Elite Mathematics
Edexcel International A Level

IAL MECHANICS 1 · WME01

WME01/01 January 2020 Worked Solutions

January 2020 paper rebuilt with the worked solution after each question.

7	2019–2026	Single-paper solutions
paper questions	18 exam sessions	checked by Dr Eslam



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PAST PAPER

WME01/01 January 2020

January 2020

7	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WME01/01 JANUARY 2020

Question 1

8 marks | Momentum, Impulse & Collisions

1. Two particles, P and Q, of mass m_1 and m_2 respectively, are moving on a smooth horizontal plane. The particles are moving towards each other in opposite directions along the same straight line when they collide directly. Immediately before the collision, both particles are moving with speed u.

The direction of motion of each particle is reversed by the collision.

Immediately after the collision, the speed of Q is $\frac{1}{3}u$.

(a) Find, in terms of m_2 and u, the magnitude of the impulse exerted by P on Q in the collision.

(3)

(b) Find, in terms of m_1, m_2 and u, the speed of P immediately after the collision.

(3)

(c) Hence show that $m_2 > \frac{3}{4}\ m_1$

(2)

WORKED SOLUTION

Question 1 · WME01/01 January 2020

1 Choose a positive direction

Take P's original direction as positive. Before collision, $v_P = u$ and $v_Q = -u$. After collision, Q has velocity $\frac{u}{3}$ because its direction is reversed.

2 Find the impulse on Q

Impulse on Q equals change in momentum: $I = m_2\left(\frac{u}{3} - (-u)\right) = m_2\left(\frac{4u}{3}\right) = \frac{4}{3}m_2u$.

3 Use conservation of momentum

Let the speed of P after collision be v. Since P reverses direction, its velocity is $-v$. Hence $m_1u - m_2u = -m_1v + \frac{1}{3}m_2u$.

4 Solve for P's speed

Rearrange: $m_1v = \frac{4}{3}m_2u - m_1u$, so $v = \frac{(4m_2 - 3m_1)u}{3m_1}$.

5 **Use the fact that speed is positive**

Since $u > 0$ and $m_1 > 0$, the numerator must be positive: $4m_2 - 3m_1 > 0$. Therefore $m_2 > \frac{3}{4}m_1$.

✓ **FINAL ANSWER**

$(a)\ I = \frac{4}{3}m_2 u.$ $(b)\ \text{speed of P} = \frac{(4m_2 - 3m_1)u}{3m_1}.$ $(c)\ m_2 > \frac{3}{4}m_1.$

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Question 2

7 marks | Moments

2.

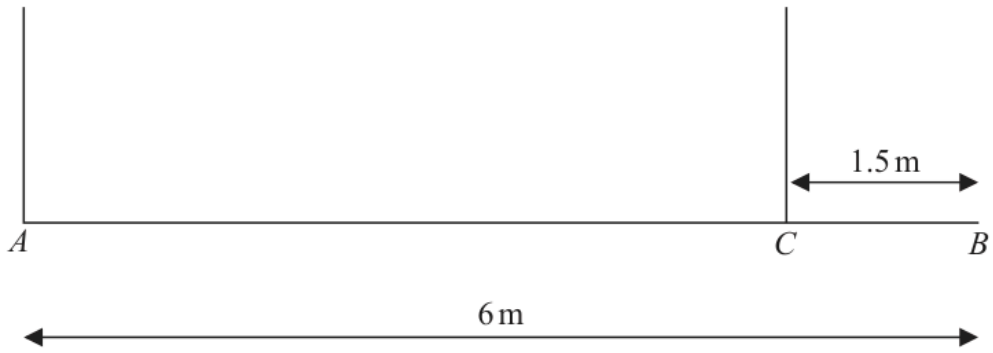


Figure 1

A non-uniform beam AB has length 6 m and weight W newtons. The beam is supported in equilibrium in a horizontal position by two vertical ropes, one attached to the beam at A and the other attached to the beam at C, where $CB = 1.5$ m, as shown in Figure 1.

The centre of mass of the beam is 2.625 m from A.

The ropes are modelled as light strings. The beam is modelled as a non-uniform rod.

Given that the tension in the rope attached at C is 20 N greater than the tension in the rope attached at A,

(a) find the value of W.

(6)

(b) State how you have used the fact that the beam is modelled as a rod.

(1)

WORKED SOLUTION

Question 2 · WME01/01 January 2020

1 Set the tensions

Let the tension at A be T. The tension at C is then $T + 20$. Since $CB = 1.5$ m and $AB = 6$ m, $AC = 4.5$ m.

2 Use vertical equilibrium

The upward tensions balance the weight: $T + (T + 20) = W$, so $2T + 20 = W$.

3 Take moments about A

The centre of mass is 2.625 m from A. Taking moments about A gives $4.5(T + 20) = 2.625W$.

4 **Solve for W**

From the moments equation, $T + 20 = \dfrac{2.625}{4.5}W = \dfrac{7}{12}W$. Also $T + 20 = \dfrac{W + 20}{2}$. Hence $\dfrac{W + 20}{2} = \dfrac{7W}{12}$, so $6W + 120 = 7W$ and $W = 120$ N.

5 **State the modelling use**

Modelling the beam as a rod means it stays straight and rigid, so the distances used in the moments equations do not change.

✓ **FINAL ANSWER**

$(a)\ W = 120\ \text{N}.$ (b) The rod model keeps the beam straight and rigid, so the stated distances along the beam are fixed.

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Question 3

11 marks | Constant Acceleration in 1D

3. A particle, P, is projected vertically upwards with speed U from a fixed point O. At the instant when P reaches its greatest height H above O, a second particle, Q, is projected with speed $\frac{1}{2}U$ vertically upwards from O.

(a) Find H in terms of U and g.

(2)

(b) Find, in terms of U and g, the time between the instant when Q is projected and the instant when the two particles collide.

(6)

(c) Find where the two particles collide.

(3)

WORKED SOLUTION

Question 3 · WME01/01 January 2020

1 Find the greatest height

For P, at greatest height the speed is zero. Use $v^2 = u^2 + 2as$ with $u = U$, $v = 0$, $a = -g$ and $s = H$: $0 = U^2 - 2gH$. Hence $H = \dfrac{U^2}{2g}$.

2 Start timing when Q is projected

At this instant P is at height H and has zero velocity. After t seconds, P has fallen $\frac{1}{2}gt^2$, so its height above O is $H - \frac{1}{2}gt^2$.

3 Write Q's height

Q is projected upwards from O with speed $\frac{1}{2}U$, so its height after t seconds is $\frac{1}{2}Ut - \frac{1}{2}gt^2$.

4 Equate the heights

At collision, $H - \frac{1}{2}gt^2 = \frac{1}{2}Ut - \frac{1}{2}gt^2$. The gravity terms cancel, so $H = \frac{1}{2}Ut$.

5 Find the time

Substitute $H = \dfrac{U^2}{2g}$: $\dfrac{U^2}{2g} = \frac{1}{2}Ut$. Therefore $t = \dfrac{U}{g}$.

6 **Find the collision position**

Substitute $t = \dfrac{U}{g}$ into P's height: $H - \dfrac{1}{2}g\left(\dfrac{U}{g}\right)^2 = \dfrac{U^2}{2g} - \dfrac{U^2}{2g} = 0$. Thus they collide at O.

✓ **FINAL ANSWER**

$(a)\ H = \dfrac{U^2}{2g}.$ $(b)\ t = \dfrac{U}{g}.$ (c) The particles collide at O.

Question 4

10 marks | Forces

4.

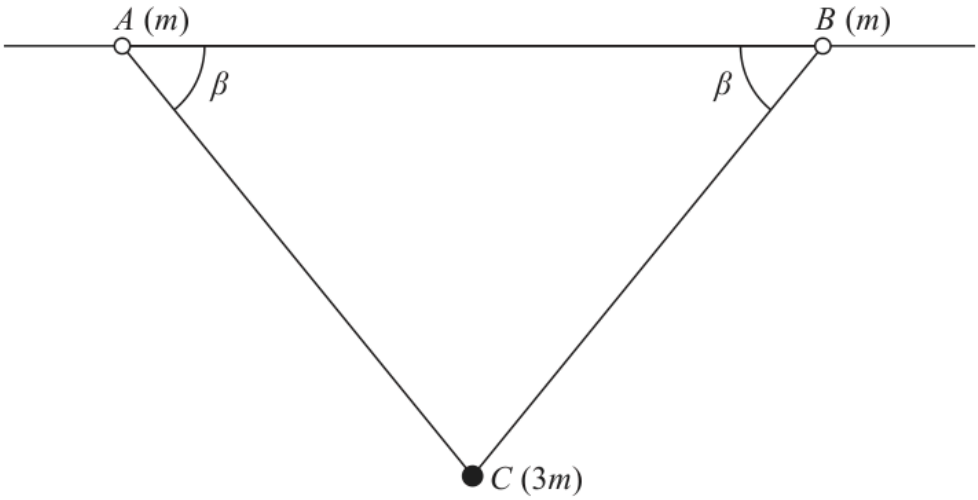


Figure 2

Two identical small rings, A and B, each of mass m, are threaded onto a rough horizontal wire. The rings are connected by a light inextensible string. A particle C of mass $3m$ is attached to the midpoint of the string. The particle C hangs in equilibrium below the wire with angle $BAC = \beta$, as shown in Figure 2.

The tension in each of the parts, AC and BC, of the string is T

(a) By considering particle C, find T in terms of m, g and β

(2)

(b) Find, in terms of m and g, the magnitude of the normal reaction between the wire and A.

(3)

The coefficient of friction between each ring and the wire is $\frac{4}{5}$

The two rings, A and B, are on the point of sliding along the wire towards each other.

(c) Find the value of $\tan\beta$

(5)

1 Resolve forces on C

The two tensions are equal and symmetric. Their vertical components support the weight of C, so $2T\sin\beta = 3mg$. Hence $T = \dfrac{3mg}{2\sin\beta}$.

2 Resolve vertically for ring A

For ring A, the normal reaction balances its own weight and the downward vertical component of the string tension: $R = mg + T\sin\beta$.

3 Find the normal reaction

From $2T\sin\beta = 3mg$, we have $T\sin\beta = \dfrac{3}{2}mg$. Therefore $R = mg + \dfrac{3}{2}mg = \dfrac{5}{2}mg$.

4 Use limiting friction

The rings are just about to slide towards each other, so friction is limiting. With $\mu = \dfrac{4}{5}$, $F = \mu R = \dfrac{4}{5}\cdot\dfrac{5}{2}mg = 2mg$.

5 Resolve horizontally for a ring

The horizontal component of the tension is balanced by friction: $T\cos\beta = F = 2mg$.

6 Find tan beta

Substitute $T = \dfrac{3mg}{2\sin\beta}$: $\dfrac{3mg}{2\sin\beta}\cos\beta = 2mg$. Hence $\dfrac{3}{2}\cot\beta = 2$, so $\cot\beta = \dfrac{4}{3}$ and $\tan\beta = \dfrac{3}{4}$.

FINAL ANSWER

$(a)\ T = \dfrac{3mg}{2\sin\beta}.\quad (b)\ R = \dfrac{5}{2}mg.\quad (c)\ \tan\beta = \dfrac{3}{4}.$

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Question 5

10 marks | Kinematics Graphs

5. A car travels at a constant speed of $40\,\text{m s}^{-1}$ in a straight line along a horizontal racetrack. At time $t = 0$, the car passes a motorcyclist who is at rest. The motorcyclist immediately sets off to catch up with the car.

The motorcyclist accelerates at $4\,\text{m s}^{-2}$ for 15 s and then accelerates at $1\,\text{m s}^{-2}$ for a further T seconds until he catches up with the car.

(a) Sketch, on the same axes, the speed-time graph for the motion of the car and the speed-time graph for the motion of the motorcyclist, from time $t = 0$ to the instant when the motorcyclist catches up with the car.

(2)

At the instant when $t = t_1$ seconds, the car and the motorcyclist are moving at the same speed.

(b) Find the value of t_1

(2)

(c) Show that $T^2 + kT - 300 = 0$, where k is a constant to be found.

(6)

WORKED SOLUTION

Question 5 · WME01/01 January 2020

1 Describe the graph

The car's graph is a horizontal line at $40\ \text{m s}^{-1}$. The motorcyclist's graph starts at zero, rises with gradient 4 until $t = 15$, then rises with smaller gradient 1 for a further T seconds.

2 Find the equal-speed time

Before $t = 15$, the motorcyclist's speed is $4t$. Set this equal to the car's speed: $4t_1 = 40$, so $t_1 = 10$ s.

3 Find the motorcyclist's speed at 15 seconds

After the first 15 seconds, the motorcyclist's speed is $4(15) = 60\ \text{m s}^{-1}$.

4 Find the final speed at catch-up

During the next T seconds the acceleration is $1\ \text{m s}^{-2}$, so the speed at catch-up is $60 + T$.

5 **Equate distances**

Motorcyclist distance $= \frac{1}{2}(15)(60) + \frac{1}{2}[60 + (60 + T)]T$. Car distance $= 40(15 + T)$.

6 **Simplify the equation**

$450 + 60T + \frac{1}{2}T^2 = 600 + 40T$. Hence $\frac{1}{2}T^2 + 20T - 150 = 0$, so $T^2 + 40T - 300 = 0$ and $k = 40$.

✓ **FINAL ANSWER**

(a) Car: horizontal line at 40; motorcyclist: two rising straight-line sections ending at the catch-up time.

(b) $t_1 = 10\ \text{s}$. (c) $T^2 + 40T - 300 = 0$, so $k = 40$.

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Question 6

11 marks | Working with Vectors

6. A force $\mathbf{F}$ is given by $\mathbf{F} = (10\mathbf{i} + \mathbf{j})$ N.

(a) Find the exact value of the magnitude of $\mathbf{F}$.

(2)

(b) Find, in degrees, the size of the angle between the direction of $\mathbf{F}$ and the direction of the vector $(\mathbf{i} + \mathbf{j})$.

(4)

The resultant of the force $\mathbf{F}$ and the force $(-15\mathbf{i} + a\mathbf{j})$ N, where a is a constant, is parallel to, but in the opposite direction to, the vector $(2\mathbf{i} - 3\mathbf{j})$.

(c) Find the value of a.

(5)

WORKED SOLUTION

Question 6 · WME01/01 January 2020

1 Find the magnitude

For $\mathbf{F} = 10\mathbf{i} + \mathbf{j}$, $|\mathbf{F}| = \sqrt{10^2 + 1^2} = \sqrt{101}$ N.

2 Find the direction angle of F

The angle that $\mathbf{F}$ makes with the positive $\mathbf{i}$ direction is α, where $\tan\alpha = \frac{1}{10}$. Thus $\alpha = 5.71°$ approximately.

3 Compare with i plus j

The vector $\mathbf{i} + \mathbf{j}$ makes an angle of $45°$ with the positive $\mathbf{i}$ direction.

4 Find the angle between the vectors

So the required angle is $45° - 5.71° = 39.29°$, approximately $39.3°$.

5 Write the resultant

The resultant of $10\mathbf{i} + \mathbf{j}$ and $-15\mathbf{i} + a\mathbf{j}$ is $-5\mathbf{i} + (a+1)\mathbf{j}$.

6 Use the parallel opposite direction

Parallel but opposite to $2\mathbf{i} - 3\mathbf{j}$ means the resultant is a positive multiple of $-2\mathbf{i} + 3\mathbf{j}$.

7 **Solve for a**

Let $-5\mathbf{i} + (a+1)\mathbf{j} = \lambda(-2\mathbf{i} + 3\mathbf{j})$. From the $\mathbf{i}$ component, $-5 = -2\lambda$, so $\lambda = 2.5$. Then $a + 1 = 3(2.5) = 7.5$, hence $a = 6.5$.

✓ **FINAL ANSWER**

$(a)\ |\mathbf{F}| = \sqrt{101}\ \text{N}.$ $(b)\ 39.3°$ **approximately.** $(c)\ a = 6.5.$

Question 7

18 marks | Newton's Second Law

7.

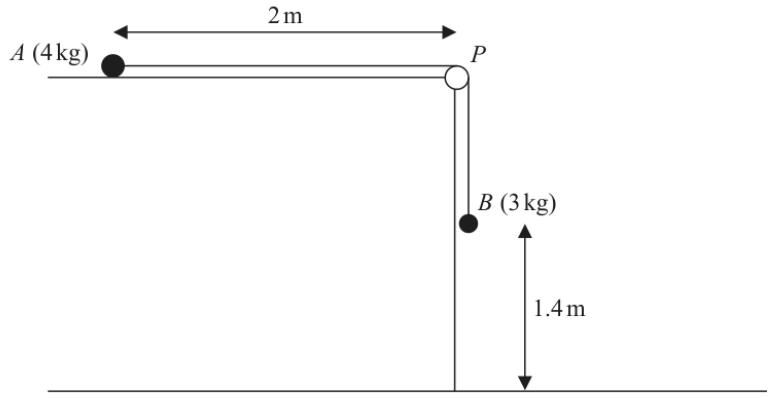


Figure 3

A particle A of mass 4 kg is held at rest on a rough horizontal table. Particle A is attached to one end of a string that passes over a pulley P. The pulley is fixed at the edge of the table. The other end of the string is attached to a particle B, of mass 3 kg, which hangs freely below P.

The part of the string from A to P is perpendicular to the edge of the table and A, P and B all lie in the same vertical plane.

The string is modelled as being light and inextensible and the pulley is modelled as being small, smooth and light.

The system is released from rest with the string taut. At the instant of release, A is 2 m from the edge of the table and B is 1.4 m above a horizontal floor, as shown in Figure 3.

After descending with constant acceleration for 2 seconds, B hits the floor and does not rebound.

(a) Show that the acceleration of A before B hits the floor is $0.7\,\text{m s}^{-2}$

(2)

(b) State which of the modelling assumptions you have used in order to answer part (a).

(1)

(c) Find the magnitude of the resultant force exerted on the pulley by the string.

(4)

The coefficient of friction between A and the table is μ.

(d) Find the value of μ.

(6)

(e) Determine, by calculation, whether or not A reaches the pulley.

(5)

(Total 18 marks)

1 **Use B's descent**

Before B hits the floor, it descends 1.4 m from rest in 2 s. Use $s = ut + \frac{1}{2}at^2$: $1.4 = \frac{1}{2}a(2)^2$. Hence $a = 0.7\ \text{m s}^{-2}$.

2 **Name the modelling assumption**

The string is inextensible, so A and B have the same magnitude of acceleration while the string is taut.

3 **Find the tension**

For B, taking downward as positive: $3g - T = 3(0.7)$. Thus $T = 3g - 2.1 = 27.3$ N.

4 **Find the resultant force on the pulley**

The pulley is pulled by two perpendicular tensions of magnitude T. Therefore the resultant is $\sqrt{T^2 + T^2} = T\sqrt{2} = 27.3\sqrt{2} = 38.6$ N approximately.

5 **Find the friction on A**

For A before B hits the floor, $T - F = 4(0.7)$. Hence $F = 27.3 - 2.8 = 24.5$ N.

6 **Find mu**

On the horizontal table, $R = 4g = 39.2$ N and $F = \mu R$. So $\mu = \dfrac{24.5}{39.2} = 0.625$.

7 **Find A's speed when B hits the floor**

At this instant, $v = at = 0.7(2) = 1.4\ \text{m s}^{-1}$. A has also moved 1.4 m towards the pulley.

8 **Find the extra stopping distance**

After B hits the floor, A is slowed only by friction. Its deceleration is $\mu g = 0.625(9.8) = 6.125\ \text{m s}^{-2}$. Use $0 = (1.4)^2 - 2(6.125)s$, giving $s = 0.16$ m.

9 **Compare with the distance to the pulley**

A's total distance is $1.4 + 0.16 = 1.56$ m, which is less than 2 m. Therefore A does not reach the pulley.

✓ **FINAL ANSWER**

$(a)\ a = 0.7\ \text{m s}^{-2}$. (b) Inextensibility of the string.

$(c)\ 38.6$ N. $(d)\ \mu = 0.625$. (e) A does not reach the pulley.