Elite Mathematics
Edexcel International A Level

IAL PURE 3 · WMA13

WMA13/01 January 2026 Worked Solutions

January 2026 paper rebuilt with the worked solution after each question.

10	2020–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA13/01 January 2026

January 2026

10	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA13/01 JANUARY 2026

Question 1

6 marks | Numerical Methods

1. $$g(x) = 3x - 5 + 5\sin\left(x^2\right)$$

where x is measured in radians.

(a) Show that $g(x) = 0$ has a root α in the interval $[0.7, 0.8]$

(2)

(b) Show that the equation $g(x) = 0$ can be written in the form

$$x = \left[\arcsin\left(1 - px\right)\right]^{\frac{1}{2}}$$

where p is a constant to be found.

(1)

Using the iteration formula

$$x_{n+1} = \left[\arcsin\left(1 - px_n\right)\right]^{\frac{1}{2}}$$

with the value of p found in part (b) and starting with $x_1 = 0.7$

(c) (i) find, to 4 decimal places, the value of x_2

(ii) find, by repeated iteration, the value of α, giving your answer to 4 decimal places.

(3)

WORKED SOLUTION

Question 1 · WMA13/01 January 2026

1 **Part (a)**

Let $g(x) = 3x - 5 + 5\sin(x^2)$. Using radians,

$$g(0.7) = -0.546870559\ldots, \qquad g(0.8) = 0.385977207\ldots.$$

Since g is continuous and changes sign, a root α lies between 0.7 and 0.8.

2 **Part (b)**

$$3x - 5 + 5\sin(x^2) = 0 \quad \Longrightarrow \quad \sin(x^2) = 1 - \frac{3}{5}x.$$

On the stated root interval, $0.49 < x^2 < 0.64 < \pi/2$, so the principal inverse sine and positive square root give

$$x = \sqrt{\arcsin(1 - \tfrac{3}{5}x)}, \qquad p = \frac{3}{5}.$$

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3 **Part (c)(i)**

Starting with $x_1 = 0.7$,

$$x_2 = \sqrt{\arcsin(1 - 0.6(0.7))} = 0.786593091\ldots = 0.7866.$$

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4 **Part (c)(ii)**

Continue $x_{n+1} = \sqrt{\arcsin(1 - 0.6x_n)}$ in radians, retaining the full calculator values. The iterates converge to $0.758924378\ldots$, so

$$\alpha = 0.7589 \quad (4 \text{ decimal places}).$$

✓ **FINAL ANSWER**

(a) $a root \backslash alpha lies between 0.7 and 0.8\$\$$ (b) $p = \frac{3}{5}$ (c)(i) 0.7866 (c)(ii) $\alpha = 0.7589$

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Question 2

6 marks | Differentiation Techniques

2. (i) Given $y = x^2 \ln 2x$, find in simplest form, $\dfrac{dy}{dx}$

(2)

(ii) Given

$$f(x) = \frac{e^{2x}}{(4x+3)^2} \qquad x > -\frac{3}{4}$$

show that

$$f'(x) = \frac{2e^{2x}(ax+b)}{(4x+3)^3}$$

where a and b are constants to be found.

(4)

WORKED SOLUTION

Question 2 · WMA13/01 January 2026

1 Part (i)

Use the product rule and $\frac{d}{dx}\ln(2x) = 1/x$:

$$\frac{dy}{dx} = 2x\ln(2x) + x^2\frac{1}{x} = 2x\ln(2x) + x = x\big(2\ln(2x)+1\big).$$

2 Part (ii)

Write $f(x) = e^{2x}(4x+3)^{-2}$. Product and chain rules give

$$f'(x) = 2e^{2x}(4x+3)^{-2} - 8e^{2x}(4x+3)^{-3}.$$

Factoring out $2e^{2x}(4x+3)^{-3}$,

$$f'(x) = \frac{2e^{2x}\big((4x+3)-4\big)}{(4x+3)^3} = \frac{2e^{2x}(4x-1)}{(4x+3)^3}.$$

Thus $a = 4$ and $b = -1$.

FINAL ANSWER

(i) $x(2\ln(2x)+1)$ (ii) $a = 4,\ b = -1$

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Question 3

10 marks | Integration

3. (i) Find, in simplest form,

$$\int 2(3x+5)^7 \, dx$$

(2)

(ii) $$f(x) = \frac{4x^3 + x^2 - 7x + 14}{(x-1)(x+2)} \qquad x > 1$$

(a) Write $f(x)$ in the form

$$ax + b + \frac{c}{x-1}$$

where a, b and c are constants to be found.

(4)

(b) Hence, using algebraic integration, find

$$\int_3^9 f(x) \, dx$$

writing the answer in simplest form.

(4)

WORKED SOLUTION

Question 3 · WMA13/01 January 2026

1 **Part (i)**

Increase the power to 8 and divide by both 8 and the inner derivative 3:

$$\int 2(3x+5)^7 \, dx = \frac{(3x+5)^8}{12} + C.$$

2 **Part (ii)(a)**

The denominator is $x^2 + x - 2$. The first quotient term is $4x$. Subtracting $4x(x^2 + x - 2)$ leaves $-3x^2 + x + 14$. The next quotient term is -3. Subtracting $-3(x^2 + x - 2)$ leaves $4x + 8$. Thus polynomial division gives

$$4x^3 + x^2 - 7x + 14 = (4x - 3)(x^2 + x - 2) + 4x + 8.$$

Since $4x + 8 = 4(x + 2)$,

$$f(x) = 4x - 3 + \frac{4(x+2)}{(x-1)(x+2)} = 4x - 3 + \frac{4}{x-1}.$$

Hence $a = 4$, $b = -3$, $c = 4$.

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3 **Part (ii)(b)**

For $x > 1$, an antiderivative is $2x^2 - 3x + 4\ln(x-1)$, so

$$\int_3^9 f(x)\,dx = \left[2x^2 - 3x + 4\ln(x-1)\right]_3^9 = 135 + 4\ln 8 - 9 - 4\ln 2.$$

Thus the exact integral is $126 + 8\ln 2$.

✓ **FINAL ANSWER**

(i) $\frac{(3x+5)^8}{12} + C$ (ii)(a) $a = 4,\ b = -3,\ c = 4$ (ii)(b) $126 + 8\ln 2$

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Question 4

6 marks | Log Graphs & Exponential Models

4. The resting heart rate, h beats per minute, of a particular group of mammals, can be modelled by the equation

$$\log_{10} h = 2.4 - 0.25 \log_{10} m$$

where m kg is the mass of the mammal.

The mass of one of these mammals is 3 kg.

According to the model,

(a) find the resting heart rate of this mammal. Give your answer to 3 significant figures.

(2)

(b) Show that the equation of the model can be written in the form

$$h = \frac{p}{m^q}$$

giving each of the values of the constants p and q to 3 significant figures where appropriate.

(3)

(c) With reference to the model, interpret the value of the constant p.

(1)

WORKED SOLUTION

Question 4 · WMA13/01 January 2026

1 **Part (a)**

At $m = 3$,

$$\log_{10} h = 2.4 - 0.25 \log_{10} 3, \qquad h = 10^{2.4 - 0.25 \log_{10} 3} = 190.862095\ldots.$$

The predicted heart rate is $\mathbf{191}$ beats per minute (3 significant figures).

2 Part (b)

Using the power law of logarithms,

$$\log_{10} h = \log_{10}(10^{2.4}) - \log_{10}(m^{0.25}) = \log_{10}\left(\frac{10^{2.4}}{m^{0.25}}\right).$$

Therefore

$$h = \frac{10^{2.4}}{m^{0.25}}, \qquad p = 10^{2.4} = 251 \text{ (3 s.f.)}, \qquad q = 0.25.$$

The model, with its coefficient rounded as requested, is

$$h = \frac{251}{m^{0.25}}.$$

3 Part (c)

When $m = 1$, $h = p$. Thus p is the model's predicted heart rate, in beats per minute, for a mammal of mass 1 kg.

FINAL ANSWER

(a) 191 beats per minute (b) $p = 10^{2.4} = 251 \text{ (3 s.f.)}, \qquad q = 0.25$ (c) $predicted heart rate, in beats per minute, for a mammal of mass 1 kg$

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Question 5

8 marks | Functions, Composition & Inverses

5. The function f is defined by

$$\mathrm{f} : x \to \frac{2x+16}{x-4} \qquad x \neq 4$$

(a) Find $\mathrm{ff}(x)$, giving your answer as a single fraction in its simplest form.

(3)

(b) Find $\mathrm{f}^{-1}(12)$

(2)

The function g is defined by

$$\mathrm{g} : x \to \ln x \qquad x > 0 \quad x \neq \mathrm{e}^4$$

(c) Find the exact values of a such that

$$\mathrm{fg}(a) = \mathrm{g}(a)$$

(3)

WORKED SOLUTION

Question 5 · WMA13/01 January 2026

1 **Part (a)**

Substitute $f(x)$ into f and multiply numerator and denominator by $x-4$:

$$ff(x) = \frac{2\left(\frac{2x+16}{x-4}\right)+16}{\frac{2x+16}{x-4}-4} = \frac{4x+32+16x-64}{2x+16-4x+16} = \frac{10x-16}{16-x}.$$

The composition requires $x \neq 4$ and $f(x) \neq 4$, hence also $x \neq 16$.

2 **Part (b)**

To find $f^{-1}(12)$, solve $f(x) = 12$:

$$2x+16 = 12(x-4) \quad \Longrightarrow \quad 10x = 64.$$

Thus $f^{-1}(12) = \frac{32}{5}$.

3

Part (c)

Put $u = \ln a$. Then $fg(a) = g(a)$ means $f(u) = u$, where $u \neq 4$:

$$\frac{2u+16}{u-4} = u \quad \Longrightarrow \quad u^2 - 6u - 16 = 0 \quad \Longrightarrow \quad (u-8)(u+2) = 0.$$

Therefore $a = e^8$ or $a = e^{-2}$. Both are positive and neither is e^4, so both satisfy the domain restrictions.

FINAL ANSWER

(a) $\frac{10x-16}{16-x}, \quad x \neq 4, 16$ (b) $f^{-1}(12) = \frac{32}{5}$ (c) $a = e^8$ or e^{-2}

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Question 6

7 marks | Trig Identities & Exact Values

6. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\cos(\theta - 60°) = 3\sin\theta$$

can be written in the form

$$\tan\theta = \frac{1}{6}\left(3+\sqrt{3}\right)$$

(4)

(b) Hence solve, for $0 \leqslant x < 180°$, the equation

$$2\cos(2x - 70°) = 3\sin(2x - 10°)$$

giving your solutions to one decimal place.

(3)

WORKED SOLUTION

Question 6 · WMA13/01 January 2026

1 Part (a)

Expanding the compound angle,

$$2\left(\cos\theta\cos 60° + \sin\theta\sin 60°\right) = 3\sin\theta,$$

so $\cos\theta + \sqrt{3}\sin\theta = 3\sin\theta$ and $\cos\theta = (3-\sqrt{3})\sin\theta$. If $\cos\theta = 0$, this equation would require $\sin\theta = 0$, which is impossible. We may therefore divide by $\cos\theta$:

$$\tan\theta = \frac{1}{3-\sqrt{3}} = \frac{3+\sqrt{3}}{9-3} = \frac{3+\sqrt{3}}{6}.$$

2 Part (b)

Set $\theta = 2x - 10°$, so $2x - 70° = \theta - 60°$ and $-10° \leq \theta < 350°$.

$$\theta = \arctan\left(\frac{3+\sqrt{3}}{6}\right) + 180°n = 38.2619662\ldots° + 180°n.$$

The admissible values are $38.2619662\ldots°$ and $218.2619662\ldots°$. Hence $x = (\theta + 10°)/2$, giving $x = 24.1°$ or $114.1°$ (1 decimal place).

FINAL ANSWER

(a) $\frac{3+\sqrt{3}}{6}$ (b) $x = 24.1° \text{ or } 114.1°$

Question 7

10 marks | Modulus

7.

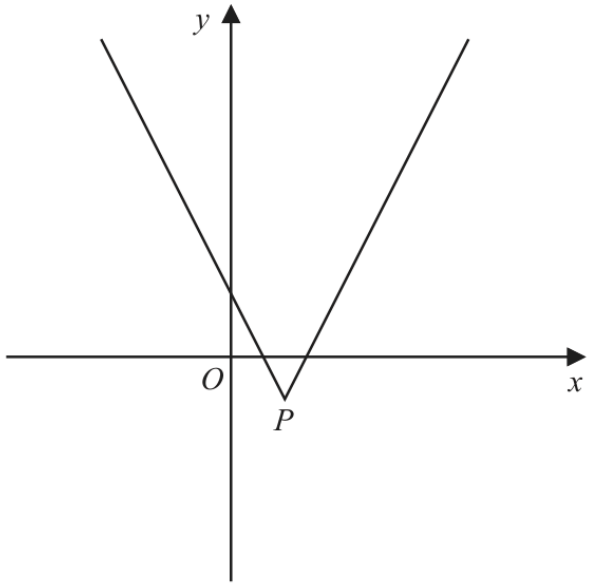


Figure 1

Figure 1 shows a sketch of part of the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = |3x - 11| - 4$$

The vertex of the graph is at the point P, as shown in Figure 1.

(a) State the coordinates of P.

(2)

(b) Solve the equation $\mathrm{f}(x) = 8$

(3)

The line l has equation $y = mx$, where m is a constant.

Given that l intersects the graph of $y = \mathrm{f}(x)$ at exactly one point,

(c) find the possible values of m.

(3)

The graph with equation $y = \mathrm{f}(x)$ is transformed onto the graph with equation $y = a\,\mathrm{f}(x - b)$, where a and b are constants.

Given that the vertex of the graph with equation $y = a\,\mathrm{f}(x - b)$ is $(5, 16)$

(d) find the value of a and the value of b.

(2)

1 Part (a)

At the vertex, $3x - 11 = 0$, so $P = (\frac{11}{3}, -4)$.

2 Part (b)

$$|3x - 11| - 4 = 8 \quad \Longrightarrow \quad |3x - 11| = 12.$$

Thus $3x - 11 = 12$ or $3x - 11 = -12$, giving $x = \frac{23}{3}$ or $x = -\frac{1}{3}$.

3 Part (c)

Write the two branches explicitly:

$$f(x) = \begin{cases} 7 - 3x, & x \le \frac{11}{3}, \\ 3x - 15, & x \ge \frac{11}{3}. \end{cases}$$

On the left branch, $mx = 7 - 3x$ gives $x = 7/(m + 3)$ for $m \neq -3$. This lies on the left branch precisely when $m < -3$ or $m \ge -12/11$. On the right branch, $mx = 3x - 15$ gives $x = 15/(3 - m)$ for $m \neq 3$. This lies on the right branch precisely when $-12/11 \le m < 3$. At $m = -12/11$, the two branch intersections coincide at the vertex, so there is only one distinct point. For $m < -3$ or $m \ge 3$, only the left intersection exists. At $m = -3$ there is no intersection; at $m = 3$ there is one. Consequently the complete answer is

$$m < -3 \quad \text{or} \quad m = -\frac{12}{11} \quad \text{or} \quad m \ge 3.$$

4 Part (d)

Under $y = af(x - b)$, the vertex becomes $(\frac{11}{3} + b, -4a)$. Equating this with $(5, 16)$ gives

$$b = 5 - \frac{11}{3} = \frac{4}{3}, \qquad a = \frac{16}{-4} = -4.$$

FINAL ANSWER

(a) $P = (\frac{11}{3}, -4)$ (b) $x = \frac{23}{3}$ or $-\frac{1}{3}$ (c) $m < -3 \quad \text{or} \quad m = -\frac{12}{11} \quad \text{or} \quad m \ge 3$ (d) $b = 5 - \frac{11}{3} = \frac{4}{3}, \qquad a = \frac{16}{-4} = -4$

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Question 8

4 marks | Differentiation Techniques

8. A curve has equation

$$x = e^{2\tan y} \qquad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

Show that

$$\frac{dy}{dx} = \frac{a}{x\left(b + (\ln x)^n\right)}$$

where a, b and n are integers to be found.

(4)

WORKED SOLUTION

Question 8 · WMA13/01 January 2026

1 **Solution**

\paragraph{Solution} Differentiate $x = e^{2\tan y}$ with respect to y:

$$\frac{dx}{dy} = 2\sec^2 y\, e^{2\tan y} = 2x\sec^2 y.$$

This is positive on the stated interval, so

$$\frac{dy}{dx} = \frac{1}{2x\sec^2 y}.$$

Taking logarithms of the original relation gives $\ln x = 2\tan y$. Using $\sec^2 y = 1 + \tan^2 y$,

$$\frac{dy}{dx} = \frac{1}{2x\left(1 + \frac{(\ln x)^2}{4}\right)} = \frac{2}{x\left(4 + (\ln x)^2\right)}.$$

Hence $a = 2$, $b = 4$ and $n = 2$.

FINAL ANSWER

(main) $a = 2,\ b = 4,\ n = 2$

Question 9

9 marks | Trig Equations & R-Form

9. $$\mathrm{h}(x) = 8 + 3\sin x(\cos x - 2\sin x) \quad 0 \leqslant x \leqslant 2\pi$$

(a) Write $\mathrm{h}(x)$ in the form

$$P + R\sin(2x + \alpha)$$

where P, R and α are positive constants, $0 < \alpha < \frac{\pi}{2}$

Give the exact values of P and R and give the value of α, in radians, to 3 decimal places.

(6)

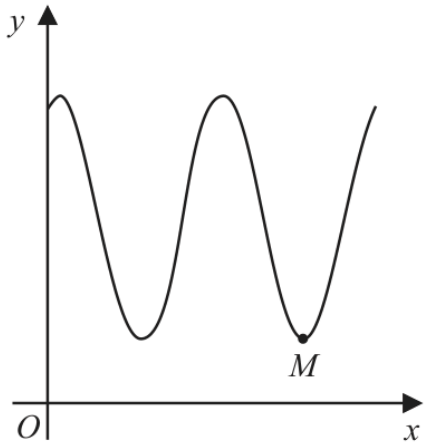


Figure 2

Figure 2 is a sketch of the curve with equation $y = \mathrm{h}(x)$

(b) State the range of h.

(1)

The point M, shown in Figure 2, is the minimum point on the curve with the largest x coordinate.

(c) Find the x coordinate of M.

(Solutions based entirely on calculator technology are not acceptable.)

(2)

1 Part (a)

Use $2\sin x\cos x = \sin 2x$ and $2\sin^2 x = 1 - \cos 2x$:

$$h(x) = 8 + 3\sin x\cos x - 6\sin^2 x = 5 + \frac{3}{2}\sin 2x + 3\cos 2x.$$

Expanding $R\sin(2x+\alpha)$ and comparing coefficients gives

$$R\cos\alpha = \frac{3}{2}, \qquad R\sin\alpha = 3.$$

Therefore

$$P = 5, \qquad R = \sqrt{\frac{9}{4} + 9} = \frac{3\sqrt{5}}{2}, \qquad \alpha = \arctan(2) = 1.107 \text{ radians (3 d.p.)}.$$

Thus $h(x) = 5 + \frac{3\sqrt{5}}{2}\sin(2x+\alpha)$ with the exact angle $\alpha = \arctan(2)$.

2 Part (b)

The given interval includes two complete periods. The range is

$$\left[5 - \frac{3\sqrt{5}}{2},\ 5 + \frac{3\sqrt{5}}{2}\right].$$

3 Part (c)

At a minimum, $\sin(2x+\alpha) = -1$, so

$$2x + \alpha = \frac{3\pi}{2} + 2\pi n, \qquad x = \frac{3\pi}{4} - \frac{\alpha}{2} + \pi n.$$

On $0 \le x \le 2\pi$, $n = 0$ and $n = 1$ give the two minima. The larger coordinate is

$$x_M = \frac{7\pi}{4} - \frac{1}{2}\arctan(2) = 4.944212785\ldots.$$

$$x_M = 4.94 \quad (3 \text{ significant figures}).$$

FINAL ANSWER

(a) $P = 5,\ R = \frac{3\sqrt{5}}{2},\ \alpha = \arctan 2$ (b) $\left[5 - \frac{3\sqrt{5}}{2},\ 5 + \frac{3\sqrt{5}}{2}\right]$ (c) $x_M = 4.94$

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Question 10

9 marks | Log Graphs & Exponential Models

10. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Two populations of insects are being studied over the same period of time.

The number of insects N_A in population A, t years after the start of the study, is modelled by the equation

$$N_A = 8000 + 900\mathrm{e}^{0.2t}$$

According to the model,

(a) find the rate of growth in the number of insects in population A exactly 5 years after the start of the study.

(2)

The number of insects N_B in population B, t years after the start of the study, is modelled by the equation

$$N_B = 8000 + P\mathrm{e}^{kt}$$

where P and k are positive constants.

Given that

- there are 10 000 insects in population B at the start of the study
- there are 11 570 insects in population B exactly 4 years after the start of the study
- there are the same number of insects in population A and population B after T years

find, according to the models,

(b) the exact value of P and the value of k to 3 decimal places,

(4)

(c) the value of T, giving your answer to one decimal place.

(3)

WORKED SOLUTION

Question 10 · WMA13/01 January 2026

1 **Part (a)**

$$\frac{dN_A}{dt} = 900(0.2)e^{0.2t} = 180e^{0.2t}.$$

At $t = 5$, the growth rate is $180e = 489.290729\ldots$ insects per year.

2 Part (b)

At $t = 0$, $10000 = 8000 + P$, so $P = 2000$ exactly. Using the population at $t = 4$,

$$11570 = 8000 + 2000e^{4k} \quad \Longrightarrow \quad e^{4k} = \frac{3570}{2000} = 1.785.$$

Taking natural logarithms,

$$k = \frac{\ln 1.785}{4} = 0.144854604\ldots = 0.145 \quad (3\text{ d.p.}).$$

3 Part (c)

At equality, $8000 + 900e^{0.2T} = 8000 + 2000e^{kT}$, hence

$$e^{(0.2-k)T} = \frac{20}{9}, \qquad T = \frac{\ln(20/9)}{0.2 - k}.$$

Use the unrounded value $k = \ln(1.785)/4$:

$$T = 14.480042784\ldots = 14.5 \text{ years (1 decimal place)}.$$

FINAL ANSWER

(a) $180e$ insects per year (b) $P = 2000,\ k = 0.145\ (3\text{ d.p.})$ (c) 14.5 years