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Elite Mathematics
Edexcel International A Level

IAL MECHANICS 1 · WME01

WME01/01 January 2022 Worked Solutions

January 2022 paper rebuilt with the worked solution after each question.

8	2019–2026	Single-paper solutions
paper questions	18 exam sessions	checked by Dr Eslam



Dr Eslam Ahmed
Assistant Lecturer, Faculty of Engineering, Cairo University
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PAST PAPER

WME01/01 January 2022

January 2022

8	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WME01/01 JANUARY 2022

Question 1

6 marks | Forces

1.

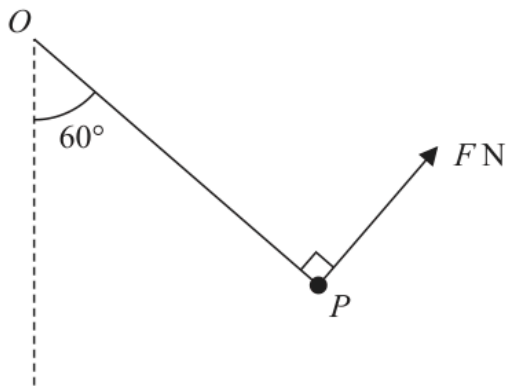


Figure 1

A particle P of weight 5 N is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. The particle P is held in equilibrium by a force of magnitude F newtons. The direction of this force is perpendicular to the string and OP makes an angle of $60°$ with the vertical, as shown in Figure 1.

Find

(a) the value of F

(3)

(b) the tension in the string.

(3)

WORKED SOLUTION

Question 1 · WME01/01 January 2022

1 Resolve perpendicular to the string

The force F is perpendicular to the string. The weight is 5 N vertically downward, and the string makes $60°$ with the vertical.

2 Find F

The component of the weight perpendicular to the string is $5\cos 30°$, so $F = 5\cos 30° = \dfrac{5\sqrt{3}}{2}$ N.

3 Resolve along the string

The tension balances the component of the weight along the string. This component is $5\sin 30°$.

4 **Find T**

$$T = 5\sin 30^\circ = \frac{5}{2}\ \text{N}.$$

✓ **FINAL ANSWER**

$(a)\ F = \dfrac{5\sqrt{3}}{2}\ \text{N} \approx 4.3\ \text{N}.$ $(b)\ T = \dfrac{5}{2}\ \text{N}.$

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Question 2

7 marks | Momentum, Impulse & Collisions

2. A particle P has mass km and a particle Q has mass m. The particles are moving towards each other in opposite directions along the same straight line when they collide directly.

Immediately before the collision, P has speed $3u$ and Q has speed u.

As a result of the collision, the direction of motion of each particle is reversed and the speed of each particle is halved.

(a) Find the value of k.

(4)

(b) Find, in terms of m and u, the magnitude of the impulse exerted on Q in the collision.

(3)

WORKED SOLUTION

Question 2 · WME01/01 January 2022

1 Choose a positive direction

Take the original direction of P as positive. Before the collision, P has velocity $3u$ and Q has velocity $-u$.

2 Write the velocities after collision

After collision, each direction is reversed and each speed is halved. So P has velocity $-\frac{3}{2}u$ and Q has velocity $\frac{1}{2}u$.

3 Use conservation of momentum

$$km(3u) + m(-u) = km\left(-\frac{3}{2}u\right) + m\left(\frac{1}{2}u\right).$$

4 Find k

Divide by mu: $3k - 1 = -\frac{3}{2}k + \frac{1}{2}$. Hence $\frac{9}{2}k = \frac{3}{2}$, so $k = \frac{1}{3}$.

5 Find impulse on Q

Impulse on Q equals change in momentum of Q: $I = m\left(\frac{1}{2}u - (-u)\right) = \frac{3}{2}mu$.

6 State the magnitude

The magnitude of the impulse is $\frac{3}{2}mu$.

FINAL ANSWER

$(a)\ k = \frac{1}{3}.\quad (b)\ I = \frac{3}{2}mu.$

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Question 3

7 marks | Moments

3.

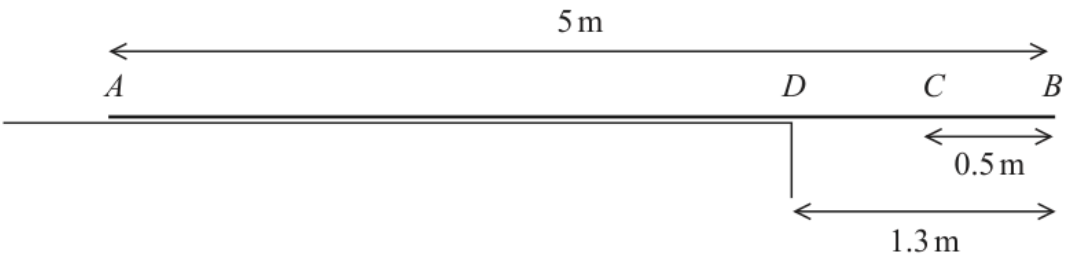


Figure 2

A beam $ADCB$ has length 5 m. The beam lies on a horizontal step with the end A on the step and the end B projecting over the edge of the step. The edge of the step is at the point D where $DB = 1.3$ m, as shown in Figure 2.

When a small boy of mass 30 kg stands on the beam at C, where $CB = 0.5$ m, the beam is on the point of tilting.

The boy is modelled as a particle and the beam is modelled as a uniform rod.

(a) Find the mass of the beam.

(3)

A block of mass X kg is now placed on the beam at A.

The block is modelled as a particle.

(b) Find the smallest value of X that will enable the boy to stand on the beam at B without the beam tilting.

(3)

(c) State how you have used the modelling assumption that the block is a particle in your calculations.

(1)

WORKED SOLUTION

Question 3 · WME01/01 January 2022

1 **Set the distances from D**

The beam length is 5 m and $DB = 1.3$ m, so D is 3.7 m from A. The centre of mass of the uniform beam is at its midpoint, so it is 1.2 m to the left of D. Point C is 0.8 m to the right of D.

2 **Use the first tilting condition**

When the boy stands at C, the beam is about to tilt about D. Let the mass of the beam be M kg.

3 **Take moments about D**

$Mg(1.2) = 30g(0.8)$. Hence $1.2M = 24$ and $M = 20\ \text{kg}$.

4 **Set up the second tilting condition**

With the boy at B, his moment about D is $30g(1.3)$. The block at A is 3.7 m to the left of D, and the beam's weight acts 1.2 m to the left of D.

5 **Take moments about D again**

$Xg(3.7) + 20g(1.2) = 30g(1.3)$.

6 **Find X**

$3.7X + 24 = 39$, so $3.7X = 15$ and $X = \dfrac{150}{37} = 4.05\ldots\ \text{kg}$.

7 **Use the particle model**

The block is modelled as a particle, so its mass is treated as acting at one point, A.

✓ **FINAL ANSWER**

(a) $\text{beam mass} = 20\ \text{kg}$. (b) $X = \dfrac{150}{37}\ \text{kg} \approx 4.05\ \text{kg}$. (c) The block's mass is treated as concentrated at A.

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Question 4

8 marks | Constant Acceleration in 1D

4. At time $t = 0$, a small ball is projected vertically upwards from a point A which is 24.5 m above the ground. The ball first comes to instantaneous rest at the point B, where $AB = 19.6$ m and first hits the ground at time $t = T$ seconds.

The ball is modelled as a particle moving freely under gravity.

(a) Find the value of T.

(6)

(b) Sketch a speed-time graph for the motion of the ball from $t = 0$ to $t = T$ seconds.

(No further calculations are needed in order to draw this sketch.)

(2)

WORKED SOLUTION

Question 4 · WME01/01 January 2022

1 Find the initial speed

At the highest point B, the velocity is 0 and $AB = 19.6$ m. Use $v^2 = u^2 + 2as$ upward: $0 = u^2 - 2g(19.6)$.

2 Calculate u

$u^2 = 2(9.8)(19.6) = 384.16$, so $u = 19.6\ \text{m s}^{-1}$.

3 Use displacement to the ground

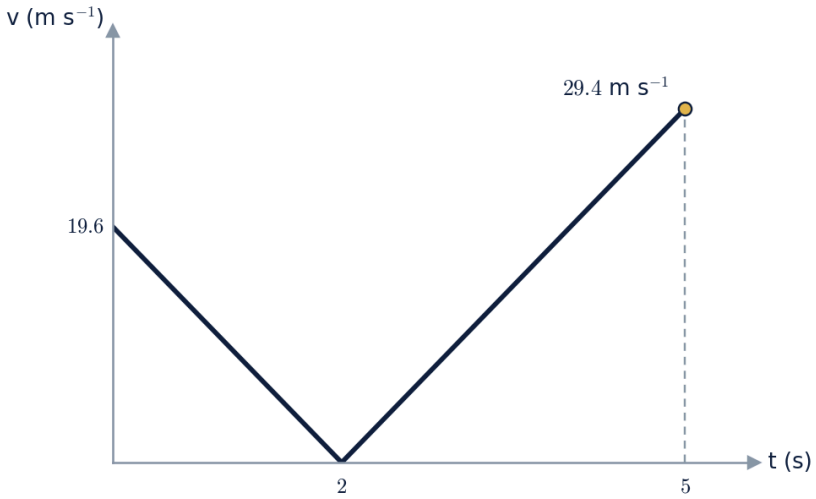
From A to the ground, taking upward as positive, displacement is -24.5 m. Use $s = ut + \frac{1}{2}at^2$: $-24.5 = 19.6T - 4.9T^2$.

4 Solve for T

$4.9T^2 - 19.6T - 24.5 = 0$. Divide by 4.9: $T^2 - 4T - 5 = 0$, so $(T - 5)(T + 1) = 0$. The positive time is $T = 5$ s.

5 **Describe the speed-time graph**

The speed starts at 19.6, decreases linearly to 0 at $t = 2$, then increases linearly until the ball hits the ground at $t = 5$. The final speed is 29.4, so the second line is longer than the first.



FINAL ANSWER

(a) $T = 5$ s. (b) Speed-time graph is V-shaped, from speed 19.6 at $t = 0$ to 0 at $t = 2$, then to speed 29.4 at $t = 5$.

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Question 5

8 marks | Resolving Forces, Inclined Planes

5.

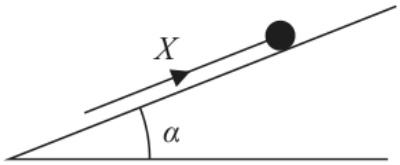


Figure 3

A particle of mass m rests in equilibrium on a fixed rough plane under the action of a force of magnitude X. The force acts up a line of greatest slope of the plane, as shown in Figure 3.

The plane is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$

The coefficient of friction between the particle and the plane is μ.

- When $X = 2P$, the particle is on the point of sliding up the plane.
- When $X = P$, the particle is on the point of sliding down the plane.

Find the value of μ.

(8)

WORKED SOLUTION

Question 5 · WME01/01 January 2022

1 Use the trig ratios

Since $\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$.

2 Resolve perpendicular to the plane

The force X acts along the plane, so the normal reaction is $R = mg\cos\alpha = \frac{4}{5}mg$.

3 Use friction

The limiting friction is $F = \mu R = \frac{4}{5}\mu mg$.

4 Use the sliding-up condition

When $X = 2P$, the particle is about to slide up, so friction acts down the plane. Hence

$2P = mg\sin\alpha + F = \frac{3}{5}mg + \frac{4}{5}\mu mg$.

5 **Use the sliding-down condition**

When $X = P$, the particle is about to slide down, so friction acts up the plane. Hence $P + F = mg\sin\alpha$, so $P = \frac{3}{5}mg - \frac{4}{5}\mu mg$.

6 **Eliminate P**

Double the second equation: $2P = \frac{6}{5}mg - \frac{8}{5}\mu mg$. This equals the first expression for $2P$.

7 **Solve for mu**

$\frac{6}{5}mg - \frac{8}{5}\mu mg = \frac{3}{5}mg + \frac{4}{5}\mu mg$. Cancel mg and multiply by 5: $6 - 8\mu = 3 + 4\mu$. Hence $3 = 12\mu$ and $\mu = \frac{1}{4}$.

✓ **FINAL ANSWER**

$\mu = \frac{1}{4}$.

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Question 6

12 marks | Working with Vectors

6. [*In this question* **i** *and* **j** *are horizontal unit vectors.*]

A particle P of mass 2 kg moves under the action of two forces, $(p\mathbf{i} + q\mathbf{j})$N and $(2q\mathbf{i} + p\mathbf{j})$N, where p and q are constants.

Given that the acceleration of P is $(\mathbf{i} - \mathbf{j})\,\text{m s}^{-2}$

(a) find the value of p and the value of q.

(5)

(b) Find the size of the angle between the direction of the acceleration and the vector **j**.

(2)

At time $t = 0$, the velocity of P is $(3\mathbf{i} - 4\mathbf{j})\,\text{m s}^{-1}$

At $t = T$ seconds, P is moving in the direction of the vector $(11\mathbf{i} - 13\mathbf{j})$.

(c) Find the value of T.

(5)

WORKED SOLUTION

Question 6 · WME01/01 January 2022

1 Use F = ma

The total force is $(p\mathbf{i} + q\mathbf{j}) + (2q\mathbf{i} + p\mathbf{j}) = (p + 2q)\mathbf{i} + (p + q)\mathbf{j}$. Since the mass is 2 kg and acceleration is $\mathbf{i} - \mathbf{j}$, the total force is $2\mathbf{i} - 2\mathbf{j}$.

2 Equate components

$p + 2q = 2$ and $p + q = -2$.

3 Find p and q

Subtract the equations to get $q = 4$. Then $p + 4 = -2$, so $p = -6$.

4 Find the angle with j

The acceleration is $\mathbf{i} - \mathbf{j}$. Its dot product with $\mathbf{j}$ is -1, so the angle with the positive $\mathbf{j}$ direction is obtuse. Since the components are equal in magnitude, the angle is $135°$.

5 Write the velocity at time T

$\mathbf{v} = (3\mathbf{i} - 4\mathbf{j}) + T(\mathbf{i} - \mathbf{j}) = (3 + T)\mathbf{i} + (-4 - T)\mathbf{j}$.

6 **Use the direction vector**

The velocity is parallel to $11\mathbf{i} - 13\mathbf{j}$, so $\dfrac{3+T}{11} = \dfrac{-4-T}{-13}$.

7 **Solve for T**

$13(3+T) = 11(4+T)$, so $39 + 13T = 44 + 11T$. Hence $2T = 5$ and $T = 2.5$.

✓ **FINAL ANSWER**

$(a)\ p = -6,\ q = 4.\quad (b)\ 135°.\quad (c)\ T = 2.5.$

Question 7

13 marks | Resolving Forces, Inclined Planes

7.

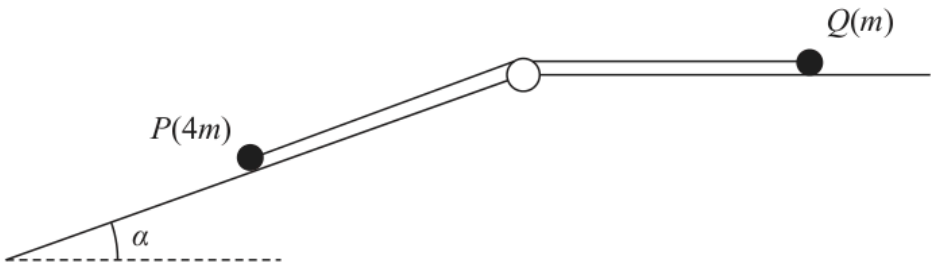


Figure 4

A particle P of mass $4m$ lies on the surface of a fixed rough inclined plane.

The plane is inclined to the horizontal at an angle α where $\tan\alpha = \frac{3}{4}$

The particle P is attached to one end of a light inextensible string.

The string passes over a small smooth pulley that is fixed at the top of the plane. The other end of the string is attached to a particle Q of mass m which lies on a smooth horizontal plane.

The string lies along the horizontal plane and in the vertical plane that contains the pulley and a line of greatest slope of the inclined plane.

The system is released from rest with the string taut, as shown in Figure 4, and P moves down the plane.

The coefficient of friction between P and the plane is $\frac{1}{4}$

For the motion before Q reaches the pulley

(a) write down an equation of motion for Q,

(1)

(b) find, in terms of m and g, the tension in the string,

(7)

(c) find the magnitude of the force exerted on the pulley by the string.

(4)

(d) State where in your working you have used the information that the string is light.

(1)

1 **Write the equation for Q**

Q lies on a smooth horizontal plane. The only horizontal force on Q is the tension, so $T = ma$.

2 **Use the trig ratios**

Since $\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$.

3 **Find friction on P**

For P, $R = 4mg\cos\alpha = 4mg \cdot \frac{4}{5} = \frac{16mg}{5}$. With coefficient $\frac{1}{4}$, friction is $F = \frac{1}{4}R = \frac{4mg}{5}$.

4 **Apply Newtons second law to P**

P moves down the plane. Taking down the plane as positive: $4mg\sin\alpha - T - F = 4ma$.

5 **Substitute values**

$4mg \cdot \frac{3}{5} - T - \frac{4mg}{5} = 4ma$, so $\frac{8mg}{5} - T = 4ma$.

6 **Use T = ma**

From Q, $a = \frac{T}{m}$. Substitute into the equation for P: $\frac{8mg}{5} - T = 4T$.

7 **Find T**

$\frac{8mg}{5} = 5T$, so $T = \frac{8}{25}mg$.

8 **Find the pulley resultant**

The pulley is pulled by two tensions of magnitude T: one horizontal and one along the inclined string. Their resultant has components $T - T\cos\alpha$ horizontally and $T\sin\alpha$ vertically.

9 **Calculate the resultant**

Magnitude $= \sqrt{(T - T\cos\alpha)^2 + (T\sin\alpha)^2} = T\sqrt{(1 - \frac{4}{5})^2 + (\frac{3}{5})^2} = \frac{T}{5}\sqrt{10}$. With $T = \frac{8}{25}mg$, the resultant is $\frac{8\sqrt{10}}{125}mg$.

10 **Use the light-string model**

The string is light, so the tension can be treated as the same on the P side and the Q side of the pulley.

FINAL ANSWER

$(a)\ T = ma.$ $(b)\ T = \frac{8}{25}mg.$ $(c)\ \textbf{resultant} = \frac{8\sqrt{10}}{125}mg.$ (d) Same tension was used on both sides of the pulley.

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Question 8

14 marks | Working with Vectors

8. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin.*]

A ship A moves with constant velocity $(3\mathbf{i} - 10\mathbf{j})\,\text{km h}^{-1}$

At time t hours, the position vector of A is $\mathbf{r}$ km.

At time $t = 0$, A is at the point with position vector $(13\mathbf{i} + 5\mathbf{j})$ km.

(a) Find $\mathbf{r}$ in terms of t.

(2)

Another ship B moves with constant velocity $(15\mathbf{i} + 14\mathbf{j})\,\text{km h}^{-1}$

At time $t = 0$, B is at the point with position vector $(3\mathbf{i} - 5\mathbf{j})$ km.

(b) Show that, at time t hours,

$$\overrightarrow{AB} = \left[(12t - 10)\mathbf{i} + (24t - 10)\mathbf{j}\right]\text{km}$$

(4)

Given that the two ships do not change course,

(c) find the shortest distance between the two ships,

(6)

(d) find the bearing of ship B from ship A when the ships are closest.

(2)

(Total 14 marks)

WORKED SOLUTION

Question 8 · WME01/01 January 2022

1 Write the position of A

$\mathbf{r} = (13\mathbf{i} + 5\mathbf{j}) + t(3\mathbf{i} - 10\mathbf{j}) = (13 + 3t)\mathbf{i} + (5 - 10t)\mathbf{j}$.

2 Write the position of B

$\mathbf{s} = (3\mathbf{i} - 5\mathbf{j}) + t(15\mathbf{i} + 14\mathbf{j}) = (3 + 15t)\mathbf{i} + (-5 + 14t)\mathbf{j}$.

3 Find AB

$\overrightarrow{AB} = \mathbf{s} - \mathbf{r} = [3 + 15t - (13 + 3t)]\mathbf{i} + [-5 + 14t - (5 - 10t)]\mathbf{j}$.

4 **Show the expression**

This simplifies to $\overrightarrow{AB} = (12t - 10)\mathbf{i} + (24t - 10)\mathbf{j}$ km.

5 **Use distance squared**

The square of the distance is $d^2 = (12t - 10)^2 + (24t - 10)^2$.

6 **Expand and minimise**

$d^2 = 720t^2 - 720t + 200$. This quadratic is minimum at $t = \dfrac{720}{2 \cdot 720} = \dfrac{1}{2}$.

7 **Find the shortest distance**

At $t = \dfrac{1}{2}$, $\overrightarrow{AB} = (-4)\mathbf{i} + 2\mathbf{j}$, so the shortest distance is $\sqrt{(-4)^2 + 2^2} = \sqrt{20} = 2\sqrt{5}\ \text{km}$.

8 **Find the bearing**

At closest approach, B is west and north of A. The angle west of north satisfies $\tan\theta = \dfrac{4}{2} = 2$, so $\theta = 63.4°$. Bearing $= 360° - 63.4° = 296.6°$, which is $297°$.

✓ **FINAL ANSWER**

$(a)\ \mathbf{r} = (13 + 3t)\mathbf{i} + (5 - 10t)\mathbf{j}$. $(b)\ \overrightarrow{AB} = (12t - 10)\mathbf{i} + (24t - 10)\mathbf{j}$. $(c)\ 2\sqrt{5}\ \text{km} \approx 4.5\ \text{km}$. $(d)\ 297°$

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