so $5T = (p+1)mg$.

4 Take moments about A

The particle at A has no moment about A. Hence $4T(0.6) + T(1.8) = mg(1.0)$.

5 Find T

$2.4T + 1.8T = mg$, so $4.2T = mg$ and $T = \dfrac{5mg}{21}$.

6 Find p

Substitute into $5T = (p+1)mg$: $5 \cdot \dfrac{5mg}{21} = (p+1)mg$. Thus $p + 1 = \dfrac{25}{21}$ and $p = \dfrac{4}{21}$.

7 Use the tilting condition

When the particle of mass qm is at A and the beam is on the point of tilting, the rope at D is just slack, so its tension is zero.

8 Take moments about C

About C, the particle at A has moment $qmg(0.6)$ and the beam weight has moment $mg(0.4)$ in the opposite direction. Thus $qmg(0.6) = mg(0.4)$, giving $q = \dfrac{2}{3}$.

9 Use the modelling assumption

The beam is uniform, so its centre of mass is at the midpoint of the beam and its weight acts there.

✓ FINAL ANSWER

$(a)\ p = \dfrac{4}{21}$. $(b)\ q = \dfrac{2}{3}$. (c) The beam weight acts at its midpoint.

Question 5

11 marks | Kinematics Graphs

5.

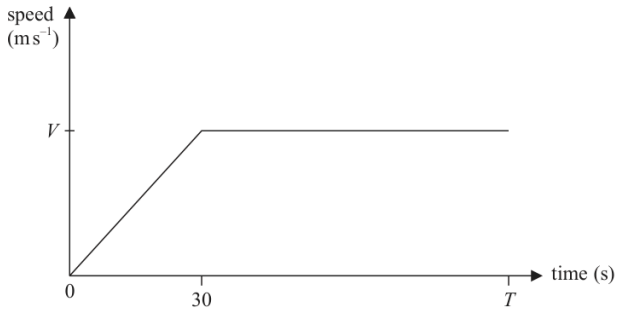


Figure 2

The speed-time graph in Figure 2 illustrates the motion of a car travelling along a straight horizontal road.
At time $t = 0$, the car starts from rest and accelerates uniformly for 30 s until it reaches a speed of V m s^{-1}
The car then travels at a constant speed of V m s^{-1} until time $t = T$ seconds.

(a) Show that the distance travelled by the car between $t = 0$ and $t = T$ seconds is $V(T - 15)$ metres.

(2)

A motorbike also travels along the same road.

- The motorbike starts from rest at time $t = 10$ s and accelerates uniformly for 40 s
- The acceleration of the motorbike is the **same** as the acceleration of the car
- The motorbike then travels at a constant speed for a further 10 s before decelerating uniformly until it reaches a speed of V m s^{-1} at time T seconds

(b) On Figure 2, sketch a speed-time graph for the motion of the motorbike.

[*If you need to redraw your sketch, there is a copy of Figure 2 on page 15.*]

(2)

(c) Show that the constant speed of the motorbike is $\frac{4V}{3}$ m s^{-1}

(2)

At time $t = T$ seconds, the distance travelled by each vehicle is the same.

(d) Find the value of T

(5)

Only use this copy of Figure 2 if you need to redraw your sketch.

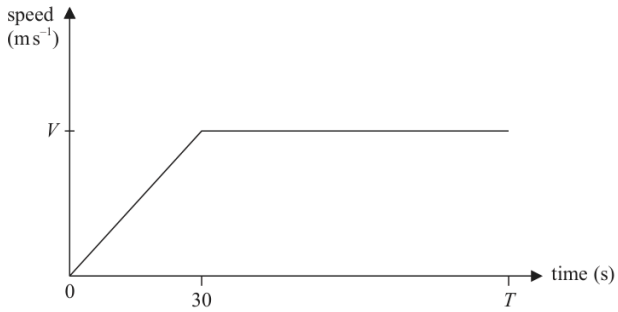


Figure 2

(Total for Question 5 is 11 marks)

1 Find the car distance

The car graph is a triangle from 0 to 30 plus a rectangle from 30 to T. Distance $= \frac{1}{2}(30)V + V(T-30)$.

2 Simplify the car distance

$\frac{1}{2}(30)V + V(T-30) = 15V + VT - 30V = V(T-15)$, as required.

3 Describe the motorbike graph

The motorbike starts at $(10, 0)$, accelerates with the same gradient as the car until $t = 50$, then travels at constant speed until $t = 60$, then decelerates linearly to (T, V).

4 Find the motorbike constant speed

The car acceleration is $\frac{V}{30}$. The motorbike accelerates for 40 s, so its speed is $40 \cdot \frac{V}{30} = \frac{4V}{3}$.

5 Find the motorbike distance

Its distance is $\frac{1}{2}(40)\left(\frac{4V}{3}\right) + 10\left(\frac{4V}{3}\right) + \frac{1}{2}\left(\frac{4V}{3} + V\right)(T-60)$.

6 Simplify the motorbike distance

This becomes $\frac{80V}{3} + \frac{40V}{3} + \frac{7V}{6}(T-60) = 40V + \frac{7V}{6}(T-60) = \frac{7VT}{6} - 30V$.

7 Equate the two distances

At time T, the distances are equal: $V(T-15) = \frac{7VT}{6} - 30V$. Divide by V to get $T - 15 = \frac{7T}{6} - 30$.

8 Find T

Multiplying by 6 gives $6T - 90 = 7T - 180$, so $T = 90$.

FINAL ANSWER

(a) car distance $= V(T-15)$.

(b) Motorbike graph starts at $t = 10$, reaches $\frac{4V}{3}$ at $t = 50$, stays constant to $t = 60$, then falls to V at $t = T$.

(c) constant speed $= \frac{4V}{3}$. (d) $T = 90$.

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Question 6

7 marks | Forces

6.

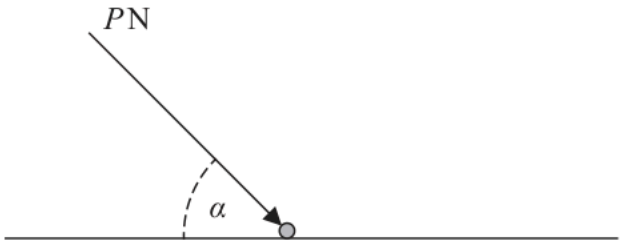


Figure 3

A particle of weight W newtons lies at rest on a rough horizontal surface, as shown in Figure 3.
A force of magnitude P newtons is applied to the particle.

The force acts at an angle α to the horizontal, where $\tan\alpha = \frac{4}{3}$

The coefficient of friction between the particle and the surface is $\frac{1}{4}$

Given that the particle does not move, show that

$$P \leqslant \frac{5W}{8}$$

(7)

WORKED SOLUTION

Question 6 · WME01/01 May/June 2023

1 Use the trig ratios

Given $\tan\alpha = \frac{4}{3}$, use $\sin\alpha = \frac{4}{5}$ and $\cos\alpha = \frac{3}{5}$.

2 Resolve vertically

The applied force increases the normal reaction in this model, so $R = W + P\sin\alpha$.

3 Resolve horizontally

For the particle not to move, friction must be able to balance the horizontal component of the applied force, so $P\cos\alpha \leq F_{\max}$.

4 Use limiting friction

Since $\mu = \frac{1}{4}$, $F_{\max} = \frac{1}{4}R = \frac{1}{4}(W + P\sin\alpha)$.

5 **Substitute the exact trig values**

Thus $P \cdot \frac{3}{5} \leq \frac{1}{4}\left(W + P \cdot \frac{4}{5}\right)$.

6 **Rearrange the inequality**

Multiplying by 20 gives $12P \leq 5W + 4P$. Hence $8P \leq 5W$, so $P \leq \frac{5}{8}W$.

✓ **FINAL ANSWER**

$P \leq \frac{5}{8}W$.

Question 7

11 marks | Newton's Second Law

7.

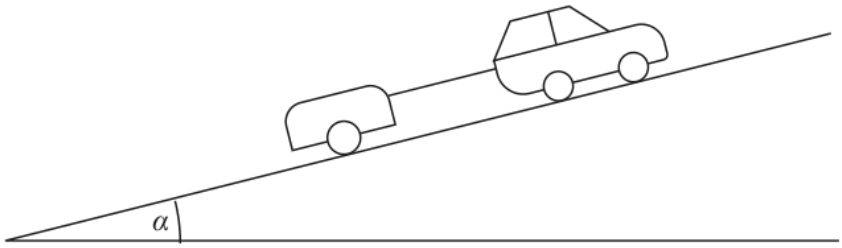


Figure 4

A car of mass 1200 kg is towing a trailer of mass 600 kg up a straight road, as shown in Figure 4.

The road is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{12}$

The driving force produced by the engine of the car is 3000 N.

The car moves with acceleration $0.75\,\text{m}\,\text{s}^{-2}$

The non-gravitational resistance to motion of

- the **car** is modelled as a constant force of magnitude $2R$ newtons
- the **trailer** is modelled as a constant force of magnitude R newtons

The car and the trailer are modelled as particles.
The tow bar between the car and trailer is modelled as a light rod that is parallel to the direction of motion.

Using the model,

(a) show that the value of R is 60

(4)

(b) find the tension in the tow bar.

(3)

When the car and trailer are moving at a speed of $12\,\text{m}\,\text{s}^{-1}$, the tow bar breaks.

Given that the non-gravitational resistance to motion of the trailer remains unchanged,

(c) use the model to find the further distance moved by the trailer before it first comes to rest.

(4)

1 **Use the whole system**

For car plus trailer, the total mass is 1800 kg and the total resistance is $2R + R = 3R$. Taking up the slope as positive: $3000 - 1800g\sin\alpha - 3R = 1800(0.75)$.

2 **Show R is 60**

Since $\sin\alpha = \dfrac{1}{12}$, $1800g\sin\alpha = 1800 \cdot 9.8 \cdot \dfrac{1}{12} = 1470$. So $3000 - 1470 - 3R = 1350$, giving $180 = 3R$ and $R = 60$ N.

3 **Apply Newtons second law to the trailer**

For the trailer while it is being towed, $T - 600g\sin\alpha - R = 600(0.75)$.

4 **Find the tow-bar tension**

$T - 600 \cdot 9.8 \cdot \dfrac{1}{12} - 60 = 450$, so $T - 490 - 60 = 450$ and $T = 1000$ N.

5 **Find the trailer acceleration after the break**

After the tow bar breaks, the trailer is still moving up the slope, but the component of weight and resistance act down the slope. Taking up the slope as positive: $-600g\sin\alpha - 60 = 600a$.

6 **Simplify the acceleration**

$-490 - 60 = 600a$, so $a = -\dfrac{550}{600} = -\dfrac{11}{12}$ m s^{-2}.

7 **Find the stopping distance**

With initial speed 12 m s^{-1} and final speed 0, use $v^2 = u^2 + 2as$: $0 = 12^2 + 2\left(-\dfrac{11}{12}\right)d$. Hence $d = \dfrac{864}{11} = 78.5\ldots$ m.

✓ **FINAL ANSWER**

(a) $R = 60$ N. (b) $T = 1000$ N. (c) $d = 78.5$ m approximately.

Question 8

9 marks | Working with Vectors

8.

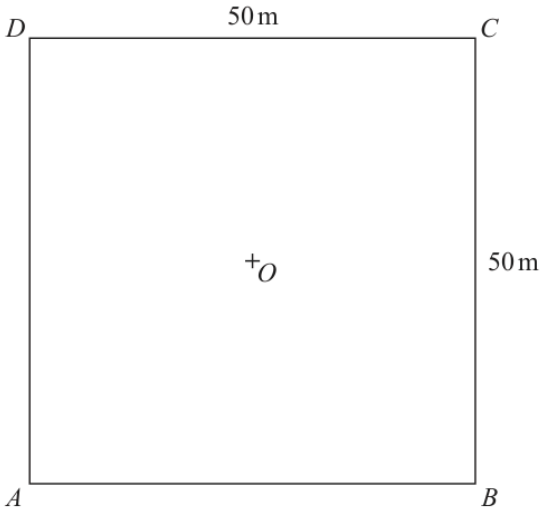


Figure 5

A square floor space $ABCD$, with centre O, is modelled as a flat horizontal surface measuring 50 m by 50 m, as shown in Figure 5.
The horizontal unit vectors **i** and **j** are in the direction of $\overrightarrow{AB}$ and $\overrightarrow{AD}$ respectively.

All position vectors are given relative to O.

A small robot R is programmed to travel across the floor at a constant velocity.

- At time $t = 0$, R is at the point with position vector $(-2\mathbf{i} + \mathbf{j})$ m
- At time $t = 11$ s, R is at the point with position vector $(9\mathbf{i} + 23\mathbf{j})$ m
- At time t seconds, the position vector of R is **r** metres

(a) Find, in terms of t, **i** and **j**, an expression for **r**

(3)

A second robot S is at the point C.

- At time $t = 0$, S leaves C and moves with constant velocity $(-\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$
- At time t seconds, the position vector of S is **s** metres

(b) Write down, in terms of t, **i** and **j**, an expression for **s**

(1)

(c) Show that

$$\overrightarrow{SR} = [(2t - 27)\mathbf{i} + (3t - 24)\mathbf{j}]\text{ m}$$

(2)

(d) Find the time when the distance between R and S is a minimum.

(3)

1 Find the velocity of R

R moves from $-2\mathbf{i}+\mathbf{j}$ to $9\mathbf{i}+23\mathbf{j}$ in 11 s. Its displacement is $11\mathbf{i}+22\mathbf{j}$, so its velocity is $\mathbf{i}+2\mathbf{j}$.

2 Write r

$\mathbf{r}=(-2\mathbf{i}+\mathbf{j})+t(\mathbf{i}+2\mathbf{j})=(t-2)\mathbf{i}+(2t+1)\mathbf{j}$.

3 Write s

Point C has position vector $25\mathbf{i}+25\mathbf{j}$. Since S moves with velocity $-\mathbf{i}-\mathbf{j}$, $\mathbf{s}=(25-t)\mathbf{i}+(25-t)\mathbf{j}$.

4 Find SR vector

$\overrightarrow{SR}=\mathbf{r}-\mathbf{s}=[(t-2)-(25-t)]\mathbf{i}+[(2t+1)-(25-t)]\mathbf{j}$.

5 Simplify SR

Therefore $\overrightarrow{SR}=(2t-27)\mathbf{i}+(3t-24)\mathbf{j}$ m.

6 Minimise the distance squared

The distance squared is $d^2=(2t-27)^2+(3t-24)^2$.

7 Expand the quadratic

$d^2=4t^2-108t+729+9t^2-144t+576=13t^2-252t+1305$.

8 Find the minimum time

The quadratic is minimum at $t=\dfrac{252}{2\cdot 13}=\dfrac{126}{13}=9.69\ldots$ s.

FINAL ANSWER

$(a)\ \mathbf{r}=(t-2)\mathbf{i}+(2t+1)\mathbf{j}$. $(b)\ \mathbf{s}=(25-t)\mathbf{i}+(25-t)\mathbf{j}$. $(c)\ \overrightarrow{SR}=(2t-27)\mathbf{i}+(3t-24)\mathbf{j}$. $(d)\ t=\dfrac{126}{13}\ \text{s}\approx 9.7\ \text{s}$.