Elite Mathematics
Edexcel International A Level

IAL MECHANICS 1 · WME01

WME01/01 January 2021 Worked Solutions

January 2021 paper rebuilt with the worked solution after each question.

8	2019–2026	Single-paper solutions
paper questions	18 exam sessions	checked by Dr Eslam



Dr Eslam Ahmed
Assistant Lecturer, Faculty of Engineering, Cairo University
eliteigcse.com · free for every student

PAST PAPER

WME01/01 January 2021

January 2021

8	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WME01/01 JANUARY 2021

Question 1

6 marks | Constant Acceleration in 1D

1. A small stone is projected vertically upwards with speed $20\,\text{m s}^{-1}$ from a point O which is 5 m above horizontal ground. The stone is modelled as a particle moving freely under gravity.

Find

(a) the speed of the stone at the instant when it is 2 m above the ground,

(2)

(b) the total time between the instant when the stone is projected from O and the instant when it first strikes the ground.

(4)

WORKED SOLUTION

Question 1 · WME01/01 January 2021

1 Find displacement for part a

The stone starts 5 m above the ground. When it is 2 m above the ground, its displacement from O is -3 m.

2 Use SUVAT for speed

Taking upward as positive, $u = 20$, $a = -9.8$ and $s = -3$. Use $v^2 = u^2 + 2as$:
$v^2 = 20^2 + 2(-9.8)(-3) = 458.8$.

3 State the speed

$v = 21.4\,\text{m s}^{-1}$ to 3 significant figures.

4 Set up the ground equation

At the ground, displacement from O is -5 m. Use $s = ut + \frac{1}{2}at^2$: $-5 = 20t - 4.9t^2$.

5 Solve for time

$4.9t^2 - 20t - 5 = 0$. The positive root is $t = 4.3179\ldots$, so the total time is 4.32 s.

FINAL ANSWER

$(a)\ 21.4\,\text{m s}^{-1}$. $(b)\ 4.32$ s approximately.

WME01/01 JANUARY 2021

Question 2

6 marks | Momentum, Impulse & Collisions

2. Two particles, P and Q, have masses $2m$ and m respectively. The particles are moving towards each other in opposite directions along the same straight line on a smooth horizontal plane. The particles collide directly.

Immediately before the collision, the speed of P is $3u$ and the speed of Q is $2u$.

The magnitude of the impulse exerted on Q by P in the collision is $5mu$.

Find

(a) the speed of P immediately after the collision,

(3)

(b) the speed of Q immediately after the collision.

(3)

WORKED SOLUTION

Question 2 · WME01/01 January 2021

1 Choose a positive direction

Take P's original direction as positive. Before collision, P has velocity $3u$ and Q has velocity $-2u$.

2 Use impulse on P

The impulse on P is opposite to P's original motion and has magnitude $5mu$. Let P's velocity after collision be v. Then $-5mu = 2m(v - 3u)$.

3 Find P speed

Divide by m: $-5u = 2v - 6u$, so $2v = u$ and $v = \frac{u}{2}$.

4 Use impulse on Q

The impulse on Q is in P's original direction. Let Q's velocity after collision be w. Then $5mu = m(w - (-2u))$.

5 Find Q speed

$5u = w + 2u$, so $w = 3u$. Therefore the speed of Q is $3u$.

FINAL ANSWER

(a) speed of P $= \frac{u}{2}$. (b) speed of Q $= 3u$.

WME01/01 JANUARY 2021

Question 3

9 marks | Forces

3.

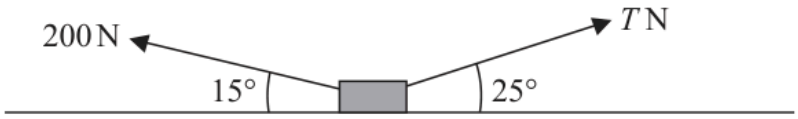


Figure 1

A parcel of mass 20 kg is at rest on a rough horizontal floor. The coefficient of friction between the parcel and the floor is 0.3

Two forces, both acting in the same vertical plane, of magnitudes 200 N and T N are applied to the parcel. The line of action of the 200 N force makes an angle of 15° with the horizontal and the line of action of the T N force makes an angle of 25° with the horizontal, as shown in Figure 1. The parcel is modelled as a particle P.

Find the smallest value of T for which P remains in equilibrium.

(9)

WORKED SOLUTION

Question 3 · WME01/01 January 2021

1 Resolve vertically

For vertical equilibrium, $R + 200\sin 15° + T\sin 25° = 20g$.

2 Resolve horizontally

For the smallest T, friction is limiting and balances the remaining horizontal tendency. Horizontally, $200\cos 15° - T\cos 25° - F = 0$.

3 Use limiting friction

$F = 0.3R$.

4 Eliminate R and F

From the vertical equation, $R = 20g - 200\sin 15° - T\sin 25°$. Substitute into $F = 0.3R$ and then into the horizontal equation.

5 Solve for T

$200\cos 15° - T\cos 25° - 0.3(20g - 200\sin 15° - T\sin 25°) = 0$. Solving gives $T = 192.3\ldots$ N, so the smallest value is about 192 N.

FINAL ANSWER

$T = 192\ \text{N}$ approximately.

WME01/01 JANUARY 2021

Question 4

6 marks | Moments

4.

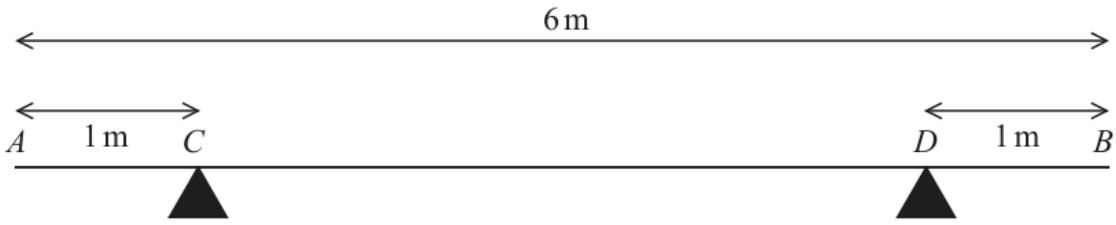


Figure 2

A metal girder AB has weight W newtons and length 6 m. The girder rests in a horizontal position on two supports C and D where $AC = DB = 1$ m, as shown in Figure 2.

When a force of magnitude 900 N is applied vertically upwards to the girder at A, the girder is about to tilt about D.

When a force of magnitude 1500 N is applied vertically upwards to the girder at B, the girder is about to tilt about C.

The girder is modelled as a non-uniform rod whose centre of mass is a distance x metres from A.

Find the value of x.

(6)

WORKED SOLUTION

Question 4 · WME01/01 January 2021

1 Set support positions

The girder has length 6 m, with C 1 m from A and D 1 m from B. So D is 5 m from A.

2 Use tilting about D

When the 900 N upward force is applied at A, the girder is about to tilt about D. Taking moments about D gives $900(5) = W(5 - x)$.

3 Use tilting about C

When the 1500 N upward force is applied at B, the girder is about to tilt about C. Taking moments about C gives $1500(5) = W(x - 1)$.

4 **Solve the equations**

The equations are $W(5-x)=4500$ and $W(x-1)=7500$. Add them: $W[(5-x)+(x-1)]=12000$, so $4W=12000$ and $W=3000$.

5 **Find x**

Use $W(x-1)=7500$: $3000(x-1)=7500$, so $x-1=2.5$ and $x=3.5$ m.

✓ **FINAL ANSWER**

$x=3.5\ \text{m}$.

WME01/01 JANUARY 2021

Question 5

7 marks | Working with Vectors

5. A particle is acted upon by two forces $\mathbf{F}$ and $\mathbf{G}$. The force $\mathbf{F}$ has magnitude 8 N and acts in a direction with a bearing of 240°. The force $\mathbf{G}$ has magnitude 10 N and acts due South.

Given that $\mathbf{R} = \mathbf{F} + \mathbf{G}$, find

(i) the magnitude of $\mathbf{R}$,

(ii) the direction of $\mathbf{R}$, giving your answer as a bearing to the nearest degree.

(7)

WORKED SOLUTION

Question 5 · WME01/01 January 2021

1 Resolve F into components

A bearing of $240°$ is $60°$ west of south. Hence $\mathbf{F} = (-8\sin 60°)\mathbf{i} + (-8\cos 60°)\mathbf{j} = -4\sqrt{3}\mathbf{i} - 4\mathbf{j}$.

2 Add G

The force $\mathbf{G}$ is due south with magnitude 10, so $\mathbf{G} = -10\mathbf{j}$. Therefore $\mathbf{R} = \mathbf{F} + \mathbf{G} = -4\sqrt{3}\mathbf{i} - 14\mathbf{j}$.

3 Find the magnitude

$|\mathbf{R}| = \sqrt{(-4\sqrt{3})^2 + (-14)^2} = \sqrt{48 + 196} = \sqrt{244}\ \text{N}$.

4 Find the bearing angle

The resultant points south-west. The angle west of south satisfies $\tan\theta = \dfrac{4\sqrt{3}}{14} = \dfrac{2\sqrt{3}}{7}$.

5 State the bearing

$\theta = 26.3°$. Bearing $= 180° + 26.3° = 206.3°$, so the bearing is $206°$ to the nearest degree.

FINAL ANSWER

$(i)\ |\mathbf{R}| = \sqrt{244}\ \text{N} \approx 15.6\ \text{N}. \quad (ii)\ \text{bearing} = 206°.$

WME01/01 JANUARY 2021

Question 6

12 marks | Working with Vectors

6. Two girls, Agatha and Brionie, are roller skating inside a large empty building. The girls are modelled as particles.

At time $t = 0$, Agatha is at the point with position vector $(11\mathbf{i} + 11\mathbf{j})$ m and Brionie is at the point with position vector $(7\mathbf{i} + 16\mathbf{j})$ m. The position vectors are given relative to the door, O, and $\mathbf{i}$ and $\mathbf{j}$ are horizontal perpendicular unit vectors.

Agatha skates with constant velocity $(3\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$

Brionie skates with constant velocity $(4\mathbf{i} - 2\mathbf{j})\,\text{m s}^{-1}$

(a) Find the position vector of Agatha at time t seconds.

(2)

At time $t = 6$ seconds, Agatha passes through the point P.

(b) Show that Brionie also passes through P and find the value of t when this occurs.

(4)

At time t seconds, Agatha is at the point A and Brionie is at the point B.

(c) Show that $\overrightarrow{AB} = [(t - 4)\mathbf{i} + (5 - t)\mathbf{j}]\,\text{m}$

(2)

(d) Find the distance between the two girls when they are closest together.

(4)

WORKED SOLUTION

Question 6 · WME01/01 January 2021

1 Write Agatha position

$\mathbf{r}_A = (11\mathbf{i} + 11\mathbf{j}) + t(3\mathbf{i} - \mathbf{j}) = (11 + 3t)\mathbf{i} + (11 - t)\mathbf{j}$.

2 Find point P

At $t = 6$, Agatha is at $(11 + 18)\mathbf{i} + (11 - 6)\mathbf{j} = 29\mathbf{i} + 5\mathbf{j}$.

3 Write Brionie position

$\mathbf{r}_B = (7\mathbf{i} + 16\mathbf{j}) + t(4\mathbf{i} - 2\mathbf{j}) = (7 + 4t)\mathbf{i} + (16 - 2t)\mathbf{j}$.

4 **Show Brionie passes P**

Set $7 + 4t = 29$ to get $t = 5.5$. Also $16 - 2t = 5$ gives $t = 5.5$. So Brionie also passes through P at $t = 5.5$ s.

---

5 **Find AB vector**

$\overrightarrow{AB} = \mathbf{r}_B - \mathbf{r}_A = [7 + 4t - (11 + 3t)]\mathbf{i} + [16 - 2t - (11 - t)]\mathbf{j}$.

---

6 **Simplify AB**

So $\overrightarrow{AB} = (t - 4)\mathbf{i} + (5 - t)\mathbf{j}$ m.

---

7 **Minimise distance squared**

$AB^2 = (t - 4)^2 + (5 - t)^2 = 2(t - 4.5)^2 + 0.5$.

---

8 **State the minimum distance**

The minimum occurs when $t = 4.5$, giving $AB^2 = 0.5$. Hence the minimum distance is $\sqrt{0.5} = 0.707\ldots$ m.

✓ **FINAL ANSWER**

$(a)\ \mathbf{r}_A = (11 + 3t)\mathbf{i} + (11 - t)\mathbf{j}$. $(b)\ t = 5.5$ s. $(c)\ \overrightarrow{AB} = (t - 4)\mathbf{i} + (5 - t)\mathbf{j}$. (d) minimum distance $= \sqrt{0.5}$ m ≈ 0.71 m .

WME01/01 JANUARY 2021

Question 7

12 marks | Kinematics Graphs

7. A helicopter is hovering at rest above horizontal ground at the point H. A parachutist steps out of the helicopter and immediately falls vertically and freely under gravity from rest for 2.5 s. His parachute then opens and causes him to immediately decelerate at a constant rate of $3.9\,\text{m s}^{-2}$ for T seconds ($T < 6$), until his speed is reduced to $V\,\text{m s}^{-1}$. He then moves with this constant speed $V\,\text{m s}^{-1}$ until he hits the ground. While he is decelerating, he falls a distance of 73.75 m. The total time between the instant when he leaves H and the instant when he hits the ground is 20 s.

The parachutist is modelled as a particle.

(a) Find the speed of the parachutist at the instant when his parachute opens.

(1)

(b) Sketch a speed-time graph for the motion of the parachutist from the instant when he leaves H to the instant when he hits the ground.

(2)

(c) Find the value of T.

(5)

(d) Find, to the nearest metre, the height of the point H above the ground.

(4)

WORKED SOLUTION

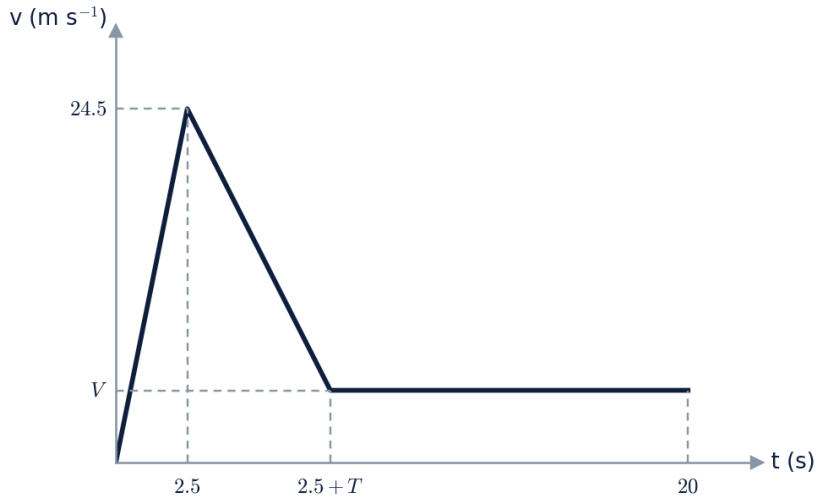
Question 7 · WME01/01 January 2021

1 **Find speed when parachute opens**

The parachutist falls freely from rest for 2.5 s, so $v = gt = 9.8(2.5) = 24.5\ \text{m s}^{-1}$.

2 **Describe the graph**

The speed-time graph starts at 0, rises linearly to 24.5 at $t = 2.5$, then decreases linearly with smaller gradient for T seconds, then stays horizontal at speed V until $t = 20$.



3 **Use the deceleration distance**

During the deceleration phase, initial speed is 24.5, acceleration is -3.9, time is T, and distance is 73.75.

4 **Form the equation for T**

$73.75 = 24.5T + \frac{1}{2}(-3.9)T^2 = 24.5T - 1.95T^2$.

5 **Choose the valid T**

Solving gives $T = 5$ or $T = 7.56\ldots$. Since $T < 6$, take $T = 5$.

6 **Find V**

$V = 24.5 - 3.9(5) = 5\ \mathrm{m\,s^{-1}}$.

7 **Find the total height**

Height is the total area under the speed-time graph: first triangle $\frac{1}{2}(2.5)(24.5) = 30.625$, deceleration distance 73.75, and final rectangle $(20 - 2.5 - 5)(5) = 62.5$.

8 **Round the height**

Total height $= 30.625 + 73.75 + 62.5 = 166.875$ m, which rounds to 167 m.

✓ **FINAL ANSWER**

$(a)\ 24.5\ \mathrm{m\,s^{-1}}$. (b) Speed-time graph rises to 24.5 at $t = 2.5$, falls to V at $t = 2.5 + T$, then is horizontal to $t = 20$.

$(c)\ T = 5$. $(d)\ H = 167\ \mathrm{m}$ to the nearest metre.

Question 8

17 marks | Resolving Forces, Inclined Planes

8.

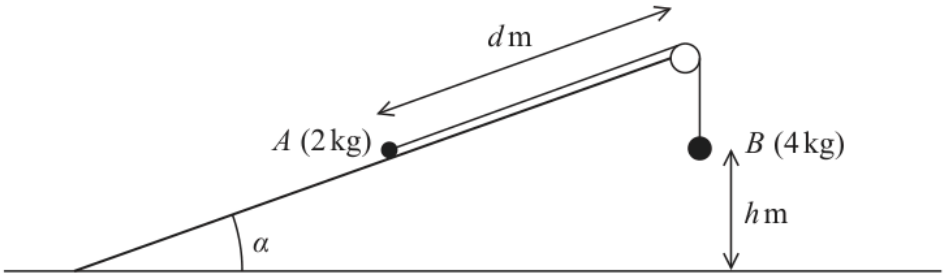


Figure 3

Two particles, A and B, have masses 2 kg and 4 kg respectively. The particles are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough plane. The plane is inclined to the horizontal ground at an angle α where $\tan\alpha = \frac{3}{4}$. The particle A is held at rest on the plane at a distance d metres from the pulley. The particle B hangs freely at rest, vertically below the pulley, at a distance h metres above the ground, as shown in Figure 3. The part of the string between A and the pulley is parallel to a line of greatest slope of the plane. The coefficient of friction between A and the plane is $\frac{1}{4}$

The system is released from rest with the string taut and B descends.

(a) Find the tension in the string as B descends.

(9)

On hitting the ground, B immediately comes to rest.

Given that A comes to rest before reaching the pulley,

(b) find, in terms of h, the range of possible values of d.

(7)

(c) State one physical factor, other than air resistance, that could be taken into account to make the model described above more realistic.

(1)

(Total 17 marks)

1 **Use the trig ratios**

Since $\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$.

2 **Find friction on A**

For A, $R = 2g\cos\alpha = \frac{8g}{5}$. With coefficient $\frac{1}{4}$, friction is $F = \frac{1}{4}R = \frac{2g}{5}$.

3 **Equation for A**

A moves up the plane. Taking up the plane as positive: $T - 2g\sin\alpha - F = 2a$.

4 **Simplify A's equation**

$T - 2g\cdot\frac{3}{5} - \frac{2g}{5} = 2a$, so $T - \frac{8g}{5} = 2a$.

5 **Equation for B**

B descends. Taking downward as positive: $4g - T = 4a$.

6 **Solve for T**

From A, $T = 2a + \frac{8g}{5}$. From B, $T = 4g - 4a$. Equating gives $6a = \frac{12g}{5}$, so $a = \frac{2g}{5}$. Then $T = 4g - 4\left(\frac{2g}{5}\right) = \frac{12g}{5}$ N.

7 **Find speed when B reaches the ground**

Before B hits the ground, both particles move distance h from rest with acceleration $\frac{2g}{5}$. Thus $v^2 = 2\left(\frac{2g}{5}\right)h = \frac{4gh}{5}$.

8 **Find extra distance after B stops**

After B hits the ground, A continues up the plane. The forces down the plane on A are $2g\sin\alpha + F = \frac{6g}{5} + \frac{2g}{5} = \frac{8g}{5}$, so its deceleration is $\frac{4g}{5}$.

9 **Calculate extra distance**

Let the extra distance be s. Then $0 = \frac{4gh}{5} - 2\left(\frac{4g}{5}\right)s$, so $s = \frac{h}{2}$.

10 **Find the range for d**

A travels total distance $h + \frac{h}{2} = \frac{3h}{2}$ before stopping. Since it comes to rest before reaching the pulley, $d > \frac{3h}{2}$.

11 **Name a more realistic factor**

A more realistic model could include friction at the pulley.

FINAL ANSWER

$(a)\ T = \dfrac{12g}{5}\ \text{N} = 23.5\ \text{N}.$ $(b)\ d > \dfrac{3h}{2}.$ (c) A more realistic model could include friction at the pulley.