Elite Mathematics
Edexcel International A Level

IAL PURE 3 · WMA13

WMA13/01 May/June 2026 Worked Solutions

May/June 2026 paper rebuilt with the worked solution after each question.

9	2020–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA13/01 May/June 2026

May/June 2026

9	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA13/01 MAY/JUNE 2026

Question 1

8 marks | Numerical Methods

1. A curve has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{2}{3}x^3 - 3x^2 + 5x + \mathrm{e}^{2x}(1 - 2x) \qquad x \in \mathbb{R}$$

(a) Find $\mathrm{f}'(x)$ giving the answer in simplest form.

(3)

The equation $\mathrm{f}'(x) = 0$ has a single root, α

(b) Show that α lies in the interval $[0, 1]$

(2)

Given that the equation $\mathrm{f}'(x) = 0$ can be written as

$$x = \frac{2x^2 + 5}{4\mathrm{e}^{2x} + 6}$$

(c) use the iteration formula

$$x_{n+1} = \frac{2x_n^{\ 2} + 5}{4\mathrm{e}^{2x_n} + 6} \qquad \text{with } x_1 = 0$$

to find, to 4 decimal places,

(i) the value of x_3

(ii) the value of α

(3)

WORKED SOLUTION

Question 1 · WMA13/01 May/June 2026

1 **Part (a)**

Apply the product rule to the exponential term:

$$f'(x) = 2x^2 - 6x + 5 + 2e^{2x}(1 - 2x) - 2e^{2x} = \boxed{2x^2 - 6x + 5 - 4xe^{2x}}.$$

2 Part (b)

The derivative is continuous on $[0,1]$, and

$$f'(0) = 5 > 0, \qquad f'(1) = 1 - 4e^2 \approx -28.5562 < 0.$$

The change of sign therefore places a zero of f' between 0 and 1. Since the zero is unique, $\boxed{\alpha \in (0,1)}$, and hence it is in $[0,1]$.

3 Part (c)(i)

Start at $x_1 = 0$, retaining unrounded values in each substitution:

$$x_2 = \frac{2(0)^2 + 5}{4e^0 + 6} = 0.5, \qquad x_3 = \frac{2(0.5)^2 + 5}{4e^1 + 6} = 0.3259620992\ldots.$$

Thus $\boxed{x_3 = 0.3260}$ to 4 decimal places.

4 Part (c)(ii)

Continuing the supplied iteration gives

n	4	5	6	7	8	9
x_n	0.381116669	0.363053595	0.368908652	0.367004338	0.367623025	0.367421950

$$x_{10} = 0.3674872922, \quad x_{11} = 0.3674660574, \quad x_{12} = 0.3674729582, \quad x_{13} = 0.3674707156.$$

Further iteration stabilises at $0.367471265\ldots$, so $\boxed{\alpha = 0.3675}$ to 4 decimal places.

FINAL ANSWER

(a) $2x^2 - 6x + 5 - 4xe^{2x}$ (b) $\alpha \in (0,1)$ (c)(i) $x_3 = 0.3260$ (c)(ii) $\alpha = 0.3675$

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Question 2

9 marks | Functions, Composition & Inverses

2. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The functions f and g are defined by

$$f(x) = 3 + 4e^{2x} \qquad x \in \mathbb{R}$$

$$g(x) = \sqrt{x+5} \qquad x \in \mathbb{R} \quad x > -5$$

(a) State the range of f

(1)

(b) Find g^{-1}

(3)

(c) Find $gf\left(\ln\sqrt{7}\right)$

(2)

(d) Find the value of a, $a > 0$, for which

$$fg(a^2 - 5) = g(59)$$

giving the answer in the form $\ln p$ where p is an irrational constant.

(3)

WORKED SOLUTION

Question 2 · WMA13/01 May/June 2026

1 **Part (a)**

Since $e^{2x} > 0$ for every real x, the range is $\boxed{(3, \infty)}$.

2 **Part (b)**

Put $y = \sqrt{x+5}$. The original domain is $x > -5$, so $y > 0$. Squaring gives $y^2 = x + 5$, hence $x = y^2 - 5$. Interchanging the input and output,

$$\boxed{g^{-1}(x) = x^2 - 5, \qquad x > 0}.$$

3 **Part (c)**

Evaluate the inside function first:

$$f(\ln\sqrt{7}) = 3 + 4e^{2\ln\sqrt{7}} = 3 + 4(7) = 31, \qquad gf(\ln\sqrt{7}) = g(31) = \sqrt{36} = \boxed{6}.$$

4 **Part (d)**

Because $a > 0$, $g(a^2 - 5) = \sqrt{a^2} = a$, while $g(59) = 8$. Thus

$$3 + 4e^{2a} = 8 \quad \Longrightarrow \quad e^{2a} = \frac{5}{4} \quad \Longrightarrow \quad a = \frac{1}{2}\ln\frac{5}{4} = \boxed{\ln\frac{\sqrt{5}}{2}}.$$

Here $p = \sqrt{5}/2$ is irrational and greater than 1, so the answer satisfies $a > 0$.

✓ **FINAL ANSWER**

(a) $(3, \infty)$ (b) $g^{-1}(x) = x^2 - 5, \qquad x > 0$ (d) $\ln\frac{\sqrt{5}}{2}$ (c) 6

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Question 3

8 marks | Differentiation Techniques

3. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{4x^2 - 14x + 15}{x - 3} \qquad x < 3$$

(a) Find $\mathrm{f}'(x)$, giving the answer in the form

$$\frac{ax^2 + bx + c}{(x-3)^2}$$

where a, b and c are constants.

(4)

(b) Hence find the coordinates of the turning point of C.

(2)

The point $P\left(\frac{5}{2}, -10\right)$ lies on C.

(c) Find the coordinates of the point to which P is mapped, when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation $y = \mathrm{f}(2x) + 3$

(2)

WORKED SOLUTION

Question 3 · WMA13/01 May/June 2026

1 **Part (a)**

Using the quotient rule,

$$f'(x) = \frac{(x-3)(8x-14) - (4x^2 - 14x + 15)}{(x-3)^2} = \boxed{\frac{4x^2 - 24x + 27}{(x-3)^2}}.$$

Thus $a = 4$, $b = -24$ and $c = 27$.

2 Part (b)

A stationary point requires

$$4x^2 - 24x + 27 = (2x - 3)(2x - 9) = 0.$$

The roots are $3/2$ and $9/2$. Reject $9/2$ because the stated domain is $x < 3$. At $x = 3/2$, $f(x) = -2$, so the turning point is $\boxed{(3/2, -2)}$. The derivative changes from positive to negative there, confirming a maximum.

3 Part (c)

The inside factor 2 halves the horizontal coordinate; adding 3 raises the vertical coordinate by 3. Therefore

$$(x, y) \mapsto (x/2, y + 3), \qquad (5/2, -10) \mapsto \boxed{(5/4, -7)}.$$

FINAL ANSWER

(a) $\frac{4x^2 - 24x + 27}{(x-3)^2}$ (b) $(3/2, -2)$ (c) $(5/4, -7)$

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Question 4

5 marks | Integration

4. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Using algebraic integration, find the exact value of

$$\int_{\frac{\pi}{18}}^{\frac{\pi}{12}} (\sin 3x + \cos 3x)^2 \, \mathrm{d}x$$

(5)

WORKED SOLUTION

Question 4 · WMA13/01 May/June 2026

1 **Solution**

First expand and use the double-angle identity:

$$(\sin 3x + \cos 3x)^2 = \sin^2 3x + \cos^2 3x + 2\sin 3x \cos 3x = 1 + \sin 6x.$$

Hence

$$\begin{aligned}\int_{\pi/18}^{\pi/12} (\sin 3x + \cos 3x)^2 \, dx &= \left[x - \frac{1}{6}\cos 6x\right]_{\pi/18}^{\pi/12} \\ &= \left(\frac{\pi}{12} - \frac{1}{6}\cos\frac{\pi}{2}\right) - \left(\frac{\pi}{18} - \frac{1}{6}\cos\frac{\pi}{3}\right) \\ &= \frac{\pi}{12} - \frac{\pi}{18} + \frac{1}{12} = \boxed{\frac{\pi}{36} + \frac{1}{12}}.\end{aligned}$$

✓ **FINAL ANSWER**

$\frac{\pi}{36} + \frac{1}{12}$

Question 5

7 marks | Log Graphs & Exponential Models

5.

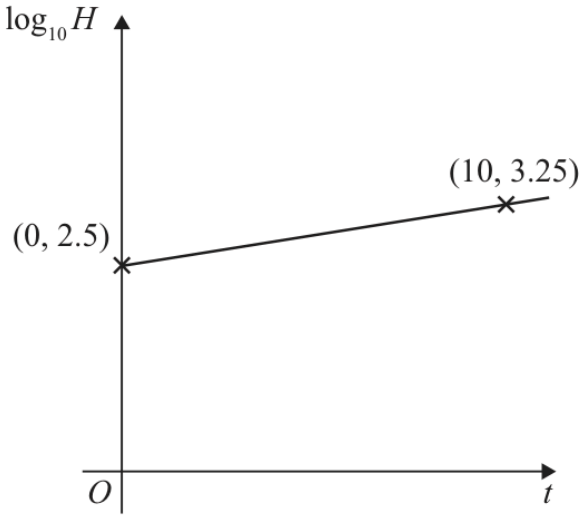


Figure 1

The recycling of household waste in a particular town is being monitored.

The number of households in the town, H, recycling their household waste, t years after monitoring began, is modelled by the equation

$$H = ab^t$$

where a and b are constants.

The line in Figure 1 shows the linear relationship between t and $\log_{10} H$

The line passes through the points (0, 2.5) and (10, 3.25)

Using this information,

(a) find, to 3 significant figures, the value of a and the value of b

(4)

(b) With reference to the model, interpret the value of the constant b

(1)

(c) Find the value of $\frac{dH}{dt}$ when $t = 5$, giving the answer to 2 significant figures.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(2)

1 Part (a)

Taking base-10 logarithms gives

$$\log_{10} H = \log_{10} a + t \log_{10} b.$$

The vertical intercept is 2.5 and the gradient is $\dfrac{3.25 - 2.5}{10 - 0} = 0.075$. Therefore

$$a = 10^{2.5} = 316.227766\ldots, \qquad b = 10^{0.075} = 1.188502227\ldots.$$

To 3 significant figures, $\boxed{a = 316, \quad b = 1.19}$.

2 Part (b)

Each year the model multiplies the number of recycling households by b. This means an increase of approximately $\boxed{19\% \text{ per year}}$ (about 18.85% using the unrounded value).

3 Part (c)

Retain the unrounded constants:

$$H = 10^{2.5 + 0.075t}, \qquad \frac{dH}{dt} = 0.075 \ln(10)\, 10^{2.5 + 0.075t}.$$

At $t = 5$ this is $129.5021421\ldots$, hence

$$\boxed{\frac{dH}{dt} = 1.3 \times 10^2 \text{ households per year}}$$

to 2 significant figures.

FINAL ANSWER

(a) $a = 316, \quad b = 1.19$ (b) 19% per year (c) $\frac{dH}{dt} = 1.3 \times 10^2$ households per year

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Question 6

6 marks | Differentiation Techniques

6. The curve C has equation

$$x = \left(3\sin 2y - \frac{6y}{\pi}\right)^2$$

Given that

- the point P with y coordinate $\frac{\pi}{4}$ lies on C
- the tangent to C at P cuts the x-axis at the point Q

use calculus to find the x coordinate of Q

(6)

WORKED SOLUTION

Question 6 · WMA13/01 May/June 2026

1 **Solution**

At $y = \pi/4$,

$$x = \left(3\sin\frac{\pi}{2} - \frac{6(\pi/4)}{\pi}\right)^2 = \left(3 - \frac{3}{2}\right)^2 = \frac{9}{4}.$$

Differentiate x with respect to y using the chain rule:

$$\frac{dx}{dy} = 2\left(3\sin 2y - \frac{6y}{\pi}\right)\left(6\cos 2y - \frac{6}{\pi}\right).$$

At P, $dx/dy = 2(3/2)(-6/\pi) = -18/\pi$. Since this is nonzero, the tangent gradient is $\frac{dy}{dx} = -\frac{\pi}{18}$, and its equation is

$$y - \frac{\pi}{4} = -\frac{\pi}{18}\left(x - \frac{9}{4}\right).$$

Put $y = 0$ for its intersection with the x-axis:

$$x - \frac{9}{4} = \frac{18}{4}, \qquad \boxed{x_Q = \frac{27}{4}}.$$

FINAL ANSWER

$x_Q = \frac{27}{4}$

Question 7

12 marks | Log Graphs & Exponential Models

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The selling price of a car is monitored over a 15-year period.

The selling price P_C of the car, in thousands of pounds, t years after monitoring began, is modelled by the equation

$$P_C = 7 + 16e^{-0.1t} \qquad 0 \leqslant t \leqslant 15$$

Given that when $t = T$ the selling price of the car is £15 000

(a) find, according to the model, the value of T. Give the answer to 2 decimal places.

(3)

(b) Find the value of $\frac{dP_C}{dt}$ when $t = 5$. Give the answer to 2 decimal places.

(2)

The selling price of a van is monitored over the same 15-year period.

The selling price P_V of the van, in thousands of pounds, t years after monitoring began, is modelled by the equation

$$P_V = 4 + me^{-0.2t} \qquad 0 \leqslant t \leqslant 15$$

where m is a constant.

Use the equations of these models to answer parts (c) and (d).

Given that $P_V = 36$ when $t = 0$

(c) (i) state the value of m

(ii) find the difference between the initial selling price of the car and the initial selling price of the van.

(3)

When $t = \alpha$, the selling price of the car is equal to the selling price of the van.

(d) Find, using algebra and showing your working, the value of α, giving the answer to 3 significant figures.

(4)

1 Part (a)

Prices in the model are in thousands of pounds, so

$$15 = 7 + 16e^{-0.1T} \quad \Longrightarrow \quad e^{-0.1T} = \frac{1}{2}.$$

Taking logarithms gives $T = 10\ln 2 = 6.9314718\ldots$, hence $\boxed{T = 6.93 \text{ years}}$.

2 Part (b)

$$\frac{dP_C}{dt} = -1.6e^{-0.1t}, \qquad \left.\frac{dP_C}{dt}\right|_{t=5} = -1.6e^{-0.5} = -0.970449\ldots.$$

Thus the answer is $\boxed{-0.97 \text{ thousand pounds per year}}$ to 2 decimal places.

3 Part (c)(i)

At $t = 0$, $36 = 4 + m$, so $\boxed{m = 32}$.

4 Part (c)(ii)

The initial car price is $7 + 16 = 23$ thousand pounds. The difference is $36 - 23 = 13$ thousand pounds, or $\boxed{\pounds 13\,000}$.

5 Part (d)

Equating the models with $m = 32$ gives

$$7 + 16e^{-0.1\alpha} = 4 + 32e^{-0.2\alpha}.$$

Let $u = e^{-0.1\alpha} > 0$. Then $32u^2 - 16u - 3 = 0$, so

$$u = \frac{16 \pm \sqrt{256 + 384}}{64} = \frac{2 \pm \sqrt{10}}{8}.$$

Reject the negative root because an exponential is positive. Hence

$$\alpha = -10\ln\left(\frac{2 + \sqrt{10}}{8}\right) = 4.3806365\ldots, \qquad \boxed{\alpha = 4.38 \text{ years}}.$$

This value is inside the model interval $0 \le t \le 15$.

FINAL ANSWER

(a) $T = 6.93$ years (b) -0.97 thousand pounds per year (c)(i) $m = 32$ (c)(ii) $\pounds 13\,000$ (d) $\alpha = 4.38$ years

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Question 8

9 marks | Trig Identities & Exact Values

8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{\sec\theta\,\operatorname{cosec}\theta}{4\operatorname{cosec}4\theta} \equiv \cos 2\theta$$

(2)

(b) Hence show that the equation

$$4(5\cos^2\theta - 9)\operatorname{cosec}4\theta = 3(1+\tan^2 2\theta)\sec\theta\operatorname{cosec}\theta$$

can be written as

$$a\cos^2 2\theta + b\cos 2\theta + c = 0$$

where a, b and c are integers to be found.

(3)

(c) Hence, solve for $0 < x < 180°$, $x \neq (45n)°$, $n \in \mathbb{N}$, the equation

$$4(5\cos^2 x - 9)\operatorname{cosec}4x = 3(1+\tan^2 2x)\sec x\operatorname{cosec}x$$

giving the answers to one decimal place.

(4)

WORKED SOLUTION

Question 8 · WMA13/01 May/June 2026

1 **Part (a)**

For angles where the original expression is defined,

$$\frac{\sec\theta\ \operatorname{cosec}\theta}{4\operatorname{cosec}4\theta} = \frac{\sin 4\theta}{4\sin\theta\cos\theta} = \frac{2\sin 2\theta\cos 2\theta}{2\sin 2\theta} = \boxed{\cos 2\theta}.$$

2 Part (b)

Divide the given equation by $4\operatorname{cosec} 4\theta$ and use (a):

$$5\cos^2\theta - 9 = 3(1+\tan^2 2\theta)\cos 2\theta = \frac{3}{\cos 2\theta}.$$

Use $\cos^2\theta = (1+\cos 2\theta)/2$ and multiply by $2\cos 2\theta$:

$$(5\cos 2\theta - 13)\cos 2\theta = 6.$$

Therefore

$$\boxed{5\cos^2 2\theta - 13\cos 2\theta - 6 = 0}, \qquad a = 5,\ b = -13,\ c = -6.$$

The divisions are valid on the domain of the original equation.

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3 Part (c)

Set $u = \cos 2x$:

$$5u^2 - 13u - 6 = (5u+2)(u-3) = 0.$$

Reject $u = 3$ because cosine lies between -1 and 1. Thus $\cos 2x = -2/5$. Since $0 < 2x < 360°$,

$$2x = 113.5781785\ldots° \quad \text{or} \quad 246.4218215\ldots°.$$

Hence $\boxed{x = 56.8° \text{ or } 123.2°}$ to one decimal place. Neither is an excluded multiple of $45°$; these are all the solutions in the interval.

✓ FINAL ANSWER

(a) $\cos 2\theta$ (b) $5\cos^2 2\theta - 13\cos 2\theta - 6 = 0$ (c) $x = 56.8° \text{ or } 123.2°$

Question 9

11 marks | Modulus

9.

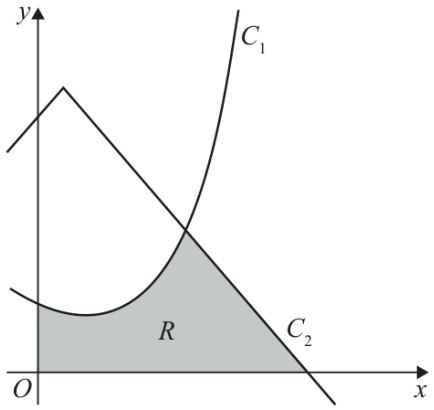


Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(a) Find $\int \left(\frac{11}{2x+5} + 4e^{x-3} \right) dx$

(2)

Figure 2 shows a sketch of part of the curve C_1 and part of the graph C_2
The curve C_1 has equation

$$y = \frac{11}{2x+5} + 4e^{x-3}$$

The graph C_2 has equation

$$y = a - |bx - 1|$$

where a and b are positive integers.

(b) State, in terms of a and/or b, the coordinates of the maximum point of C_2

(2)

Given that

- C_1 intersects C_2 at $x = 3$
- C_2 crosses the y-axis at $(0, 9)$

(c) find the value of a and the value of b

(3)

The region R, shown shaded in Figure 2, is bounded by C_1, C_2, the x-axis and the y-axis.

(d) Find the area of R, giving the answer to 3 significant figures.

(4)

1 Part (a)

$$\int\left(\frac{11}{2x+5}+4e^{x-3}\right)dx=\boxed{\frac{11}{2}\ln|2x+5|+4e^{x-3}+C}.$$

The absolute value allows both intervals on which the integrand is defined.

2 Part (b)

The largest value of $a-|bx-1|$ occurs when $bx-1=0$. The maximum point is therefore $\boxed{(1/b,a)}$.

3 Part (c)

At $x=0$, $9=a-1$, so $\boxed{a=10}$. At the intersection $x=3$, the first curve has height $11/11+4=5$. Thus $10-|3b-1|=5$, giving $3b-1=5$ or -5. These give $b=2$ or $-4/3$; only the positive integer is allowed, so $\boxed{b=2}$.

4 Part (d)

For $x\geq 3$, the required branch of C_2 is $y=10-(2x-1)=11-2x$. Its x-intercept is $x=11/2$. The shaded area is the area under C_1 from 0 to 3 plus the triangle from 3 to $11/2$:

$$\begin{aligned}A&=\int_0^3\left(\frac{11}{2x+5}+4e^{x-3}\right)dx+\frac{1}{2}\left(\frac{11}{2}-3\right)5\\&=\left[\frac{11}{2}\ln(2x+5)+4e^{x-3}\right]_0^3+\frac{25}{4}\\&=\frac{11}{2}\ln\frac{11}{5}+4(1-e^{-3})+\frac{25}{4}=14.3873672\ldots.\end{aligned}$$

Hence $\boxed{A=14.4\text{ square units}}$ to 3 significant figures.

FINAL ANSWER

(a) $\frac{11}{2}\ln|2x+5|+4e^{x-3}+C$ (b) $(1/b,a)$ (c) $a=10$, $b=2$ (d) $A=14.4$ square units