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Elite Mathematics
Edexcel International A Level

IAL MECHANICS 1 · WME01

WME01/01 October 2022 Worked Solutions

October 2022 paper rebuilt with the worked solution after each question.

8	2019–2026	Single-paper solutions
paper questions	18 exam sessions	checked by Dr Eslam



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PAST PAPER

WME01/01 October 2022

October 2022

8	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

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Question 1

5 marks | Momentum, Impulse & Collisions

1. A railway truck S of mass 20 tonnes is moving along a straight horizontal track when it collides with another railway truck T of mass 30 tonnes which is at rest. Immediately before the collision the speed of S is $4\,\text{m s}^{-1}$
As a result of the collision, the two railway trucks join together.

Find

(a) the common speed of the railway trucks immediately after the collision,

(2)

(b) the magnitude of the impulse exerted on S in the collision, stating the units of your answer.

(3)

WORKED SOLUTION

Question 1 · WME01/01 October 2022

1 Use conservation of momentum

The trucks join together, so they move with a common speed v. Using tonnes consistently,

$20(4) + 30(0) = (20 + 30)v$.

2 Find the common speed

$80 = 50v$, so $v = 1.6\ \text{m s}^{-1}$.

3 Use change in momentum of S

Truck S has mass $20\,000$ kg. Its speed changes from 4 to $1.6\ \text{m s}^{-1}$.

4 Find the impulse magnitude

$I = 20000|1.6 - 4| = 20000(2.4) = 48000\ \text{N s}$.

✓ **FINAL ANSWER**

$(a)\ v = 1.6\ \text{m s}^{-1}$. $(b)\ I = 48000\ \text{N s}$.

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Question 2

6 marks | Moments

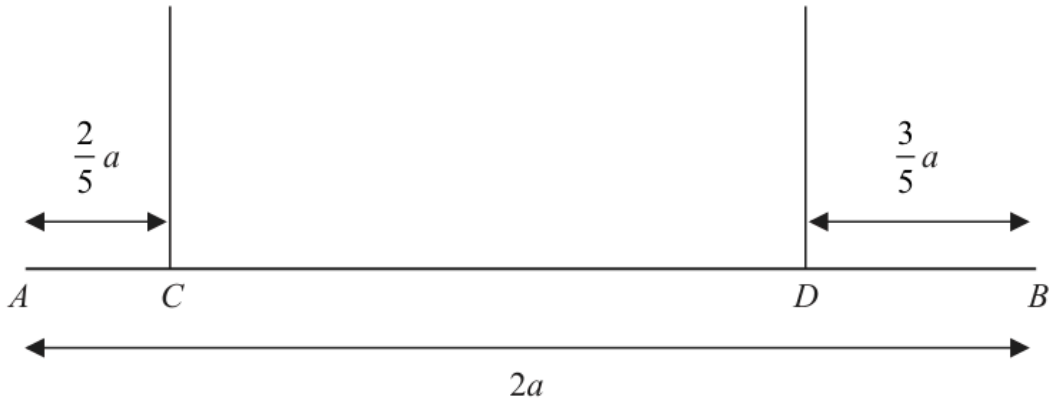


Figure 1

A uniform rod AB has length $2a$ and mass M. The rod is held in equilibrium in a horizontal position by two vertical light strings which are attached to the rod at C and D, where $AC = \frac{2}{5}a$ and $DB = \frac{3}{5}a$, as shown in Figure 1.

A particle P is placed on the rod at B.

The rod remains horizontal and in equilibrium.

(a) Find, in terms of M, the largest possible mass of the particle P

(3)

Given that the mass of P is $\frac{1}{2}M$

(b) find, in terms of M and g, the tension in the string that is attached to the rod at C.

(3)

WORKED SOLUTION

Question 2 · WME01/01 October 2022

1 Set the distances

The rod length is $2a$. The centre of mass is at the midpoint, distance a from A. Also $AC = \frac{2a}{5}$ and $AD = 2a - \frac{3a}{5} = \frac{7a}{5}$.

2 Use the largest mass condition

For the largest possible mass at B, the string at C is just slack and the rod is about to tilt about D.

3 **Take moments about D**

Let the particle mass be X. The distance from D to B is $\frac{3a}{5}$ and the distance from D to the rod's centre of mass is $\frac{2a}{5}$. Hence $Xg\left(\frac{3a}{5}\right) = Mg\left(\frac{2a}{5}\right)$.

4 **Find the largest mass**

Cancel $ga/5$: $3X = 2M$, so $X = \frac{2M}{3}$.

5 **Use the given mass**

Now the particle has mass $\frac{1}{2}M$. Let the tensions at C and D be T_C and T_D.

6 **Take moments about D**

About D, $T_C(a) + \frac{1}{2}Mg\left(\frac{3a}{5}\right) = Mg\left(\frac{2a}{5}\right)$.

7 **Find the tension at C**

$T_C a + \frac{3}{10}Mga = \frac{2}{5}Mga$, so $T_C = \frac{1}{10}Mg$.

✓ **FINAL ANSWER**

(a) largest mass $= \frac{2M}{3}$. $\quad (b)\ T_C = \frac{1}{10}Mg$.

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Question 3

11 marks | Resolving Forces, Inclined Planes

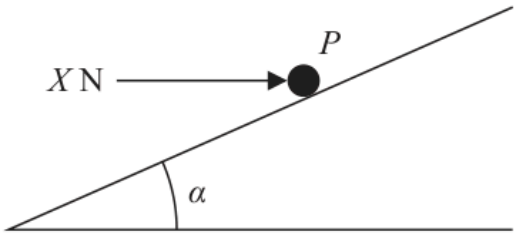


Figure 2

A rough plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$

A particle P of mass 2 kg is held in equilibrium on the plane by a horizontal force of magnitude X newtons, as shown in Figure 2. The force acts in a vertical plane which contains a line of greatest slope of the inclined plane.

(a) Show that when $X = 14.7$ there is no frictional force acting on P

(3)

The coefficient of friction between P and the plane is 0.5

(b) Find the smallest possible value of X.

(8)

WORKED SOLUTION

Question 3 · WME01/01 October 2022

1 Use the trig ratios

Since $\tan \alpha = \frac{3}{4}$, $\sin \alpha = \frac{3}{5}$ and $\cos \alpha = \frac{4}{5}$.

2 Check the no-friction case

If there is no friction, the component of the horizontal force up the plane balances the component of weight down the plane: $X \cos \alpha = 2g \sin \alpha$.

3 Substitute X = 14.7

$14.7 \cdot \frac{4}{5} = 11.76$, and $2(9.8) \cdot \frac{3}{5} = 11.76$. The components balance exactly, so the frictional force is zero.

4 **Set the direction for smallest X**

For the smallest possible X, the particle is about to slide down the plane, so friction acts up the plane.

5 **Resolve perpendicular to the plane**

The horizontal force increases the normal reaction, so $R = 2g\cos\alpha + X\sin\alpha$.

6 **Use limiting friction**

With $\mu = 0.5$, the limiting friction is $F = 0.5R = 0.5(2g\cos\alpha + X\sin\alpha)$.

7 **Resolve along the plane**

At the limiting case, $X\cos\alpha + F = 2g\sin\alpha$.

8 **Solve for X**

Substitute the exact trig values: $\frac{4}{5}X + 0.5\left(2g\cdot\frac{4}{5} + X\cdot\frac{3}{5}\right) = 2g\cdot\frac{3}{5}$. This gives $\frac{11}{10}X + \frac{4g}{5} = \frac{6g}{5}$, so $X = \frac{4g}{11} = 3.56\ldots\ \text{N}$.

✓ **FINAL ANSWER**

(a) When $X = 14.7$, $F = 0$.

(b) $X_{\min} = \frac{4g}{11} = 3.56\ldots\ \text{N}$.

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Question 4

6 marks | Newton's Second Law

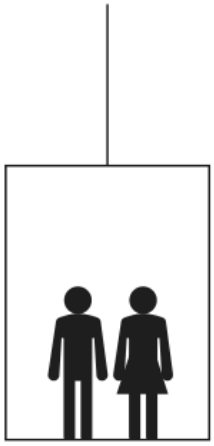


Figure 3

Two children, Alan and Bhavana, are standing on the horizontal floor of a lift, as shown in Figure 3.

The lift has mass 250 kg. The lift is raised vertically upwards with constant acceleration by a vertical cable which is attached to the top of the lift. The cable is modelled as being light and inextensible. While the lift is accelerating upwards, the tension in the cable is 3616 N.

As the lift accelerates upwards, the floor of the lift exerts a force of magnitude 565 N on Alan and a force of magnitude 226 N on Bhavana.

Air resistance is modelled as being negligible and Alan and Bhavana are modelled as particles.

(a) By considering the forces acting on the lift only, find the acceleration of the lift.

(3)

(b) Find the mass of Alan.

(3)

WORKED SOLUTION

Question 4 · WME01/01 October 2022

1 **Consider the lift only**

For the lift, the upward force is the cable tension 3616 N. Downward forces on the lift are its own weight $250g$ and the contact forces from Alan and Bhavana, 565 N and 226 N.

2 **Apply Newtons second law**

Taking upward as positive: $3616 - 250g - 565 - 226 = 250a$.

3 **Find the acceleration**

$3616 - 2450 - 565 - 226 = 250a$, so $375 = 250a$ and $a = 1.5\ \text{m s}^{-2}$.

4 **Consider Alan**

Let Alan's mass be m_A. The upward force on Alan is 565 N and his weight is $m_A g$.

5 **Find Alan's mass**

$565 - m_A g = m_A(1.5)$, so $565 = m_A(9.8 + 1.5) = 11.3 m_A$. Hence $m_A = 50\ \text{kg}$.

✓ **FINAL ANSWER**

$(a)\ a = 1.5\ \text{m s}^{-2}$. $\quad (b)\ \text{mass of Alan} = 50\ \text{kg}$.

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Question 5

9 marks | Constant Acceleration in 1D

5. A small ball is projected vertically upwards with speed $29.4\,\text{m s}^{-1}$ from a point A which is $19.6\,\text{m}$ above horizontal ground.

The ball is modelled as a particle moving freely under gravity until it hits the ground. It is assumed that the ball does not rebound.

(a) Find the distance travelled by the ball while its speed is less than $14.7\,\text{m s}^{-1}$

(3)

(b) Find the time for which the ball is moving with a speed of more than $29.4\,\text{m s}^{-1}$

(3)

(c) Sketch a speed-time graph for the motion of the ball from the instant when it is projected from A to the instant when it hits the ground. Show clearly where your graph meets the axes.

(3)

WORKED SOLUTION

Question 5 · WME01/01 October 2022

1 Find one side of the slow-speed interval

The speed is less than $14.7\ \text{m s}^{-1}$ near the highest point. From speed 14.7 to speed 0, use $v^2 = u^2 + 2as$: $0 = 14.7^2 - 2gs$.

2 Find the total distance

This gives $s = 11.025$ m on the way up. The same distance occurs on the way down before the speed reaches 14.7 again, so the total distance is $2s = 22.05\ \text{m}$, about $22.1\ \text{m}$.

3 Interpret speed greater than 29.4

The ball has speed 29.4 again when it passes A on the way down. It is faster than 29.4 only after passing A downward until it hits the ground.

4 Find that time

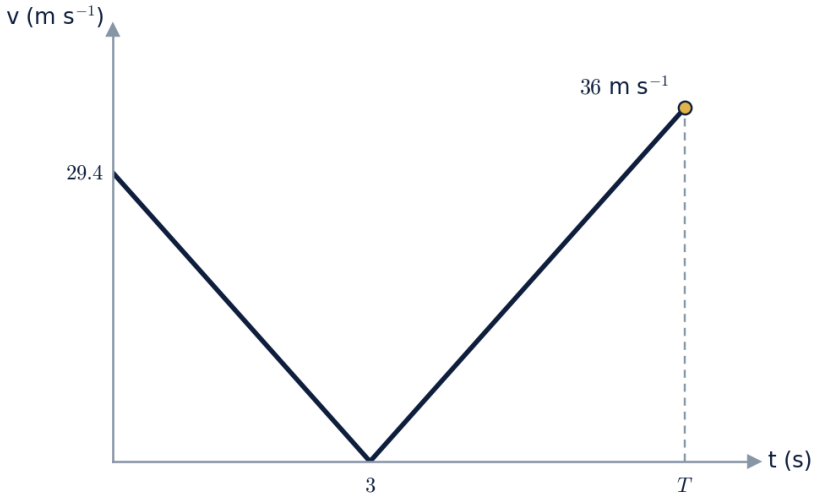
From A to the ground, taking downward as positive, $19.6 = 29.4t + \frac{1}{2}gt^2$.

5 Solve the time equation

$19.6 = 29.4t + 4.9t^2$, giving the positive root $t = 0.606\ldots\ \text{s}$.

6 Describe the speed-time graph

The graph starts at speed 29.4 when $t = 0$, decreases linearly to speed 0 at $t = 3$, then increases linearly until the ball hits the ground. The final speed is greater than 29.4, so the right-hand arm ends above the starting height on the speed axis.



FINAL ANSWER

(a) 22.1 m approximately. (b) 0.606 s approximately. (c) V-shaped speed-time graph through $(0, 29.4)$ and $(3, 0)$, ending above 29.4.

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Question 6

9 marks | Working with Vectors

6. [*In this question,* **i** *and* **j** *are horizontal unit vectors.*]

A particle A of mass 0.5 kg is at rest on a smooth horizontal plane.

At time $t = 0$, two forces, $\mathbf{F}_1 = (-3\mathbf{i} + 2\mathbf{j})$ N and $\mathbf{F}_2 = (p\mathbf{i} + q\mathbf{j})$ N, where p and q are constants, are applied to A.

Given that A moves in the direction of the vector $(\mathbf{i} - 2\mathbf{j})$,

(a) show that $2p + q - 4 = 0$

(4)

Given that $p = 5$

(b) find the speed of A at time $t = 4$ seconds.

(5)

WORKED SOLUTION

Question 6 · WME01/01 October 2022

1 Find the resultant force

$\mathbf{F} = (-3\mathbf{i} + 2\mathbf{j}) + (p\mathbf{i} + q\mathbf{j}) = (p - 3)\mathbf{i} + (q + 2)\mathbf{j}$.

2 Use the direction of motion

Since the particle starts from rest, its motion is in the direction of the resultant force. This direction is $\mathbf{i} - 2\mathbf{j}$, so $\dfrac{q+2}{p-3} = -2$.

3 Show the relation

$q + 2 = -2(p - 3) = -2p + 6$, hence $2p + q - 4 = 0$.

4 Find q when p = 5

Substitute $p = 5$ into $2p + q - 4 = 0$: $10 + q - 4 = 0$, so $q = -6$.

5 Find the resultant force

The resultant force is $(5 - 3)\mathbf{i} + (-6 + 2)\mathbf{j} = 2\mathbf{i} - 4\mathbf{j}$ N.

6 Find acceleration

The mass is 0.5 kg, so $\mathbf{a} = \dfrac{2\mathbf{i} - 4\mathbf{j}}{0.5} = 4\mathbf{i} - 8\mathbf{j}$.

7 **Find velocity at t = 4**

Starting from rest, $\mathbf{v} = \mathbf{a}t = (4\mathbf{i} - 8\mathbf{j})4 = 16\mathbf{i} - 32\mathbf{j}$.

8 **Find the speed**

Speed $= \sqrt{16^2 + (-32)^2} = \sqrt{1280} = 16\sqrt{5}\ \mathrm{m\,s^{-1}}$, about $36\ \mathrm{m\,s^{-1}}$.

✓ **FINAL ANSWER**

$(a)\ 2p + q - 4 = 0.$ $(b)\ \text{speed} = 16\sqrt{5}\ \mathrm{m\,s^{-1}} \approx 36\ \mathrm{m\,s^{-1}}$.

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Question 7

13 marks | Newton's Second Law

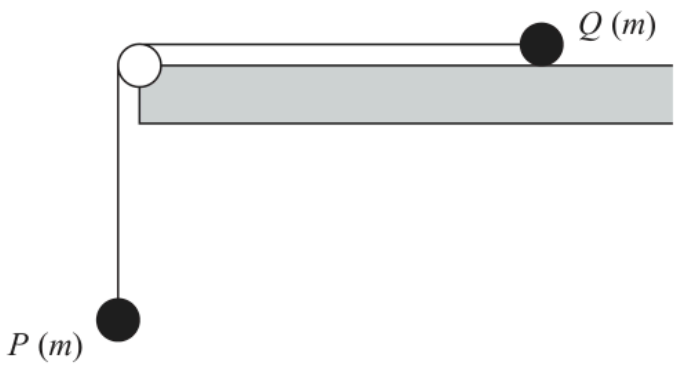


Figure 4

A particle P of mass m is attached to one end of a light inextensible string. Another particle Q, also of mass m, is attached to the other end of the string. The string passes over a small smooth pulley which is fixed at the edge of a rough horizontal table. Particle Q is held at rest on the table and particle P hangs vertically below the pulley with the string taut, as shown in Figure 4.

The pulley, P and Q all lie in the same vertical plane.

The coefficient of friction between Q and the table is μ, where $\mu < 1$

Particle Q is released from rest.

The tension in the string before Q hits the pulley is kmg, where k is a constant.

(a) Find k in terms of μ.

(7)

Given that Q is initially a distance d from the pulley,

(b) find, in terms of d, g and μ, the time taken by Q, after release, to reach the pulley.

(4)

(c) Describe what would happen if $\mu \geqslant 1$, giving a reason for your answer.

(2)

WORKED SOLUTION

Question 7 · WME01/01 October 2022

1 **Write the friction on Q**

For Q on the horizontal table, $R = mg$, so the friction is $F = \mu mg$.

2 **Apply Newtons second law to P**

P moves downward and Q moves towards the pulley. For P, taking downward as positive: $mg - T = ma$.

3 **Apply Newtons second law to Q**

For Q, taking motion towards the pulley as positive: $T - \mu mg = ma$.

4 **Use T = kmg**

From P, $a = g(1 - k)$. From Q, $a = g(k - \mu)$.

5 **Find k**

Equate the two accelerations: $1 - k = k - \mu$. Hence $2k = 1 + \mu$ and $k = \dfrac{1 + \mu}{2}$.

6 **Find the acceleration**

$a = g(1 - k) = g\left(1 - \dfrac{1 + \mu}{2}\right) = \dfrac{g(1 - \mu)}{2}$.

7 **Use motion from rest**

Q starts from rest and travels distance d, so $d = \dfrac{1}{2}at^2 = \dfrac{1}{2} \cdot \dfrac{g(1 - \mu)}{2}t^2$.

8 **Find the time**

$d = \dfrac{g(1 - \mu)t^2}{4}$, so $t = \sqrt{\dfrac{4d}{g(1 - \mu)}}$.

9 **Consider mu at least 1**

If $\mu \geq 1$, the maximum available friction on Q is at least mg, so it can balance the pull caused by the hanging particle. The system would not start moving.

FINAL ANSWER

$(a)\ k = \dfrac{1 + \mu}{2}$. $(b)\ t = \sqrt{\dfrac{4d}{g(1 - \mu)}}$. (c) If $\mu \geq 1$, the system would not move because friction can balance the pull from P.

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Question 8

16 marks | Working with Vectors

8. [*In this question,* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two ships, A and B, are moving with constant velocities.
The velocity of A is $(3\mathbf{i} + 12\mathbf{j})\,\text{km h}^{-1}$ and the velocity of B is $(p\mathbf{i} + q\mathbf{j})\,\text{km h}^{-1}$

(a) Find the speed of A.

(2)

The ships are modelled as particles.
At 12 noon, A is at the point with position vector $(-9\mathbf{i} + 6\mathbf{j})$ km and B is at the point with position vector $(16\mathbf{i} + 6\mathbf{j})$ km.

At time t hours after 12 noon,

$$\overrightarrow{AB} = [(25 - 12t)\mathbf{i} - 9t\mathbf{j}]\,\text{km}$$

(b) Find the value of p and the value of q.

(7)

(c) Find the bearing of A from B when the ships are 15 km apart, giving your answer to the nearest degree.

(7)

(Total 16 marks)

WORKED SOLUTION

Question 8 · WME01/01 October 2022

1 Find the speed of A

$|\mathbf{v}_A| = \sqrt{3^2 + 12^2} = \sqrt{153} = 3\sqrt{17}\,\text{km h}^{-1}$.

2 Write A position

At time t, $\mathbf{a} = (-9\mathbf{i} + 6\mathbf{j}) + t(3\mathbf{i} + 12\mathbf{j}) = (-9 + 3t)\mathbf{i} + (6 + 12t)\mathbf{j}$.

3 Write B position

$\mathbf{b} = (16\mathbf{i} + 6\mathbf{j}) + t(p\mathbf{i} + q\mathbf{j}) = (16 + pt)\mathbf{i} + (6 + qt)\mathbf{j}$.

4 Use AB vector

$\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = [25 + (p - 3)t]\mathbf{i} + [(q - 12)t]\mathbf{j}$.

5 **Compare components**

Given $\overrightarrow{AB} = (25 - 12t)\mathbf{i} - 9t\mathbf{j}$, compare coefficients: $p - 3 = -12$ and $q - 12 = -9$.

6 **Find p and q**

Therefore $p = -9$ and $q = 3$.

7 **Find the time when the distance is 15**

Use $|\overrightarrow{AB}| = 15$: $(25 - 12t)^2 + (-9t)^2 = 15^2$.

8 **Solve the distance equation**

This gives $225t^2 - 600t + 400 = 0$, so $(3t - 4)^2 = 0$ and $t = \dfrac{4}{3}$.

9 **Find BA at this time**

At $t = \dfrac{4}{3}$, $\overrightarrow{AB} = 9\mathbf{i} - 12\mathbf{j}$. Therefore $\overrightarrow{BA} = -9\mathbf{i} + 12\mathbf{j}$.

10 **Convert to a bearing**

A is north-west of B. The angle west of north satisfies $\tan\theta = \dfrac{9}{12}$, so $\theta = 36.9°$. Bearing $= 360° - 36.9° = 323°$ to the nearest degree.

✓ **FINAL ANSWER**

$(a)\ 3\sqrt{17}\ \text{km h}^{-1}$. $(b)\ p = -9,\ q = 3$. $(c)\ \text{bearing} = 323°$.