Elite Mathematics
Edexcel International A Level

IAL MECHANICS 1 · WME01

WME01/01 May/June 2021 Worked Solutions

May/June 2021 paper rebuilt with the worked solution after each question.

8	2019–2026	Single-paper solutions
paper questions	18 exam sessions	checked by Dr Eslam



Dr Eslam Ahmed
Assistant Lecturer, Faculty of Engineering, Cairo University
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PAST PAPER

WME01/01 May/June 2021

May/June 2021

8	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WME01/01 MAY/JUNE 2021

Question 1

7 marks | Momentum, Impulse & Collisions

1. A particle P has mass $3m$ and a particle Q has mass $5m$. The particles are moving towards each other in opposite directions along the same straight line on a smooth horizontal surface. The particles collide directly.

Immediately before the collision the speed of P is ku, where k is a constant, and the speed of Q is $2u$.

Immediately after the collision the speed of P is u and the speed of Q is $3u$.

The direction of motion of Q is reversed by the collision.

(a) Find, in terms of m and u, the magnitude of the impulse exerted on Q by P in the collision.

(2)

(b) Find the two possible values of k.

(5)

WORKED SOLUTION

Question 1 · WME01/01 May/June 2021

1 Choose a positive direction

Take P's original direction as positive. Q initially has velocity $-2u$. Since Q reverses direction and then has speed $3u$, its final velocity is $3u$.

2 Find the impulse on Q

Impulse equals change in momentum: $I = 5m(3u - (-2u)) = 25mu$.

3 Use conservation of momentum

Before collision, total momentum is $3m(ku) + 5m(-2u) = 3mku - 10mu$.

4 Case 1: P continues in its original direction

If P has final velocity u, final momentum is $3m(u) + 5m(3u) = 18mu$. So $3mku - 10mu = 18mu$, giving $3k = 28$ and $k = \frac{28}{3}$.

5 Case 2: P reverses direction

If P has final velocity $-u$, final momentum is $3m(-u) + 5m(3u) = 12mu$. So $3mku - 10mu = 12mu$, giving $3k = 22$ and $k = \frac{22}{3}$.

FINAL ANSWER

$(a)\ I = 25mu. \quad (b)\ k = \frac{28}{3}$ or $\frac{22}{3}$.

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Question 2

8 marks | Constant Acceleration in 1D

2. A car moves along a straight horizontal road with constant acceleration $a\,\text{m s}^{-2}$ where $a > 0$

The car is modelled as a particle.

At time $t = 0$, the car passes point A and is moving with speed $u\,\text{m s}^{-1}$

In the first three seconds after passing A the car travels 20 m.

In the fourth second after passing A the car travels 10 m.

The speed of the car as it passes point B is $20\,\text{m s}^{-1}$

Find the time taken for the car to travel from A to B.

(8)

WORKED SOLUTION

Question 2 · WME01/01 May/June 2021

1 Use the first three seconds

Using $s = ut + \frac{1}{2}at^2$, the first 3 seconds give $20 = 3u + \frac{1}{2}a(3^2) = 3u + \frac{9}{2}a$.

2 Use the fourth second

The distance in the fourth second is $u + a(4 - \frac{1}{2}) = u + \frac{7}{2}a$. Hence $10 = u + \frac{7}{2}a$.

3 Solve for a and u

From $10 = u + \frac{7}{2}a$, $u = 10 - \frac{7}{2}a$. Substitute into $20 = 3u + \frac{9}{2}a$:
$20 = 30 - \frac{21}{2}a + \frac{9}{2}a = 30 - 6a$. Thus $a = \frac{5}{3}$.

4 Find u

$u = 10 - \frac{7}{2} \cdot \frac{5}{3} = 10 - \frac{35}{6} = \frac{25}{6}$.

5 Use the speed at B

At B, speed is 20. Use $v = u + at$: $20 = \frac{25}{6} + \frac{5}{3}t$.

6 **Find the time**

$\frac{5}{3}t = 20 - \frac{25}{6} = \frac{95}{6}$, so $t = \frac{95}{10} = 9.5\ \text{s}$.

FINAL ANSWER

$t = 9.5\ \text{s}$.

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Question 3

9 marks | Working with Vectors

3. [*In this question* **i** *and* **j** *are perpendicular horizontal unit vectors.*]

Three forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$, are given by

$$\mathbf{F}_1 = (5\mathbf{i} + 2\mathbf{j})\text{N} \quad \mathbf{F}_2 = (-3\mathbf{i} + \mathbf{j})\text{N} \quad \mathbf{F}_3 = (a\mathbf{i} + b\mathbf{j})\text{N}$$

where a and b are constants.

The forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a particle P of mass 4 kg.

Given that P rests in equilibrium on a smooth horizontal surface under the action of these three forces,

(a) find the size of the angle between the direction of $\mathbf{F}_3$ and the direction of $-\mathbf{j}$.

(4)

The force $\mathbf{F}_3$ is now removed and replaced by the force $\mathbf{F}_4$ given by $\mathbf{F}_4 = \lambda(\mathbf{i} + 3\mathbf{j})\text{N}$, where λ is a positive constant.

When the three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_4$ act on P, the acceleration of P has magnitude $3.25\ \text{m s}^{-2}$

(b) Find the value of λ.

(5)

WORKED SOLUTION

Question 3 · WME01/01 May/June 2021

1 Find F3 from equilibrium

Equilibrium gives $\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} = \mathbf{0}$. Now $\mathbf{F_1} + \mathbf{F_2} = (5\mathbf{i} + 2\mathbf{j}) + (-3\mathbf{i} + \mathbf{j}) = 2\mathbf{i} + 3\mathbf{j}$.

2 Write F3

So $\mathbf{F_3} = -2\mathbf{i} - 3\mathbf{j}$.

3 Find the angle with -j

The vector $-\mathbf{j}$ points vertically downward. For $\mathbf{F_3} = -2\mathbf{i} - 3\mathbf{j}$, the angle from $-\mathbf{j}$ satisfies $\tan\theta = \frac{2}{3}$.

4 State the angle

$\theta = 33.7°$, so the angle is about $34°$.

5 **Find the new resultant force**

With $\mathbf{F}_4 = \lambda(\mathbf{i} + 3\mathbf{j})$, the resultant is $(2 + \lambda)\mathbf{i} + (3 + 3\lambda)\mathbf{j}$.

6 **Use the acceleration magnitude**

The mass is 4 kg and acceleration magnitude is 3.25, so the resultant force magnitude is $4(3.25) = 13$ N.

7 **Form the equation**

$(2 + \lambda)^2 + (3 + 3\lambda)^2 = 13^2$.

8 **Solve for lambda**

Expanding gives $10\lambda^2 + 22\lambda - 156 = 0$. The roots are 3 and -5.2, and since λ is positive, $\lambda = 3$.

✓ **FINAL ANSWER**

(a) $34°$ approximately. (b) $\lambda = 3$.

Question 4

6 marks | Newton's Second Law

4.

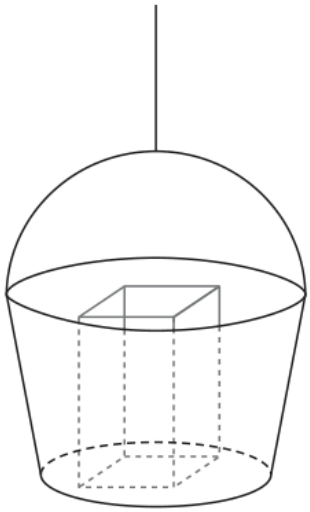


Figure 1

Figure 1 shows a large bucket used by a crane on a building site to move materials between the ground and the top of the building. The mass of the bucket is 15 kg.

The bucket is attached to a vertical cable with the bottom of the bucket horizontal. The cable is modelled as light and inextensible.

When the bucket is on the ground, a bag of cement of mass 25 kg is placed in the bucket.

The bucket with the bag of cement moves vertically upwards with constant acceleration $0.2\,\text{m}\,\text{s}^{-2}$. Air resistance is modelled as being negligible.

(a) Find the tension in the cable.

(3)

At the top of the building, the bag of cement is removed. A box of tools of mass 12 kg is now placed in the bucket.

Later on the bucket with the box of tools is moving vertically downwards with constant deceleration $0.1\,\text{m}\,\text{s}^{-2}$. Air resistance is again modelled as being negligible.

(b) Find the magnitude of the normal reaction between the bucket and the box of tools.

(3)

1 Find total mass going up

The bucket and cement have total mass $15 + 25 = 40$ kg and acceleration $0.2\ \mathrm{m\,s^{-2}}$ upward.

2 Apply Newtons second law

Taking upward as positive: $T - 40g = 40(0.2)$.

3 Find the tension

$T - 392 = 8$, so $T = 400\ \mathrm{N}$.

4 Interpret the downward deceleration

The bucket and tools are moving downward but decelerating, so their acceleration is upward with magnitude $0.1\ \mathrm{m\,s^{-2}}$.

5 Consider the box of tools

For the 12 kg box, upward force is the normal reaction R, and downward force is weight $12g$.

6 Find R

$R - 12g = 12(0.1)$. Thus $R = 117.6 + 1.2 = 118.8\ \mathrm{N}$, about $119\ \mathrm{N}$.

✓ **FINAL ANSWER**

$(a)\ T = 400\ \mathrm{N}$. $(b)\ R = 119\ \mathrm{N}$ approximately.

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Question 5

9 marks | Working with Vectors

5. [*In this question* **i** *and* **j** *are perpendicular horizontal unit vectors.*]

A particle P is moving with constant acceleration. At 2 pm, the velocity of P is $(3\mathbf{i} + 5\mathbf{j})$ km h^{-1} and at 2.30 pm the velocity of P is $(\mathbf{i} + 7\mathbf{j})$ km h^{-1}

At time T hours after 2 pm, P is moving in the direction of the vector $(-\mathbf{i} + 2\mathbf{j})$

(a) Find the value of T.

(6)

Another particle, Q, has velocity $\mathbf{v}_Q$ km h^{-1} at time t hours after 2 pm, where

$$\mathbf{v}_Q = (-4 - 2t)\mathbf{i} + (\mu + 3t)\mathbf{j}$$

and μ is a constant.

Given that there is an instant when the velocity of P is equal to the velocity of Q,

(b) find the value of μ.

(3)

WORKED SOLUTION

Question 5 · WME01/01 May/June 2021

1 Find P acceleration

From 2:00 to 2:30 is 0.5 hours. The velocity changes from $3\mathbf{i} + 5\mathbf{j}$ to $\mathbf{i} + 7\mathbf{j}$.

2 Calculate acceleration

$\mathbf{a} = \dfrac{(\mathbf{i} + 7\mathbf{j}) - (3\mathbf{i} + 5\mathbf{j})}{0.5} = -4\mathbf{i} + 4\mathbf{j}$.

3 Write P velocity

At time t hours after 2:00, $\mathbf{v}_P = (3 - 4t)\mathbf{i} + (5 + 4t)\mathbf{j}$.

4 Use the direction vector

At time T, the velocity is parallel to $-\mathbf{i} + 2\mathbf{j}$, so $\dfrac{5 + 4T}{3 - 4T} = \dfrac{2}{-1} = -2$.

5 Find T

$5 + 4T = -2(3 - 4T) = -6 + 8T$. Hence $11 = 4T$ and $T = \dfrac{11}{4}$ hours.

6 **Equate i-components for P and Q**

For equal velocities, $3 - 4t = -4 - 2t$. This gives $7 = 2t$, so $t = 3.5$.

7 **Equate j-components**

At $t = 3.5$, P's j-component is $5 + 4(3.5) = 19$. Q's j-component is $\mu + 3(3.5) = \mu + 10.5$.

8 **Find mu**

$\mu + 10.5 = 19$, so $\mu = 8.5$.

✓ **FINAL ANSWER**

$(a)\ T = \dfrac{11}{4}\ \text{h}.$ $(b)\ \mu = 8.5.$

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Question 6

13 marks | Resolving Forces, Inclined Planes

6. A fixed rough plane is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$

A particle of mass 6 kg is projected with speed 5 m s^{-1} from a point A on the plane, up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is $\frac{1}{4}$

(a) Find the magnitude of the frictional force acting on the particle as it moves up the plane.

(3)

The particle comes to instantaneous rest at the point B.

(b) Find the distance AB.

(5)

The particle now slides down the plane from B. At the instant when the particle passes through the point C on the plane, the speed of the particle is again 5 m s^{-1}

(c) Find the distance BC.

(5)

WORKED SOLUTION

Question 6 · WME01/01 May/June 2021

1 Use the trig ratios

Since $\tan\theta = \frac{5}{12}$, $\sin\theta = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$.

2 Find the normal reaction

$R = 6g\cos\theta = 6g \cdot \frac{12}{13} = \frac{72g}{13}$.

3 Find friction

With $\mu = \frac{1}{4}$, $F = \frac{1}{4}R = \frac{18g}{13} = 13.6\ \text{N}$ approximately.

4 Find acceleration while moving up

While moving up, both friction and the component of weight act down the plane. Taking up the plane as positive: $6a = -6g\sin\theta - F$.

5 **Simplify the upward acceleration**

$6a = -6g \cdot \frac{5}{13} - \frac{18g}{13} = -\frac{48g}{13}$, so $a = -\frac{8g}{13}$.

6 **Find AB**

Use $v^2 = u^2 + 2as$ with $u = 5$ and $v = 0$: $0 = 25 + 2\left(-\frac{8g}{13}\right)AB$. Hence $AB = 2.07\ldots\ \text{m}$.

7 **Find acceleration while sliding down**

When sliding down, friction acts up the plane. Taking down the plane as positive:

$6a = 6g\sin\theta - F = \frac{30g}{13} - \frac{18g}{13} = \frac{12g}{13}$. Thus $a = \frac{2g}{13}$.

8 **Find BC**

Starting from rest at B and reaching speed 5 at C: $25 = 2\left(\frac{2g}{13}\right)BC$. Hence $BC = 8.29\ldots\ \text{m}$.

✓ **FINAL ANSWER**

$(a)\ F = 13.6\ \text{N}$. $(b)\ AB = 2.07\ \text{m} \approx 2.1\ \text{m}$. $(c)\ BC = 8.29\ \text{m} \approx 8.3\ \text{m}$.

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Question 7

10 marks | Moments

7.

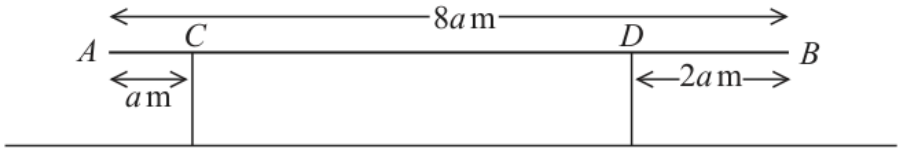


Figure 2

A non-uniform beam AB, of mass 60 kg and length $8a$ metres, rests in equilibrium in a horizontal position on two vertical supports. One support is at C, where $AC = a$ metres and the other support is at D, where $DB = 2a$ metres, as shown in Figure 2.

The magnitude of the normal reaction between the beam and the support at D is three times the magnitude of the normal reaction between the beam and the support at C.

By modelling the beam as a non-uniform rod whose centre of mass is at a distance x metres from A,

(a) find an expression for x in terms of a.

(5)

A box of mass M kg is placed on the beam at E, where $AE = 2a$ metres.

The beam remains in equilibrium in a horizontal position.

The magnitude of the normal reaction between the beam and the support at C is now equal to the magnitude of the normal reaction between the beam and the support at D.

By modelling the box as a particle,

(b) find the value of M.

(5)

WORKED SOLUTION

Question 7 · WME01/01 May/June 2021

1 Set the support positions

The beam length is $8a$. C is at distance a from A, and D is at distance $6a$ from A because $DB = 2a$.

2 Name the reactions

Let the reaction at C be R. The reaction at D is $3R$.

3 **Use vertical equilibrium**

$R + 3R = 60g$, so $4R = 60g$ and $R = 15g$.

4 **Take moments about A**

The beam's weight acts at distance x from A. Hence $60gx = R(a) + 3R(6a)$.

5 **Find x**

Substitute $R = 15g$: $60gx = 15ga + 45g(6a) = 285ga$. Thus $x = \dfrac{285a}{60} = \dfrac{19a}{4}$.

6 **Use equal reactions with the box**

When the box is added, let each support reaction be S. Vertical equilibrium gives $2S = (60 + M)g$.

7 **Take moments about A again**

The box is at $AE = 2a$. Moments about A give $60g\left(\dfrac{19a}{4}\right) + Mg(2a) = S(a) + S(6a) = 7aS$.

8 **Solve for M**

Substitute $S = \dfrac{(60 + M)g}{2}$ and cancel ag: $285 + 2M = \dfrac{7}{2}(60 + M) = 210 + \dfrac{7}{2}M$. Hence $75 = \dfrac{3}{2}M$ and $M = 50$.

✓ **FINAL ANSWER**

$(a)\ x = \dfrac{19a}{4}.$ $\quad (b)\ M = 50.$

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Question 8

13 marks | Kinematics Graphs

8. Two trams, tram A and tram B, run on parallel straight horizontal tracks. Initially the two trams are at rest in the depot and level with each other.

At time $t = 0$, tram A starts to move. Tram A moves with constant acceleration $2\,\text{m s}^{-2}$ for 5 seconds and then continues to move along the track at constant speed.

At time $t = 20$ seconds, tram B starts from rest and moves in the same direction as tram A. Tram B moves with constant acceleration $3\,\text{m s}^{-2}$ for 4 seconds and then continues to move along the track at constant speed.

The trams are modelled as particles.

(a) Sketch, on the same axes, a speed-time graph for the motion of tram A and a speed-time graph for the motion of tram B, from $t = 0$ to the instant when tram B overtakes tram A.

(3)

At the instant when the two trams are moving with the same speed, tram A is d metres in front of tram B.

(b) Find the value of d.

(5)

(c) Find the distance of the trams from the depot at the instant when tram B overtakes tram A.

(5)

(Total 13 marks)

WORKED SOLUTION

Question 8 · WME01/01 May/June 2021

1 **Describe tram A graph**

Tram A starts at $t = 0$, accelerates at $2\ \text{m s}^{-2}$ for 5 s, reaching speed $10\ \text{m s}^{-1}$, then continues at this constant speed.

2 **Describe tram B graph**

Tram B starts at $t = 20$, accelerates at $3\ \text{m s}^{-2}$ for 4 s, reaching speed $12\ \text{m s}^{-1}$, then continues at this constant speed. Its graph crosses A's distance later.

3 **Find when speeds are equal**

Tram A's constant speed is 10. Tram B reaches speed 10 after $\frac{10}{3}$ s of acceleration, so the time from $t = 0$ is $20 + \frac{10}{3} = \frac{70}{3}$ s.

4 **Distance of A then**

A travels $\frac{1}{2}(5)(10) = 25$ m in its acceleration phase, then $10\left(\frac{70}{3} - 5\right) = \frac{550}{3}$ m. Total $= \frac{625}{3}$ m.

5 **Distance of B then**

B is still accelerating from rest, so its distance is $\frac{1}{2}\left(\frac{10}{3}\right)(10) = \frac{50}{3}$ m.

6 **Find d**

$d = \frac{625}{3} - \frac{50}{3} = \frac{575}{3}$ m.

7 **Set overtake distances equal**

For $t > 24$, A's distance is $25 + 10(t - 5) = 10t - 25$. B's distance is $\frac{1}{2}(4)(12) + 12(t - 24) = 12t - 264$.

8 **Find the overtake time**

Set $10t - 25 = 12t - 264$. Then $2t = 239$ and $t = 119.5$ s.

9 **Find the distance from the depot**

Using A's distance, $10(119.5) - 25 = 1170$ m.

✓ **FINAL ANSWER**

(a) Tram A reaches speed 10 at $t = 5$; tram B starts at $t = 20$, reaches speed 12 at $t = 24$, then later overtakes A.

(b) $d = \frac{575}{3}\ \text{m} \approx 192\ \text{m}.$ (c) $1170\ \text{m}.$