Elite Mathematics
Edexcel International A Level

IAL MECHANICS 1 · WME01

WME01/01 May/June 2022 Worked Solutions

May/June 2022 paper rebuilt with the worked solution after each question.

8	2019–2026	Single-paper solutions
paper questions	18 exam sessions	checked by Dr Eslam



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PAST PAPER

WME01/01 May/June 2022

May/June 2022

8	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WME01/01 MAY/JUNE 2022

Question 1

4 marks | Momentum, Impulse & Collisions

1. Two particles, P and Q, are moving towards each other in opposite directions along the same straight line when they collide directly. Immediately before the collision the speed of Q is $2u$. The mass of Q is $3m$ and the magnitude of the impulse exerted by P on Q in the collision is $4mu$.

Find

(a) the speed of Q immediately after the collision,

(3)

(b) the direction of motion of Q immediately after the collision.

(1)

WORKED SOLUTION

Question 1 · WME01/01 May/June 2022

1 Use impulse as change in momentum

Take the original direction of Q as positive. Q has mass $3m$ and initial velocity $2u$. Let its velocity after collision be v.

2 Set up the impulse equation

The impulse has magnitude $4mu$ and acts opposite to Q's original motion, so $4mu = 3m(2u - v)$.

3 Find the speed

Cancel m: $4u = 6u - 3v$. Thus $3v = 2u$ and $v = \dfrac{2u}{3}$.

4 State the direction

Since v is positive in Q's original direction, Q continues in the same direction after the collision.

FINAL ANSWER

(a) speed $= \dfrac{2u}{3}$. (b) same direction as before the collision.

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Question 2

7 marks | Constant Acceleration in 1D

2. A motorbike is moving with constant acceleration along a straight horizontal road.

The motorbike passes a point P and 10 seconds later passes a point Q.

The speed of the motorbike as it passes Q is $28\,\text{m s}^{-1}$

Given that $PQ = 220\,\text{m}$,

(a) find the acceleration of the motorbike,

(3)

(b) find the distance travelled by the motorbike during the fifth second after passing P

(4)

WORKED SOLUTION

Question 2 · WME01/01 May/June 2022

1 Find the speed at P

Let the speed at P be u. Over the 10 seconds from P to Q, the speed changes uniformly from u to 28, and the distance is 220 m.

2 Use average speed

$220 = \dfrac{u + 28}{2} \cdot 10 = 5(u + 28)$. Hence $u + 28 = 44$ and $u = 16\,\text{m s}^{-1}$.

3 Find the acceleration

Use $v = u + at$: $28 = 16 + 10a$, so $a = 1.2\,\text{m s}^{-2}$.

4 Write the distance from P after t seconds

$s = 16t + \dfrac{1}{2}(1.2)t^2 = 16t + 0.6t^2$.

5 Find the distance in the fifth second

The fifth second is from $t = 4$ to $t = 5$. $s(5) = 16(5) + 0.6(25) = 95$ and $s(4) = 16(4) + 0.6(16) = 73.6$.

6 Subtract

Distance in the fifth second $= 95 - 73.6 = 21.4\,\text{m}$.

FINAL ANSWER

$(a)\ a = 1.2\ \mathrm{m\,s^{-2}}.\quad (b)\ 21.4\ \mathrm{m}.$

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Question 3

7 marks | Newton's Second Law

3. A tractor of mass 6 tonnes is dragging a large block of mass 2 tonnes along rough horizontal ground. The cable connecting the tractor to the block is horizontal and parallel to the direction of motion.

The cable is modelled as being light and inextensible.

The driving force of the tractor is 7400 N and the resistance to the motion of the tractor is 200 N. The resistance to the motion of the block is R newtons, where R is a constant.

Given that the tension in the cable is 6000 N and the tractor is accelerating,

(a) find the value of R.

(6)

(b) State how you have used the fact that the cable is modelled as being inextensible.

(1)

WORKED SOLUTION

Question 3 · WME01/01 May/June 2022

1 Apply Newtons second law to the tractor

The tractor has mass 6000 kg. Its forward driving force is 7400 N, and the backward forces are 200 N resistance and 6000 N tension.

2 Find the acceleration

$7400 - 200 - 6000 = 6000a$, so $1200 = 6000a$ and $a = 0.2\ \text{m s}^{-2}$.

3 Apply Newtons second law to the block

The block has mass 2000 kg. The forward force is the tension 6000 N and the resistance is R N.

4 Find R

$6000 - R = 2000(0.2) = 400$. Therefore $R = 5600$ N.

5 Use the inextensible cable model

The cable is inextensible, so the tractor and block move together with the same acceleration.

FINAL ANSWER

$(a)\ R = 5600$ N. (b) The tractor and block have the same acceleration.

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Question 4

12 marks | Forces

4.

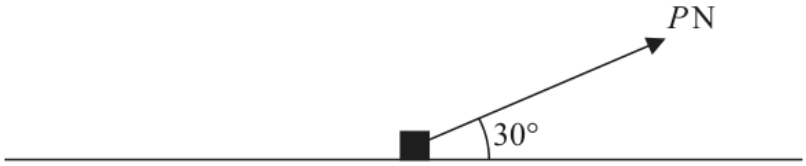


Figure 1

A small block of mass 5 kg lies at rest on a rough horizontal plane.

The coefficient of friction between the block and the plane is $\frac{3}{7}$

A force of magnitude P newtons is applied to the block in a direction which makes an angle of 30° with the plane, as shown in Figure 1.

The block is modelled as a particle.

Given that $P = 14$

(a) find the magnitude of the frictional force exerted on the block by the plane and describe what happens to the block, justifying your answer.

(6)

The value of P is now changed so that the block is on the point of slipping along the plane.

(b) Find the value of P

(6)

WORKED SOLUTION

Question 4 · WME01/01 May/June 2022

1 Find the normal reaction when P = 14

Resolve vertically. The upward component of the applied force is $14\sin 30°$, so $R + 14\sin 30° = 5g$.

2 Calculate R

$R + 7 = 49$, so $R = 42$ N.

3 Find the maximum friction

$F_{\max} = \mu R = \frac{3}{7}(42) = 18$ N.

4 **Compare with the horizontal pull**

The horizontal component of the applied force is $14\cos 30° = 12.1\ldots\ \text{N}$. Since this is less than the maximum possible friction, the block does not move.

5 **State the actual friction**

For equilibrium, the actual friction equals the horizontal pull, so $F = 12.1\ \text{N}$ approximately.

6 **Set up the limiting case**

When the block is on the point of slipping, friction is limiting. Resolve vertically: $R + P\sin 30° = 5g$, so $R = 5g - P\sin 30°$.

7 **Resolve horizontally**

At the limiting case, $P\cos 30° = \frac{3}{7}R = \frac{3}{7}(5g - P\sin 30°)$.

8 **Find P**

Substitute $g = 9.8$, $\sin 30° = \frac{1}{2}$: $P\cos 30° = \frac{3}{7}(49 - \frac{1}{2}P)$. Solving gives $P = 19.4\ldots\ \text{N}$.

✓ **FINAL ANSWER**

$(a)\ F = 12.1\ \text{N}$ approximately, and the block does not move. $(b)\ P = 19.4\ \text{N}$ approximately

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Question 5

9 marks | Moments

5.

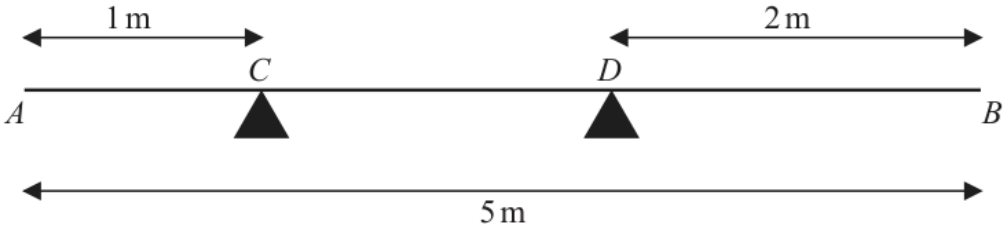


Figure 2

A uniform rod AB has length 5 m and mass 5 kg. The rod rests in equilibrium in a horizontal position on two supports C and D, where $AC = 1$ m and $DB = 2$ m, as shown in Figure 2.

A particle of mass 10 kg is placed on the rod at A and a particle of mass M kg is placed on the rod at B. The rod remains horizontal and in equilibrium.

(a) Find, in terms of M, the magnitude of the reaction on the rod at C.

(3)

(b) Find, in terms of M, the magnitude of the reaction on the rod at D.

(3)

(c) Hence, or otherwise, find the range of possible values of M.

(3)

WORKED SOLUTION

Question 5 · WME01/01 May/June 2022

1 Set the positions

Take distances from A. Then C is at 1 m, D is at 3 m, the rod's centre of mass is at 2.5 m, and B is at 5 m.

2 Find RC by moments about D

Taking moments about D: $R_C(2) + Mg(2) = 10g(3) + 5g(0.5)$.

3 Simplify RC

$2R_C + 2Mg = 30g + 2.5g = 32.5g$, so $R_C = 16.25g - Mg = (16.25 - M)g$.

4 Find RD by moments about C

Taking moments about C: $10g(1) + R_D(2) = 5g(1.5) + Mg(4)$.

5 **Simplify RD**

$10g + 2R_D = 7.5g + 4Mg$, so $2R_D = (4M - 2.5)g$ and $R_D = (2M - 1.25)g$.

6 **Use non-negative reactions**

For the rod to rest on both supports, both reactions must be non-negative.

7 **Find the range**

$R_C \geq 0$ gives $16.25 - M \geq 0$, so $M \leq 16.25$. Also $R_D \geq 0$ gives $2M - 1.25 \geq 0$, so $M \geq 0.625$. Therefore $0.625 \leq M \leq 16.25$.

✓ **FINAL ANSWER**

$(a)\ R_C = (16.25 - M)g.$ $(b)\ R_D = (2M - 1.25)g.$ $(c)\ 0.625 \leq M \leq 16.25.$

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Question 6

6 marks | Working with Vectors

6. A particle P is moving with constant acceleration.

At time $t = 1$ second, P has velocity $(-\mathbf{i} + 4\mathbf{j})\,\mathrm{m\,s^{-1}}$

At time $t = 4$ seconds, P has velocity $(5\mathbf{i} - 8\mathbf{j})\,\mathrm{m\,s^{-1}}$

Find the speed of P at time $t = 3.5$ seconds.

(6)

WORKED SOLUTION

Question 6 · WME01/01 May/June 2022

1 Find the acceleration

The velocity changes from $-\mathbf{i} + 4\mathbf{j}$ at $t = 1$ to $5\mathbf{i} - 8\mathbf{j}$ at $t = 4$. The time difference is 3 s.

2 Calculate acceleration

$$\mathbf{a} = \frac{(5\mathbf{i} - 8\mathbf{j}) - (-\mathbf{i} + 4\mathbf{j})}{3} = \frac{6\mathbf{i} - 12\mathbf{j}}{3} = 2\mathbf{i} - 4\mathbf{j}.$$

3 Find velocity at t = 3.5

From $t = 1$ to $t = 3.5$ is 2.5 s, so $\mathbf{v} = (-\mathbf{i} + 4\mathbf{j}) + 2.5(2\mathbf{i} - 4\mathbf{j})$.

4 Simplify the velocity

$\mathbf{v} = -\mathbf{i} + 4\mathbf{j} + 5\mathbf{i} - 10\mathbf{j} = 4\mathbf{i} - 6\mathbf{j}$.

5 Find the speed

Speed $= \sqrt{4^2 + (-6)^2} = \sqrt{52} = 2\sqrt{13}\,\mathrm{m\,s^{-1}}$, about $7.2\,\mathrm{m\,s^{-1}}$.

FINAL ANSWER

$\text{speed} = \sqrt{52} = 2\sqrt{13}\,\mathrm{m\,s^{-1}} \approx 7.2\,\mathrm{m\,s^{-1}}$.

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Question 7

13 marks | Kinematics Graphs

7. Two small children, Ajaz and Beth, are running a 100 m race along a straight horizontal track.

They both start from rest, leaving the start line at the same time.

Ajaz accelerates at $0.8\,\text{m s}^{-2}$ up to a speed of $4\,\text{m s}^{-1}$ and then maintains this speed until he crosses the finish line.

Beth accelerates at $1\,\text{m s}^{-2}$ for T seconds and then maintains a constant speed until she crosses the finish line.

Ajaz and Beth cross the finish line at the same time.

(a) Sketch, on the same axes, a speed-time graph for each child, from the instant when they leave the start line to the instant when they cross the finish line.

(3)

(b) Find the time taken by Ajaz to complete the race.

(4)

(c) Find the value of T

(4)

(d) Find the difference in the speeds of the two children as they cross the finish line.

(2)

WORKED SOLUTION

Question 7 · WME01/01 May/June 2022

1 Describe Ajaz graph

Ajaz starts from rest, accelerates linearly to speed $4\ \text{m s}^{-1}$, then continues with a horizontal speed-time line at speed 4.

2 Describe Beth graph

Beth starts from rest, has a steeper straight acceleration line for T seconds, then a horizontal line. Both graphs finish at the same final time.

3 Find Ajaz acceleration time

Ajaz reaches speed 4 with acceleration 0.8, so the time is $\dfrac{4}{0.8} = 5$ s.

4 Find Ajaz total time

Distance during acceleration is $\dfrac{1}{2}(5)(4) = 10$ m. The remaining 90 m at speed 4 takes 22.5 s, so Ajaz's total time is 27.5 s.

5 **Set Beth distance equation**

Beth accelerates for T seconds to speed T, then runs at speed T until time 27.5. Her distance is $\frac{1}{2}T^2 + T(27.5 - T)$.

6 **Solve for T**

$100 = \frac{1}{2}T^2 + 27.5T - T^2 = 27.5T - \frac{1}{2}T^2$. Thus $T^2 - 55T + 200 = 0$. The valid root is $T = 3.915\ldots$ s.

7 **Find the finish speeds**

Ajaz finishes at $4\ \mathrm{m\,s^{-1}}$. Beth's constant speed is $T = 3.915\ldots\ \mathrm{m\,s^{-1}}$.

8 **Find the difference**

Difference $= 4 - 3.915\ldots = 0.085\ \mathrm{m\,s^{-1}}$ approximately.

✓ **FINAL ANSWER**

(a) Ajaz reaches speed 4 then runs horizontally; Beth has a steeper initial line to speed T, then horizontal, both ending at the same time.

(b) 27.5 s. (c) $T = 3.92$ s approximately. (d) $0.085\ \mathrm{m\,s^{-1}}$ approximately.

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Question 8

17 marks | Working with Vectors

8. [*In this question,* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two boats, P and Q, are moving with constant velocities.

The velocity of P is $15\mathbf{i}\,\text{m s}^{-1}$ and the velocity of Q is $(20\mathbf{i} - 20\mathbf{j})\,\text{m s}^{-1}$

(a) Find the direction in which Q is travelling, giving your answer as a bearing.

(2)

The boats are modelled as particles.

At time $t = 0$, P is at the origin O and Q is at the point with position vector $200\mathbf{j}$ m.

At time t seconds, the position vector of P is $\mathbf{p}$ m and the position vector of Q is $\mathbf{q}$ m.

(b) Show that

$$\overrightarrow{PQ} = [5t\mathbf{i} + (200 - 20t)\mathbf{j}]\,\text{m}$$

(5)

(c) Find the bearing of P from Q when $t = 10$

(2)

(d) Find the distance between P and Q when Q is north east of P

(5)

(e) Find the times when P and Q are 200 m apart.

(3)

(Total 17 marks)

TOTAL FOR PAPER 75 MARKS

WORKED SOLUTION

Question 8 · WME01/01 May/June 2022

1 Find the bearing of Q

Q has velocity $20\mathbf{i} - 20\mathbf{j}$, so it travels south-east with equal east and south components. Its bearing is $135°$.

2 Write P position

P starts at the origin and has velocity $15\mathbf{i}$, so $\mathbf{p} = 15t\mathbf{i}$.

3 Write Q position

Q starts at $200\mathbf{j}$ and has velocity $20\mathbf{i} - 20\mathbf{j}$, so $\mathbf{q} = 20t\mathbf{i} + (200 - 20t)\mathbf{j}$.

4 **Show PQ**

$\overrightarrow{PQ} = \mathbf{q} - \mathbf{p} = (20t - 15t)\mathbf{i} + (200 - 20t)\mathbf{j} = 5t\mathbf{i} + (200 - 20t)\mathbf{j}$.

5 **Find the bearing at t = 10**

At $t = 10$, $\overrightarrow{PQ} = 50\mathbf{i}$. This means Q is east of P, so P is west of Q. The bearing of P from Q is $270°$.

6 **Use the north-east condition**

Q is north-east of P when the i and j components of $\overrightarrow{PQ}$ are equal and positive. So $5t = 200 - 20t$.

7 **Find the distance then**

The equation gives $t = 8$. Then $\overrightarrow{PQ} = 40\mathbf{i} + 40\mathbf{j}$, so the distance is $\sqrt{40^2 + 40^2} = 40\sqrt{2}\ \text{m}$.

8 **Set the 200 m distance equation**

For distance 200 m, $(5t)^2 + (200 - 20t)^2 = 200^2$.

9 **Solve for t**

This simplifies to $425t^2 - 8000t = 0$, so $t(425t - 8000) = 0$. Hence $t = 0$ or $t = \dfrac{8000}{425} = \dfrac{320}{17}$ s.

✓ **FINAL ANSWER**

$(a)\ 135°$. $(b)\ \overrightarrow{PQ} = 5t\mathbf{i} + (200 - 20t)\mathbf{j}$. $(c)\ 270°$. $(d)\ 40\sqrt{2}\ \text{m}$. $(e)\ t = 0\ \text{s or}\ \dfrac{320}{17}\ \text{s}$.