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Elite Mathematics
Edexcel International A Level

IAL MECHANICS 1 · WME01

WME01/01 October 2021 Worked Solutions

October 2021 paper rebuilt with the worked solution after each question.

8	2019–2026	Single-paper solutions
paper questions	18 exam sessions	checked by Dr Eslam



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PAST PAPER

WME01/01 October 2021

October 2021

8	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

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Question 1

6 marks | Moments

1.

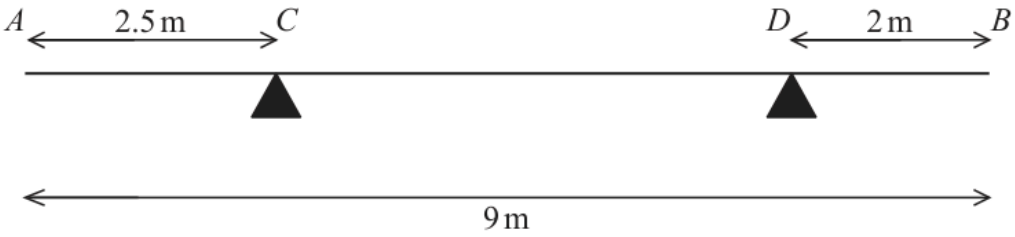


Figure 1

A non-uniform rod AB has length 9 m and mass M kg.

The rod rests in equilibrium in a horizontal position on two supports, one at C where $AC = 2.5$ m and the other at D where $DB = 2$ m, as shown in Figure 1.

The magnitude of the force acting on the rod at D is twice the magnitude of the force acting on the rod at C.

The centre of mass of the rod is d metres from A.

Find the value of d.

(6)

WORKED SOLUTION

Question 1 · WME01/01 October 2021

1 Set the support positions

C is 2.5 m from A. Since $DB = 2$ m and $AB = 9$ m, D is 7 m from A.

2 Name the reactions

Let the reaction at C be R. The reaction at D is twice this, so it is $2R$.

3 Use vertical equilibrium

$R + 2R = Mg$, so $3R = Mg$.

4 Take moments about A

$R(2.5) + 2R(7) = Mgd$.

5 Substitute Mg = 3R

$2.5R + 14R = 3Rd$. Hence $16.5R = 3Rd$.

6 **Find d**

Cancel R: $d = \dfrac{16.5}{3} = 5.5\ \text{m}$.

FINAL ANSWER

$d = 5.5\ \text{m}$.

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Question 2

10 marks | Momentum, Impulse & Collisions

2. A particle P of mass $2m$ is moving on a rough horizontal plane when it collides directly with a particle Q of mass $4m$ which is at rest on the plane. The speed of P immediately before the collision is $3u$. The speed of Q immediately after the collision is $2u$.

(a) Find, in terms of u, the speed of P immediately after the collision.

(3)

(b) State clearly the direction of motion of P immediately after the collision.

(1)

Following the collision, Q comes to rest after travelling a distance $\frac{6u^2}{g}$ along the plane.

The coefficient of friction between Q and the plane is μ.

(c) Find the value of μ.

(6)

WORKED SOLUTION

Question 2 · WME01/01 October 2021

1 Use conservation of momentum

Take P's original direction as positive. Before collision, P has velocity $3u$ and Q is at rest. After collision, Q has velocity $2u$. Let P's velocity be v.

2 Find P velocity

$2m(3u) = 2mv + 4m(2u)$. Thus $6mu = 2mv + 8mu$, so $2mv = -2mu$ and $v = -u$.

3 State speed and direction

The speed of P is u. The negative sign shows that P moves in the opposite direction after the collision.

4 Find the deceleration of Q

After collision, Q starts with speed $2u$ and stops after distance $\frac{6u^2}{g}$. Use $v^2 = u^2 + 2as$:

$0 = (2u)^2 + 2a\left(\frac{6u^2}{g}\right)$.

5 Simplify acceleration

$0 = 4u^2 + \frac{12au^2}{g}$, so $a = -\frac{g}{3}$. The deceleration magnitude is $\frac{g}{3}$.

6 **Use friction on Q**

For Q, $R = 4mg$, so friction is $\mu R = 4\mu mg$. This friction produces deceleration: $4\mu mg = 4m \cdot \dfrac{g}{3}$.

7 **Find mu**

Cancel $4mg$: $\mu = \dfrac{1}{3}$.

✓ **FINAL ANSWER**

(a) speed of $\mathrm{P} = u$. (b) P reverses direction. (c) $\mu = \dfrac{1}{3}$.

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Question 3

10 marks | Constant Acceleration in 1D

3. A car is moving at a constant speed of $25\,\text{m s}^{-1}$ along a straight horizontal road.

The car is modelled as a particle.

At time $t = 0$, the car is at the point A and the driver sees a road sign 48 m ahead.

Let t seconds be the time that elapses after the car passes A.

In a **first** model, the car is assumed to decelerate uniformly at $6\,\text{m s}^{-2}$ from A until the car reaches the road sign.

(a) Use this first model to find the speed of the car as it reaches the sign.

(2)

The road sign indicates that the speed limit immediately after the sign is $13\,\text{m s}^{-1}$.

In a **second** model, the car is assumed to decelerate uniformly at $6\,\text{m s}^{-2}$ from A until it reaches a speed of $13\,\text{m s}^{-1}$. The car then maintains this speed until it reaches the road sign.

(b) Use this second model to find the value of t at which the car reaches the sign.

(4)

In a **third** model, the car is assumed to move with constant speed $25\,\text{m s}^{-1}$ from A until time $t = 0.2$, the car then decelerates uniformly at $6\,\text{m s}^{-2}$ until it reaches a speed of $13\,\text{m s}^{-1}$. The car then maintains this speed until it reaches the road sign.

(c) Use this third model to find the value of t at which the car reaches the sign.

(4)

WORKED SOLUTION

Question 3 · WME01/01 October 2021

1 Use the first model

From A to the sign, $u = 25$, $a = -6$ and $s = 48$. Use $v^2 = u^2 + 2as$.

2 Find the speed at the sign

$v^2 = 25^2 + 2(-6)(48) = 625 - 576 = 49$, so $v = 7\,\text{m s}^{-1}$.

3 Second model: time to reach 13

Use $v = u + at$: $13 = 25 - 6t$, so $t = 2$ s to slow to $13\,\text{m s}^{-1}$.

4 Second model: distance while slowing

Distance while slowing is $\dfrac{25 + 13}{2} \cdot 2 = 38$ m.

5 **Second model: finish time**

The remaining distance is $48 - 38 = 10$ m at speed 13. Total time $= 2 + \dfrac{10}{13} = \dfrac{36}{13}$ s.

6 **Third model: first 0.2 seconds**

The car first travels at $25\ \mathrm{m\,s^{-1}}$ for 0.2 s, covering 5 m.

7 **Third model: slowing distance**

It then takes another 2 s to slow from 25 to 13, covering 38 m as before. Total covered before constant-speed part is $5 + 38 = 43$ m.

8 **Third model: finish time**

The remaining distance is $48 - 43 = 5$ m at speed 13. Total time $= 0.2 + 2 + \dfrac{5}{13} = \dfrac{168}{65}$ s.

✓ **FINAL ANSWER**

$(a)\ 7\ \mathrm{m\,s^{-1}}$. $(b)\ t = \dfrac{36}{13}\ \mathrm{s} \approx 2.77\ \mathrm{s}$. $(c)\ t = \dfrac{168}{65}\ \mathrm{s} \approx 2.58\ \mathrm{s}$.

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Question 4

8 marks | Working with Vectors

4. The position vector, $\mathbf{r}$ metres, of a particle P at time t seconds, relative to a fixed origin O, is given by

$$\mathbf{r} = (t - 3)\mathbf{i} + (1 - 2t)\mathbf{j}$$

(a) Find, to the nearest degree, the size of the angle between $\mathbf{r}$ and the vector $\mathbf{j}$, when $t = 2$

(3)

(b) Find the values of t for which the distance of P from O is 2.5 m.

(5)

WORKED SOLUTION

Question 4 · WME01/01 October 2021

1 Find r when t = 2

$\mathbf{r} = (2 - 3)\mathbf{i} + (1 - 2(2))\mathbf{j} = -\mathbf{i} - 3\mathbf{j}$.

2 Find the angle with j

The vector is in the third quadrant. The angle between $\mathbf{r}$ and the positive $\mathbf{j}$ direction satisfies $\tan\alpha = \frac{1}{3}$ from the vertical comparison.

3 State the angle

$\alpha = 18.4°$ from the negative $\mathbf{j}$ direction, so the size of the angle with positive $\mathbf{j}$ is $180° - 18.4° = 161.6°$, which is $162°$.

4 Use the distance condition

Distance from O is 2.5, so $(t - 3)^2 + (1 - 2t)^2 = 2.5^2$.

5 Form the quadratic

$t^2 - 6t + 9 + 1 - 4t + 4t^2 = 6.25$, so $5t^2 - 10t + 3.75 = 0$.

6 Solve

Multiply by 4: $20t^2 - 40t + 15 = 0$, or $4t^2 - 8t + 3 = 0$. Hence $(2t - 1)(2t - 3) = 0$, so $t = \frac{1}{2}$ or $t = \frac{3}{2}$.

FINAL ANSWER

$(a)\ 162°. \quad (b)\ t = \frac{1}{2} \text{ or } t = \frac{3}{2}.$

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Question 5

10 marks | Forces

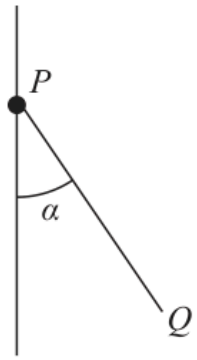


Figure 2

A small bead of mass 0.2 kg is attached to the end P of a light rod PQ. The bead is threaded onto a fixed vertical rough wire.

The bead is held in equilibrium with the rod PQ inclined to the wire at an angle α, where $\tan\alpha = \frac{4}{3}$, as shown in Figure 2.

The thrust in the rod is T newtons.

The bead is modelled as a particle.

(a) Find the magnitude and direction of the friction force acting on the bead when $T = 2.5$

(3)

The coefficient of friction between the bead and the wire is μ.

Given that the greatest possible value of T is 6.125

(b) find the value of μ.

(7)

WORKED SOLUTION

Question 5 · WME01/01 October 2021

1 **Use the trig ratios**

Since $\tan\alpha = \frac{4}{3}$, $\sin\alpha = \frac{4}{5}$ and $\cos\alpha = \frac{3}{5}$.

2 **Resolve vertically when T = 2.5**

The vertical component of the thrust is $2.5\cos\alpha = 2.5 \cdot \frac{3}{5} = 1.5$ N upward. The weight is $0.2g = 1.96$ N downward.

3 **Find friction and direction**

For equilibrium, friction must supply the missing upward force: $F = 1.96 - 1.5 = 0.46\ \text{N}$. So friction acts upwards.

4 **Use greatest T**

When $T = 6.125$, the vertical component of the thrust is larger than the weight, so friction acts downward at the limiting value.

5 **Find normal reaction**

The normal reaction is the horizontal component of the thrust:
$R = 6.125\sin\alpha = 6.125 \cdot \frac{4}{5} = 4.9\ \text{N}$.

6 **Find limiting friction**

Vertical equilibrium gives $0.2g + F = 6.125\cos\alpha$. Thus $1.96 + F = 6.125 \cdot \frac{3}{5} = 3.675$, so $F = 1.715$ N.

7 **Find mu**

$F = \mu R$, so $\mu = \frac{1.715}{4.9} = 0.35$.

✓ **FINAL ANSWER**

$(a)\ F = 0.46\ \text{N}$ upwards. $(b)\ \mu = 0.35$.

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Question 6

5 marks | Constant Acceleration in 1D

6.

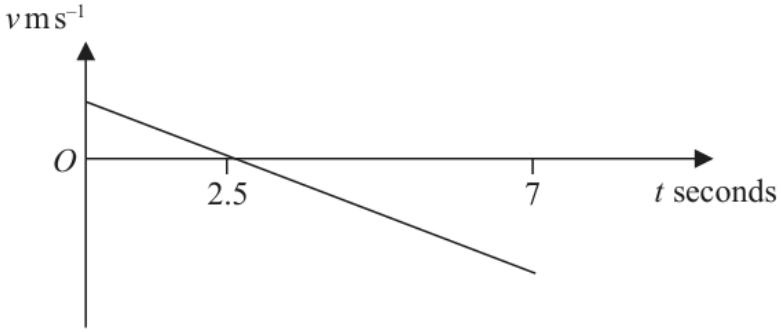


Figure 3

A small ball is thrown vertically upwards at time $t = 0$ from a point A which is above horizontal ground. The ball hits the ground 7 s later.

The ball is modelled as a particle moving freely under gravity.

The velocity-time graph shown in Figure 3 represents the motion of the ball for $0 \leqslant t \leqslant 7$

(a) Find the speed with which the ball is thrown.

(2)

(b) Find the height of A above the ground.

(3)

WORKED SOLUTION

Question 6 · WME01/01 October 2021

1 Use the velocity-time graph

The velocity becomes zero at $t = 2.5$ s. Since acceleration is $-g = -9.8\ \text{m s}^{-2}$, the initial velocity is $u = 9.8(2.5)$.

2 Find the initial speed

$u = 24.5\ \text{m s}^{-1}$.

3 Use displacement to the ground

From launch to $t = 7$, the displacement is the area under the velocity-time graph. Algebraically, $s = ut - \frac{1}{2}gt^2$ with $t = 7$.

4 **Calculate displacement**

$s = 24.5(7) - \frac{1}{2}(9.8)(7^2) = 171.5 - 240.1 = -68.6$ m.

5 **State the height**

The negative sign shows the ground is below A, so the height of A above the ground is $\mathbf{68.6\ m}$.

✓ **FINAL ANSWER**

$(a)\ 24.5\ \mathrm{m\,s^{-1}}$. $(b)\ 68.6\ \mathrm{m}$ approximately.

Question 7

14 marks | Resolving Forces, Inclined Planes

7.

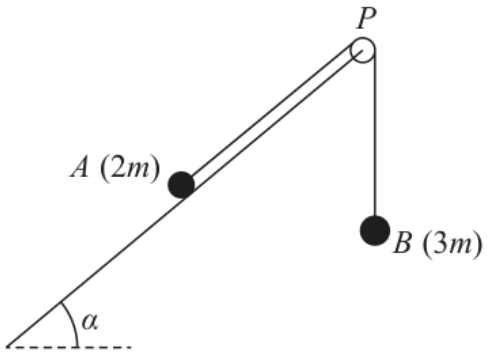


Figure 4

One end of a light inextensible string is attached to a particle A of mass $2m$. The other end of the string is attached to a particle B of mass $3m$. The string passes over a small, smooth, light pulley P which is fixed at the top of a rough inclined plane. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

Particle A is held at rest on the plane with the string taut and B hanging freely below P, as shown in Figure 4. The section of the string AP is parallel to a line of greatest slope of the plane.

The coefficient of friction between A and the plane is $\frac{1}{2}$

Particle A is released and begins to move up the plane.

For the motion before A reaches the pulley,

(a) (i) write down an equation of motion for A,

(ii) write down an equation of motion for B,

(4)

(b) find, in terms of g, the acceleration of A,

(5)

(c) find the magnitude of the force exerted on the pulley by the string.

(4)

(d) State how you have used the information that P is a smooth pulley.

(1)

1 **Equation for A**

A moves up the plane, so friction and the component of weight act down the plane. Taking up the plane as positive: $T - 2mg\sin\alpha - F = 2ma$.

2 **Equation for B**

B moves downward. Taking downward as positive: $3mg - T = 3ma$.

3 **Find friction on A**

Since $\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$. The normal reaction on A is $R = 2mg\cos\alpha = \frac{8mg}{5}$, so $F = \frac{1}{2}R = \frac{4mg}{5}$.

4 **Simplify A's equation**

$T - 2mg\cdot\frac{3}{5} - \frac{4mg}{5} = 2ma$, so $T - 2mg = 2ma$.

5 **Solve with B's equation**

From B, $T = 3mg - 3ma$. Substitute into $T - 2mg = 2ma$: $3mg - 3ma - 2mg = 2ma$. Thus $mg = 5ma$ and $a = \frac{g}{5}$.

6 **Find the tension**

Using $3mg - T = 3m\left(\frac{g}{5}\right)$ gives $T = 3mg - \frac{3mg}{5} = \frac{12mg}{5}$.

7 **Find the pulley resultant**

The two tensions are one vertical and one along the plane. Components of the resultant are $T\cos\alpha$ horizontally and $T + T\sin\alpha$ vertically.

8 **Calculate the magnitude**

Resultant $= T\sqrt{\cos^2\alpha + (1+\sin\alpha)^2} = T\sqrt{(\frac{4}{5})^2 + (\frac{8}{5})^2} = \frac{4\sqrt{5}}{5}T$. With $T = \frac{12mg}{5}$, this is $\frac{48\sqrt{5}}{25}mg$.

9 **Use the smooth pulley information**

The pulley is smooth, so the tension is the same on both sides of the pulley.

✓ **FINAL ANSWER**

$(a)(i)\ T - 2mg\sin\alpha - F = 2ma.$ $(a)(ii)\ 3mg - T = 3ma.$ $(b)\ a = \frac{g}{5}.$ $(c)\ \text{resultant} = \frac{48\sqrt{5}}{25}mg.$ (d) The tension is the same on both sides of the pulley.

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Question 8

12 marks | Working with Vectors

8. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin.*]

At 7 am a ship leaves a port and moves with constant velocity. The position vector of the port is $(-2\mathbf{i} + 9\mathbf{j})$ km.

At 7.36 am the ship is at the point with position vector $(4\mathbf{i} + 6\mathbf{j})$ km.

(a) Show that the velocity of the ship is $(10\mathbf{i} - 5\mathbf{j})\,\text{km h}^{-1}$

(2)

(b) Find the position vector of the ship t hours after leaving port.

(2)

At 8.48 am a passenger on the ship notices that a lighthouse is due east of the ship.

At 9 am the same passenger notices that the lighthouse is now north east of the ship.

(c) Find the position vector of the lighthouse.

(4)

(d) Find the position vector of the ship when it is due south of the lighthouse.

(4)

(Total 12 marks)

WORKED SOLUTION

Question 8 · WME01/01 October 2021

1 Convert the time

From 7:00 to 7:36 is 36 minutes, which is 0.6 hours.

2 Find velocity

Displacement $= (4\mathbf{i} + 6\mathbf{j}) - (-2\mathbf{i} + 9\mathbf{j}) = 6\mathbf{i} - 3\mathbf{j}$. Divide by 0.6: $\mathbf{v} = 10\mathbf{i} - 5\mathbf{j}$.

3 Write the ship position

$\mathbf{r} = (-2\mathbf{i} + 9\mathbf{j}) + t(10\mathbf{i} - 5\mathbf{j}) = (-2 + 10t)\mathbf{i} + (9 - 5t)\mathbf{j}$.

4 Use 8:48

At 8:48, $t = 1.8$. The ship position is $(-2 + 18)\mathbf{i} + (9 - 9)\mathbf{j} = 16\mathbf{i}$. The lighthouse is due east, so it has the same j-component, 0.

5 **Use 9:00**

At 9:00, $t = 2$. The ship position is $18\mathbf{i} - \mathbf{j}$. The lighthouse is north-east of the ship, so its east and north displacements from the ship are equal.

---

6 **Find the lighthouse position**

Since the lighthouse has j-component 0, from $18\mathbf{i} - \mathbf{j}$ it is 1 km north, so it is also 1 km east. Hence its position is $19\mathbf{i}$.

---

7 **Find when the ship is due south**

Due south of the lighthouse means the ship has i-component 19. Set $-2 + 10t = 19$, so $t = 2.1$.

---

8 **Find the ship position**

At $t = 2.1$, $\mathbf{r} = (-2 + 21)\mathbf{i} + (9 - 10.5)\mathbf{j} = 19\mathbf{i} - 1.5\mathbf{j}$.

✓ **FINAL ANSWER**

$(a)\ \mathbf{v} = 10\mathbf{i} - 5\mathbf{j}\ \text{km h}^{-1}$. $(b)\ \mathbf{r} = (-2 + 10t)\mathbf{i} + (9 - 5t)\mathbf{j}$. $(c)\ \text{lighthouse} = 19\mathbf{i}$. $(d)\ \text{ship position} = 19\mathbf{i} - 1.5\mathbf{j}$.