Elite Mathematics
Edexcel International A Level

IAL PURE 3 · WMA13

WMA13/01 May/June 2025 Worked Solutions

May/June 2025 paper rebuilt with the worked solution after each question.

10	2020–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA13/01 May/June 2025

May/June 2025

10	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA13/01 MAY/JUNE 2025

Question 1

8 marks | Functions, Composition & Inverses

1: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The functions f and g are defined by

$$\mathrm{f}(x) = \frac{2x}{3x+1} \qquad x \in \mathbb{R} \quad x \geqslant 0$$

$$\mathrm{g}(x) = 4 - x^2 \qquad x \in \mathbb{R} \quad x \geqslant 0$$

(a) Find the value of $\mathrm{gf}(1)$

(2)

(b) Find the range of f

(2)

(c) Find $\mathrm{f}^{-1}(x)$

(2)

(d) Solve $\mathrm{f}^{-1}(x) = \mathrm{f}(x)$

(2)

WORKED SOLUTION

Question 1 · WMA13/01 May/June 2025

Part (a)

First apply f, then g:

$$f(1) = \frac{2}{3+1} = \frac{1}{2}, \qquad gf(1) = 4 - \left(\frac{1}{2}\right)^2 = \boxed{\frac{15}{4}}.$$

Part (b)

For $x \geq 0$,

$$f(x) = \frac{2}{3} - \frac{2}{3(3x+1)}.$$

Thus $f(0) = 0$ and $f(x) < 2/3$. Also $f'(x) = 2/(3x+1)^2 > 0$, so the range is

$$\boxed{0 \leq f(x) < \frac{2}{3}}.$$

3 Part (c)

$$y = \frac{2x}{3x+1} \implies 3xy + y = 2x \implies x = \frac{y}{2-3y}.$$

Therefore

$$\boxed{f^{-1}(x) = \frac{x}{2-3x}}, \qquad 0 \le x < \frac{2}{3}.$$

---

4 Part (d)

On the common domain $0 \le x < 2/3$, both denominators below are nonzero:

$$\frac{x}{2-3x} = \frac{2x}{3x+1} \implies x(3x+1) = 2x(2-3x) \implies 3x(3x-1) = 0.$$

Both candidates lie in the common domain, giving

$$\boxed{x = 0 \quad \text{or} \quad x = \frac{1}{3}}.$$

FINAL ANSWER

(a) $\frac{15}{4}$ (b) $0 \le f(x) < \frac{2}{3}$ (c) $f^{-1}(x) = \frac{x}{2-3x}$ (d) $x = 0 \quad \text{or} \quad x = \frac{1}{3}$

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Question 2

7 marks | Trig Equations & R-Form

2: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

$$\mathrm{f}(x) = 7\cos x - 24\sin x$$

(a) Express $\mathrm{f}(x)$ in the form $R\cos(x + \alpha)$ where R and α are constants, $R > 0$

and $0 < \alpha < \dfrac{\pi}{2}$

Give the exact value of R and give the value of α, in radians, to 3 decimal places.

(3)

$$\mathrm{g}(x) = \frac{5}{90 - 3\mathrm{f}(2x)}$$

(b) Using the answer to part (a), find

(i) the minimum value of $\mathrm{g}(x)$, giving your answer as a fully simplified fraction,

(ii) the smallest positive value of x for which this minimum value occurs, giving your answer to 3 decimal places.

(4)

WORKED SOLUTION

Question 2 · WMA13/01 May/June 2025

1 **Part (a)**

Expand $R\cos(x+\alpha) = R\cos\alpha\cos x - R\sin\alpha\sin x$. Comparing coefficients gives

$$R\cos\alpha = 7, \qquad R\sin\alpha = 24.$$

Hence

$$R = \sqrt{7^2 + 24^2} = \boxed{25}, \qquad \alpha = \arctan\frac{24}{7} = 1.287002\ldots = \boxed{1.287 \text{ radians}}.$$

Thus $f(x) = 25\cos(x+\alpha)$, using the unrounded α in subsequent calculations.

2 **Part (i)**

Since $-25 \le f(2x) \le 25$, the positive denominator $90 - 3f(2x)$ lies between 15 and 165. The smallest reciprocal occurs at the largest denominator:

$$\boxed{g_{\min} = \frac{5}{165} = \frac{1}{33}}.$$

---

3 **Part (ii)**

The minimum requires $f(2x) = -25$, so

$$\cos(2x + \alpha) = -1, \qquad 2x + \alpha = \pi + 2n\pi.$$

Since $0 < \alpha < \pi/2$, the first positive solution uses $n = 0$:

$$x = \frac{\pi - \arctan(24/7)}{2} = 0.927295\ldots = \boxed{0.927}.$$

✓ **FINAL ANSWER**

(a) 25, 1.287 radians (i) $g_{\min} = \frac{5}{165} = \frac{1}{33}$ (ii) 0.927 (b)(i) $g_{\min} = \frac{5}{165} = \frac{1}{33}$ (b)(ii) $\boxed{0.927}$

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Question 3

5 marks | Log Graphs & Exponential Models

3:

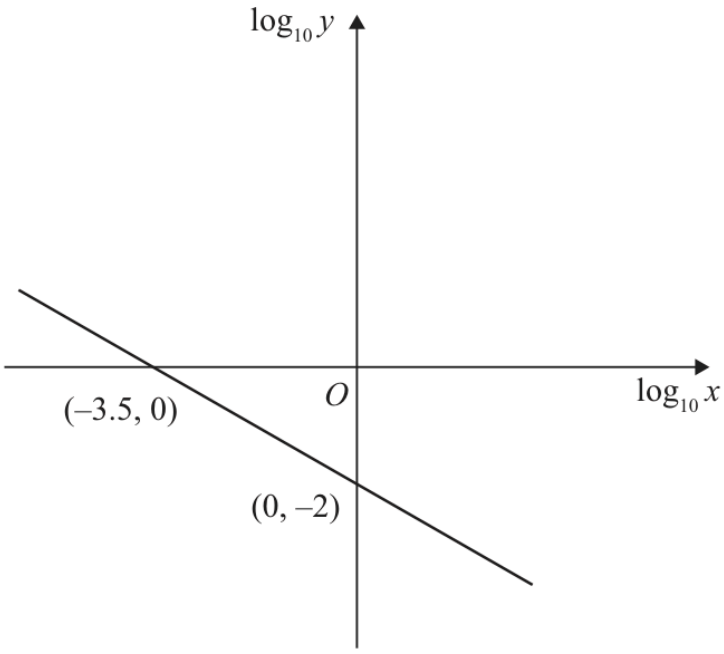


Figure 1

Figure 1 shows a linear relationship between $\log_{10} y$ and $\log_{10} x$

The line passes through the points (–3.5, 0) and (0, –2) as shown.

(a) Find an equation linking $\log_{10} y$ with $\log_{10} x$

(2)

(b) Hence, or otherwise, express y in the form px^q where p and q are rational constants.

(3)

WORKED SOLUTION

Question 3 · WMA13/01 May/June 2025

1 **Part (a)**

The gradient in the logarithmic coordinates is

$$m = \frac{-2-0}{0-(-3.5)} = -\frac{4}{7}.$$

Using the vertical intercept -2,

$$\boxed{\log_{10} y = -\frac{4}{7}\log_{10} x - 2}.$$

2 **Part (b)**

For $x > 0$, the logarithm laws give

$$\log_{10} y = \log_{10}(x^{-4/7}) + \log_{10} \frac{1}{100} = \log_{10}\left(\frac{1}{100}x^{-4/7}\right).$$

Therefore

$$\boxed{y = \frac{1}{100}x^{-4/7}}, \qquad \boxed{p = \frac{1}{100}, \quad q = -\frac{4}{7}}.$$

✓ **FINAL ANSWER**

(a) $\log_{10} y = -\frac{4}{7}\log_{10} x - 2$ (b) $y = \frac{1}{100}x^{-4/7}$, $p = \frac{1}{100}$, $\quad q = -\frac{4}{7}$

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Question 4

5 marks | Rational Algebra

4: The function f is defined by

$$f(x) = \frac{49x}{x^2 + x - 12} + \frac{7x}{x + 4} \qquad x > 3$$

(a) Show that

$$f(x) = \frac{7x}{x - 3} \qquad x > 3$$

(3)

(b) Hence find $f'(x)$ giving your answer in simplest form.

(2)

WORKED SOLUTION

Question 4 · WMA13/01 May/June 2025

1 Part (a)

Factor $x^2 + x - 12 = (x + 4)(x - 3)$ and combine the fractions:

$$f(x) = \frac{49x + 7x(x - 3)}{(x + 4)(x - 3)} = \frac{7x(x + 4)}{(x + 4)(x - 3)} = \boxed{\frac{7x}{x - 3}}.$$

The cancellation is valid because $x > 3$. In particular, the numerator simplifies as $49x + 7x^2 - 21x = 7x^2 + 28x = 7x(x + 4)$.

2 Part (b)

Rewrite $f(x) = 7 + 21/(x - 3)$. Differentiating gives

$$\boxed{f'(x) = -\frac{21}{(x - 3)^2}}.$$

FINAL ANSWER

(a) $\frac{7x}{x-3}$ (b) $f'(x) = -\frac{21}{(x-3)^2}$

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Question 5

4 marks | Integration

5: Find

(i) $\int \sin^2 3x \, dx$

(2)

(ii) $\int x(x^2+4)^{\frac{3}{2}} dx$

(2)

WORKED SOLUTION

Question 5 · WMA13/01 May/June 2025

1 **Part (i)**

Use $\sin^2(3x) = (1-\cos 6x)/2$:

$$\int \sin^2(3x)\,dx = \frac{1}{2}\int (1-\cos 6x)\,dx = \boxed{\frac{x}{2} - \frac{\sin 6x}{12} + C}.$$

2 **Part (ii)**

The derivative of $(x^2+4)^{5/2}$ is $5x(x^2+4)^{3/2}$. Therefore

$$\int x(x^2+4)^{3/2}\,dx = \boxed{\frac{1}{5}(x^2+4)^{5/2} + C}.$$

For example, differentiating this answer gives

$$\frac{1}{5}\cdot\frac{5}{2}(x^2+4)^{3/2}\cdot 2x = x(x^2+4)^{3/2},$$

which recovers the integrand. Each indefinite integral has its own arbitrary constant.

FINAL ANSWER

(i) $\frac{x}{2} - \frac{\sin 6x}{12} + C$ (ii) $\frac{1}{5}(x^2+4)^{5/2} + C$

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Question 6

8 marks | Log Graphs & Exponential Models

6: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The temperature, θ°C, of a computer processor, t minutes after the computer is switched off, is modelled by the equation

$$\theta = 21 + Ae^{-kt}$$

where A and k are positive constants.

Given that the temperature of the processor was 75°C when the computer was switched off,

(a) find the value of A.

(2)

Given also that it takes 5 minutes for the temperature of the processor to decrease from 75°C to 25°C,

(b) find the value of k, giving your answer to 3 significant figures.

(3)

At time T minutes, the temperature of the processor is decreasing at a rate of 9°C per minute.

(c) Find the value of T according to the model, giving your answer to 2 decimal places.

(3)

WORKED SOLUTION

Question 6 · WMA13/01 May/June 2025

1 **Part (a)**

At $t = 0$, $75 = 21 + A$, so $\boxed{A = 54}$.

2 **Part (b)**

At $t = 5$, $25 = 21 + 54e^{-5k}$, hence

$$e^{-5k} = \frac{2}{27}, \qquad k = \frac{1}{5}\ln\frac{27}{2} = 0.520537937\ldots = \boxed{0.521}.$$

3 Part (c)

Differentiate the temperature model:

$$\frac{d\theta}{dt} = -54ke^{-kt}.$$

A decrease of $9\,^\circ\text{C}$ per minute means $d\theta/dt = -9$, not $+9$. Thus

$$-54ke^{-kT} = -9 \implies e^{-kT} = \frac{1}{6k} \implies T = \frac{\ln(6k)}{k}.$$

Using the unrounded value of k,

$$T = 2.187865\ldots = \boxed{2.19 \text{ minutes}}.$$

FINAL ANSWER

(a) $A = 54$ (b) 0.521 (c) 2.19 minutes

Question 7

9 marks | Numerical Methods

7: A continuous curve has equation

$$y = e^{-x^2}\sin 3x \qquad 0 \leqslant x \leqslant \frac{\pi}{3}$$

The curve has a stationary point at the point P.

(a) Show, using calculus, that the x coordinate of P is a solution of the equation

$$x = \frac{1}{3}\arctan\left(\frac{3}{2x}\right)$$

(4)

Using the iteration formula

$$x_{n+1} = \frac{1}{3}\arctan\left(\frac{3}{2x_n}\right) \qquad x_1 = 0.4$$

(b) find the value of

(i) x_2

(ii) x_4

giving your answers to 4 decimal places.

(3)

(c) Using a suitable interval and a suitable function which should be stated, show that the x coordinate of P is 0.430 correct to 3 decimal places.

(2)

1 Part (a)

Apply the product and chain rules:

$$\frac{dy}{dx} = e^{-x^2}\left(3\cos 3x - 2x\sin 3x\right).$$

Since $e^{-x^2} > 0$, a stationary point satisfies

$$3\cos 3x = 2x\sin 3x.$$

Neither endpoint $x = 0, \pi/3$ is stationary. In the interior $\sin 3x > 0$ and $x > 0$, so $\cos 3x > 0$ at a stationary point and $0 < 3x < \pi/2$. Therefore division and the principal inverse tangent are valid:

$$\tan 3x = \frac{3}{2x} \quad \Longrightarrow \quad \boxed{x = \frac{1}{3}\arctan\left(\frac{3}{2x}\right)}.$$

2 Part (b)

Work in radians and retain calculator precision between iterations:

$$x_2 = \frac{1}{3}\arctan\frac{3}{0.8} = 0.4367313117\ldots,$$

$$x_3 = 0.4291581882\ldots, \qquad x_4 = 0.4307116849\ldots.$$

Thus (i) $\boxed{x_2 = 0.4367}$ and (ii) $\boxed{x_4 = 0.4307}$.

3 Part (c)

Let $h(x) = 3\cos 3x - 2x\sin 3x$, which is continuous. At the rounding boundaries,

$$h(0.4295) = 0.0106861\ldots > 0, \qquad h(0.4305) = -0.000598027\ldots < 0.$$

The sign change places a root strictly between 0.4295 and 0.4305. On $0 < x < \pi/6$, $h'(x) = -11\sin 3x - 6x\cos 3x < 0$, so the stationary root is unique; for $\pi/6 \le x \le \pi/3$, $h(x) < 0$. Hence the x coordinate of P is $\boxed{0.430 \text{ to } 3 \text{ decimal places}}$.

FINAL ANSWER

(a) $x = \frac{1}{3}\arctan\left(\frac{3}{2x}\right)$ (b) $x_2 = 0.4367$, $x_4 = 0.4307$ (c) 0.430 to 3 decimal places

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Question 8

8 marks | Trig Identities & Exact Values

8: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\tan 3x \equiv \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} \qquad x \neq (2n+1)\frac{\pi}{6} \qquad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$

$$\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = 2\sec^2 3\theta - 8$$

giving your answers to 2 decimal places.

(5)

WORKED SOLUTION

Question 8 · WMA13/01 May/June 2025

1 **Part (a)**

Put $s = \sin x$ and $c = \cos x$. Using the addition and double-angle formulae,

$$\sin 3x = \sin 2x \cos x + \cos 2x \sin x = 3sc^2 - s^3,$$

$$\cos 3x = \cos 2x \cos x - \sin 2x \sin x = c^3 - 3s^2c.$$

Where the stated expressions are defined, $c \neq 0$ and $\cos 3x \neq 0$. Dividing numerator and denominator by c^3 gives

$$\tan 3x = \frac{3sc^2 - s^3}{c^3 - 3s^2c} = \boxed{\frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}}.$$

2 Part (b)

By part (a), the equation is $\tan 3\theta = 2\sec^2 3\theta - 8$. Set $u = \tan 3\theta$ and use $\sec^2 3\theta = 1 + u^2$:

$$u = 2(1 + u^2) - 8 \implies 2u^2 - u - 6 = 0 \implies (2u + 3)(u - 2) = 0.$$

Thus $\tan 3\theta = 2$ or $\tan 3\theta = -3/2$. The transformed interval is $0 < 3\theta < 3\pi/2$. Its three allowed angles are

$$3\theta = \arctan 2, \quad \pi - \arctan\frac{3}{2}, \quad \pi + \arctan 2.$$

Consequently

$$\theta = 0.36904957\ldots, \quad 0.71959964\ldots, \quad 1.41624712\ldots,$$

$$\boxed{\theta = 0.37,\ 0.72,\ 1.42 \text{ radians}}.$$

None lies at an excluded pole of either side.

FINAL ANSWER

(a) $\frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$ (b) $\theta = 0.37,\ 0.72,\ 1.42$ radians

Question 9

11 marks | Differentiation Techniques

9:

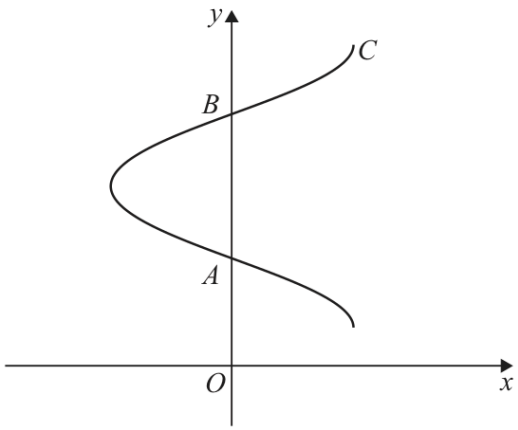


Figure 2

Figure 2 shows a sketch of the curve C with equation

$$x = \frac{2}{3}\sin\left(3y + \frac{\pi}{4}\right) \qquad \frac{\pi}{12} < y < \frac{3\pi}{4}$$

The curve intersects the y-axis at the points A and B as shown.

(a) Find the exact value of the y coordinate of

- point A
- point B

(3)

(b) Show that

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{p - qx^2}$$

where p and q are integers to be found.

(4)

The **normal** to C at A and the **tangent** to C at B intersect at the point D.

Using

- the answer to part (b)
- the sketch of curve C in Figure 2

(c) find, in simplest form, the exact x coordinate of D.

(4)

1 Part (a)

On the y axis, $x = 0$, so $\sin(3y + \pi/4) = 0$. Since $\pi/12 < y < 3\pi/4$, we have $\pi/2 < 3y + \pi/4 < 5\pi/2$. Therefore the possible angles are π and 2π, giving

$$\boxed{y_A = \frac{\pi}{4}, \qquad y_B = \frac{7\pi}{12}}.$$

2 Part (b)

Differentiate x with respect to y, then take the reciprocal:

$$\frac{dx}{dy} = 2\cos(3y + \pi/4), \qquad \frac{dy}{dx} = \frac{1}{2\cos(3y + \pi/4)}.$$

Using $\sin(3y + \pi/4) = 3x/2$,

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{4(1 - \sin^2(3y + \pi/4))} = \boxed{\frac{1}{4 - 9x^2}}.$$

Thus $\boxed{p = 4,\ q = 9}$. The derivative formula applies where the tangent is not vertical.

3 Part (c)

At $x = 0$, part (b) gives $|dy/dx| = 1/2$. The sketch shows a negative tangent gradient at A and a positive one at B. The normal at A therefore has gradient 2, while the tangent at B has gradient $1/2$:

$$y = 2x + \frac{\pi}{4}, \qquad y = \frac{x}{2} + \frac{7\pi}{12}.$$

At their intersection,

$$\frac{3}{2}x = \frac{7\pi}{12} - \frac{\pi}{4} = \frac{\pi}{3} \quad \Longrightarrow \quad \boxed{x_D = \frac{2\pi}{9}}.$$

FINAL ANSWER

(a) $y_A = \frac{\pi}{4}, \quad y_B = \frac{7\pi}{12}$ (b) $\frac{1}{4-9x^2}$, $p = 4,\ q = 9$ (c) $x_D = \frac{2\pi}{9}$

Question 10

10 marks | Modulus

10:

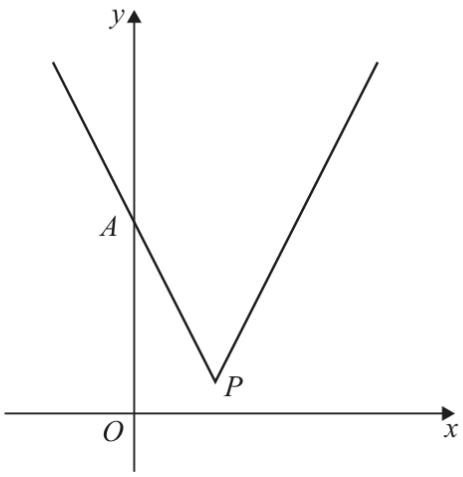


Figure 3

Figure 3 shows a sketch of part of the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = |kx - 10| + k \qquad x \in \mathbb{R}$$

and k is a positive constant.

The graph

- cuts the y-axis at the point A
- has a vertex at the point P

(a) Find, in simplest form in terms of k,

(i) the y coordinate of A

(ii) the coordinates of P

(3)

(b) Find, in terms of k, the range of values of x which satisfy

$$|kx - 10| + k \geqslant 2k$$

(3)

Given that the line with equation $y = 3x + 1$ intersects the graph of $y = \mathrm{f}(x)$ at 2 distinct points,

(c) find the range of values of k.

(4)

1 **Part (i)**

At $x = 0$, $f(0) = |-10| + k$, so $\boxed{y_A = 10 + k}$.

2 **Part (ii)**

At the vertex the expression inside the modulus vanishes:

$$kx - 10 = 0 \quad \Longrightarrow \quad \boxed{P = \left(\frac{10}{k}, k\right)}.$$

3 **Part (b)**

$$|kx - 10| + k \geq 2k \quad \Longleftrightarrow \quad |kx - 10| \geq k.$$

Since $k > 0$, the two branches give

$$kx - 10 \leq -k \quad \text{or} \quad kx - 10 \geq k,$$

$$\boxed{x \leq \frac{10}{k} - 1 \quad \text{or} \quad x \geq \frac{10}{k} + 1}.$$

4 **Part (c)**

Consider the vertical difference $d(x) = f(x) - (3x + 1)$:

$$d(x) = \begin{cases} 9 + k - (k+3)x, & x \leq 10/k, \\ (k-3)x + k - 11, & x \geq 10/k. \end{cases}$$

The left branch is strictly decreasing. If $0 < k < 3$, the right branch also decreases, so there cannot be two roots. If $k = 3$, the right branch is the nonzero constant -8, again giving no second root. For $k > 3$ the right branch increases and both outer ends tend to $+\infty$. There are two distinct roots precisely when the vertex lies below zero:

$$d(10/k) = k - 1 - \frac{30}{k} < 0.$$

Multiplying by $k > 0$ gives

$$k^2 - k - 30 = (k-6)(k+5) < 0 \quad \Longrightarrow \quad 0 < k < 6.$$

Combining with $k > 3$ yields $\boxed{3 < k < 6}$. At $k = 6$ the two roots merge at the vertex, so the endpoint is excluded.

FINAL ANSWER

(i) $y_A = 10 + k$ (ii) $P = \left(\frac{10}{k}, k\right)$ (b) $x \leq \frac{10}{k} - 1 \quad \text{or} \quad x \geq \frac{10}{k} + 1$ (c) $3 < k < 6$ (a)(i) $y_A = 10 + k$ (a)(ii) $P = \left(\frac{10}{k}, k\right)$