Elite Mathematics
Edexcel International A Level

IAL PURE 3 · WMA13

WMA13/01 January 2025 Worked Solutions

January 2025 paper rebuilt with the worked solution after each question.

10	2020–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA13/01 January 2025

January 2025

10	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA13/01 JANUARY 2025

Question 1

6 marks | Numerical Methods

1. $$f(x) = 2\sec x + 6x - 3 \qquad 0 < x < \frac{\pi}{2}$$

The equation $f(x) = 0$ has a single root α

(a) Show that $0.1 < \alpha < 0.2$

(2)

(b) Show that α is a solution of

$$x = \frac{1}{2} - \frac{1}{3\cos x}$$

(1)

The iterative formula

$$x_{n+1} = \frac{1}{2} - \frac{1}{3\cos x_n}$$

is used to find α

(c) Starting with $x_1 = 0.15$ and using the iterative formula,

(i) find, to 4 decimal places, the value of x_2

(ii) find, to 4 decimal places, the value of α

(3)

WORKED SOLUTION

Question 1 · WMA13/01 January 2025

1 **Part (a)**

Work in radians. Substituting the endpoints into $f(x) = 2\sec x + 6x - 3$ gives

$$f(0.1) = -0.3899581632\ldots < 0, \qquad f(0.2) = 0.2406776899\ldots > 0.$$

The function is continuous on $[0.1, 0.2]$ since $\cos x \neq 0$ there. The sign change therefore places a root between the endpoints. Since the given root is unique, this root is α.

$$\boxed{0.1 < \alpha < 0.2}$$

2 Part (b)

Set $f(x) = 0$ and use $\sec x = 1/\cos x$:

$$\frac{2}{\cos x} + 6x - 3 = 0 \implies 6x = 3 - \frac{2}{\cos x} \implies x = \frac{1}{2} - \frac{1}{3\cos x}.$$

Thus α, which satisfies $f(\alpha) = 0$, satisfies this rearranged equation.

$$\boxed{x = \frac{1}{2} - \frac{1}{3\cos x}}$$

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3 Part (c)(i)

With $x_1 = 0.15$ and angles in radians,

$$x_2 = \frac{1}{2} - \frac{1}{3\cos(0.15)} = 0.1628811858\ldots.$$

Round only the displayed result to four decimal places.

$$\boxed{x_2 = 0.1629}$$

---

4 Part (c)(ii)

Continuing the iteration with unrounded internal values gives

$$x_3 = 0.1621955413\ldots, \quad x_4 = 0.1622335203\ldots, \quad x_5 = 0.1622314210\ldots, \quad x_6 = 0.1622315370\ldots.$$

The iterates settle at $0.16223153\ldots$, giving the root to four decimal places.

$$\boxed{\alpha = 0.1622}$$

FINAL ANSWER

(a) $0.1 < \alpha < 0.2$ (b) $x = \frac{1}{2} - \frac{1}{3\cos x}$ (c)(i) $x_2 = 0.1629$ (c)(ii) $\alpha = 0.1622$

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Question 2

3 marks | Log Graphs & Exponential Models

2. The weed on the surface of a pond is being monitored.

The surface area of the pond covered by the weed, $A\,\text{m}^2$, is modelled by the equation

$$\log_{10} A = 1 + 0.03t$$

where t is the number of weeks after monitoring began.

Use the equation of the model to answer parts (a) and (b).

(a) Find the surface area of the pond initially covered by the weed.

(1)

After T weeks, $25\,\text{m}^2$ of the pond is covered by the weed.

(b) Find the value of T, giving your answer to 2 decimal places.

(2)

WORKED SOLUTION

Question 2 · WMA13/01 January 2025

1 **Part (a)**

At $t = 0$, $\log_{10} A = 1$, hence $A = 10^1 = 10$ square metres.

$$\boxed{A(0) = 10\ \text{m}^2}$$

2 **Part (b)**

Substitute $A = 25$ into the model:

$$\log_{10} 25 = 1 + 0.03T \quad \Longrightarrow \quad T = \frac{\log_{10} 25 - 1}{0.03} = 13.2646669557\ldots$$

$$\boxed{T = 13.26\ \text{weeks}}$$

✓ **FINAL ANSWER**

(a) $A(0) = 10\ \text{m}^2$ (b) $T = 13.26\ \text{weeks}$

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Question 3

6 marks | Applications of Differentiation

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$y = \frac{4x+1}{(x+3)^2} \qquad x \neq -3 \qquad x \in \mathbb{R}$$

Use calculus to find the range of values of x for which y is increasing.

(6)

WORKED SOLUTION

Question 3 · WMA13/01 January 2025

1 Part (main)

Apply the quotient rule, retaining the restriction $x \neq -3$:

$$\frac{dy}{dx} = \frac{4(x+3)^2 - (4x+1)2(x+3)}{(x+3)^4} = \frac{(x+3)[4(x+3) - 2(4x+1)]}{(x+3)^4} = \frac{10-4x}{(x+3)^3}.$$

The numerator vanishes at $10 - 4x = 0$, so $x = 5/2$; the denominator changes sign at the excluded value $x = -3$. For $x < -3$, the numerator is positive and denominator negative, so $y' < 0$. For $-3 < x < 5/2$, both are positive, so $y' > 0$. For $x > 5/2$, the numerator is negative and denominator positive, so $y' < 0$. Thus the positive-gradient interval is as follows.

$$\boxed{-3 < x < \frac{5}{2}}$$

FINAL ANSWER

(main) $-3 < x < \frac{5}{2}$

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Question 4

9 marks | Integration

4. Given that

$$\frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4} \equiv Ax + B + \frac{Cx + D}{x^2 + 4}$$

(a) (i) find the values of the constants A, B and C

(ii) show that $D = 0$

(4)

(b) Hence, using algebraic integration, find

$$\int_1^4 \frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4}\,\mathrm{d}x$$

giving your answer in the form $p + q\ln 2$, where p and q are integers.

(5)

WORKED SOLUTION

Question 4 · WMA13/01 January 2025

1 **Part (a)(i)**

Multiply the identity by $x^2 + 4$:

$$4x^3 + 2x^2 + 3x + 8 = (Ax + B)(x^2 + 4) + Cx + D = Ax^3 + Bx^2 + (4A + C)x + 4B + D.$$

Comparing the cubic, quadratic and linear coefficients gives $A = 4$, $B = 2$ and $4A + C = 3$, hence $C = 3 - 16 = -13$.

$$\boxed{A = 4, \quad B = 2, \quad C = -13}$$

2 **Part (a)(ii)**

The x^2 coefficients give $B = 2$, while the constant terms give $4B + D = 8$. Substitute $B = 2$: $4(2) + D = 8$, so $D = 0$.

$$\boxed{D = 0}$$

3 **Part (b)**

Using the decomposition, the integrand is $4x + 2 - 13x/(x^2+4)$. Since the derivative of x^2+4 is $2x$, an antiderivative is

$$F(x) = 2x^2 + 2x - \frac{13}{2}\ln(x^2+4).$$

Therefore

$$\int_1^4 \frac{4x^3+2x^2+3x+8}{x^2+4}\,dx = \left[2x^2+2x-\frac{13}{2}\ln(x^2+4)\right]_1^4 = 40 - \frac{13}{2}\ln 20 - \left(4 - \frac{13}{2}\ln 5\right).$$

Combine the logarithms: $36 - \frac{13}{2}\ln 4 = 36 - 13\ln 2$.

$$\boxed{36 - 13\ln 2}$$

✓ **FINAL ANSWER**

(a)(i) $A = 4, \quad B = 2, \quad C = -13$ (a)(ii) $D = 0$ (b) $36 - 13\ln 2$

Question 5

9 marks | Log Graphs & Exponential Models

5. A hot piece of metal is cooled by dropping it into water. The temperature, H°C, of the metal, t minutes after it is dropped into the water, is modelled by the equation

$$H = 280e^{-0.05t} + 24 \qquad t \geqslant 0$$

Use the equation of the model to answer parts (a) to (d).

(a) Find the initial temperature of the piece of metal.

(1)

(b) On Diagram 1, sketch the graph of H against t. On your sketch, state the equation of the asymptote to the curve.

(2)

(c) Find the value of t for which $H = 144$, giving your answer to 2 decimal places.

(*Solutions based entirely on calculator technology are not acceptable.*)

(3)

(d) Show by differentiation that

$$\frac{dH}{dt} = a + bH$$

where a and b are constants to be found.

(3)

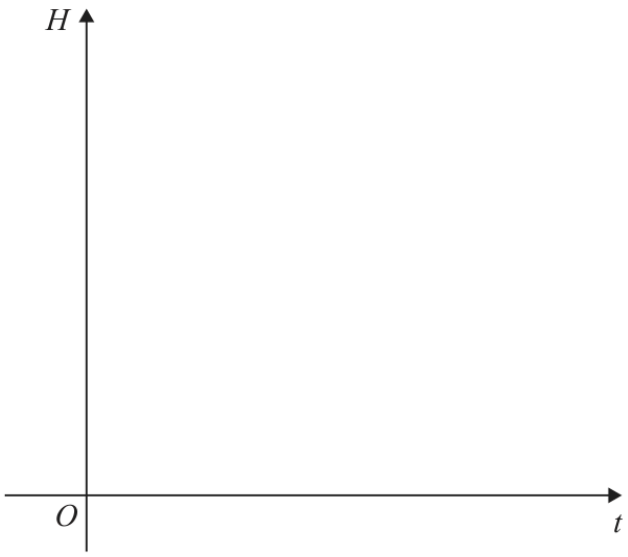


Diagram 1

1 Part (a)

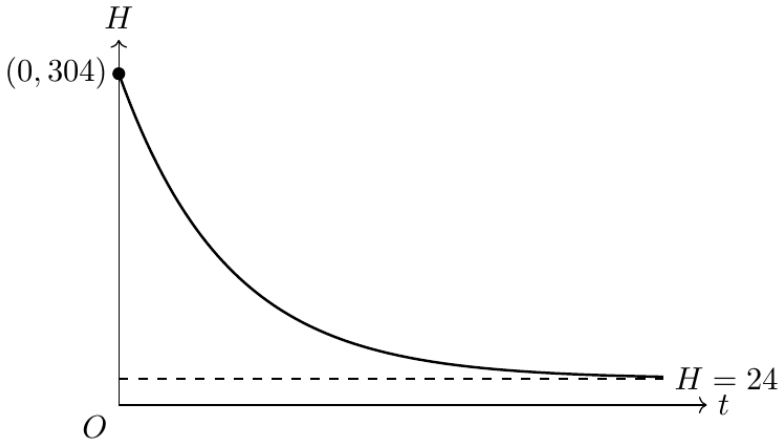
At $t = 0$, $e^{-0.05t} = 1$, so $H = 280 + 24 = 304$ degrees Celsius.

$$\boxed{H(0) = 304^\circ\text{C}}$$

2 Part (b)

The curve starts at $(0, 304)$ and decreases for $t \geq 0$. The exponential term stays positive and tends to zero, so the curve approaches $H = 24$ from above without crossing it.

$$\boxed{H = 24 \text{ is the horizontal asymptote}}$$



3 Part (c)

Substitute $H = 144$:

$$144 = 280e^{-0.05t} + 24 \implies e^{-0.05t} = \frac{120}{280} = \frac{3}{7}.$$

Take natural logarithms: $-0.05t = \ln(3/7)$. Hence
$t = -20\ln(3/7) = 20\ln(7/3) = 16.9459572077\ldots$ minutes.

$$\boxed{t = 16.95 \text{ minutes}}$$

4 Part (d)

Differentiate using the chain rule:

$$\frac{dH}{dt} = -0.05(280)e^{-0.05t} = -14e^{-0.05t}.$$

The original equation gives $280e^{-0.05t} = H - 24$, so

$$\frac{dH}{dt} = -0.05(H - 24) = 1.2 - 0.05H.$$

$$\boxed{a = 1.2, \quad b = -0.05}$$

FINAL ANSWER

(a) $H(0) = 304°\text{C}$ (b) $H = 24$ **is the horizontal asymptote** (c) $t = 16.95$ **minutes** (d) $a = 1.2, \quad b = -0.05$

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Question 6

8 marks | Functions, Composition & Inverses

6. The function f is defined by

$$\mathrm{f}(x) = \frac{4x + 3}{x - 2} \qquad x \neq 2$$

(a) Find f^{-1}

(3)

(b) Show that

$$\mathrm{ff}(x) = \frac{ax + b}{cx + d}$$

where a, b, c and d are integers to be found.

(3)

The point $P(3, 15)$ lies on the curve with equation $y = \mathrm{f}(x)$.

(c) Find the point to which P is mapped when $y = \mathrm{f}(x)$ is transformed to the curve with equation $y = 2\mathrm{f}(3x) + 8$

(2)

WORKED SOLUTION

Question 6 · WMA13/01 January 2025

1 **Part (a)**

Let $y = (4x + 3)/(x - 2)$. Then $yx - 2y = 4x + 3$, so

$$x(y - 4) = 2y + 3 \quad \Longrightarrow \quad x = \frac{2y + 3}{y - 4}.$$

Swap the input and output names. Since $f(x) = 4 + 11/(x - 2)$ can never equal 4, the inverse domain excludes 4.

$$\boxed{f^{-1}(x) = \frac{2x + 3}{x - 4}, \quad x \neq 4}$$

2 Part (b)

Substitute $f(x)$ into both occurrences of the argument:

$$ff(x) = \frac{4\left(\frac{4x+3}{x-2}\right)+3}{\left(\frac{4x+3}{x-2}\right)-2} = \frac{4(4x+3)+3(x-2)}{4x+3-2(x-2)} = \frac{19x+6}{2x+7}.$$

Thus $a=19$, $b=6$, $c=2$, $d=7$. The composite retains $x \neq 2$ from the inner function and also requires $f(x) \neq 2$, that is $x \neq -7/2$.

$$\boxed{ff(x) = \frac{19x+6}{2x+7}, \quad (a,b,c,d) = (19,6,2,7)}$$

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3 Part (c)

For $y = 2f(3x)+8$, the point mapping is $(x,y) \mapsto (x/3, 2y+8)$. Hence $(3,15) \mapsto (3/3, 2(15)+8) = (1,38)$.

$$\boxed{(1,38)}$$

FINAL ANSWER

(a) $f^{-1}(x) = \frac{2x+3}{x-4}, \quad x \neq 4$ (b) $ff(x) = \frac{19x+6}{2x+7}, \quad (a,b,c,d) = (19,6,2,7)$ (c) $(1,38)$

Question 7

8 marks | Modulus

7. Given that a and b are positive constants with $a > b$,

(a) sketch, on **separate** diagrams, the graph with equation

(i) $y = |3x - a|$

(ii) $y = |3x - a| - b$

Show on each sketch

- the coordinates of the minimum point on the graph
- the coordinates of the point at which the graph crosses the y-axis

(6)

(b) Solve the equation

$$|3x - a| - b = 5x$$

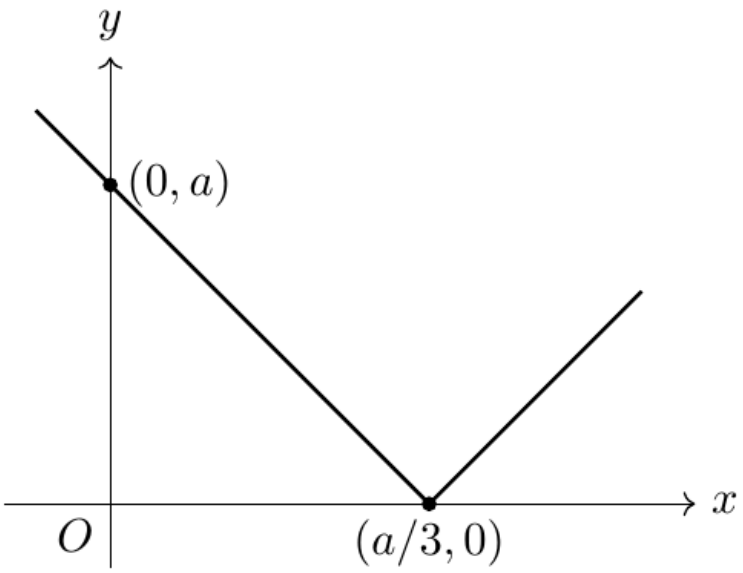
giving any solution for x in terms of a and b.

(2)

1 Part (a)(i)

For $x \leq a/3$, $y = a - 3x$; for $x \geq a/3$, $y = 3x - a$. These two straight branches form a V with minimum $(a/3, 0)$ and y-intercept $(0, a)$.

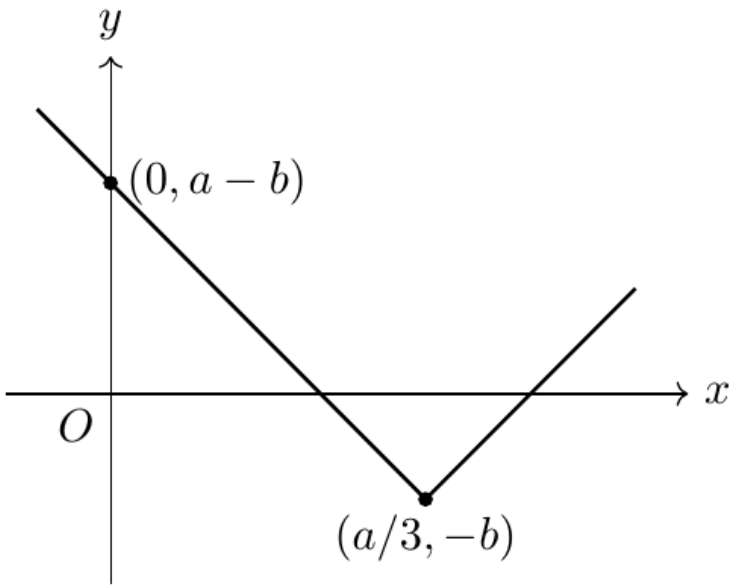
$$\boxed{\text{minimum } (a/3, 0), \quad y\text{-intercept } (0, a)}$$



2 Part (a)(ii)

Subtracting b translates the previous graph vertically down by b. The minimum becomes $(a/3, -b)$ and the y-intercept becomes $(0, a - b)$, which is above the x-axis because $a > b > 0$.

$$\boxed{\text{minimum } (a/3, -b), \quad y\text{-intercept } (0, a - b)}$$



3 Part (b)

For $x \leq a/3$, the equation becomes $a - 3x - b = 5x$, giving $x = (a-b)/8$. This is positive and less than $a/8 < a/3$, so it satisfies this branch condition. For $x \geq a/3$, the equation becomes $3x - a - b = 5x$, giving $x = -(a+b)/2 < 0$. This contradicts $x \geq a/3 > 0$ and must be rejected. Therefore only one solution remains.

$$\boxed{x = \frac{a-b}{8}}$$

FINAL ANSWER

(a)(i) $\textbf{minimum } (a/3, 0), \quad y\textbf{-intercept } (0, a)$ (a)(ii) $\textbf{minimum } (a/3, -b), \quad y\textbf{-intercept } (0, a-b)$ (b) $x = \frac{a-b}{8}$

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Question 8

9 marks | Trig Equations & R-Form

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta < \pi$

$$3\,\text{cosec}\,\theta = 8\cos\theta$$

giving your answers, in radians, to 3 significant figures.

(5)

(ii) Solve, for $0 < x < 180°$

$$\frac{\tan 2x - \tan 70°}{1 + \tan 2x \tan 70°} = -\frac{3}{8}$$

giving your answers, in degrees, to one decimal place.

(4)

WORKED SOLUTION

Question 8 · WMA13/01 January 2025

1 **Part (i)**

Use $\csc\theta = 1/\sin\theta$. Since $0 < \theta < \pi$, $\sin\theta \neq 0$ and

$$\frac{3}{\sin\theta} = 8\cos\theta \quad \Longrightarrow \quad 3 = 8\sin\theta\cos\theta = 4\sin 2\theta.$$

Thus $\sin 2\theta = 3/4$. Since $0 < 2\theta < 2\pi$, the two possibilities are

$$2\theta = \arcsin(3/4) \quad \text{or} \quad 2\theta = \pi - \arcsin(3/4).$$

They give $\theta = 0.4240310395\ldots$ and $1.1467652873\ldots$ radians.

$$\boxed{\theta = 0.424,\ 1.15 \text{ radians}}$$

2 Part (ii)

Apply the tangent subtraction identity:

$$\frac{\tan 2x - \tan 70^\circ}{1 + \tan 2x \tan 70^\circ} = \tan(2x - 70^\circ) = -\frac{3}{8}.$$

Set $u = 2x - 70^\circ$. The interval becomes $-70^\circ < u < 290^\circ$. With $\arctan(-3/8) = -20.5560452196\ldots^\circ$, the permitted values are $u = -20.5560452196\ldots^\circ$ and $159.4439547804\ldots^\circ$. Adding 70° and halving gives $x = 24.7219773902\ldots^\circ$ and $114.7219773902\ldots^\circ$. Both keep the original tangents and denominator defined.

$$\boxed{x = 24.7^\circ,\ 114.7^\circ}$$

FINAL ANSWER

(i) $\theta = 0.424,\ 1.15$ radians (ii) $x = 24.7^\circ,\ 114.7^\circ$

Question 9

8 marks | Applications of Differentiation

9.

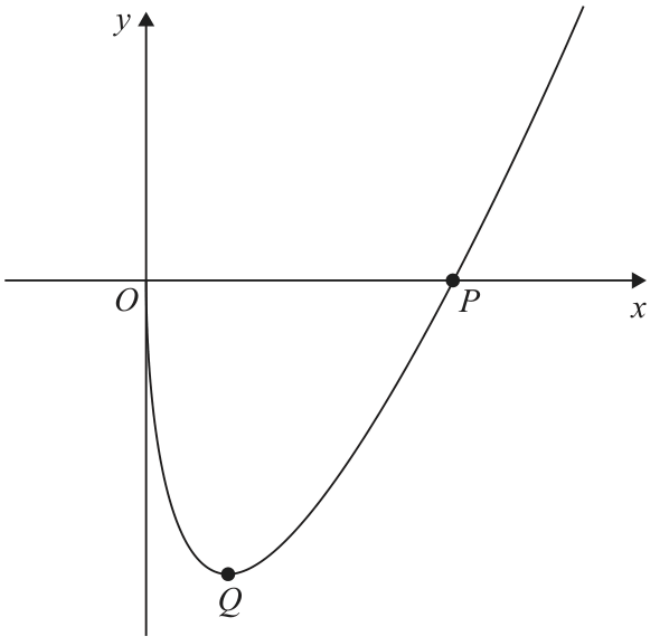


Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 6\sqrt{x}\ln(4x) \qquad x > 0$$

The curve cuts the x-axis at point P

(a) State the x coordinate of P

(1)

The point Q, shown in Figure 1, is the stationary point on C

(b) Use calculus to find the exact coordinates of Q

(5)

(c) Hence find the range of the function $\mathrm{g}(x)$ where

$$\mathrm{g}(x) = -2\,\mathrm{f}(x)$$

(2)

1 Part (a)

Since $x > 0$, $6\sqrt{x}$ is positive. Therefore $f(x) = 0$ requires $\ln(4x) = 0$, so $4x = 1$ and $x = 1/4$.

$$\boxed{x_P = \frac{1}{4}}$$

2 Part (b)

Apply the product and chain rules:

$$f'(x) = \frac{3}{\sqrt{x}}\ln(4x) + 6\sqrt{x}\frac{1}{x} = \frac{3}{\sqrt{x}}[\ln(4x) + 2].$$

At Q, set $f'(x) = 0$. Since $3/\sqrt{x} > 0$,

$$\ln(4x) + 2 = 0 \quad \Longrightarrow \quad 4x = e^{-2} \quad \Longrightarrow \quad x = \frac{1}{4e^2}.$$

Then $\sqrt{x} = 1/(2e)$ and $\ln(4x) = -2$, giving $f(x) = 6(1/(2e))(-2) = -6/e$.

$$\boxed{Q = \left(\frac{1}{4e^2}, -\frac{6}{e}\right)}$$

3 Part (c)

The factor $3/\sqrt{x}$ is positive and $\ln(4x) + 2$ increases through zero at Q, so f' changes from negative to positive: Q is the global minimum. As $x \to \infty$, $f(x) \to \infty$, hence the range of f is $[-6/e, \infty)$. Multiplication by -2 reverses the inequality and includes the maximum $12/e$.

$$\boxed{g(x) \leq \frac{12}{e}}$$

FINAL ANSWER

(a) $x_P = \frac{1}{4}$ (b) $Q = \left(\frac{1}{4e^2}, -\frac{6}{e}\right)$ (c) $g(x) \leq \frac{12}{e}$

Question 10

9 marks | Differentiation Techniques

10.

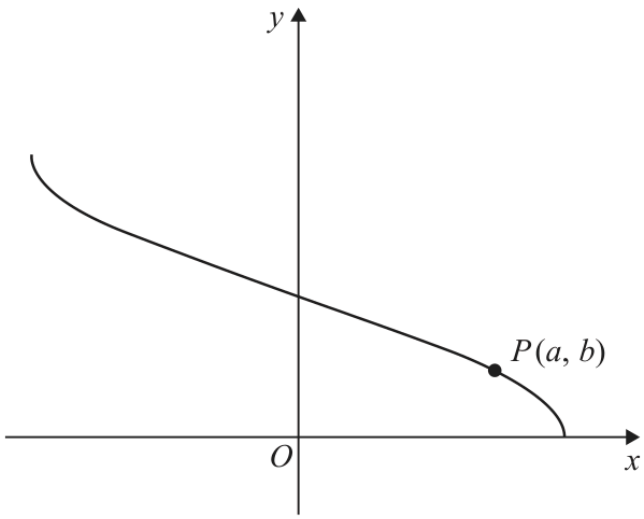


Figure 2

Figure 2 shows a sketch of the curve with equation

$$x = 3\cos 2y \qquad -3 \leqslant x \leqslant 3 \qquad 0 \leqslant y \leqslant \frac{\pi}{2}$$

(a) Find $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y.

(2)

(b) Hence show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{\sqrt{9 - x^2}}$$

where k is a constant to be found.

(3)

The point $P(a, b)$ lies on the curve and is shown in Figure 2.

Given that

- the gradient of the curve at P is $-\dfrac{1}{4}$
- both a and b are positive

(c) find the exact values of a and b.

(4)

1 **Part (a)**

Differentiate $x = 3\cos 2y$ with respect to y using the chain rule:

$$\frac{dx}{dy} = 3(-\sin 2y)\cdot 2 = -6\sin 2y.$$

$$\boxed{\frac{dx}{dy} = -6\sin 2y}$$

2 **Part (b)**

For $0 < y < \pi/2$, $\sin 2y > 0$ and

$$\frac{dy}{dx} = \frac{1}{dx/dy} = -\frac{1}{6\sin 2y}.$$

Since $\cos 2y = x/3$ and the sine is positive on this interval,

$$\sin 2y = \sqrt{1-\cos^2 2y} = \sqrt{1 - x^2/9} = \frac{\sqrt{9-x^2}}{3}.$$

Consequently $dy/dx = -1/(2\sqrt{9-x^2})$, valid for $-3 < x < 3$. The endpoint tangents are vertical; there is no finite derivative there.

$$\boxed{\frac{dy}{dx} = -\frac{1}{2\sqrt{9-x^2}}, \qquad k = -\frac{1}{2}}$$

3 **Part (c)**

Set the gradient equal to $-1/4$:

$$-\frac{1}{2\sqrt{9-a^2}} = -\frac{1}{4} \quad\Longrightarrow\quad \sqrt{9-a^2} = 2 \quad\Longrightarrow\quad a^2 = 5.$$

Because $a > 0$, take $a = \sqrt{5}$. Substitute into $a = 3\cos 2b$:

$$\cos 2b = \frac{\sqrt{5}}{3} \quad\Longrightarrow\quad b = \frac{1}{2}\arccos\left(\frac{\sqrt{5}}{3}\right).$$

Here $0 < 2b < \pi/2$ since both coordinates are positive, so the principal arccos value is the required one.

$$\boxed{a = \sqrt{5}, \qquad b = \frac{1}{2}\arccos\left(\frac{\sqrt{5}}{3}\right)}$$

FINAL ANSWER

(a) $\frac{dx}{dy} = -6\sin 2y$ (b) $\frac{dy}{dx} = -\frac{1}{2\sqrt{9-x^2}}, \qquad k = -\frac{1}{2}$ (c) $a = \sqrt{5}, \qquad b = \frac{1}{2}\arccos\left(\frac{\sqrt{5}}{3}\right)$