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Elite Mathematics
Edexcel International A Level

IAL MECHANICS 1 · WME01

WME01/01 October 2019 Worked Solutions

October 2019 paper rebuilt with the worked solution after each question.

8	2019–2026	Single-paper solutions
paper questions	18 exam sessions	checked by Dr Eslam



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PAST PAPER

WME01/01 October 2019

October 2019

8	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

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Question 1

6 marks | Momentum, Impulse & Collisions

1. Two particles, P and Q, have masses $3m$ and $2m$ respectively. The particles are connected by a light inextensible string. Initially P and Q are at rest on a smooth horizontal plane with the string slack.

Particle P is then projected along the plane directly away from Q with speed $4u$. At the same instant, particle Q is projected along the plane in the opposite direction with speed $3u$.

Find

(a) the common speed of the particles immediately after the string becomes taut,

(3)

(b) the magnitude of the impulse exerted on Q at the instant when the string becomes taut.

(3)

WORKED SOLUTION

Question 1 · WME01/01 October 2019

1 Choose a direction

Take P's initial direction as positive. Then P initially has velocity $4u$ and Q initially has velocity $-3u$.

2 Use conservation of momentum

When the string becomes taut, both particles move with common speed v. Hence

$3m(4u) + 2m(-3u) = (3m + 2m)v$.

3 Find the common speed

$12mu - 6mu = 5mv$, so $v = \dfrac{6u}{5}$.

4 Use impulse on Q

Impulse on Q equals change in momentum: $I = 2m\left(\dfrac{6u}{5} - (-3u)\right)$.

5 Find the impulse magnitude

$I = 2m\left(\dfrac{6u}{5} + \dfrac{15u}{5}\right) = 2m \cdot \dfrac{21u}{5} = \dfrac{42}{5}mu$.

FINAL ANSWER

$(a)\ v = \dfrac{6u}{5}.\quad (b)\ I = \dfrac{42}{5}mu.$

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Question 2

9 marks | Constant Acceleration in 1D

2. A small ball is released from rest from a point that is 40 m above horizontal ground. The ball bounces on the ground and rebounds vertically. Each time the ball bounces on the ground, the speed of the ball is instantaneously reduced by 50%. The ball is modelled as a particle moving freely under gravity, from the instant when it is released until it first hits the ground, and between each successive bounce.

(a) Find the time from the instant when the ball is released from rest to the instant when it hits the ground for the second time.

(5)

(b) Find the maximum height reached by the ball above the ground after the ball's third bounce.

(4)

WORKED SOLUTION

Question 2 · WME01/01 October 2019

1 Find the time to the first hit

The ball is released from rest and falls 40 m. Use $s = ut + \frac{1}{2}gt^2$: $40 = \frac{1}{2}gt_1^2$. Thus $t_1 = \sqrt{\frac{80}{g}} = \frac{20}{7}$ s.

2 Find the speed just before the first bounce

Using $v^2 = u^2 + 2gs$, $v_1^2 = 2g(40)$, so $v_1 = 28\ \mathrm{m\,s^{-1}}$.

3 Use the rebound speed

After the first bounce the speed is reduced by 50%, so the upward speed is $14\ \mathrm{m\,s^{-1}}$.

4 Find the time until the second hit

The time to go up and return to the ground is $\frac{2(14)}{g} = \frac{20}{7}$ s.

5 Find the total time

Total time to the second hit is $\frac{20}{7} + \frac{20}{7} = \frac{40}{7} = 5.71$ s approximately.

6 Find the speed after the third bounce

After each bounce, the speed is halved. The upward speeds after the first, second and third bounces are 14, 7 and $3.5\ \mathrm{m\,s^{-1}}$.

7 **Find the height after the third bounce**

At the top after the third bounce, $0 = (3.5)^2 - 2gh$. Therefore $h = \dfrac{(3.5)^2}{2g} = 0.625$ m.

✓ **FINAL ANSWER**

(a) 5.71 s approximately. (b) 0.625 m.

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Question 3

7 marks | Newton's Second Law

3. A car of mass 800 kg is towing a trailer of mass 400 kg up a straight road using a towbar. The towbar is parallel to the road and parallel to the direction of motion of the car. The road is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{7}$. The engine of the car produces a constant driving force of magnitude D newtons. The resistance to the motion of the car from non-gravitational forces is modelled as a single force of magnitude 420 N. The resistance to the motion of the trailer from non-gravitational forces is modelled as a single force of magnitude 300 N. The car and trailer are modelled as particles and the towbar is modelled as a light rod.

Given that the tension in the towbar is 2060 N, find the value of D.

(7)

WORKED SOLUTION

Question 3 · WME01/01 October 2019

1 Use the trailer to find acceleration

For the trailer, take up the road as positive. The forces along the road are tension 2060 N up the road, resistance 300 N down the road, and weight component $400g\sin\alpha$ down the road.

2 Form the trailer equation

$2060 - 300 - 400g\sin\alpha = 400a$. Since $\sin\alpha = \frac{1}{7}$, this becomes $2060 - 300 - 400g \cdot \frac{1}{7} = 400a$.

3 Find the acceleration

Using $g = 9.8$, $400g \cdot \frac{1}{7} = 560$. Hence $1200 = 400a$ and $a = 3\ \text{m s}^{-2}$.

4 Apply Newton's second law to the car

For the car: $D - 420 - 800g\sin\alpha - 2060 = 800a$.

5 Find the driving force

$D - 420 - 800g \cdot \frac{1}{7} - 2060 = 800(3)$. Since $800g \cdot \frac{1}{7} = 1120$,

$D = 420 + 1120 + 2060 + 2400 = 6000$ N.

FINAL ANSWER

$D = 6000$ N.

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Question 4

12 marks | Resolving Forces, Inclined Planes

4.

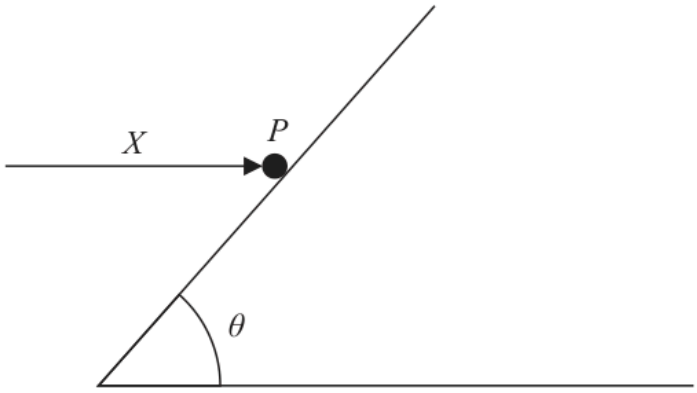


Figure 1

A particle, P, of mass km lies on a fixed rough plane. The plane is inclined to the horizontal at an acute angle θ. A horizontal force of magnitude X acts on P, as shown in Figure 1. The line of action of the force lies in the vertical plane which contains the line of greatest slope of the inclined plane that passes through P. The coefficient of friction between P and the inclined plane is μ.

When $X = mg$, the particle P is in equilibrium and on the point of sliding down the plane.

(a) Show that $\mu = \dfrac{k\tan\theta - 1}{k + \tan\theta}$

(10)

(b) Deduce that, when $k = 1$, θ must be greater than $45°$

(2)

WORKED SOLUTION

Question 4 · WME01/01 October 2019

1 **Set friction direction**

The particle is on the point of sliding down the plane, so friction acts up the plane. Also $X = mg$.

2 **Resolve perpendicular to the plane**

Both the weight kmg and the horizontal force mg press the particle into the plane. Therefore $R = kmg\cos\theta + mg\sin\theta$.

3 **Resolve parallel to the plane**

Down the plane is the component $kmg\sin\theta$. Up the plane are friction F and the component $mg\cos\theta$. Thus $F + mg\cos\theta = kmg\sin\theta$.

4 **Write friction in terms of theta**

So $F = kmg\sin\theta - mg\cos\theta$.

5 **Use limiting friction**

$$\mu = \frac{F}{R} = \frac{kmg\sin\theta - mg\cos\theta}{kmg\cos\theta + mg\sin\theta}.$$

6 **Simplify the expression**

Cancel mg and divide numerator and denominator by $\cos\theta$: $\mu = \dfrac{k\tan\theta - 1}{k + \tan\theta}$, as required.

7 **Deduce the condition when k is 1**

When $k = 1$, $\mu = \dfrac{\tan\theta - 1}{1 + \tan\theta}$. Since $\mu > 0$ and the denominator is positive, $\tan\theta - 1 > 0$.

8 **Finish the inequality**

Therefore $\tan\theta > 1$, so because θ is acute, $\theta > 45°$.

✓ **FINAL ANSWER**

$(a)\ \mu = \dfrac{k\tan\theta - 1}{k + \tan\theta}.$ $\quad (b)\ \theta > 45°.$

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Question 5

8 marks | Moments

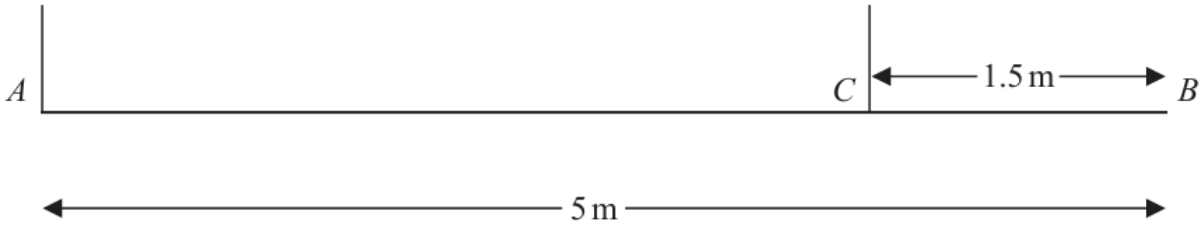


Figure 2

A non-uniform beam, AB, has length 5 m and mass 12 kg. The beam is suspended in a horizontal position by two vertical ropes. One rope is attached to the beam at A. The other rope is attached to the beam at C, where $CB = 1.5$ m, as shown in Figure 2. The distance of the centre of mass of the beam from A is 1.75 m. The beam is modelled as a non-uniform rod and the ropes are modelled as light inextensible strings.

A particle of mass M kg is now placed on the beam at B and the beam remains in equilibrium in a horizontal position.

(a) Find the largest possible value of M.

(3)

The particle at B is now removed and a particle of mass 15 kg is now placed on the beam at the point D, where $AD = x$ metres. The beam remains in equilibrium in a horizontal position.

Given that the tension in the rope attached to the beam at C is now twice the tension in the rope attached to the beam at A,

(b) find the value of x.

(5)

WORKED SOLUTION

Question 5 · WME01/01 October 2019

1 Locate the key distances

The beam is 5 m long. Since $CB = 1.5$ m, $AC = 3.5$ m. The centre of mass is 1.75 m from A, so it is also 1.75 m from C.

2 Use the largest M condition

For the largest possible M at B, the rope at A is just slack, so the beam is supported at C only.

3 **Take moments about C**

The particle at B is 1.5 m to the right of C, and the beam's weight acts 1.75 m to the left of C. Hence $Mg(1.5) = 12g(1.75)$.

4 **Find M**

$$M = \frac{12(1.75)}{1.5} = 14.$$

5 **Set the tensions for part b**

Let the tension at A be T. The tension at C is $2T$. Vertical equilibrium gives $T + 2T = 12g + 15g = 27g$, so $T = 9g$ and the tension at C is $18g$.

6 **Take moments about A**

For the new particle at $AD = x$, moments about A give $18g(3.5) = 12g(1.75) + 15gx$.

7 **Solve for x**

Cancel g: $63 = 21 + 15x$. Thus $15x = 42$ and $x = 2.8$ m.

✓ **FINAL ANSWER**

$(a)\ M = 14. \quad (b)\ x = 2.8\ \text{m}.$

Question 6

12 marks | Kinematics Graphs

6. An athlete runs a 200 m race along a straight horizontal track.

In a model of the motion of the athlete, air resistance is ignored, the athlete starts from rest at time $t = 0$ seconds and moves with uniform acceleration $0.8\,\text{m s}^{-2}$ for T seconds, reaching a speed of $V\,\text{m s}^{-1}$. She then maintains this speed until she crosses the finishing line.

The total time from when the athlete starts to when she crosses the finishing line is 30 s.

(a) Sketch a speed-time graph for the model of the motion of the athlete from the instant when she starts to the instant when she crosses the finishing line.

(2)

(b) Write down an expression for V in terms of T.

(1)

(c) Show that $T^2 - kT + 500 = 0$, where k is a constant to be found.

(4)

(d) Hence find the value of T, justifying your answer carefully.

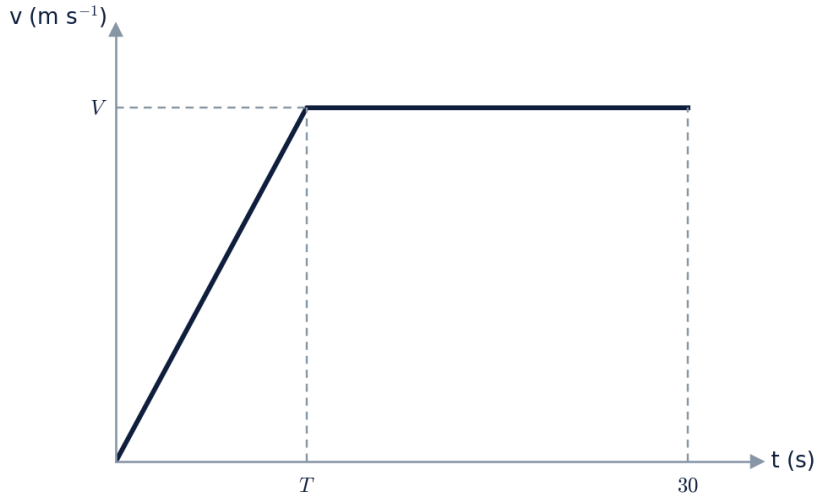
(3)

(e) Considering your speed-time graph or otherwise, state two ways, apart from including air resistance, in which the model could be made to be more realistic.

(2)

1 Describe the speed-time graph

The graph starts at $(0, 0)$, rises in a straight line with gradient 0.8 until time T and speed V, then stays horizontal at speed V until $t = 30$.



2 Write V in terms of T

Uniform acceleration gives $V = 0.8T$.

3 Use area for the distance

The distance is the area under the speed-time graph: $\frac{1}{2}TV + V(30 - T) = 200$.

4 Substitute V

Using $V = 0.8T$: $\frac{1}{2}T(0.8T) + (0.8T)(30 - T) = 200$.

5 Form the quadratic

$0.4T^2 + 24T - 0.8T^2 = 200$, so $-0.4T^2 + 24T - 200 = 0$. Multiplying by -2.5 gives $T^2 - 60T + 500 = 0$, so $k = 60$.

6 Solve and choose the valid root

$T^2 - 60T + 500 = (T - 10)(T - 50) = 0$, so $T = 10$ or 50. Since the whole race lasts 30 s, $T = 10$.

7 Give two model improvements

A more realistic model could include reaction time at the start, and could replace the instantaneous change from acceleration to constant speed with a smoother transition.

FINAL ANSWER

(a) Speed-time graph rises linearly to V at $t = T$, then is horizontal until $t = 30$.

(b) $V = 0.8T$. (c) $T^2 - 60T + 500 = 0$, so $k = 60$.

(d) $T = 10$. (e) For example, include reaction time at the start and avoid the instantaneous change to constant speed.

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Question 7

7 marks | Working with Vectors

7. Two forces, $\mathbf{F}$ and $\mathbf{G}$, act on a particle. The force $\mathbf{F}$ has magnitude $4\,\text{N}$ and acts in a direction with a bearing of $120°$ and the force $\mathbf{G}$ has magnitude $6\,\text{N}$ and acts due north.

Given that $\mathbf{P} = 2\mathbf{F} + \mathbf{G}$, find

(i) the magnitude of $\mathbf{P}$

(ii) the direction of $\mathbf{P}$, giving your answer as a bearing to the nearest degree.

(7)

WORKED SOLUTION

Question 7 · WME01/01 October 2019

1 Resolve 2F

Since $\mathbf{F}$ has magnitude 4 N on a bearing of $120°$, $2\mathbf{F}$ has magnitude 8 N on the same bearing.

2 Write the components

$2\mathbf{F} = 8\sin 120°\mathbf{i} + 8\cos 120°\mathbf{j} = 4\sqrt{3}\mathbf{i} - 4\mathbf{j}$. Also $\mathbf{G} = 6\mathbf{j}$.

3 Find P

$\mathbf{P} = 2\mathbf{F} + \mathbf{G} = 4\sqrt{3}\mathbf{i} + 2\mathbf{j}$.

4 Find the magnitude

$|\mathbf{P}| = \sqrt{(4\sqrt{3})^2 + 2^2} = \sqrt{48 + 4} = \sqrt{52}$ N.

5 Find the bearing angle

The vector is in the north-east quadrant. The angle east of north satisfies $\tan\theta = \dfrac{4\sqrt{3}}{2} = 2\sqrt{3}$.

6 State the bearing

$\theta = 73.9°$ approximately, so the bearing is $074°$ to the nearest degree.

FINAL ANSWER

$(i)\ |\mathbf{P}| = \sqrt{52}\ \text{N} \approx 7.21\ \text{N}.$ $\quad (ii)\ \text{bearing} = 074°.$

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Question 8

14 marks | Working with Vectors

8. [*In this question, the horizontal unit vectors* **i** *and* **j** *are directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two speedboats, A and B, are each moving with constant velocity. The velocity of A is $20\,\text{km h}^{-1}$ due west and the velocity of B is $40\,\text{km h}^{-1}$ on a bearing of 150°. The boats are modelled as particles.

At noon, the position vector of A is $60\mathbf{i}$ km and B is at the origin O. At time t hours after noon, the position vector of A is $\mathbf{r}$ km and the position vector of B is $\mathbf{s}$ km.

(a) Find the velocity of B in the form $(p\mathbf{i} + q\mathbf{j})\,\text{km h}^{-1}$

(3)

(b) Find expressions for $\mathbf{r}$ and $\mathbf{s}$ in terms of t, $\mathbf{i}$ and $\mathbf{j}$.

(3)

(c) Find the time, to the nearest minute, at which the distance between the boats is the same as it was at noon.

(8)

WORKED SOLUTION

Question 8 · WME01/01 October 2019

1 Resolve B's velocity

A bearing of $150°$ is $30°$ east of south. Therefore the east component is $40\sin 150° = 20$ and the north component is $40\cos 150° = -20\sqrt{3}$.

2 Write B's velocity

$\mathbf{v}_B = 20\mathbf{i} - 20\sqrt{3}\mathbf{j}\ \text{km h}^{-1}$.

3 Write A's position

Boat A starts at $60\mathbf{i}$ and travels due west at $20\ \text{km h}^{-1}$, so $\mathbf{r} = 60\mathbf{i} - 20t\mathbf{i} = (60 - 20t)\mathbf{i}$.

4 Write B's position

Boat B starts at O, so $\mathbf{s} = t(20\mathbf{i} - 20\sqrt{3}\mathbf{j}) = 20t\mathbf{i} - 20\sqrt{3}t\mathbf{j}$.

5 Find the separation vector

$\overrightarrow{AB} = \mathbf{s} - \mathbf{r} = (20t - (60 - 20t))\mathbf{i} - 20\sqrt{3}t\mathbf{j} = (40t - 60)\mathbf{i} - 20\sqrt{3}t\mathbf{j}$.

6 **Use the noon distance**

At noon the distance between the boats is 60 km. Set the later distance equal to 60: $(40t - 60)^2 + (-20\sqrt{3}t)^2 = 60^2$.

7 **Solve for t**

Expanding gives $2800t^2 - 4800t = 0$, so $t(2800t - 4800) = 0$. The later time is $t = \dfrac{4800}{2800} = \dfrac{12}{7}$ hours.

8 **Convert to clock time**

$\dfrac{12}{7}$ hours is 1 hour and $42.857\ldots$ minutes after noon, which rounds to $1{:}43$ pm, or $13{:}43$.

✓ **FINAL ANSWER**

$(a)\ \mathbf{v}_B = 20\mathbf{i} - 20\sqrt{3}\mathbf{j}\ \text{km h}^{-1}.$ $(b)\ \mathbf{r} = (60 - 20t)\mathbf{i},\ \mathbf{s} = 20t\mathbf{i} - 20\sqrt{3}t\mathbf{j}.$ $(c)\ 13{:}43$, or $1{:}43\ \text{pm}$.