Elite Mathematics
Edexcel International A Level

IAL PURE 3 · WMA13

WMA13/01 October 2025 Worked Solutions

October 2025 paper rebuilt with the worked solution after each question.

9	2020–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA13/01 October 2025

October 2025

9	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

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Question 1

9 marks | Functions, Composition & Inverses

1. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The functions f and g are defined by

$$\mathrm{f}(x) = \ln(x^2 + 3) \qquad x \in \mathbb{R}$$

$$\mathrm{g}(x) = \frac{3 + 5x}{x + 2} \qquad x \in \mathbb{R} \quad x > -2$$

(a) State the range of f

(1)

(b) Find g^{-1}

(3)

(c) Find $\mathrm{fg}(0)$

(2)

(d) Find the exact value of a for which

$$\mathrm{g}\left(\mathrm{e}^{2a}\right) = \mathrm{f}\left(\sqrt{\mathrm{e}^4 - 3}\right)$$

(3)

WORKED SOLUTION

Question 1 · WMA13/01 October 2025

1 **Part (a)**

Since $x^2 + 3 \geq 3$, with equality at $x = 0$, and the logarithm is increasing, the range of f is $[\ln 3, \infty)$.

2 **Part (b)**

Let $y = (3 + 5x)/(x + 2)$. Then

$$yx + 2y = 3 + 5x, \qquad x(y - 5) = 3 - 2y, \qquad x = \frac{3 - 2y}{y - 5}.$$

Moreover, $g(x) = 5 - 7/(x + 2)$ with $x > -2$ has range $(-\infty, 5)$. Thus

$$g^{-1}(x) = \frac{3 - 2x}{x - 5}, \qquad x < 5.$$

3 **Part (c)**

$$g(0) = \frac{3}{2}, \qquad fg(0) = \ln\left(\frac{9}{4} + 3\right) = \ln\frac{21}{4}.$$

4 **Part (d)**

$$f\left(\sqrt{e^4-3}\right)=\ln(e^4-3+3)=4.$$

Therefore

$$\frac{3+5e^{2a}}{e^{2a}+2}=4\Longrightarrow 3+5e^{2a}=4e^{2a}+8\Longrightarrow e^{2a}=5.$$

Taking natural logarithms gives $a=\frac{1}{2}\ln 5$. The input $e^{2a}=5$ satisfies the domain of g.

FINAL ANSWER

(a) $[ln(3), infinity)$ (b) $(3-2*x)/(x-5)$; $(-infinity, 5)$ (c) $ln(21/4)$ (d) $ln(5)/2$

Question 2

7 marks | Numerical Methods

2.

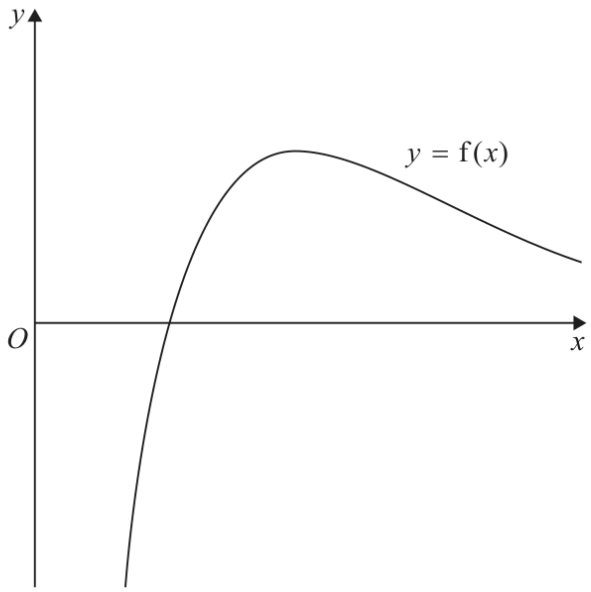


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{2x^2 + 3x - 4}{\mathrm{e}^x} - \frac{1}{x^2} \qquad x \in \mathbb{R} \quad x \neq 0$$

(a) Show that $\mathrm{f}(x) = 0$ has a root α in the interval $[1, 2]$

(2)

(b) Show that the equation $\mathrm{f}(x) = 0$ can be written in the form

$$x = \sqrt[3]{\frac{\mathrm{e}^x + 4x^2}{2x + 3}}$$

(2)

Using the iteration formula

$$x_{n+1} = \sqrt[3]{\frac{\mathrm{e}^{x_n} + 4x_n^{\,2}}{2x_n + 3}} \qquad \text{with } x_1 = 1$$

find, to 4 decimal places,

(c) (i) the value of x_3

(ii) the value of α

(3)

1 Part (a)

$$f(1) = e^{-1} - 1 = -0.632120559\ldots, \qquad f(2) = 10e^{-2} - \frac{1}{4} = 1.103352832\ldots.$$

The function is continuous on $[1, 2]$ and the endpoint values have opposite signs, so a root α lies in $(1, 2)$.

2 Part (b)

Multiplying $f(x) = 0$ by $x^2 e^x$, which is nonzero on the stated interval,

$$x^2(2x^2 + 3x - 4) = e^x \Longrightarrow 2x^4 + 3x^3 = e^x + 4x^2 \Longrightarrow x^3(2x + 3) = e^x + 4x^2.$$

As $2x + 3 > 0$ here, divide and take the real cube root:

$$x = \sqrt[3]{\frac{e^x + 4x^2}{2x + 3}}.$$

3 Part (c)(i)

Starting with $x_1 = 1$ and retaining unrounded values,

$$x_2 = \left(\frac{e + 4}{5}\right)^{1/3} = 1.103475609\ldots, \qquad x_3 = \left(\frac{e^{x_2} + 4x_2^2}{2x_2 + 3}\right)^{1/3} = 1.148358496\ldots = 1.1484.$$

4 Part (c)(ii)

Repeated iteration converges to $1.181331\ldots$, giving $\alpha = 1.1813$ to four decimal places.

FINAL ANSWER

(a) $A root lies in (1, 2) by continuity and opposite endpoint signs$ (b) Shown, as required. (c)(i) 1.1484 (c)(ii) 1.1813

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Question 3

9 marks | Log Graphs & Exponential Models

3. The share price, £V, of a company is being monitored.

A graph is drawn of $\log_{10} V$ against t, where t is the number of years after monitoring began.

The graph is a straight line passing through the points $(0, 2)$ and $(5, 2.25)$

Using this information,

(a) find an equation for the line in the form

$$\log_{10} V = mt + c$$

where m and c are constants.

(2)

(b) Write the answer to part (a) in the form

$$V = ab^t$$

where a and b are constants to be found.

Give the exact value of a and the value of b to 3 significant figures.

(3)

When $t = T$, the rate of increase in the share price of the company was £50 per year.

(c) Find the value of T, giving your answer to the nearest integer.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

WORKED SOLUTION

Question 3 · WMA13/01 October 2025

1 **Part (a)**

The gradient is $(2.25 - 2)/(5 - 0) = 0.05$ and the vertical intercept is 2. Hence $\log_{10} V = 0.05t + 2$.

2 **Part (b)**

$$V = 10^{0.05t+2} = 100(10^{0.05})^t.$$

Thus $a = 100$ exactly and $b = 10^{0.05} = 1.122018454\ldots = 1.12$ to three significant figures. In the requested rounded-parameter form, $V = 100(1.12)^t$; retain the unrounded base for later calculations.

3 Part (c)

Differentiating the unrounded model,

$$\frac{dV}{dt} = 100(0.05\ln 10)\,10^{0.05t} = 5\ln 10\,10^{0.05t}.$$

At $t = T$ this equals 50, so

$$10^{0.05T} = \frac{10}{\ln 10}, \qquad T = \frac{\ln(10/\ln 10)}{0.05\ln 10} = 12.755686226\ldots.$$

Therefore $T = 13$ years to the nearest integer.

FINAL ANSWER

(a) $log10(V) = 0.05 * t + 2$ (b) $V = 100 * (1.12)^t$ (c) 13

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Question 4

6 marks | Trig Equations & R-Form

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) $$f(x) = \sqrt{3}\sin 2x - 3\cos 2x$$

Express $f(x)$ in the form $R\sin(2x - \alpha)$, where R and α are constants,

$$R > 0 \text{ and } 0 < \alpha < \frac{\pi}{2}$$

Give the exact value of R and the exact value of α.

(3)

(b) $$g(x) = \frac{18}{f(3x) + 4\sqrt{3}} \qquad x > 0$$

Using the answer to part (a),

(i) write down the exact minimum value of $g(x)$,

(ii) find the smallest value of x for which this minimum value occurs.

You must make your method clear.

(3)

WORKED SOLUTION

Question 4 · WMA13/01 October 2025

1 **Part (a)**

Expand $R\sin(2x-\alpha) = R\cos\alpha\sin 2x - R\sin\alpha\cos 2x$. Comparison gives $R\cos\alpha = \sqrt{3}$ and $R\sin\alpha = 3$. Thus $R = \sqrt{3+9} = 2\sqrt{3}$ and $\tan\alpha = \sqrt{3}$. As $0 < \alpha < \pi/2$, $\alpha = \pi/3$ and

$$f(x) = 2\sqrt{3}\sin(2x - \pi/3).$$

2 **Part (b)(i)**

$$g(x) = \frac{18}{2\sqrt{3}\sin(6x - \pi/3) + 4\sqrt{3}}.$$

The denominator is always positive, between $2\sqrt{3}$ and $6\sqrt{3}$. The fraction is smallest when the denominator is largest:

$$\min g = \frac{18}{6\sqrt{3}} = \sqrt{3}.$$

3

Part (b)(ii)

The maximum denominator occurs when $\sin(6x - \pi/3) = 1$:

$$6x - \frac{\pi}{3} = \frac{\pi}{2} + 2\pi n \Longrightarrow x = \frac{5\pi}{36} + \frac{\pi n}{3}.$$

The smallest positive value is $x = 5\pi/36$, since $n = -1$ gives a negative value.

FINAL ANSWER

(a) $2 * sqrt(3) * sin(2 * x - pi/3)$ (b)(i) $sqrt(3)$ (b)(ii) $5 * pi/36$

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Question 5

8 marks | Log Graphs & Exponential Models

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The number of squirrels in a forest is being studied.

The number of squirrels, N, in the forest, t years after the start of the study, is modelled by the equation

$$N = \frac{4000\mathrm{e}^{0.1t}}{19 + \mathrm{e}^{0.2t}} \qquad t \geqslant 0$$

Use the equation of the model to answer parts (a), (b), (c) and (d).

(a) Find the number of squirrels in the forest at the start of the study.

(1)

(b) Find $\dfrac{\mathrm{d}N}{\mathrm{d}t}$

(2)

The number of squirrels in the forest is at a maximum when $t = T$

Using the answer to part (b),

(c) show that $\mathrm{e}^{0.2T} = A$, where A is a constant to be found.

(2)

(d) Hence find the maximum number of squirrels in the forest.

Show your working and give your answer to the nearest whole number.

(3)

WORKED SOLUTION

Question 5 · WMA13/01 October 2025

1 **Part (a)**

At $t = 0$, $N = 4000/(19 + 1) = 200$ squirrels.

2 **Part (b)**

The quotient rule gives

$$\frac{dN}{dt} = \frac{400e^{0.1t}(19 + e^{0.2t}) - 800e^{0.3t}}{(19 + e^{0.2t})^2} = \frac{400e^{0.1t}(19 - e^{0.2t})}{(19 + e^{0.2t})^2}.$$

3

Part (c)

At $t = T$, the derivative is zero. The exponential and denominator are positive, so $19 - e^{0.2T} = 0$. Therefore $e^{0.2T} = 19$ and $A = 19$. The derivative is positive before T and negative after T, confirming a maximum.

4

Part (d)

Since $e^{0.2T} = 19$, we have $e^{0.1T} = \sqrt{19}$. Substitution into the model gives

$$N_{\max} = \frac{4000\sqrt{19}}{19 + 19} = \frac{2000}{\sqrt{19}} = 458.831467741\ldots.$$

The maximum number is 459 squirrels to the nearest whole number.

✓ **FINAL ANSWER**

(a) 200 (b) $400 * exp(t/10) * (19 - exp(t/5))/(19 + exp(t/5))^2$ (c) $exp(T/5) = 19$ (d) 459

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Question 6

7 marks | Integration

6. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Given $y = \ln(2x^2 + 5)$ find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(2)

(ii) A curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{21x}{3x^2 + k} \qquad x \in \mathbb{R}$$

where k is a positive integer, $k > 1$

Given that

$$\int_1^k \mathrm{f}(x)\,\mathrm{d}x < 7\ln 8$$

find the greatest possible value of k

(5)

WORKED SOLUTION

Question 6 · WMA13/01 October 2025

1 **Part (i)**

Using the chain rule,

$$\frac{dy}{dx} = \frac{1}{2x^2 + 5}(4x) = \frac{4x}{2x^2 + 5}.$$

2 **Part (ii)**

Treat k as a constant when integrating:

$$\int_1^k \frac{21x}{3x^2+k}\,dx = \left[\frac{7}{2}\ln(3x^2+k)\right]_1^k = \frac{7}{2}\ln\left(\frac{3k^2+k}{k+3}\right).$$

The stated inequality therefore implies

$$\ln\left(\frac{3k^2+k}{k+3}\right) < 2\ln 8 = \ln 64.$$

Since the exponential is increasing and $k+3>0$,

$$3k^2+k < 64(k+3) \Longrightarrow k^2-21k-64<0.$$

The roots are $(21\pm\sqrt{697})/2$, so the positive upper bound is $k<(21+\sqrt{697})/2 = 23.700378782\ldots$. With $k>1$ an integer, the greatest possible value is $k=23$.

FINAL ANSWER

(i) $4*x/(2*x^2+5)$ (ii) 23

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Question 7

7 marks | Applications of Differentiation

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A curve C has equation

$$y = 2x\mathrm{e}^{x^2+(3k-2)x}$$

where k is a constant.

Given that C has two distinct turning points, find the range of possible values of k.

(7)

WORKED SOLUTION

Question 7 · WMA13/01 October 2025

1 Solution

\paragraph{Solution} Put $u = x^2 + (3k-2)x$, so $u' = 2x + 3k - 2$. By the product and chain rules,

$$\frac{dy}{dx} = 2e^u + 2xe^u(2x + 3k - 2) = 2e^u\big(2x^2 + (3k-2)x + 1\big).$$

As $2e^u > 0$, stationary points satisfy

$$2x^2 + (3k-2)x + 1 = 0.$$

For two distinct real roots, the discriminant must be positive:

$$(3k-2)^2 - 8 > 0 \iff |3k-2| > 2\sqrt{2}.$$

Consequently

$$k < \frac{2-2\sqrt{2}}{3} \quad \text{or} \quad k > \frac{2+2\sqrt{2}}{3}.$$

For these values the quadratic changes sign at both simple roots, so both stationary points are genuine turning points. Equality gives a repeated root with no derivative sign change and is excluded.

FINAL ANSWER

(main) $(-infinity, (2 - 2 * sqrt(2))/3) union ((2 + 2 * sqrt(2))/3, infinity)$

Question 8

13 marks | Modulus

8.

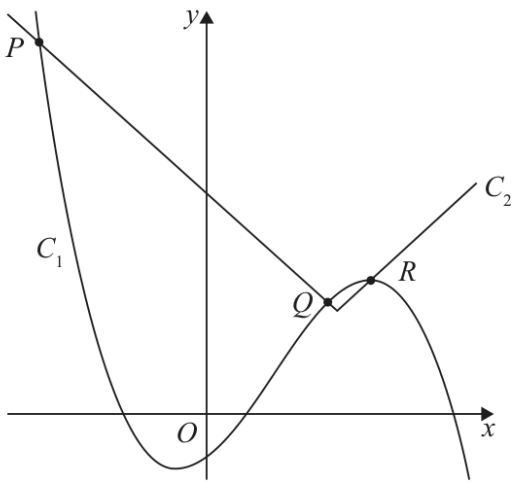


Figure 2

Figure 2 shows a sketch of the graph C_1 with equation

$$y = -2x^3 + 5x^2 + 4x - 3$$

and a sketch of the graph C_2 with equation

$$y = a + |5x + b|$$

where a and b are constants.

Graphs C_1 and C_2 intersect at point P, point Q and point R, as shown in Figure 2.

Given that P has coordinates $(-2, 25)$

(a) show that

$$a = 15 + b$$

(2)

Given also that R has coordinates $(2, 9)$

(b) find the value of a and the value of b

(3)

Using the answer to part (b),

(c) state the coordinates of the vertex of C_2

(2)

(d) Find, using algebra, the coordinates of Q. Show each stage of your working.

(*Solutions relying on calculator technology are not acceptable.*)

(6)

1 Part (a)

Point P lies on the descending branch of C_2, where $5x + b < 0$. At $P = (-2, 25)$,

$$25 = a - (-10 + b) = a + 10 - b,$$

hence $a = 15 + b$.

2 Part (b)

Point $R = (2, 9)$ lies on the ascending branch, so

$$9 = a + (10 + b) \Longrightarrow a + b = -1.$$

Combining with $a - b = 15$ gives $2a = 14$, hence $a = 7$ and $b = -8$.

3 Part (c)

The vertex occurs at $5x - 8 = 0$, giving $(x, y) = (8/5, 7)$.

4 Part (d)

The diagram places Q on the descending branch $y = 7 - (5x - 8) = 15 - 5x$, with $x < 8/5$. Equate this with the cubic:

$$-2x^3 + 5x^2 + 4x - 3 = 15 - 5x \Longrightarrow 2x^3 - 5x^2 - 9x + 18 = 0.$$

Since $x = -2$ is the known intersection P, divide by $x + 2$:

$$2x^3 - 5x^2 - 9x + 18 = (x + 2)(2x^2 - 9x + 9) = (x + 2)(2x - 3)(x - 3).$$

The candidates are $-2, 3/2, 3$. Reject $x = 3$ because it is outside the descending branch, and $x = -2$ is P, not Q. Thus $x_Q = 3/2$ and $y_Q = 15 - 5(3/2) = 15/2$, giving $Q = (3/2, 15/2)$.

FINAL ANSWER

(a) Shown, as required. (b) 7; -8 (c) $(8/5, 7)$ (d) $(3/2, 15/2)$

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Question 9

9 marks | Trig Identities & Exact Values

9. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Express

$$6\sin^2\theta\cot 2\theta + 4\sin\theta\cos\theta$$

in terms of $\sin 2\theta$ and $\cos 2\theta$ only.

(3)

(b) Hence show that the equation

$$3\cot 2\theta - 14 = 6\sin^2\theta\cot 2\theta + 4\sin\theta\cos\theta$$

can be written in the form

$$5\sin^2 2\theta + 14\sin 2\theta - 3 = 0$$

(3)

(c) Hence solve, for $0 < x < 90°$, the equation

$$3\cot 2x - 14 = 6\sin^2 x\cot 2x + 4\sin x\cos x$$

giving your answers to one decimal place.

(3)

WORKED SOLUTION

Question 9 · WMA13/01 October 2025

1 **Part (a)**

Use $2\sin^2\theta = 1 - \cos 2\theta$, $\cot 2\theta = \cos 2\theta / \sin 2\theta$, and $2\sin\theta\cos\theta = \sin 2\theta$:

$$6\sin^2\theta\cot 2\theta + 4\sin\theta\cos\theta = \frac{3(1-\cos 2\theta)\cos 2\theta}{\sin 2\theta} + 2\sin 2\theta.$$

This is in terms of $\sin 2\theta$ and $\cos 2\theta$ only, on the original domain $\sin 2\theta \neq 0$.

2

Part (b)

Substitute the preceding expression and multiply by $\sin 2\theta$:

$$3\cos 2\theta - 14\sin 2\theta = 3\cos 2\theta - 3\cos^2 2\theta + 2\sin^2 2\theta.$$

Cancel $3\cos 2\theta$ and use $\cos^2 2\theta = 1 - \sin^2 2\theta$:

$$-14\sin 2\theta = -3 + 5\sin^2 2\theta.$$

Hence $5\sin^2 2\theta + 14\sin 2\theta - 3 = 0$.

3

Part (c)

Let $s = \sin 2x$. Then $(5s - 1)(s + 3) = 0$, so $s = 1/5$ or $s = -3$. The latter is impossible. As $0 < 2x < 180°$,

$$2x = \arcsin(1/5) = 11.53695903\ldots° \quad \text{or} \quad 180° - 11.53695903\ldots°.$$

Therefore $x = 5.8°$ or $84.2°$ to one decimal place. Both have nonzero $\sin 2x$, so the original cotangent is defined.

✓ **FINAL ANSWER**

(a) $3 * (1 - cos(2 * theta)) * cos(2 * theta)/sin(2 * theta) + 2 * sin(2 * theta)$ (b) Shown, as required. (c) $5.8 degrees$; $84.2 degrees$