Elite Mathematics
Edexcel International A Level

IAL PURE 3 · WMA13

WMA13/01 January 2020 Worked Solutions

January 2020 paper rebuilt with the worked solution after each question.

9	2020–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA13/01 January 2020

January 2020

9	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

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Question 1

6 marks | Log Graphs & Exponential Models

1. A population of a rare species of toad is being studied.

The number of toads, N, in the population, t years after the start of the study, is modelled by the equation

$$N = \frac{900e^{0.12t}}{2e^{0.12t} + 1} \qquad t \geqslant 0, t \in \mathbb{R}$$

According to this model,

(a) calculate the number of toads in the population at the start of the study,

(1)

(b) find the value of t when there are 420 toads in the population, giving your answer to 2 decimal places.

(4)

(c) Explain why, according to this model, the number of toads in the population can never reach 500

(1)

WORKED SOLUTION

Question 1 · WMA13/01 January 2020

1 **Part (a)**

At the start of the study, $t = 0$, so

$$N(0) = \frac{900e^0}{2e^0 + 1} = \frac{900}{3} = \boxed{300}.$$

There are initially 300 toads.

2 Part (b)

Set $N = 420$ and clear the denominator:

$$\begin{aligned} 420 &= \frac{900e^{0.12t}}{2e^{0.12t}+1}, \\ 420(2e^{0.12t}+1) &= 900e^{0.12t}, \\ 420 &= 60e^{0.12t}, \\ e^{0.12t} &= 7. \end{aligned}$$

Taking natural logarithms gives

$$0.12t = \ln 7 \quad \Longrightarrow \quad t = \frac{\ln 7}{0.12} = 16.2159179\ldots.$$

Thus $\boxed{t = 16.22 \text{ years}}$ to 2 decimal places.

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3 Part (c)

Divide the numerator and denominator by $e^{0.12t}$:

$$N = \frac{900}{2 + e^{-0.12t}}.$$

Since $e^{-0.12t} > 0$ for every finite $t \geq 0$, the denominator is greater than 2. Hence

$$N < \frac{900}{2} = 450 < 500.$$

The population approaches 450 as t increases without bound, so it can never reach 500 according to this model.

FINAL ANSWER

(a) 300 (b) $t = 16.22 \text{ years}$ (c) $N < 450 < 500, so the population can never reach 500$

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Question 2

8 marks | Functions, Composition & Inverses

2. The function f and the function g are defined by

$$f(x) = \frac{12}{x+1} \qquad x > 0, x \in \mathbb{R}$$

$$g(x) = \frac{5}{2}\ln x \qquad x > 0, x \in \mathbb{R}$$

(a) Find, in simplest form, the value of $fg(e^2)$

(2)

(b) Find f^{-1}

(3)

(c) Hence, or otherwise, find all real solutions of the equation

$$f^{-1}(x) = f(x)$$

(3)

WORKED SOLUTION

Question 2 · WMA13/01 January 2020

1 **Part (a)**

First evaluate the inner function:

$$g(e^2) = \frac{5}{2}\ln(e^2) = \frac{5}{2} \times 2 = 5.$$

Then apply f to this result:

$$\boxed{fg(e^2) = f(5) = \frac{12}{5+1} = 2.}$$

2 Part (b)

Put $y = f(x)$ and rearrange to make x the subject:

$$y = \frac{12}{x+1} \quad \Longrightarrow \quad y(x+1) = 12 \quad \Longrightarrow \quad x = \frac{12}{y} - 1.$$

For $x > 0$, the denominator $x + 1 > 1$, so $0 < f(x) < 12$. Conversely, every y with $0 < y < 12$ gives $x = 12/y - 1 > 0$. Thus the range of f, and hence the domain of its inverse, is $(0, 12)$. Interchanging x and y gives

$$\boxed{f^{-1}(x) = \frac{12}{x} - 1, \qquad 0 < x < 12.}$$

3 Part (c)

Both functions must be defined, so $0 < x < 12$. Substitute the formula for the inverse and multiply by the nonzero quantity $x(x+1)$:

$$\begin{aligned} \frac{12}{x} - 1 &= \frac{12}{x+1}, \\ (12 - x)(x+1) &= 12x, \\ 12 + 11x - x^2 &= 12x, \\ x^2 + x - 12 &= 0, \\ (x+4)(x-3) &= 0. \end{aligned}$$

The algebraic candidates are $x = -4$ and $x = 3$. Reject $x = -4$ because it lies outside $0 < x < 12$. For $x = 3$, $f^{-1}(3) = 12/3 - 1 = 3 = f(3)$, so the only real solution is

$$\boxed{x = 3.}$$

FINAL ANSWER

(a) $fg(e^2) = f(5) = \frac{12}{5+1} = 2$ (b) $f^{-1}(x) = \frac{12}{x} - 1, \qquad 0 < x < 12$ (c) $x = 3$

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Question 3

5 marks | Log Graphs & Exponential Models

3.

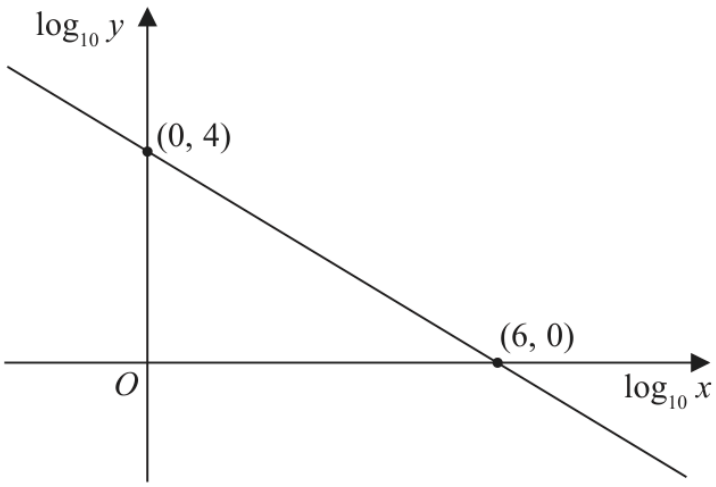


Figure 1

Figure 1 shows a linear relationship between $\log_{10} y$ and $\log_{10} x$

The line passes through the points (0, 4) and (6, 0) as shown.

(a) Find an equation linking $\log_{10} y$ with $\log_{10} x$

(2)

(b) Hence, or otherwise, express y in the form px^q, where p and q are constants to be found.

(3)

WORKED SOLUTION

Question 3 · WMA13/01 January 2020

Part (a)

The horizontal coordinate is $\log_{10} x$ and the vertical coordinate is $\log_{10} y$. The gradient of the line is

$$m = \frac{0-4}{6-0} = -\frac{2}{3}.$$

The vertical intercept is 4, so the required equation is

$$\boxed{\log_{10} y = 4 - \frac{2}{3}\log_{10} x}.$$

2 **Part (b)**

Taking powers of 10 on both sides gives

$$\begin{aligned} y &= 10^{4-\frac{2}{3}\log_{10} x} \\ &= 10^{4}\left(10^{\log_{10} x}\right)^{-2/3} \\ &= 10000x^{-2/3}. \end{aligned}$$

Here $x > 0$, as required for $\log_{10} x$ to exist. Therefore

$$\boxed{y = 10000x^{-2/3}}, \qquad \boxed{p = 10000, \quad q = -\frac{2}{3}}.$$

FINAL ANSWER

(a) $\log_{10} y = 4 - \frac{2}{3}\log_{10} x$ (b) $y = 10000x^{-2/3}$, $p = 10000, \quad q = -\frac{2}{3}$

WMA13/01 JANUARY 2020

Question 4

11 marks | Differentiation Techniques

4. (i) $$\mathrm{f}(x) = \frac{(2x+5)^2}{x-3} \qquad x \neq 3$$

(a) Find $\mathrm{f}'(x)$ in the form $\frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are fully factorised quadratic expressions.

(b) Hence find the range of values of x for which $\mathrm{f}(x)$ is increasing.

(6)

(ii) $$\mathrm{g}(x) = x\sqrt{\sin 4x} \qquad 0 \leqslant x < \frac{\pi}{4}$$

The curve with equation $y = \mathrm{g}(x)$ has a maximum at the point M.

Show that the x coordinate of M satisfies the equation

$$\tan 4x + kx = 0$$

where k is a constant to be found.

(5)

WORKED SOLUTION

Question 4 · WMA13/01 January 2020

1 Part (i)(a)

Using the quotient rule and the chain rule,

$$\begin{aligned} f'(x) &= \frac{(x-3)\,4(2x+5) - (2x+5)^2}{(x-3)^2} \\ &= \frac{(2x+5)\{4(x-3) - (2x+5)\}}{(x-3)^2} \\ &= \boxed{\frac{(2x+5)(2x-17)}{(x-3)^2}}, \qquad x \neq 3. \end{aligned}$$

Thus $P(x) = (2x+5)(2x-17)$ and $Q(x) = (x-3)^2$ are fully factorised quadratics.

2 **Part (i)(b)**

For $x \neq 3$, the denominator $(x-3)^2$ is positive, so the sign of $f'(x)$ is the sign of $(2x+5)(2x-17)$. For an increasing function we require

$$(2x+5)(2x-17) > 0.$$

Both factors are negative when $x < -\frac{5}{2}$, and both are positive when $x > \frac{17}{2}$. Therefore the required values are

$$\boxed{x < -\frac{5}{2} \quad \text{or} \quad x > \frac{17}{2}}.$$

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3 **Part (ii)**

For $0 < x < \frac{\pi}{4}$, we have $\sin 4x > 0$. Applying the product rule and the chain rule gives

$$\begin{aligned} g'(x) &= \sqrt{\sin 4x} + x\left(\frac{1}{2\sqrt{\sin 4x}}\right) 4\cos 4x \\ &= \frac{\sin 4x + 2x\cos 4x}{\sqrt{\sin 4x}}. \end{aligned}$$

Since $g(0) = 0$ while $g(x) > 0$ in the interior, the stated maximum M does not occur at $x = 0$. Its coordinate therefore lies in $0 < x < \frac{\pi}{4}$, where g is differentiable, so $g'(x) = 0$ at M. Consequently,

$$\sin 4x + 2x\cos 4x = 0.$$

At such a point $\cos 4x \neq 0$: the only interior zero of $\cos 4x$ is $x = \frac{\pi}{8}$, where the left-hand side above is 1, not 0. Dividing by $\cos 4x$ therefore gives

$$\boxed{\tan 4x + 2x = 0}, \qquad \boxed{k = 2}.$$

FINAL ANSWER

(i)(a) $\frac{(2x+5)(2x-17)}{(x-3)^2}$ (i)(b) $x < -\frac{5}{2}$ or $x > \frac{17}{2}$ (ii) $\tan 4x + 2x = 0$, $k = 2$

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Question 5

8 marks | Trig Identities & Exact Values

5. (a) Use the substitution $t = \tan x$ to show that the equation

$$12\tan 2x + 5\cot x \sec^2 x = 0$$

can be written in the form

$$5t^4 - 24t^2 - 5 = 0$$

(4)

(b) Hence solve, for $0 \leqslant x < 360°$, the equation

$$12\tan 2x + 5\cot x \sec^2 x = 0$$

Show each stage of your working and give your answers to one decimal place.

(4)

WORKED SOLUTION

Question 5 · WMA13/01 January 2020

1 **Part (a)**

Let $t = \tan x$. For the original equation to be defined, $\sin x \neq 0$, $\cos x \neq 0$ and $\cos 2x \neq 0$; hence $t \neq 0$ and $t^2 \neq 1$. Using the double-angle and reciprocal identities,

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2t}{1-t^2}, \qquad \cot x = \frac{1}{t}, \qquad \sec^2 x = 1 + t^2.$$

Substituting into the equation gives

$$\frac{24t}{1-t^2} + \frac{5(1+t^2)}{t} = 0.$$

Multiply by the nonzero quantity $t(1-t^2)$ and expand:

$$24t^2 + 5(1+t^2)(1-t^2) = 0,$$
$$24t^2 + 5 - 5t^4 = 0.$$

Therefore

$$\boxed{5t^4 - 24t^2 - 5 = 0.}$$

2 Part (b)

Factor the quadratic in t^2:

$$(5t^2+1)(t^2-5)=0 \quad\Longrightarrow\quad t^2=-\frac{1}{5} \quad\text{or}\quad t^2=5.$$

Since t is real, $t^2\geq 0$, so discard $t^2=-1/5$. Thus

$$\tan x = t = \pm\sqrt{5}.$$

In degrees, let

$$\alpha=\tan^{-1}(\sqrt{5})=65.905157\ldots^\circ.$$

Tangent is positive in quadrants I and III and negative in quadrants II and IV. The four solutions in $0\leq x<360^\circ$ are therefore

$$x=\alpha,\quad 180^\circ-\alpha,\quad 180^\circ+\alpha,\quad 360^\circ-\alpha.$$

These have finite, nonzero tangent with $t^2=5\neq 1$, so every transformation in part (a) is reversible and all four satisfy the original equation. To one decimal place,

$$\boxed{x=65.9^\circ,\quad 114.1^\circ,\quad 245.9^\circ,\quad 294.1^\circ.}$$

FINAL ANSWER

(a) $5t^4-24t^2-5=0$ (b) $x=65.9^\circ,\quad 114.1^\circ,\quad 245.9^\circ,\quad 294.1^\circ$

Question 6

9 marks | Modulus

6.

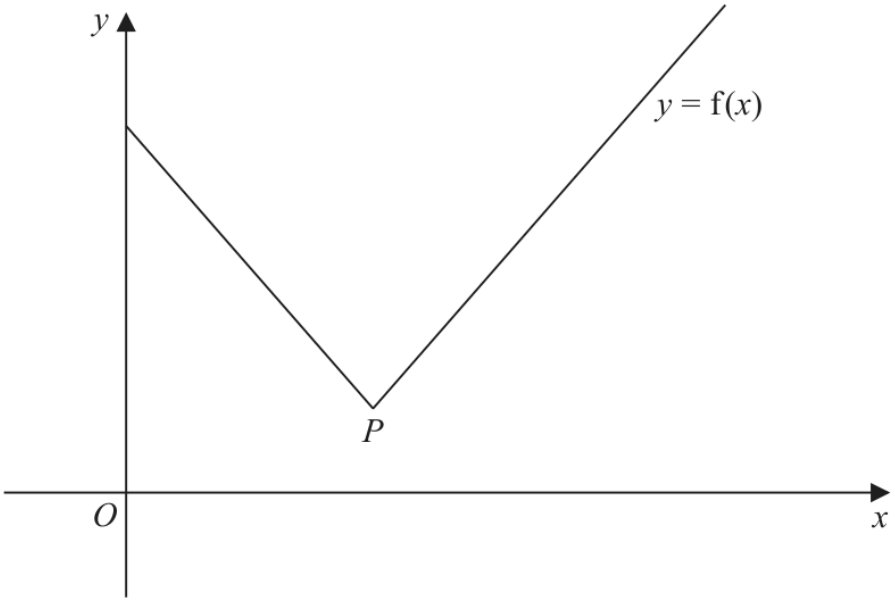


Figure 2

Figure 2 shows part of the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2|2x - 5| + 3 \qquad x \geqslant 0$$

The vertex of the graph is at point P as shown.

(a) State the coordinates of P.

(2)

(b) Solve the equation $\mathrm{f}(x) = 3x - 2$

(4)

Given that the equation

$$\mathrm{f}(x) = kx + 2$$

where k is a constant, has exactly two roots,

(c) find the range of values of k.

(3)

1 Part (a)

At the vertex, the expression inside the modulus is zero:

$$2x - 5 = 0 \quad \Longrightarrow \quad x = \frac{5}{2}, \qquad f\left(\frac{5}{2}\right) = 3.$$

Hence $\boxed{P = \left(\frac{5}{2}, 3\right)}$.

2 Part (b)

Write the function in its two branches:

$$f(x) = \begin{cases} 13 - 4x, & 0 \le x \le \frac{5}{2}, \\ 4x - 7, & x \ge \frac{5}{2}. \end{cases}$$

On the left branch,

$$13 - 4x = 3x - 2 \quad \Longrightarrow \quad 7x = 15 \quad \Longrightarrow \quad x = \frac{15}{7}.$$

This lies in $0 \le x \le \frac{5}{2}$, so it is valid. On the right branch,

$$4x - 7 = 3x - 2 \quad \Longrightarrow \quad x = 5.$$

This satisfies $x \ge \frac{5}{2}$, so it is also valid. Thus

$$\boxed{x = \frac{15}{7} \text{ or } x = 5}.$$

3

Part (c)

Each straight branch can have at most one intersection with $y = kx + 2$. Two distinct roots therefore require one intersection strictly to each side of the vertex.

For the left branch,

$$13 - 4x = kx + 2 \quad \Longrightarrow \quad x_L = \frac{11}{k+4}.$$

The condition $0 \le x_L < \frac{5}{2}$ first requires $k + 4 > 0$, then gives

$$\frac{11}{k+4} < \frac{5}{2} \quad \Longrightarrow \quad 22 < 5(k+4) \quad \Longrightarrow \quad k > \frac{2}{5}.$$

For the right branch,

$$4x - 7 = kx + 2 \quad \Longrightarrow \quad x_R = \frac{9}{4-k}.$$

The condition $x_R > \frac{5}{2}$ requires $k < 4$, and then

$$\frac{9}{4-k} > \frac{5}{2} \quad \Longrightarrow \quad 18 > 5(4-k) \quad \Longrightarrow \quad k > \frac{2}{5}.$$

At $k = \frac{2}{5}$, both intersections coincide at the vertex, giving only one root. At $k = 4$, the line is parallel to the right branch and has no intersection with that branch, again leaving only one root. Hence

$$\boxed{\frac{2}{5} < k < 4}.$$

✓ **FINAL ANSWER**

(a) $P = \left(\frac{5}{2}, 3\right)$ (b) $x = \frac{15}{7}$ or $x = 5$ (c) $\frac{2}{5} < k < 4$

Question 7

11 marks | Applications of Differentiation

7.

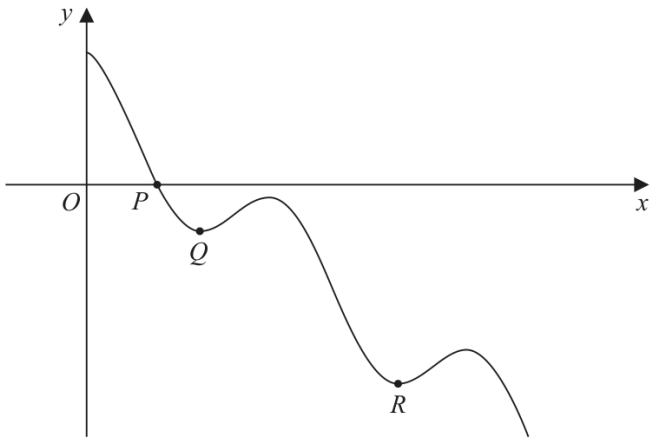


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2\cos 3x - 3x + 4 \qquad x > 0$$

where x is measured in radians.

The curve crosses the x-axis at the point P, as shown in Figure 3.

Given that the x coordinate of P is α,

(a) show that α lies between 0.8 and 0.9

(2)

The iteration formula

$$x_{n+1} = \frac{1}{3}\arccos(1.5x_n - 2)$$

can be used to find an approximate value for α.

(b) Using this iteration formula with $x_1 = 0.8$ find, to 4 decimal places, the value of

(i) x_2

(ii) x_5

(3)

The point Q and the point R are local minimum points on the curve, as shown in Figure 3.

Given that the x coordinates of Q and R are β and λ respectively, and that they are the two smallest values of x at which local minima occur,

(c) find, using calculus, the exact value of β and the exact value of λ.

(6)

1 Part (a)

Let $f(x) = 2\cos 3x - 3x + 4$, using radians. Then

$$f(0.8) = 2\cos 2.4 - 2.4 + 4 = 0.1252125\ldots > 0,$$
$$f(0.9) = 2\cos 2.7 - 2.7 + 4 = -0.5081443\ldots < 0.$$

The function is continuous on $[0.8, 0.9]$, so this change of sign shows that there is a root between the endpoints. Moreover, $f'(x) = -6\sin 3x - 3 < 0$ on this interval, so this root is unique there. Thus the crossing shown at P has $\boxed{0.8 < \alpha < 0.9}$.

2 Part (b)(i)

Using the inverse cosine in radians,

$$x_2 = \frac{1}{3}\arccos(1.5(0.8) - 2) = \frac{1}{3}\arccos(-0.8) = 0.8326971\ldots.$$

Hence $\boxed{x_2 = 0.8327}$ to 4 decimal places.

3 Part (b)(ii)

Continue the iteration, retaining the full calculator values at each step:

$$x_3 = \frac{1}{3}\arccos(1.5x_2 - 2) = 0.8067674\ldots,$$
$$x_4 = \frac{1}{3}\arccos(1.5x_3 - 2) = 0.8271196\ldots,$$
$$x_5 = \frac{1}{3}\arccos(1.5x_4 - 2) = 0.8110216\ldots.$$

Therefore $\boxed{x_5 = 0.8110}$ to 4 decimal places.

4 Part (c)

Differentiate the curve equation:

$$\frac{dy}{dx} = -6\sin 3x - 3, \qquad \frac{d^2y}{dx^2} = -18\cos 3x.$$

At a stationary point,

$$-6\sin 3x - 3 = 0 \quad \Longrightarrow \quad \sin 3x = -\frac{1}{2}.$$

For $x > 0$, the stationary points therefore belong to the two families

$$3x = \frac{7\pi}{6} + 2n\pi \quad \text{or} \quad 3x = \frac{11\pi}{6} + 2n\pi, \qquad n = 0, 1, 2, \ldots.$$

In the first family, $\cos 3x = -\frac{\sqrt{3}}{2}$, giving $y'' = 9\sqrt{3} > 0$, so these are local minima. In the second family, $\cos 3x = \frac{\sqrt{3}}{2}$ and $y'' = -9\sqrt{3} < 0$, so these are local maxima and are excluded. Thus all positive minimum coordinates are

$$x = \frac{7\pi}{18} + \frac{2n\pi}{3}, \qquad n = 0, 1, 2, \ldots.$$

The two smallest are obtained with $n = 0$ and $n = 1$:

$$\boxed{\beta = \frac{7\pi}{18}}, \qquad \boxed{\lambda = \frac{19\pi}{18}}.$$

FINAL ANSWER

(a) $0.8 < \alpha < 0.9$ (b)(i) $x_2 = 0.8327$ (b)(ii) $x_5 = 0.8110$ (c) $\beta = \frac{7\pi}{18}$, $\lambda = \frac{19\pi}{18}$

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Question 8

10 marks | Integration

8. (i) Find, using algebraic integration, the exact value of

$$\int_3^{42} \frac{2}{3x-1}\,\mathrm{d}x$$

giving your answer in simplest form.

(4)

(ii) $$\mathrm{h}(x) = \frac{2x^3 - 7x^2 + 8x + 1}{(x-1)^2} \qquad x > 1$$

Given $\mathrm{h}(x) = Ax + B + \frac{C}{(x-1)^2}$ where A, B and C are constants to be found, find

$$\int \mathrm{h}(x)\,\mathrm{d}x$$

(6)

WORKED SOLUTION

Question 8 · WMA13/01 January 2020

1 **Part (i)**

Since $3x - 1 > 0$ throughout $3 \le x \le 42$, an antiderivative is

$$\int \frac{2}{3x-1}\,dx = \frac{2}{3}\ln(3x-1) + K.$$

The factor $1/3$ compensates for the derivative of $3x - 1$. Thus

$$\begin{aligned}\int_3^{42} \frac{2}{3x-1}\,dx &= \left[\frac{2}{3}\ln(3x-1)\right]_3^{42} \\ &= \frac{2}{3}\left(\ln 125 - \ln 8\right) \\ &= \frac{2}{3}\ln\left(\frac{125}{8}\right) = \frac{2}{3}\ln\left(\left(\frac{5}{2}\right)^3\right).\end{aligned}$$

Using $\ln(a^3) = 3\ln a$ for $a > 0$, the exact value in simplest form is

$$\boxed{2\ln\left(\frac{5}{2}\right).}$$

2 **Part (ii)**

Multiply the given decomposition by $(x-1)^2$:

$$\begin{aligned} 2x^3 - 7x^2 + 8x + 1 &= (Ax + B)(x-1)^2 + C \\ &= Ax^3 + (B - 2A)x^2 + (A - 2B)x + (B + C). \end{aligned}$$

Comparing coefficients gives

$$A = 2, \qquad B - 2A = -7 \implies B = -3, \qquad B + C = 1 \implies C = 4.$$

The coefficient of x also agrees, since $A - 2B = 2 - 2(-3) = 8$. Hence

$$\boxed{A = 2, \qquad B = -3, \qquad C = 4,} \qquad h(x) = 2x - 3 + 4(x-1)^{-2}.$$

Integrating each term,

$$\begin{aligned} \int h(x)\,dx &= \int \left(2x - 3 + 4(x-1)^{-2}\right) dx \\ &= x^2 - 3x - 4(x-1)^{-1} + K. \end{aligned}$$

Therefore, for $x > 1$,

$$\boxed{\int h(x)\,dx = x^2 - 3x - \frac{4}{x-1} + K,}$$

where K is an arbitrary constant of integration, distinct from the coefficient $C = 4$.

FINAL ANSWER

(i) $2\ln\left(\frac{5}{2}\right)$ (ii) $A = 2, \qquad B = -3, \qquad C = 4$, $\int h(x)\,dx = x^2 - 3x - \frac{4}{x-1} + K$

Question 9

7 marks | Trig Equations & R-Form

9. $$f(\theta) = 5\cos\theta - 4\sin\theta \qquad \theta \in \mathbb{R}$$

(a) Express $f(\theta)$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of R and give the value of α, in radians, to 3 decimal places.

(3)

The curve with equation $y = \cos\theta$ is transformed onto the curve with equation $y = f(\theta)$ by a sequence of two transformations.

Given that the first transformation is a stretch and the second a translation,

(b) (i) describe fully the transformation that is a stretch,

(ii) describe fully the transformation that is a translation.

(2)

Given

$$g(\theta) = \frac{90}{4 + (f(\theta))^2} \qquad \theta \in \mathbb{R}$$

(c) find the range of g.

(2)

1 Part (a)

Expanding the required form gives

$$R\cos(\theta+\alpha) = R\cos\alpha\cos\theta - R\sin\alpha\sin\theta.$$

Comparing coefficients with $5\cos\theta - 4\sin\theta$,

$$R\cos\alpha = 5, \qquad R\sin\alpha = 4.$$

Squaring and adding, using $R > 0$, gives

$$R^2 = 5^2 + 4^2 = 41 \quad \Longrightarrow \quad \boxed{R = \sqrt{41}}.$$

Also, since $0 < \alpha < \frac{\pi}{2}$,

$$\tan\alpha = \frac{4}{5}, \qquad \alpha = \arctan\left(\frac{4}{5}\right) = 0.6747409\ldots \quad \Longrightarrow \quad \boxed{\alpha = 0.675 \text{ radians (3 d.p.)}}.$$

Thus the exact form is

$$\boxed{f(\theta) = \sqrt{41}\cos\left(\theta + \arctan\frac{4}{5}\right)}.$$

2 Part (b)(i)

A stretch parallel to the y-axis with scale factor $\boxed{\sqrt{41}}$, taking $y = \cos\theta$ to $y = \sqrt{41}\cos\theta$.

3 Part (b)(ii)

A translation by the vector

$$\boxed{\begin{pmatrix} -\alpha \\ 0 \end{pmatrix}} = \begin{pmatrix} -\arctan(4/5) \\ 0 \end{pmatrix} \approx \begin{pmatrix} -0.675 \\ 0 \end{pmatrix}.$$

This moves the curve α units to the left, giving $y = \sqrt{41}\cos(\theta+\alpha)$.

4

Part (c)

Since θ can be any real number,

$$-\sqrt{41} \le f(\theta) \le \sqrt{41} \quad \Longrightarrow \quad 0 \le (f(\theta))^2 \le 41.$$

Hence $4 \le 4 + (f(\theta))^2 \le 45$. Taking reciprocals of these positive bounds and multiplying by 90 gives

$$\boxed{2 \le g(\theta) \le \frac{45}{2}}.$$

Both endpoints are attained: the minimum occurs when $f(\theta) = \pm\sqrt{41}$, and the maximum when $f(\theta) = 0$.

FINAL ANSWER

(a) $R = \sqrt{41}$, $\alpha = 0.675$ radians (3 d.p.), $f(\theta) = \sqrt{41}\cos\left(\theta + \arctan\frac{4}{5}\right)$ (b)(i) $\sqrt{41}$ (b)(ii) $\begin{pmatrix} -\alpha \\ 0 \end{pmatrix}$ (c) $2 \le g(\theta) \le \frac{45}{2}$