Elite Mathematics
Edexcel International A Level

IAL MECHANICS 1 · WME01

WME01/01 January 2023 Worked Solutions

January 2023 paper rebuilt with the worked solution after each question.

8	2019–2026	Single-paper solutions
paper questions	18 exam sessions	checked by Dr Eslam



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PAST PAPER

WME01/01 January 2023

January 2023

8	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

Question 1

10 marks | Kinematics Graphs

1. A train travels along a straight horizontal track between two stations A and B.

The train starts from rest at station A and accelerates uniformly for T seconds until it reaches a speed of $20\,\text{m s}^{-1}$

The train then travels at a constant speed of $20\,\text{m s}^{-1}$ for 3 minutes before decelerating uniformly until it comes to rest at station B.

The magnitude of the acceleration of the train is twice the magnitude of the deceleration.

(a) On the axes below, sketch a speed–time graph to illustrate the motion of the train as it moves from station A to station B.



If you need to redraw your graph, use the axes on page 3

(3)

Stations A and B are 4.8 km apart.

(b) Find the value of T

(5)

(c) Find the acceleration of the train during the first T seconds of its motion.

(2)

Only use these axes if you need to redraw your graph.



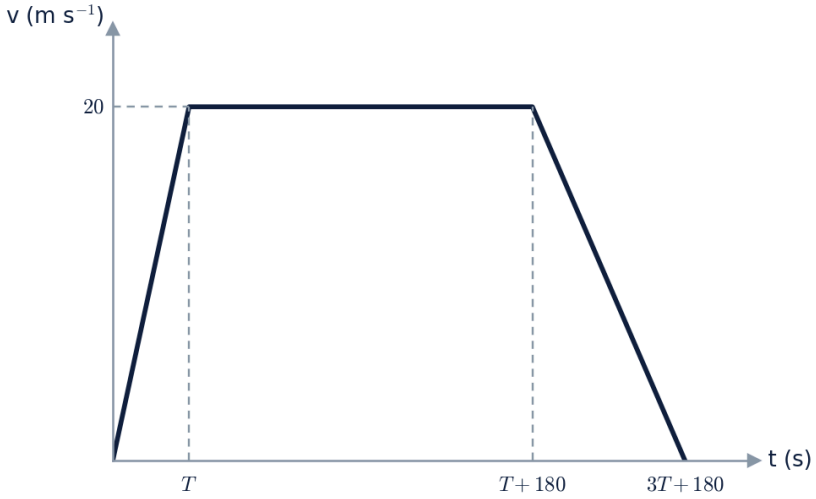
(Total for Question 1 is 10 marks)

1 Relate the acceleration and deceleration times

The train reaches $20\ \mathrm{m\,s^{-1}}$ in T seconds. Since the acceleration magnitude is twice the deceleration magnitude, the time needed to decelerate from 20 to 0 is $2T$.

2 Describe the graph

The speed-time graph rises linearly from $(0, 0)$ to $(T, 20)$, stays horizontal at speed 20 from T to $T + 180$, then falls linearly to zero at $3T + 180$.



3 Use area under the graph

The distance from A to B is $4.8\ \mathrm{km} = 4800\ \mathrm{m}$. The area is $\frac{1}{2}(T)(20) + 20(180) + \frac{1}{2}(2T)(20)$.

4 Solve for T

$4800 = 10T + 3600 + 20T$, so $1200 = 30T$ and $T = 40\ \mathrm{s}$.

5 Find the acceleration

During the first T seconds, $20 = aT$. With $T = 40$, $a = \frac{20}{40} = 0.5\ \mathrm{m\,s^{-2}}$.

✓ FINAL ANSWER

(a) Speed-time graph with times T, $T + 180$, $3T + 180$ and top speed 20.

$(b)\ T = 40\ \mathrm{s}$. $(c)\ a = 0.5\ \mathrm{m\,s^{-2}}$.

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Question 2

8 marks | Momentum, Impulse & Collisions

2. Two particles, A and B, are moving in a straight line in opposite directions towards each other on a smooth horizontal surface when they collide directly.

Particle A has mass $3m$ kg and particle B has mass m kg.

Immediately before the collision, both particles have a speed of $1.5\,\text{m s}^{-1}$

Immediately after the collision, the direction of motion of A is unchanged and the difference between the speed of A and speed of B is $1\,\text{m s}^{-1}$

(a) Find (i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision.

(5)

(b) Find, in terms of m, the magnitude of the impulse exerted on B in the collision.

(3)

WORKED SOLUTION

Question 2 · WME01/01 January 2023

1 Choose a positive direction

Take the original direction of A as positive. Before the collision, A has velocity 1.5 and B has velocity -1.5.

2 Use the speed difference

After the collision, A keeps the same direction. Let A's speed be v. The valid motion has B moving in A's direction with speed $v+1$.

3 Use conservation of momentum

$3m(1.5)+m(-1.5)=3mv+m(v+1)$.

4 Find the two speeds

$4.5m-1.5m=3mv+mv+m$, so $3=4v+1$. Hence $v=0.5$, and B's speed is $v+1=1.5$.

5 Find the impulse on B

For B, impulse equals change in momentum: $I=m(1.5-(-1.5))=3m$.

6 State the magnitude

The magnitude of the impulse is $3m\ \text{N s}$.

FINAL ANSWER

$(a)(i)$ speed of $\mathrm{A} = 0.5\ \mathrm{m\,s^{-1}}$. $(a)(ii)$ speed of $\mathrm{B} = 1.5\ \mathrm{m\,s^{-1}}$. $(b)\ I = 3m\ \mathrm{N\,s}$.

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Question 3

10 marks | Working with Vectors

3. A particle P is moving with constant acceleration $(-4\mathbf{i} + \mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, P has velocity $(14\mathbf{i} - 5\mathbf{j})\,\text{m s}^{-1}$

(a) Find the speed of P at time $t = 2$ seconds.

(3)

(b) Find the size of the angle between the direction of $\mathbf{i}$ and the direction of motion of P at time $t = 2$ seconds.

(3)

At time $t = T$ seconds, P is moving in the direction of vector $(2\mathbf{i} - 3\mathbf{j})$

(c) Find the value of T

(4)

WORKED SOLUTION

Question 3 · WME01/01 January 2023

1 Find the velocity at t = 2

$\mathbf{v} = (14\mathbf{i} - 5\mathbf{j}) + 2(-4\mathbf{i} + \mathbf{j}) = 6\mathbf{i} - 3\mathbf{j}$.

2 Find the speed

The speed is $\sqrt{6^2 + (-3)^2} = \sqrt{45} = 3\sqrt{5}\,\text{m s}^{-1}$.

3 Find the angle with i

The velocity $6\mathbf{i} - 3\mathbf{j}$ is below the positive $\mathbf{i}$ direction. The acute angle satisfies $\tan\theta = \dfrac{3}{6}$.

4 State the angle

$\theta = 26.6°$, so the size of the angle is $27°$ to the nearest degree.

5 Write the velocity at time T

$\mathbf{v}(T) = (14 - 4T)\mathbf{i} + (-5 + T)\mathbf{j}$.

6 Use the given direction

The direction is parallel to $2\mathbf{i} - 3\mathbf{j}$, so the component ratio must match: $\dfrac{14 - 4T}{2} = \dfrac{-5 + T}{-3}$.

7 Solve for T

$-3(14 - 4T) = 2(-5 + T)$, so $-42 + 12T = -10 + 2T$. Hence $10T = 32$ and $T = 3.2$.

FINAL ANSWER

$(a)\ 3\sqrt{5}\ \mathrm{m\,s^{-1}} \approx 6.7\ \mathrm{m\,s^{-1}}.\quad (b)\ 27^\circ.\quad (c)\ T = 3.2.$

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Question 4

8 marks | Moments

4.

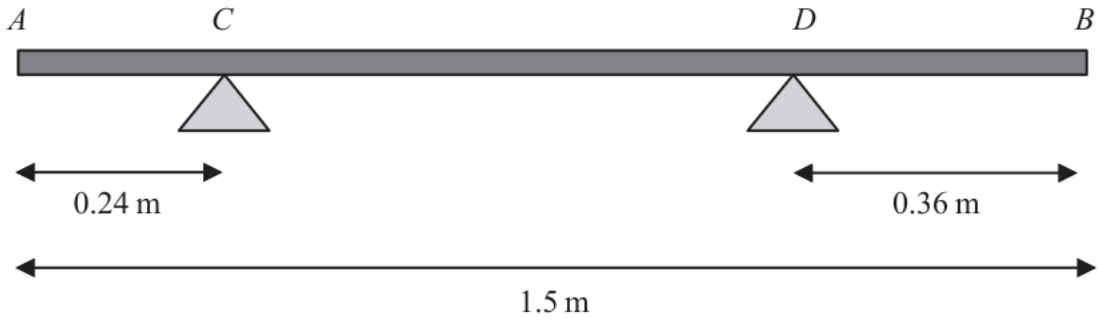


Figure 1

A branch AB, of length 1.5 m, rests horizontally in equilibrium on two supports.

The two supports are at the points C and D, where $AC = 0.24\,\text{m}$ and $DB = 0.36\,\text{m}$, as shown in Figure 1.

When a force of 150 N is applied vertically upwards at B, the branch is on the point of tilting about C.

When a force of 225 N is applied vertically downwards at B, the branch is on the point of tilting about D.

The branch is modelled as a non-uniform rod AB of weight W newtons.

The distance from the point C to the centre of mass of the rod is x metres.

Use the model to find

(i) the value of W

(ii) the value of x

(8)

WORKED SOLUTION

Question 4 · WME01/01 January 2023

1 Set the distances

The branch has length 1.5 m. Since $AC = 0.24$ m and $DB = 0.36$ m, $CB = 1.26$ m and $CD = 0.90$ m. The centre of mass is x m from C.

2 Use the first tilting condition

With the 150 N upward force at B, the branch is on the point of tilting about C, so the support at D has zero reaction.

3 **Take moments about C**

The upward force at B balances the weight moment: $150(1.26) = Wx$. Thus $Wx = 189$.

4 **Use the second tilting condition**

With the 225 N downward force at B, the branch is on the point of tilting about D, so the support at C has zero reaction.

5 **Take moments about D**

The distance from D to the centre of mass is $0.90 - x$. Hence $225(0.36) = W(0.90 - x)$.

6 **Solve the simultaneous equations**

From the second equation, $81 = 0.90W - Wx$. Since $Wx = 189$, $81 = 0.90W - 189$, so $0.90W = 270$ and $W = 300\ \text{N}$.

7 **Find x**

Using $Wx = 189$, $300x = 189$, so $x = 0.63\ \text{m}$.

✓ **FINAL ANSWER**

$(i)\ W = 300\ \text{N}.\quad (ii)\ x = 0.63\ \text{m}.$

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Question 5

9 marks | Constant Acceleration in 1D

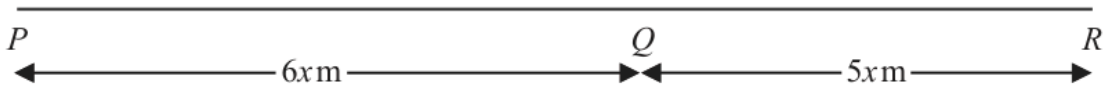


Figure 2

Three points P, Q and R are on a horizontal road where PQR is a straight line.

The point Q is between P and R, with $PQ = 6x$ metres and $QR = 5x$ metres, as shown in Figure 2.

A vehicle moves along the road from P to Q with constant acceleration.

The vehicle is modelled as a particle.

At time $t = 0$, the vehicle passes P with speed $u\,\text{m s}^{-1}$

At time $t = 12$ s, the vehicle passes Q with speed $2u\,\text{m s}^{-1}$

Using the model,

(a) show that $x = 3u$

(2)

As the vehicle passes Q, the acceleration of the vehicle changes instantaneously to $1.5\,\text{m s}^{-2}$

The vehicle continues to move with a constant acceleration of $1.5\,\text{m s}^{-2}$ and passes R with speed $3u\,\text{m s}^{-1}$

Using the model,

(b) find the value of u,

(3)

(c) find the distance travelled by the vehicle during the first 14 seconds after passing P

(4)

WORKED SOLUTION

Question 5 · WME01/01 January 2023

1 **Use average speed from P to Q**

From P to Q, the speed changes uniformly from u to $2u$ over 12 s. The average speed is $\frac{u + 2u}{2} = \frac{3u}{2}$.

2 **Show x = 3u**

Distance $PQ = 6x$, so $6x = \frac{3u}{2} \cdot 12 = 18u$. Hence $x = 3u$.

3 **Use Q to R**

The distance $QR = 5x = 15u$. The speed changes from $2u$ to $3u$ with acceleration $1.5\ \mathrm{m\,s^{-2}}$.

4 **Find u**

Use $v^2 = u^2 + 2as$: $(3u)^2 = (2u)^2 + 2(1.5)(15u)$. Thus $9u^2 = 4u^2 + 45u$, so $5u^2 = 45u$ and $u = 9$.

5 **Find the first 12 seconds distance**

In the first 12 seconds, the distance is $PQ = 6x = 18u = 18(9) = 162$ m.

6 **Find the next 2 seconds distance**

From $t = 12$ to $t = 14$, the vehicle starts this part at speed $2u = 18$ and accelerates at 1.5. Distance $= 18(2) + \frac{1}{2}(1.5)(2^2) = 36 + 3 = 39$ m.

7 **Add the distances**

Total distance in the first 14 seconds is $162 + 39 = 201\ \mathrm{m}$.

✓ **FINAL ANSWER**

$(a)\ x = 3u.$ $(b)\ u = 9.$ $(c)\ 201\ \mathrm{m}.$

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Question 6

8 marks | Forces

6.

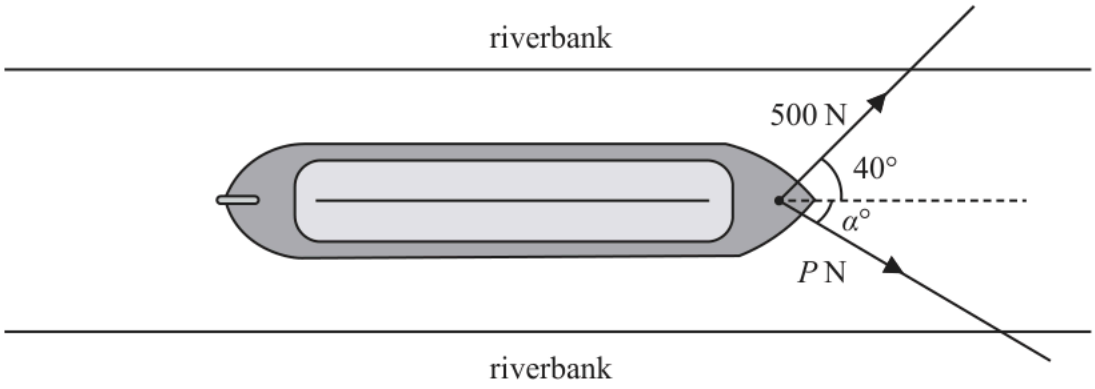


Figure 3

A boat is pulled along a river at a constant speed by two ropes.

The banks of the river are parallel and the boat travels horizontally in a straight line, parallel to the riverbanks.

- The tension in the first rope is 500 N acting at an angle of 40° to the direction of motion, as shown in Figure 3.
- The tension in the second rope is P newtons, acting at an angle of $\alpha°$ to the direction of motion, also shown in Figure 3.
- The resistance to motion of the boat as it moves through the water is a constant force of magnitude 900 N

The boat is modelled as a particle. The ropes are modelled as being light and lying in a horizontal plane.

Use the model to find

(i) the value of α

(ii) the value of P

(8)

WORKED SOLUTION

Question 6 · WME01/01 January 2023

1 **Resolve perpendicular to the motion**

The boat travels straight, so the sideways components balance: $500 \sin 40° = P \sin \alpha$.

2 **Resolve along the motion**

The speed is constant, so the forward components balance the resistance:
$500\cos 40^\circ + P\cos\alpha = 900$.

3 **Eliminate P**

From the two equations, $P\sin\alpha = 500\sin 40^\circ$ and $P\cos\alpha = 900 - 500\cos 40^\circ$.

4 **Find alpha**

Divide the sine equation by the cosine equation: $\tan\alpha = \dfrac{500\sin 40^\circ}{900 - 500\cos 40^\circ}$. This gives $\alpha = 31.9^\circ$, so $\alpha = 32^\circ$ approximately.

5 **Find P**

Use $P\sin\alpha = 500\sin 40^\circ$: $P = \dfrac{500\sin 40^\circ}{\sin 31.9^\circ} = 608.7\ldots\ \text{N}$. Hence $P \approx 610\ \text{N}$.

✓ **FINAL ANSWER**

$(i)\ \alpha = 32^\circ$ **approximately.** $\quad (ii)\ P = 610\ \text{N}$ **approximately**.

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Question 7

7 marks | Newton's Second Law

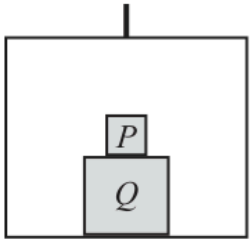


Figure 4

A simple lift operates by means of a vertical cable which is attached to the top of the lift.
The lift has mass m
A box Q is placed on the floor of the lift.
A box P is placed directly on top of box Q, as shown in Figure 4.

The cable is modelled as being light and inextensible and air resistance is modelled as being negligible.

The tension in the cable is $\frac{42mg}{5}$

The lift and its contents move vertically upwards with acceleration $\frac{2g}{5}$

Using the model,

(a) find, in terms of m, the combined mass of boxes P and Q

(4)

During the motion of the lift, the force exerted on box P by box Q is $\frac{14mg}{5}$

Using the model,

(b) find, in terms of m, the mass of box P

(3)

WORKED SOLUTION

Question 7 · WME01/01 January 2023

1 Define the box mass

Let the combined mass of boxes P and Q be M. The lift itself has mass m, so the total moving mass is $m + M$.

2 Apply Newtons second law to the whole system

Taking upward as positive, $\frac{42}{5}mg - (m+M)g = (m+M)\frac{2g}{5}$.

3 **Find M**

Move the weight term to the right: $\frac{42}{5}mg = (m+M)\left(g+\frac{2g}{5}\right) = \frac{7g}{5}(m+M)$. Hence $42m = 7(m+M)$ and $M = 5m$.

4 **Let the mass of P be p**

For box P alone, the upward force from Q is $\frac{14}{5}mg$, while its weight is pg. The acceleration is still $\frac{2g}{5}$ upward.

5 **Apply Newtons second law to P**

$\frac{14}{5}mg - pg = p\frac{2g}{5}$.

6 **Find p**

Divide by g: $\frac{14}{5}m = p + \frac{2p}{5} = \frac{7p}{5}$. Therefore $p = 2m$.

✓ **FINAL ANSWER**

(a) combined mass of P and Q $= 5m$. (b) mass of P $= 2m$.

Question 8

15 marks | Resolving Forces, Inclined Planes

8.

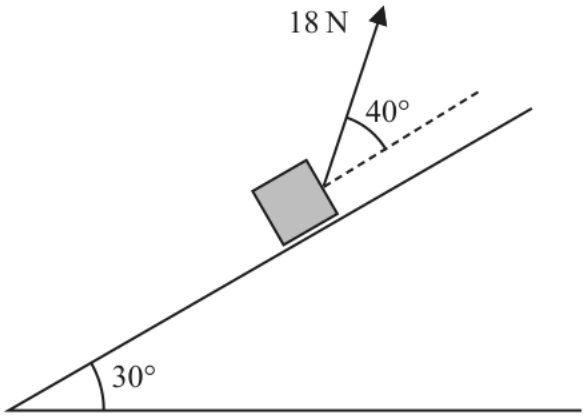


Figure 5

A parcel of mass 2 kg is pulled up a rough inclined plane by the action of a constant force.

The force has magnitude 18 N and acts at an angle of 40° to the plane.

The line of action of the force lies in a vertical plane containing a line of greatest slope of the inclined plane.

The plane is inclined at an angle of 30° to the horizontal, as shown in Figure 5.

The coefficient of friction between the plane and the parcel is 0.3

The parcel is modelled as a particle P

(a) Find the acceleration of P

(8)

The points A and B lie on a line of greatest slope of the plane, where $AB = 5\,\text{m}$ and B is above A. Particle P passes through A with speed $2\,\text{m s}^{-1}$ in the direction AB.

(b) Find the speed of P as it passes through B.

(3)

The force of 18 N is removed at the instant P passes through B. As a result, P comes to rest at the point C.

(c) Determine whether P will remain at rest at C. You must show all stages of your working clearly.

(4)

1 Resolve perpendicular to the plane

The force of 18 N is angled away from the plane, so $R + 18\sin 40° = 2g\cos 30°$. Hence $R = 2g\cos 30° - 18\sin 40°$.

2 Use friction

The parcel moves up the plane, so friction acts down the plane. With $\mu = 0.3$, $F = 0.3R$.

3 Resolve parallel to the plane

Taking up the plane as positive: $18\cos 40° - F - 2g\sin 30° = 2a$.

4 Substitute for R and F

$18\cos 40° - 0.3(2g\cos 30° - 18\sin 40°) - 2g\sin 30° = 2a$.

5 Find the acceleration

Using $g = 9.8$, this gives $a = 1.18\ldots\ \text{m s}^{-2}$ up the plane.

6 Find the speed at B

From A to B, $u = 2$, $s = 5$ and $a = 1.18\ldots$. Use $v^2 = u^2 + 2as$: $v^2 = 2^2 + 2(1.18\ldots)(5)$.

7 State the speed

$v = 3.98\ldots\ \text{m s}^{-1}$, about $4.0\ \text{m s}^{-1}$.

8 Check rest after the force is removed

At C, with the pulling force removed, $R = 2g\cos 30°$. The maximum friction is $0.3R = 0.3(2g\cos 30°) = 5.09\ldots$ N.

9 Compare with the downslope weight component

The component of weight down the plane is $2g\sin 30° = 9.8$ N. Since $9.8 > 5.09\ldots$, friction is not large enough to hold P at rest, so P will not remain at rest at C.

FINAL ANSWER

$(a)\ a = 1.18\ \text{m s}^{-2}$ up the plane. $(b)\ v = 3.98\ \text{m s}^{-1} \approx 4.0\ \text{m s}^{-1}$. (c) P will not remain at rest at C.