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Elite Mathematics
Edexcel International A Level

IAL PURE 2 · WMA12

WMA12 Pure 2 Expertise Questions

Questions 6 and above, grouped by topic with cross-topic placements where useful.

163	2019–2026	Questions
topic placements	19 exam sessions	checked by Dr Eslam



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TOPIC

Proof

Every past-paper question on this topic, October 2019 to January 2026.

Question 9

4 marks | Proof

9. (a) Prove that for all positive values of x and y,

$$\frac{x+y}{2} \geqslant \sqrt{xy}$$

(3)

(b) Prove by counter-example that this inequality does not hold when x and y are both negative.

(1)

Question 10

5 marks | Proof

10. (i) Prove by counter example that the statement

"if p is a prime number then $2p + 1$ is also a prime number"

is not true.

(1)

(ii) Use proof by exhaustion to prove that if n is an integer then

$$5n^2 + n + 12$$

is always even.

(4)

(Total 5 marks)

Question 10

4 marks | Proof

10. A student was asked to prove by exhaustion that

if n is an integer then $2n^2 + n + 1$ is **not** divisible by 3

The start of the student's proof is shown in the box below.

Consider the case when $n = 3k$ $2n^2 + n + 1 = 18k^2 + 3k + 1 = 3(6k^2 + k) + 1$ which is not divisible by 3

Complete this proof.

(4)

Question 8

5 marks | Proof

8. (i) A student writes the following statement:

"When a and b are consecutive **prime** numbers, $a^2 + b^2$ is never a multiple of 10"

Prove by counter example that this statement is **not** true.

(2)

(ii) Given that x and y are even integers greater than 0 and less than 6, prove by exhaustion, that

$$1 < x^2 - \frac{xy}{4} < 15$$

(3)

Question 8

6 marks | Proof

8. (i) Use a counter example to show that the following statement is **false**

"$n^2 + 3n + 1$ is prime for all $n \in \mathbb{N}$"

(2)

(ii) Use algebra to prove by exhaustion that for all $n \in \mathbb{N}$

"$n^2 - 2$ is **not** a multiple of 4"

(4)

Question 11

6 marks | Proof

11. (i) Prove by counter example that the statement

"If n is a prime number then $3^n + 2$ is also a prime number."

is false.

(2)

(ii) Use proof by exhaustion to prove that if m is an integer that is **not** divisible by 3, then

$$m^2 - 1$$

is divisible by 3

(4)

Question 8

5 marks | Proof

8. (i) A student states

"If x and y are irrational numbers, $x \neq y$, then xy is also irrational."

Show, by counter example, that this statement is not always true.

(1)

(ii) Prove, using algebra, that for all odd integers n, the value of the expression

$$n^3 + 3n + 2$$

is always even but never a multiple of 4

(4)

Question 10

4 marks | Proof

10: **In this question you must show detailed reasoning.**

Use algebra to prove by exhaustion that,

for all positive integers m that are **not** multiples of 3, the value of

$$m^2 + 3m + 2$$

is always a multiple of 3

(4)

Question 6

8 marks | Proof

6. (i) Given that p and q are consecutive odd numbers, where $p > q > 0$, prove that

$$p^2 - q^2$$

is a multiple of 8

(4)

(ii) The curve C has equation

$$y = x^3 + 12x^2 + 49x + 2$$

Prove that C has no stationary points.

(4)

Question 7

8 marks | Proof | Main topic: Applications of Differentiation

7. The height of a river above a fixed point on the riverbed was monitored over a 7-day period.

The height of the river, H metres, t days after monitoring began, was given by

$$H = \frac{\sqrt{t}}{20}(20 + 6t - t^2) + 17 \qquad 0 \leqslant t \leqslant 7$$

Given that H has a stationary value at $t = \alpha$

(a) use calculus to show that α satisfies the equation

$$5\alpha^2 - 18\alpha - 20 = 0$$

(5)

(b) Hence find the value of α, giving your answer to 3 decimal places.

(1)

(c) Use further calculus to prove that H is a maximum at this value of α.

(2)

Question 7

6 marks | Proof | Main topic: Circles

7. The circle C_1 has equation

$$x^2 + y^2 + 8x - 10y = 29$$

(a) (i) Find the coordinates of the centre of C_1

(ii) Find the exact value of the radius of C_1

(3)

In part (b) you must show detailed reasoning.

The circle C_2 has equation

$$(x-5)^2 + (y+8)^2 = 52$$

(b) Prove that the circles C_1 and C_2 neither touch nor intersect.

(3)

Question 9

10 marks | Proof | Main topic: Laws of Logarithms

9. **In this question you must show detailed reasoning.**
Solutions relying on calculator technology are not acceptable.

(i) Solve

$$2\log_3(4x+5) - \log_3(x+3) = 2$$

(5)

(ii) Given that $a > 0$, $b > 0$ and

$$\log_{10} a + \log_{10} b = \log_{10}(a+b)$$

(a) prove that $a = \dfrac{b}{b-1}$

(3)

(b) Hence write down the full restriction on the value of b, giving a reason for your answer.

(2)

TOPIC

Polynomials

Every past-paper question on this topic, October 2019 to January 2026.

Question 7

10 marks | Polynomials | Main topic: Laws of Logarithms

7. (a) Given that

$$3\log_3(2x - 1) = 2 + \log_3(14x - 25)$$

show that

$$2x^3 - 3x^2 - 30x + 56 = 0$$

(4)

(b) Show that -4 is a root of this cubic equation.

(2)

(c) Hence, using algebra and showing each step of your working, solve

$$3\log_3(2x - 1) = 2 + \log_3(14x - 25)$$

(4)

Question 7

7 marks | Polynomials | Main topic: Integration

7. $$f(x) = Ax^3 + 6x^2 - 4x + B$$

where A and B are constants.

Given that

- $(x + 2)$ is a factor of $f(x)$
- $\int_3^5 f(x)\,dx = 176$

find the value of A and the value of B.

(7)

Question 10

12 marks | Polynomials | Main topic: Circles

10. The circle C has centre $X(3, 5)$ and radius r

The line l has equation $y = 2x + k$, where k is a constant.

(a) Show that l and C intersect when

$$5x^2 + (4k - 26)x + k^2 - 10k + 34 - r^2 = 0$$

(3)

Given that l is a tangent to C,

(b) show that $5r^2 = (k + p)^2$, where p is a constant to be found.

(3)

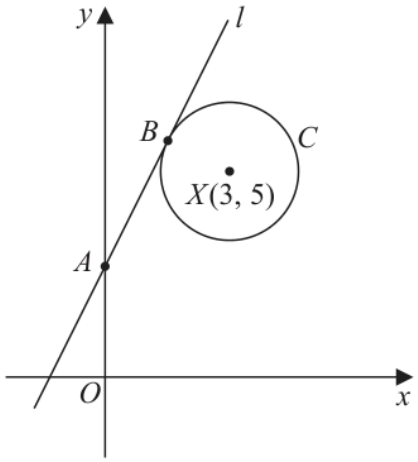


Figure 2

The line l

- cuts the y-axis at the point A
- touches the circle C at the point B

as shown in Figure 2.

Given that $AB = 2r$

(c) find the value of k

(6)

(Total 12 marks)

Question 9

8 marks | Polynomials | Main topic: Integration

9. **In this question you must show all stages of your working.**

Solutions based entirely on calculator technology are not acceptable.

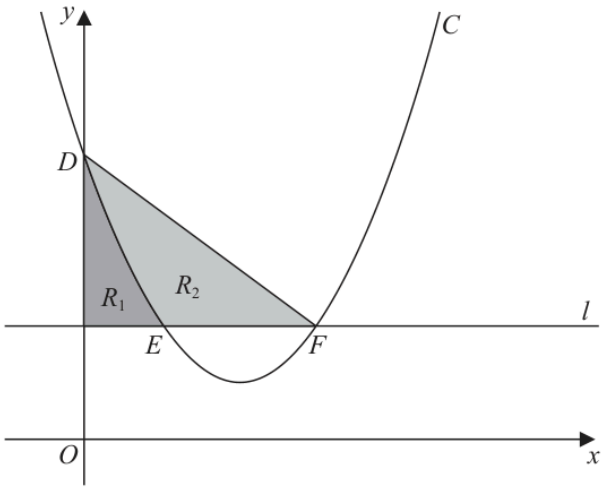


Figure 3

Figure 3 shows

- the curve C with equation $y = x^2 - 4x + 5$
- the line l with equation $y = 2$

The curve C intersects the y-axis at the point D.

(a) Write down the coordinates of D.

(1)

The curve C intersects the line l at the points E and F, as shown in Figure 3.

(b) Find the x coordinate of E and the x coordinate of F.

(2)

Shown shaded in Figure 3 is

- the region R_1 which is bounded by C, l and the y-axis
- the region R_2 which is bounded by C and the line segments EF and DF

Given that $\dfrac{\text{area of } R_1}{\text{area of } R_2} = k$, where k is a constant,

(c) use algebraic integration to find the exact value of k, giving your answer as a simplified fraction.

(5)

Question 6

8 marks | Polynomials | Main topic: Laws of Logarithms

6. (a) Given that

$$2\log_4(x+3) + \log_4 x = \log_4(4x+2) + \frac{1}{2}$$

show that

$$x^3 + 6x^2 + x - 4 = 0$$

(4)

(b) Given also that -1 is a root of the equation

$$x^3 + 6x^2 + x - 4 = 0$$

(i) use algebra to find the other two roots of the equation.

(3)

(ii) Hence solve

$$2\log_4(x+3) + \log_4 x = \log_4(4x+2) + \frac{1}{2}$$

(1)

Question 6

9 marks | Polynomials

Main topic: Applications of Differentiation

6. $f(x) = 4x^3 + px^2 + 8x + q$

where p and q are constants.

Given that

- $(2x + 3)$ is a factor of $f(x)$
- $f(x)$ has a remainder of -5 when divided by $(x + 2)$

(a) (i) show that $p = 10$

(ii) find the value of q.

(5)

(b) Hence find the range of values of x for which $f(x)$ is decreasing.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

Question 9

9 marks | Polynomials | Main topic: Laws of Logarithms

9. Given that

$$3\log_2(t+4) - 2\log_2(t-2) = 7$$

(a) verify that $t = 4$ is a solution of the above equation,

(2)

(b) show that

$$t^3 - 116t^2 + 560t - 448 = 0$$

(3)

(c) Hence, using algebra and showing your working, solve

$$3\log_2(t+4) - 2\log_2(t-2) = 7$$

giving each answer in simplest form.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

Question 8

8 marks | Polynomials | Main topic: Integration

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

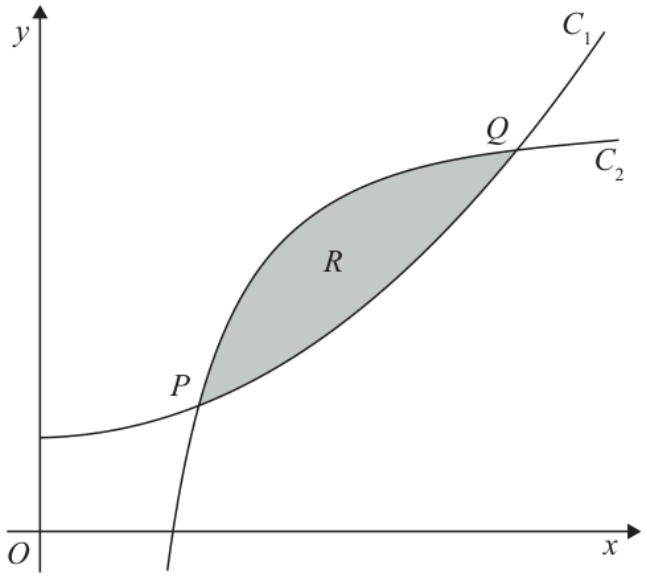


Figure 1

Figure 1 shows a sketch of part of the curve C_1 with equation

$$y = x^2 + 3 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 13 - \frac{9}{x^2} \qquad x > 0$$

The curves C_1 and C_2 intersect at the points P and Q as shown in Figure 1.

(a) Use algebra to find the x coordinate of P and the x coordinate of Q.

(4)

The finite region R, shown shaded in Figure 1, is bounded by C_1 and C_2

(b) Use algebraic integration to find the exact area of R.

(4)

TOPIC

Circles

Every past-paper question on this topic, October 2019 to January 2026.

Question 6

7 marks | Circles

6.

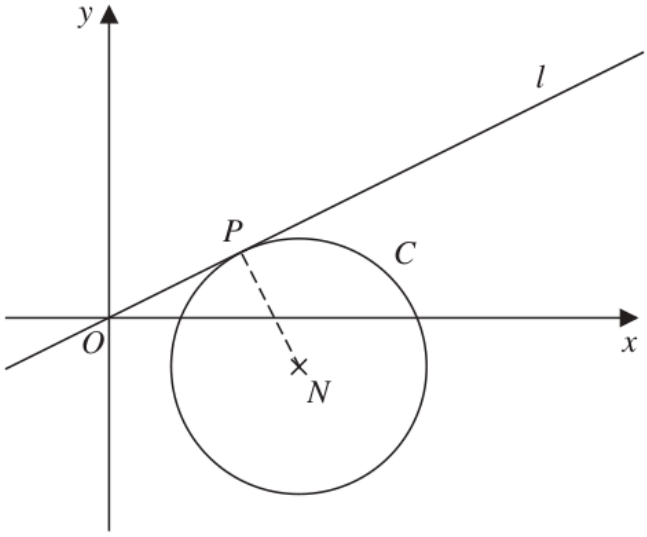


Figure 1

Figure 1 shows a sketch of a circle C with centre $N(4, -1)$.

The line l with equation $y = \frac{1}{2}x$ is a tangent to C at the point P.

Find

(a) the equation of line PN in the form $y = mx + c$, where m and c are constants,

(2)

(b) the equation of C.

(5)

Question 6

8 marks | Circles

6. The circle C has equation

$$x^2 + y^2 + 6x - 4y - 14 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C.

(3)

The line with equation $y = k$, where k is a constant, is a tangent to C.

(b) Find the possible values of k.

(2)

The line with equation $y = p$, where p is a negative constant, is a chord of C.

Given that the length of this chord is 4 units,

(c) find the value of p.

(3)

Question 9

10 marks | Circles

9. A circle C has equation

$$(x - k)^2 + (y - 2k)^2 = k + 7$$

where k is a positive constant.

(a) Write down, in terms of k,

(i) the coordinates of the centre of C,

(ii) the radius of C.

(2)

Given that the point $P(2, 3)$ lies on C

(b) (i) show that $5k^2 - 17k + 6 = 0$

(ii) hence find the possible values of k.

(3)

The tangent to the circle at P intersects the x-axis at point T.

Given that $k < 2$

(c) calculate the exact area of triangle OPT.

(5)

Question 6

7 marks | Circles

6. A circle has equation

$$x^2 - 6x + y^2 + 8y + k = 0$$

where k is a positive constant.

Given that the x-axis is a tangent to this circle,

(a) find the value of k.

(3)

The circle meets the coordinate axes at the points R, S and T.

(b) Find the exact area of the triangle RST.

(4)

Question 6

8 marks | Circles

6. (i) The circle C_1 has equation

$$x^2 + y^2 + 10x - 12y = k \qquad \text{where } k \text{ is a constant}$$

(a) Find the coordinates of the centre of C_1

(2)

(b) State the possible range in values for k.

(2)

(ii) The point $P(p, 0)$, the point $Q(-2, 10)$ and the point $R(8, -14)$ lie on a different circle, C_2

Given that

- p is a positive constant
- QR is a diameter of C_2

find the exact value of p.

(4)

Question 6

8 marks | Circles

6.

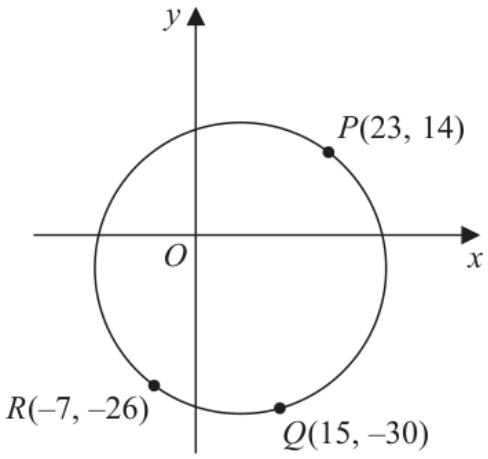


Figure 1

The points $P(23, 14)$, $Q(15, -30)$ and $R(-7, -26)$ lie on the circle C, as shown in Figure 1.

(a) Show that angle $PQR = 90°$

(2)

(b) Hence, or otherwise, find

(i) the centre of C,

(ii) the radius of C.

(3)

Given that the point S lies on C such that the distance QS is greatest,

(c) find an equation of the tangent to C at S, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

Question 10

12 marks | Circles

10. The circle C has centre $X(3, 5)$ and radius r

The line l has equation $y = 2x + k$, where k is a constant.

(a) Show that l and C intersect when

$$5x^2 + (4k - 26)x + k^2 - 10k + 34 - r^2 = 0$$

(3)

Given that l is a tangent to C,

(b) show that $5r^2 = (k + p)^2$, where p is a constant to be found.

(3)

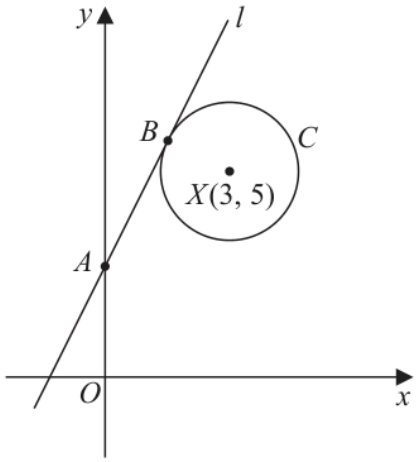


Figure 2

The line l

- cuts the y-axis at the point A
- touches the circle C at the point B

as shown in Figure 2.

Given that $AB = 2r$

(c) find the value of k

(6)

(Total 12 marks)

Question 9

12 marks | Circles

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

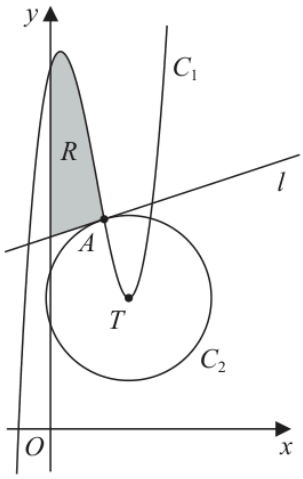


Figure 3

Figure 3 shows

- the curve C_1 with equation $y = x^3 - 5x^2 + 3x + 14$
- the circle C_2 with centre T

The point T is the minimum turning point of C_1

Using Figure 3 and calculus,

(a) find the coordinates of T

(3)

The curve C_1 intersects the circle C_2 at the point A with x coordinate 2

(b) Find an equation of the circle C_2

(3)

The line l shown in Figure 3, is the tangent to circle C_2 at A

(c) Show that an equation of l is

$$y = \frac{1}{3}x + \frac{22}{3}$$

(3)

The region R, shown shaded in Figure 3, is bounded by C_1, l and the y-axis.

(d) Find the exact area of R.

(3)

Question 6

10 marks | Circles

6. The circle C has equation

$$x^2 + y^2 + 8x - 4y = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C.

(3)

The point P lies on C.

Given that the tangent to C at P has equation $x + 2y + 10 = 0$

(b) find the coordinates of P

(4)

(c) Find the equation of the normal to C at P, giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(3)

Question 7

8 marks | Circles

7.

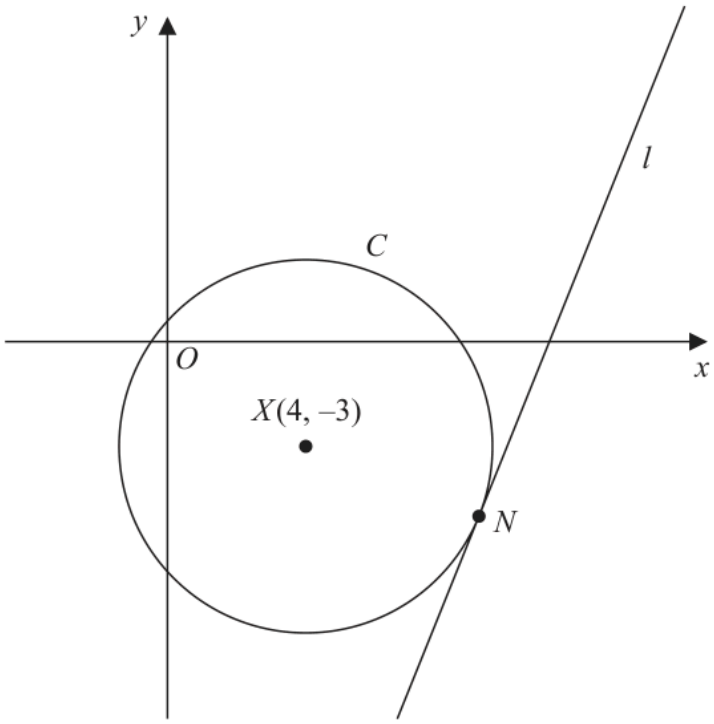


Figure 2

Figure 2 shows a sketch of

- the circle C with centre $X(4, -3)$
- the line l with equation $y = \frac{5}{2}x - \frac{55}{2}$

Given that l is the tangent to C at the point N,

(a) show that an equation for the straight line passing through X and N is

$$2x + 5y + 7 = 0$$

(3)

(b) Hence find

(i) the coordinates of N,

(ii) an equation for C.

(5)

Question 7

6 marks | Circles

7. The circle C_1 has equation

$$x^2 + y^2 + 8x - 10y = 29$$

(a) (i) Find the coordinates of the centre of C_1

(ii) Find the exact value of the radius of C_1

(3)

In part (b) you must show detailed reasoning.

The circle C_2 has equation

$$(x-5)^2 + (y+8)^2 = 52$$

(b) Prove that the circles C_1 and C_2 neither touch nor intersect.

(3)

Question 10

10 marks | Circles

10. The circle C has equation

$$x^2 + y^2 + 4x - 30y + 209 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact value of the radius of C.

(3)

The line L has equation $y = mx + 1$, where m is a constant.

Given that L is the tangent to C at the point P,

(b) show that

$$2m^2 - 7m - 22 = 0$$

(3)

(c) Hence find the possible pairs of coordinates of P.

(4)

Question 6

8 marks | Circles

6.

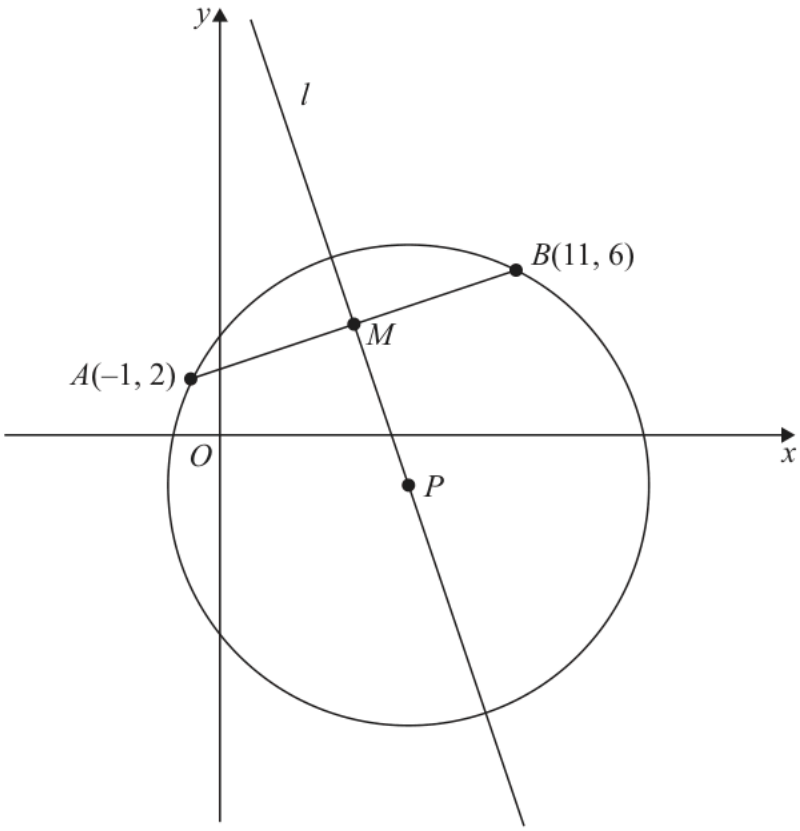


Figure 2

The point A $(-1, 2)$ and the point B $(11, 6)$ both lie on a circle with centre P.

The point M is the midpoint of AB.

Given that the line l passes through M and P, as shown in Figure 2,

(a) find an equation for l, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

Given that P has coordinates $(7, k)$, where k is a constant,

(b) find the value of k,

(1)

(c) find an equation for the circle.

(3)

Question 6

9 marks | Circles

6.

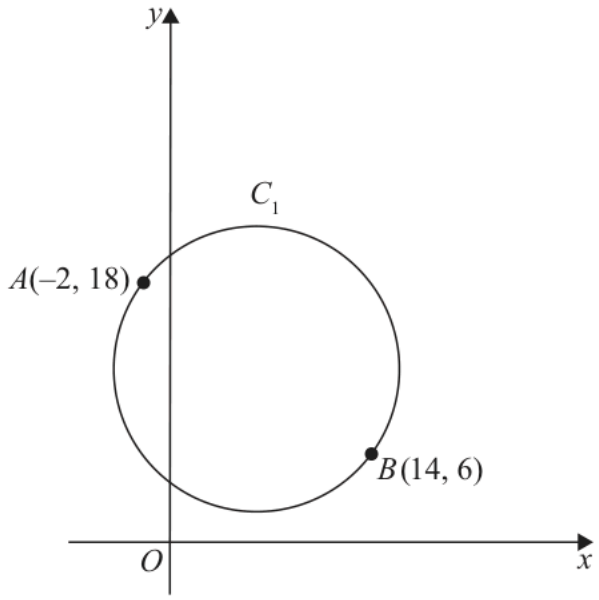


Figure 2

The points $A(-2, 18)$ and $B(14, 6)$ lie on a circle C_1 as shown in Figure 2.

Given that AB is a diameter of the circle C_1

(a) find an equation for C_1 making your method clear.

(5)

A circle C_2 has its centre at the origin.

Given that circles C_1 and C_2 touch,

(b) find possible equations for C_2

(4)

TOPIC

Binomial Expansion

Every past-paper question on this topic, October 2019 to January 2026.

Question 10

12 marks | Binomial Expansion

10. (i) (a) Find, in ascending powers of x, the 2nd, 3rd and 5th terms of the binomial expansion of

$$(3 + 2x)^6$$

(3)

For a particular value of x, these three terms form consecutive terms in a geometric series.

(b) Find this value of x.

(3)

(ii) In a **different** geometric series,

- the first term is $\sin^2\theta$
- the common ratio is $2\cos\theta$
- the sum to infinity is $\frac{8}{5}$

(a) Show that

$$5\cos^2\theta - 16\cos\theta + 3 = 0$$

(3)

(b) Hence find the exact value of the 2nd term in the series.

(3)

TOPIC

Arithmetic Sequences

Every past-paper question on this topic, October 2019 to January 2026.

Question 8

7 marks | Arithmetic Sequences

8. In a large theatre there are n rows of seats, where n is a constant.

The number of seats in the first row is a, where a is a constant.

In each subsequent row there are 4 more seats than in the previous row so that

- in the 2nd row there are $(a + 4)$ seats
- in the 3rd row there are $(a + 8)$ seats
- the number of seats in each row form an **arithmetic** sequence

Given that the **total** number of seats in the first 10 rows is 360

(a) find the value of a.

(2)

Given also that the total number of seats in the n rows is 2146

(b) show that

$$n^2 + 8n - 1073 = 0$$

(2)

(c) Hence

(i) state the number of rows of seats in the theatre,

(ii) find the maximum number of seats in any one row.

(3)

Question 8

13 marks | Arithmetic Sequences

8. (i) (a) In an **arithmetic** series the first term is a and the common difference is d.

Show that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

(3)

(b) Hence find

$$900 + 892 + 884 + \ldots + 500$$

(3)

(ii) Given that the first three terms of a **geometric** series are

$$k+4 \qquad k-2 \qquad 11-k$$

where k is a constant,

(a) show that

$$2k^2 - 11k - 40 = 0$$

(3)

Given also that this series is convergent,

(b) find the value of S_∞

(4)

Question 9

12 marks | Arithmetic Sequences | Main topic: Trigonometric Equations

9. Solutions based entirely on graphical or numerical methods are not acceptable in this question.

(i) Solve, for $0 \leqslant \theta < 180°$, the equation

$$3\sin(2\theta - 10°) = 1$$

giving your answers to one decimal place.

(4)

(ii) The first three terms of an arithmetic sequence are

$$\sin\alpha, \quad \frac{1}{\tan\alpha} \quad \text{and} \quad 2\sin\alpha$$

where α is a constant.

(a) Show that $2\cos\alpha = 3\sin^2\alpha$

(3)

Given that $\pi < \alpha < 2\pi$,

(b) find, showing all working, the value of α to 3 decimal places.

(5)

Question 8

7 marks | Arithmetic Sequences | Main topic: Sequences & Series

8. (i) An arithmetic series has first term a and common difference d.

Prove that the sum to n terms of this series is

$$\frac{n}{2}\{2a + (n-1)d\}$$

(3)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is given by

$$u_n = 5n + 3(-1)^n$$

Find the value of

(a) u_5

(1)

(b) $\sum_{n=1}^{59} u_n$

(3)

Question 10

11 marks | Arithmetic Sequences

Main topic: Modelling with Sequences & Series

10. In this question you must show detailed reasoning.

Owen wants to train for 12 weeks in preparation for running a marathon.

During the 12-week period he will run every Sunday and every Wednesday.

- On Sunday in week 1 he will run 15 km
- On Sunday in week 12 he will run 37 km

He considers two different 12-week training plans.

In training plan A, he will increase the distance he runs each Sunday by the same amount.

(a) Calculate the distance he will run on Sunday in week 5 under training plan A.

(3)

In training plan B, he will increase the distance he runs each Sunday by the same percentage.

(b) Calculate the distance he will run on Sunday in week 5 under training plan B.
Give your answer in km to one decimal place.

(3)

Owen will also run a fixed distance, x km, each Wednesday over the 12-week period.

Given that

- x is an integer
- the total distance that Owen will run on Sundays and Wednesdays over the 12 weeks will not exceed 360 km

(c) (i) find the maximum value of x, if he uses training plan A,

(ii) find the maximum value of x, if he uses training plan B.

(5)

(Total 11 marks)

Question 8

9 marks | Arithmetic Sequences | Main topic: Modelling with Sequences & Series

8. A metal post is repeatedly hit in order to drive it into the ground.

Given that

- on the 1st hit, the post is driven 100 mm into the ground
- on the 2nd hit, the post is driven an **additional** 98 mm into the ground
- on the 3rd hit, the post is driven an **additional** 96 mm into the ground
- the **additional** distances the post travels on each subsequent hit form an arithmetic sequence

(a) show that the post is driven an **additional** 62 mm into the ground with the 20th hit.

(1)

(b) Find the **total distance** that the post has been driven into the ground after 20 hits.

(2)

Given that for each subsequent hit after the 20th hit

- the **additional** distances the post travels form a geometric sequence with common ratio r
- on the 22nd hit, the post is driven an **additional** 60 mm into the ground

(c) find the value of r, giving your answer to 3 decimal places.

(2)

After a total of N hits, the post will have been driven more than 3 m into the ground.

(d) Find, showing all steps in your working, the smallest possible value of N.

(4)

Question 7

9 marks | Arithmetic Sequences

Main topic: Modelling with Sequences & Series

7. Wheat is grown on a farm.

- In year 1, the farm produced 300 tonnes of wheat.
- In year 12, the farm is predicted to produce 4 000 tonnes of wheat.

Model A assumes that the amount of wheat produced on the farm will increase by the same amount each year.

(a) Using model A, find the amount of wheat produced on the farm in year 4.
Give your answer to the nearest 10 tonnes.

(3)

Model B assumes that the amount of wheat produced on the farm will increase by the same percentage each year.

(b) Using model B, find the amount of wheat produced on the farm in year 2.
Give your answer to the nearest 10 tonnes.

(3)

(c) Calculate, according to the two models, the difference between the total amounts of wheat predicted to be produced on the farm from year 1 to year 12 inclusive.
Give your answer to the nearest 10 tonnes.

(3)

Question 7

6 marks | Arithmetic Sequences | Main topic: Modelling with Sequences & Series

7. Jem pays money into a savings scheme, A, over a period of 300 months.

Jem pays £20 into scheme A in month 1, £20.50 in month 2, £21 in month 3 and so on, so that the amounts Jem pays each month form an arithmetic sequence.

(a) Show that Jem pays £69.50 into scheme A in month 100

(1)

(b) Find the **total** amount that Jem pays into scheme A over the period of 300 months.

(2)

Kim pays money into a different savings scheme, B, over the same period of 300 months.

In a model, the amounts Kim pays into scheme B increase by the same percentage each month, so that the amounts Kim pays each month form a geometric sequence.

Given that Kim pays

- £20 into scheme B in month 1
- £250 into scheme B in month 300

(c) use the model to calculate, to the nearest £10, the difference between the total amount paid into scheme A and the total amount paid into scheme B over the period of 300 months.

(3)

Question 7

9 marks | Arithmetic Sequences | Main topic: Sequences & Series

7: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) A **geometric** series begins

$$10 + 8 + 6.4 + \ldots$$

(a) Find the sum to infinity of this series.

(2)

Given that the kth term of this series is less than 0.0005

(b) use algebra to find the smallest possible value of k.

(3)

(ii) An **arithmetic** series begins

$$850 + 843 + 836 + \ldots$$

Given that the sum of the first n terms of this series is S_n
find the greatest possible value of S_n

(4)

Question 9

10 marks | Arithmetic Sequences

Main topic: Modelling with Sequences & Series

9. A company mines lithium.

- in year 1, the company mined 12 400 tonnes of lithium
- in year 15, the company is predicted to mine 7 500 tonnes of lithium

Model *A* assumes that the mass of lithium the company mines will decrease by the same amount each year.

According to model *A*,

(a) find the mass of lithium the company will mine in year 5

(3)

(b) calculate the **total** mass of lithium the company will mine from year 1 to year 15 inclusive.

(2)

Model *B* assumes that the mass of lithium the company mines will decrease by the same percentage each year.

(c) Find, according to model *B*, the mass of lithium the company will mine in year 10. Give the answer to the nearest 10 tonnes.

(3)

According to model *B*, there is a limit to the **total** mass of lithium that can be mined.

(d) Calculate the value of this limit. Give the answer to the nearest 100 tonnes.

(2)

TOPIC

Geometric Sequences

Every past-paper question on this topic, October 2019 to January 2026.

Question 7

10 marks | Geometric Sequences

7. (i) A geometric sequence has first term 4 and common ratio 6
Given that the n^{th} term is greater than 10^{100}, find the minimum possible value of n.

(3)

(ii) A different geometric sequence has first term a and common ratio r.

Given that

- the second term of the sequence is -6
- the sum to infinity of the series is 25

(a) show that

$$25r^2 - 25r - 6 = 0$$

(3)

(b) Write down the solutions of

$$25r^2 - 25r - 6 = 0$$

(1)

Hence,

(c) state the value of r, giving a reason for your answer,

(1)

(d) find the sum of the first 4 terms of the series.

(2)

Question 6

8 marks | Geometric Sequences

6. In a geometric sequence $u_1, u_2, u_3, \ldots$

- the common ratio is r
- $u_2 + u_3 = 6$
- $u_4 = 8$

(a) Show that r satisfies

$$3r^2 - 4r - 4 = 0$$

(3)

Given that the geometric sequence has a sum to infinity,

(b) find u_1

(3)

(c) find S_∞

(2)

Question 8

7 marks | Geometric Sequences

8. A geometric sequence has first term a and common ratio r

Given that $S_\infty = 3a$

(a) show that $r = \frac{2}{3}$

(2)

Given also that

$$u_2 - u_4 = 16$$

where u_k is the k^{th} term of this sequence,

(b) find the value of S_{10} giving your answer to one decimal place.

(5)

Question 7

7 marks | Geometric Sequences

7. A geometric sequence has first term a and common ratio r, where $r > 0$

Given that

- the 3rd term is 20
- the 5th term is 12.8

(a) show that $r = 0.8$

(1)

(b) Hence find the value of a.

(2)

Given that the sum of the first n terms of this sequence is greater than 156

(c) find the smallest possible value of n.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

Question 6

9 marks | Geometric Sequences

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A software developer released an app to download.

The numbers of downloads of the app each month, in thousands, for the first three months after the app was released were

$$2k - 15 \qquad k \qquad k + 4$$

where k is a constant.

Given that the numbers of downloads each month are modelled as a geometric series,

(a) show that $k^2 - 7k - 60 = 0$

(2)

(b) predict the number of downloads in the 4th month.

(4)

The **total** number of all downloads of the app is predicted to exceed 3 million for the first time in the Nth month.

(c) Calculate the value of N according to the model.

(3)

Question 8

8 marks | Geometric Sequences

8. (i) A geometric series has first term a and common ratio r.

Prove that the sum of the first n terms of this series S_n is given by

$$S_n = \frac{a(1-r^n)}{1-r}$$

(3)

(ii) A liquid is to be stored in a barrel.

Due to evaporation, the volume of the liquid in the barrel at the end of each year is 8% less than the volume of the liquid in the barrel at the start of the year.

At the start of the first year, the barrel is filled with 150 litres of the liquid.

(a) Show that the amount of the liquid in the barrel at the end of 6 years is approximately 91 litres.

(2)

At the start of each year a new barrel is filled with 150 litres of the liquid so that, at the end of 40 years, there are 40 barrels containing the liquid.

(b) Calculate the total amount of the liquid, to the nearest litre, in the 40 barrels at the end of 40 years.

(3)

Question 10

11 marks | Geometric Sequences

Main topic: Modelling with Sequences & Series

10. In this question you must show detailed reasoning.

Owen wants to train for 12 weeks in preparation for running a marathon.

During the 12-week period he will run every Sunday and every Wednesday.

- On Sunday in week 1 he will run 15 km
- On Sunday in week 12 he will run 37 km

He considers two different 12-week training plans.

In training plan A, he will increase the distance he runs each Sunday by the same amount.

(a) Calculate the distance he will run on Sunday in week 5 under training plan A.

(3)

In training plan B, he will increase the distance he runs each Sunday by the same percentage.

(b) Calculate the distance he will run on Sunday in week 5 under training plan B.
Give your answer in km to one decimal place.

(3)

Owen will also run a fixed distance, x km, each Wednesday over the 12-week period.

Given that

- x is an integer
- the total distance that Owen will run on Sundays and Wednesdays over the 12 weeks will not exceed 360 km

(c) (i) find the maximum value of x, if he uses training plan A,

(ii) find the maximum value of x, if he uses training plan B.

(5)

(Total 11 marks)

Question 8

9 marks | Geometric Sequences

Main topic: Modelling with Sequences & Series

8. A metal post is repeatedly hit in order to drive it into the ground.

Given that

- on the 1st hit, the post is driven 100 mm into the ground
- on the 2nd hit, the post is driven an **additional** 98 mm into the ground
- on the 3rd hit, the post is driven an **additional** 96 mm into the ground
- the **additional** distances the post travels on each subsequent hit form an arithmetic sequence

(a) show that the post is driven an **additional** 62 mm into the ground with the 20th hit.

(1)

(b) Find the **total distance** that the post has been driven into the ground after 20 hits.

(2)

Given that for each subsequent hit after the 20th hit

- the **additional** distances the post travels form a geometric sequence with common ratio r
- on the 22nd hit, the post is driven an **additional** 60 mm into the ground

(c) find the value of r, giving your answer to 3 decimal places.

(2)

After a total of N hits, the post will have been driven more than 3 m into the ground.

(d) Find, showing all steps in your working, the smallest possible value of N.

(4)

Question 10

12 marks | Geometric Sequences | Main topic: Binomial Expansion

10. (i) (a) Find, in ascending powers of x, the 2nd, 3rd and 5th terms of the binomial expansion of

$$(3 + 2x)^6$$

(3)

For a particular value of x, these three terms form consecutive terms in a geometric series.

(b) Find this value of x.

(3)

(ii) In a **different** geometric series,

- the first term is $\sin^2\theta$
- the common ratio is $2\cos\theta$
- the sum to infinity is $\frac{8}{5}$

(a) Show that

$$5\cos^2\theta - 16\cos\theta + 3 = 0$$

(3)

(b) Hence find the exact value of the 2nd term in the series.

(3)

Question 7

9 marks | Geometric Sequences | Main topic: Modelling with Sequences & Series

7. Wheat is grown on a farm.

- In year 1, the farm produced 300 tonnes of wheat.
- In year 12, the farm is predicted to produce 4 000 tonnes of wheat.

Model A assumes that the amount of wheat produced on the farm will increase by the same amount each year.

(a) Using model A, find the amount of wheat produced on the farm in year 4.
Give your answer to the nearest 10 tonnes.

(3)

Model B assumes that the amount of wheat produced on the farm will increase by the same percentage each year.

(b) Using model B, find the amount of wheat produced on the farm in year 2.
Give your answer to the nearest 10 tonnes.

(3)

(c) Calculate, according to the two models, the difference between the total amounts of wheat predicted to be produced on the farm from year 1 to year 12 inclusive.
Give your answer to the nearest 10 tonnes.

(3)

Question 10

8 marks | Geometric Sequences

Main topic: Modelling with Sequences & Series

10. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The number of dormice and the number of voles on an island are being monitored.

Initially there are 2000 dormice on the island.

A model predicts that the number of dormice will increase by 3% each year, so that the numbers of dormice on the island at the end of each year form a geometric sequence.

(a) Find, according to the model, the number of dormice on the island 6 years after monitoring began. Give your answer to 3 significant figures.

(2)

The number of voles on the island is being monitored over the same period of time.

Given that

- 4 years after monitoring began there were 3690 voles on the island
- 7 years after monitoring began there were 3470 voles on the island
- the number of voles on the island at the end of each year is modelled as a geometric sequence

(b) find the equation of this model in the form

$$N = ab^t$$

where N is the number of voles, t years after monitoring began and a and b are constants. Give the value of a and the value of b to 2 significant figures.

(3)

When $t = T$, the number of dormice on the island is equal to the number of voles on the island.

(c) Find, according to the models, the value of T, giving your answer to one decimal place.

(3)

Question 8

13 marks | Geometric Sequences | Main topic: Arithmetic Sequences

8. (i) (a) In an **arithmetic** series the first term is a and the common difference is d.

Show that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

(3)

(b) Hence find

$$900 + 892 + 884 + \ldots + 500$$

(3)

(ii) Given that the first three terms of a **geometric** series are

$$k+4 \qquad k-2 \qquad 11-k$$

where k is a constant,

(a) show that

$$2k^2 - 11k - 40 = 0$$

(3)

Given also that this series is convergent,

(b) find the value of S_∞

(4)

Question 7

6 marks | Geometric Sequences

Main topic: Modelling with Sequences & Series

7. Jem pays money into a savings scheme, A, over a period of 300 months.

Jem pays £20 into scheme A in month 1, £20.50 in month 2, £21 in month 3 and so on, so that the amounts Jem pays each month form an arithmetic sequence.

(a) Show that Jem pays £69.50 into scheme A in month 100

(1)

(b) Find the **total** amount that Jem pays into scheme A over the period of 300 months.

(2)

Kim pays money into a different savings scheme, B, over the same period of 300 months.

In a model, the amounts Kim pays into scheme B increase by the same percentage each month, so that the amounts Kim pays each month form a geometric sequence.

Given that Kim pays

- £20 into scheme B in month 1
- £250 into scheme B in month 300

(c) use the model to calculate, to the nearest £10, the difference between the total amount paid into scheme A and the total amount paid into scheme B over the period of 300 months.

(3)

Question 7

9 marks | Geometric Sequences | Main topic: Sequences & Series

7: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) A **geometric** series begins

$$10 + 8 + 6.4 + \ldots$$

(a) Find the sum to infinity of this series.

(2)

Given that the kth term of this series is less than 0.0005

(b) use algebra to find the smallest possible value of k.

(3)

(ii) An **arithmetic** series begins

$$850 + 843 + 836 + \ldots$$

Given that the sum of the first n terms of this series is S_n
find the greatest possible value of S_n

(4)

Question 9

10 marks | Geometric Sequences | Main topic: Modelling with Sequences & Series

9. A company mines lithium.

- in year 1, the company mined 12 400 tonnes of lithium
- in year 15, the company is predicted to mine 7 500 tonnes of lithium

Model *A* assumes that the mass of lithium the company mines will decrease by the same amount each year.

According to model *A*,

(a) find the mass of lithium the company will mine in year 5

(3)

(b) calculate the **total** mass of lithium the company will mine from year 1 to year 15 inclusive.

(2)

Model *B* assumes that the mass of lithium the company mines will decrease by the same percentage each year.

(c) Find, according to model *B*, the mass of lithium the company will mine in year 10. Give the answer to the nearest 10 tonnes.

(3)

According to model *B*, there is a limit to the **total** mass of lithium that can be mined.

(d) Calculate the value of this limit. Give the answer to the nearest 100 tonnes.

(2)

TOPIC

Sequences & Series

Every past-paper question on this topic, October 2019 to January 2026.

Question 8

7 marks | Sequences & Series

8. (i) An arithmetic series has first term a and common difference d.

Prove that the sum to n terms of this series is

$$\frac{n}{2}\{2a + (n-1)d\}$$

(3)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is given by

$$u_n = 5n + 3(-1)^n$$

Find the value of

(a) u_5

(1)

(b) $\sum_{n=1}^{59} u_n$

(3)

Question 8

7 marks | Sequences & Series

8. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = 2(a_n + 3)^2 - 7$$
$$a_1 = p - 3$$

where p is a constant.

(a) Find an expression for a_2 in terms of p, giving your answer in simplest form.

(1)

Given that $\sum_{n=1}^{3} a_n = p + 15$

(b) find the possible values of a_2

(6)

Question 11

8 marks | Sequences & Series

11. A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = b - au_n$$

$$u_1 = 3$$

where a and b are constants.

(a) Find, in terms of a and b,

(i) u_2

(ii) u_3

(2)

Given

- $\sum_{n=1}^{3} u_n = 153$
- $b = a + 9$

(b) show that

$$a^2 - 5a - 66 = 0$$

(3)

(c) Hence find the larger possible value of u_2

(3)

Question 7

9 marks | Sequences & Series

7: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) A **geometric** series begins

$$10 + 8 + 6.4 + \ldots$$

(a) Find the sum to infinity of this series.

(2)

Given that the kth term of this series is less than 0.0005

(b) use algebra to find the smallest possible value of k.

(3)

(ii) An **arithmetic** series begins

$$850 + 843 + 836 + \ldots$$

Given that the sum of the first n terms of this series is S_n
find the greatest possible value of S_n

(4)

Question 7

7 marks | Sequences & Series | Main topic: Laws of Logarithms

7. Given $\log_a b = k$, find, in simplest form in terms of k,

(i) $\log_a\left(\frac{\sqrt{a}}{b}\right)$ **(2)**

(ii) $\frac{\log_a a^2 b}{\log_a b^3}$ **(2)**

(iii) $\sum_{n=1}^{50}(k + \log_a b^n)$ **(3)**

Question 7

10 marks | Sequences & Series | Main topic: Geometric Sequences

7. (i) A geometric sequence has first term 4 and common ratio 6
Given that the n^{th} term is greater than 10^{100}, find the minimum possible value of n.

(3)

(ii) A different geometric sequence has first term a and common ratio r.

Given that

- the second term of the sequence is -6
- the sum to infinity of the series is 25

(a) show that

$$25r^2 - 25r - 6 = 0$$

(3)

(b) Write down the solutions of

$$25r^2 - 25r - 6 = 0$$

(1)

Hence,

(c) state the value of r, giving a reason for your answer,

(1)

(d) find the sum of the first 4 terms of the series.

(2)

Question 6

8 marks | Sequences & Series | Main topic: Geometric Sequences

6. In a geometric sequence $u_1, u_2, u_3, \ldots$

- the common ratio is r
- $u_2 + u_3 = 6$
- $u_4 = 8$

(a) Show that r satisfies

$$3r^2 - 4r - 4 = 0$$

(3)

Given that the geometric sequence has a sum to infinity,

(b) find u_1

(3)

(c) find S_∞

(2)

Question 8

7 marks | Sequences & Series | Main topic: Geometric Sequences

8. A geometric sequence has first term a and common ratio r

Given that $S_\infty = 3a$

(a) show that $r = \frac{2}{3}$

(2)

Given also that

$$u_2 - u_4 = 16$$

where u_k is the k^{th} term of this sequence,

(b) find the value of S_{10} giving your answer to one decimal place.

(5)

Question 7

7 marks | Sequences & Series | Main topic: Geometric Sequences

7. A geometric sequence has first term a and common ratio r, where $r > 0$

Given that

- the 3rd term is 20
- the 5th term is 12.8

(a) show that $r = 0.8$

(1)

(b) Hence find the value of a.

(2)

Given that the sum of the first n terms of this sequence is greater than 156

(c) find the smallest possible value of n.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

Question 8

7 marks | Sequences & Series | Main topic: Arithmetic Sequences

8. In a large theatre there are n rows of seats, where n is a constant.

The number of seats in the first row is a, where a is a constant.

In each subsequent row there are 4 more seats than in the previous row so that

- in the 2nd row there are $(a + 4)$ seats
- in the 3rd row there are $(a + 8)$ seats
- the number of seats in each row form an **arithmetic** sequence

Given that the **total** number of seats in the first 10 rows is 360

(a) find the value of a.

(2)

Given also that the total number of seats in the n rows is 2146

(b) show that

$$n^2 + 8n - 1073 = 0$$

(2)

(c) Hence

(i) state the number of rows of seats in the theatre,

(ii) find the maximum number of seats in any one row.

(3)

TOPIC

Modelling with Sequences & Series

Every past-paper question on this topic, October 2019 to January 2026.

Question 10

11 marks | Modelling with Sequences & Series

10. In this question you must show detailed reasoning.

Owen wants to train for 12 weeks in preparation for running a marathon.

During the 12-week period he will run every Sunday and every Wednesday.

- On Sunday in week 1 he will run 15 km
- On Sunday in week 12 he will run 37 km

He considers two different 12-week training plans.

In training plan A, he will increase the distance he runs each Sunday by the same amount.

(a) Calculate the distance he will run on Sunday in week 5 under training plan A.

(3)

In training plan B, he will increase the distance he runs each Sunday by the same percentage.

(b) Calculate the distance he will run on Sunday in week 5 under training plan B.
Give your answer in km to one decimal place.

(3)

Owen will also run a fixed distance, x km, each Wednesday over the 12-week period.

Given that

- x is an integer
- the total distance that Owen will run on Sundays and Wednesdays over the 12 weeks will not exceed 360 km

(c) (i) find the maximum value of x, if he uses training plan A,

(ii) find the maximum value of x, if he uses training plan B.

(5)

(Total 11 marks)

Question 8

9 marks | Modelling with Sequences & Series

8. A metal post is repeatedly hit in order to drive it into the ground.

Given that

- on the 1st hit, the post is driven 100 mm into the ground
- on the 2nd hit, the post is driven an **additional** 98 mm into the ground
- on the 3rd hit, the post is driven an **additional** 96 mm into the ground
- the **additional** distances the post travels on each subsequent hit form an arithmetic sequence

(a) show that the post is driven an **additional** 62 mm into the ground with the 20th hit.

(1)

(b) Find the **total distance** that the post has been driven into the ground after 20 hits.

(2)

Given that for each subsequent hit after the 20th hit

- the **additional** distances the post travels form a geometric sequence with common ratio r
- on the 22nd hit, the post is driven an **additional** 60 mm into the ground

(c) find the value of r, giving your answer to 3 decimal places.

(2)

After a total of N hits, the post will have been driven more than 3 m into the ground.

(d) Find, showing all steps in your working, the smallest possible value of N.

(4)

Question 9

9 marks | Modelling with Sequences & Series

9. A scientist is using carbon-14 dating to determine the age of some wooden items.

The equation for carbon-14 dating an item is given by

$$N = k\lambda^t$$

where

- N grams is the amount of carbon-14 **currently** present in the item
- k grams was the **initial** amount of carbon-14 present in the item
- t is the number of years since the item was made
- λ is a constant, with $0 < \lambda < 1$

(a) Sketch the graph of N against t for $k = 1$

(2)

Given that it takes 5700 years for the amount of carbon-14 to reduce to half its initial value,

(b) show that the value of the constant λ is 0.999878 to 6 decimal places.

(2)

Given that Item A

- is known to have had 15 grams of carbon-14 present initially
- is thought to be 3250 years old

(c) calculate, to 3 significant figures, how much carbon-14 the equation predicts is currently in Item A.

(2)

Item B is known to have initially had 25 grams of carbon-14 present, but only 18 grams now remain.

(d) Use algebra to calculate the age of Item B to the nearest 100 years.

(3)

Question 7

9 marks | Modelling with Sequences & Series

7. Wheat is grown on a farm.

- In year 1, the farm produced 300 tonnes of wheat.
- In year 12, the farm is predicted to produce 4 000 tonnes of wheat.

Model *A* assumes that the amount of wheat produced on the farm will increase by the same amount each year.

(a) Using model *A*, find the amount of wheat produced on the farm in year 4.
Give your answer to the nearest 10 tonnes.

(3)

Model *B* assumes that the amount of wheat produced on the farm will increase by the same percentage each year.

(b) Using model *B*, find the amount of wheat produced on the farm in year 2.
Give your answer to the nearest 10 tonnes.

(3)

(c) Calculate, according to the two models, the difference between the total amounts of wheat predicted to be produced on the farm from year 1 to year 12 inclusive.
Give your answer to the nearest 10 tonnes.

(3)

Question 10

8 marks | Modelling with Sequences & Series

10. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The number of dormice and the number of voles on an island are being monitored.

Initially there are 2000 dormice on the island.

A model predicts that the number of dormice will increase by 3% each year, so that the numbers of dormice on the island at the end of each year form a geometric sequence.

(a) Find, according to the model, the number of dormice on the island 6 years after monitoring began. Give your answer to 3 significant figures.

(2)

The number of voles on the island is being monitored over the same period of time.

Given that
- 4 years after monitoring began there were 3690 voles on the island
- 7 years after monitoring began there were 3470 voles on the island
- the number of voles on the island at the end of each year is modelled as a geometric sequence

(b) find the equation of this model in the form

$$N = ab^t$$

where N is the number of voles, t years after monitoring began and a and b are constants. Give the value of a and the value of b to 2 significant figures.

(3)

When $t = T$, the number of dormice on the island is equal to the number of voles on the island.

(c) Find, according to the models, the value of T, giving your answer to one decimal place.

(3)

Question 7

6 marks | Modelling with Sequences & Series

7. Jem pays money into a savings scheme, A, over a period of 300 months.

Jem pays £20 into scheme A in month 1, £20.50 in month 2, £21 in month 3 and so on, so that the amounts Jem pays each month form an arithmetic sequence.

(a) Show that Jem pays £69.50 into scheme A in month 100

(1)

(b) Find the **total** amount that Jem pays into scheme A over the period of 300 months.

(2)

Kim pays money into a different savings scheme, B, over the same period of 300 months.

In a model, the amounts Kim pays into scheme B increase by the same percentage each month, so that the amounts Kim pays each month form a geometric sequence.

Given that Kim pays

- £20 into scheme B in month 1
- £250 into scheme B in month 300

(c) use the model to calculate, to the nearest £10, the difference between the total amount paid into scheme A and the total amount paid into scheme B over the period of 300 months.

(3)

Question 9

10 marks | Modelling with Sequences & Series

9. A company mines lithium.

- in year 1, the company mined 12 400 tonnes of lithium
- in year 15, the company is predicted to mine 7 500 tonnes of lithium

Model *A* assumes that the mass of lithium the company mines will decrease by the same amount each year.

According to model *A*,

(a) find the mass of lithium the company will mine in year 5

(3)

(b) calculate the **total** mass of lithium the company will mine from year 1 to year 15 inclusive.

(2)

Model *B* assumes that the mass of lithium the company mines will decrease by the same percentage each year.

(c) Find, according to model *B*, the mass of lithium the company will mine in year 10. Give the answer to the nearest 10 tonnes.

(3)

According to model *B*, there is a limit to the **total** mass of lithium that can be mined.

(d) Calculate the value of this limit. Give the answer to the nearest 100 tonnes.

(2)

Question 7

6 marks | Modelling with Sequences & Series | Main topic: Integration

7.

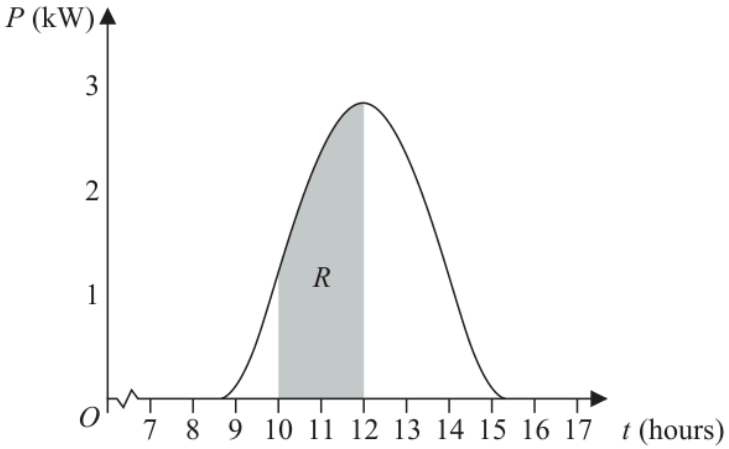


Figure 1

Solar panels are installed on the roof of a building.

The power, P, produced on a particular day, in kW, can be modelled by the equation

$$P = 0.95 + 2^{t-12} + 2^{12-t} - (t-12)^2 \qquad 8.5 \leqslant t \leqslant 15.2$$

where t is the time in hours after midnight. The graph of P against t is shown in Figure 1.

A table of values of t and P is shown below, with the values of P given to 4 significant figures where appropriate.

Time, t (hours)	10	10.5	11	11.5	12
Power, P (kW)		1.882	2.45		2.95

(a) Use the given equation to complete the table, giving the values of P to 4 significant figures where appropriate.

(2)

The amount of energy, in kWh, produced between 10:00 and 12:00 can be found by calculating the area of region R, shown shaded in Figure 1.

(b) Use the trapezium rule, with all the values of P in the completed table, to find an estimate for the amount of energy produced between 10:00 and 12:00. Give your answer to 2 decimal places.

(4)

Question 6

9 marks | Modelling with Sequences & Series | Main topic: Geometric Sequences

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A software developer released an app to download.

The numbers of downloads of the app each month, in thousands, for the first three months after the app was released were

$$2k - 15 \qquad k \qquad k + 4$$

where k is a constant.

Given that the numbers of downloads each month are modelled as a geometric series,

(a) show that $k^2 - 7k - 60 = 0$

(2)

(b) predict the number of downloads in the 4th month.

(4)

The **total** number of all downloads of the app is predicted to exceed 3 million for the first time in the Nth month.

(c) Calculate the value of N according to the model.

(3)

Question 6

6 marks | Modelling with Sequences & Series | Main topic: Integration

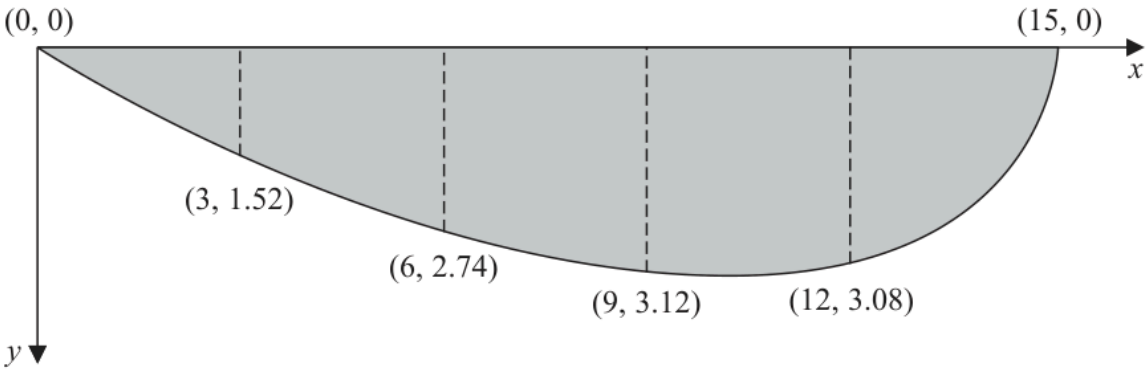


Figure 1

A river is being studied.

At one particular place, the river is 15 m wide.

The depth, y metres, of the river is measured at a point x metres from one side of the river.

Figure 1 shows a plot of the cross-section of the river and the coordinate values (x, y)

(a) Use the trapezium rule with all the y values given in Figure 1 to estimate the cross-sectional area of the river.

(3)

The water in the river is modelled as flowing at a constant speed of $1.5\,\text{m}\,\text{s}^{-1}$ across the whole of the cross-section.

(b) Use the model and the answer to part (a) to estimate the volume of water flowing through this section of the river each minute, giving your answer in m^3 to 2 significant figures.

(2)

Assuming the model,

(c) state, giving a reason for your answer, whether your answer for part (b) is an overestimate or an underestimate of the true volume of water flowing through this section of the river each minute.

(1)

Question 8

12 marks | Modelling with Sequences & Series | Main topic: Trigonometric Equations

8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x \leqslant \pi$, the equation

$$5\sin x \tan x + 13 = \cos x$$

giving your answer in radians to 3 significant figures.

(5)

(ii) The temperature inside a greenhouse is monitored on one particular day.

The temperature, $H°C$, inside the greenhouse, t hours after midnight, is modelled by the equation

$$H = 10 + 12\sin(kt + 18)° \qquad 0 \leqslant t < 24$$

where k is a constant.

Use the equation of the model to answer parts (a) to (c).

Given that

- the temperature inside the greenhouse was 20 °C at 6 am
- $0 < k < 20$

(a) find all possible values for k, giving each answer to 2 decimal places.

(4)

Given further that $0 < k < 10$

(b) find the maximum temperature inside the greenhouse,

(1)

(c) find the time of day at which this maximum temperature occurs.

Give your answer to the nearest minute.

(2)

Question 9

11 marks | Modelling with Sequences & Series | Main topic: Applications of Differentiation

9:

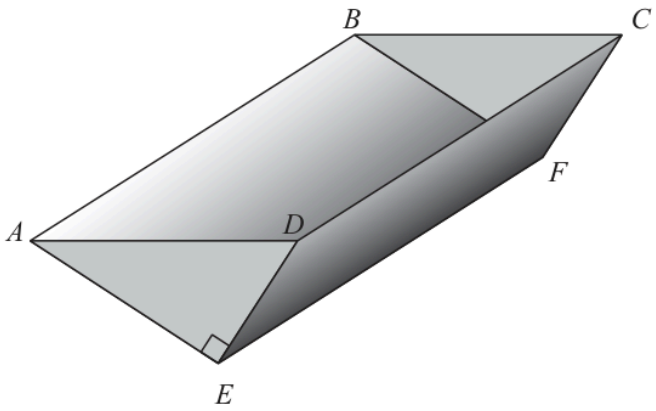


Figure 2

Figure 2 shows a sketch of an open container.

The sides $CDEF$ and $ABFE$ are rectangles.

The ends ADE and BCF are congruent (identical) right-angled triangles.

The container is made from metal of negligible thickness.

Given that

- $AE = BF = 3x$ metres
- $DE = CF = 2x$ metres
- $AB = DC = EF = L$ metres

and the capacity of the container is $12\,\text{m}^3$

(a) show that the area of metal used to make the container, $S\,\text{m}^2$, is given by

$$S = Px^2 + \frac{Q}{x}$$

where P and Q are positive integers to be found.

(4)

Given that x can vary,

(b) use algebraic calculus to find the minimum value of S, giving your answer to one decimal place.

(5)

(c) Justify that the value of S found in part (b) is a minimum.

(2)

Question 8

8 marks | Modelling with Sequences & Series | Main topic: Geometric Sequences

8. (i) A geometric series has first term a and common ratio r.

Prove that the sum of the first n terms of this series S_n is given by

$$S_n = \frac{a(1-r^n)}{1-r}$$

(3)

(ii) A liquid is to be stored in a barrel.

Due to evaporation, the volume of the liquid in the barrel at the end of each year is 8% less than the volume of the liquid in the barrel at the start of the year.

At the start of the first year, the barrel is filled with 150 litres of the liquid.

(a) Show that the amount of the liquid in the barrel at the end of 6 years is approximately 91 litres.

(2)

At the start of each year a new barrel is filled with 150 litres of the liquid so that, at the end of 40 years, there are 40 barrels containing the liquid.

(b) Calculate the total amount of the liquid, to the nearest litre, in the 40 barrels at the end of 40 years.

(3)

TOPIC

Laws of Logarithms

Every past-paper question on this topic, October 2019 to January 2026.

Question 7

7 marks | Laws of Logarithms

7. Given $\log_a b = k$, find, in simplest form in terms of k,

(i) $\log_a\left(\frac{\sqrt{a}}{b}\right)$ **(2)**

(ii) $\frac{\log_a a^2 b}{\log_a b^3}$ **(2)**

(iii) $\sum_{n=1}^{50}(k + \log_a b^n)$ **(3)**

Question 9

7 marks | Laws of Logarithms

9. (a) Sketch the curve with equation

$$y = 3 \times 4^x$$

showing the coordinates of any points of intersection with the coordinate axes.

(2)

The curve with equation $y = 6^{1-x}$ meets the curve with equation $y = 3 \times 4^x$ at the point P.

(b) Show that the x coordinate of P is $\dfrac{\log_{10} 2}{\log_{10} 24}$

(5)

Question 7

10 marks | Laws of Logarithms

7. (a) Given that

$$3\log_3(2x - 1) = 2 + \log_3(14x - 25)$$

show that

$$2x^3 - 3x^2 - 30x + 56 = 0$$

(4)

(b) Show that -4 is a root of this cubic equation.

(2)

(c) Hence, using algebra and showing each step of your working, solve

$$3\log_3(2x - 1) = 2 + \log_3(14x - 25)$$

(4)

Question 10

7 marks | Laws of Logarithms

10. Given $a = \log_2 3$

(i) write, in simplest form, in terms of a,

(a) $\log_2 9$

(b) $\log_2\left(\frac{\sqrt{3}}{16}\right)$

(3)

(ii) Solve

$$3^x \times 2^{x+4} = 6$$

giving your answer, in simplest form, in terms of a.

(4)

Question 6

8 marks | Laws of Logarithms

6. (a) Given that

$$2\log_4(x+3) + \log_4 x = \log_4(4x+2) + \frac{1}{2}$$

show that

$$x^3 + 6x^2 + x - 4 = 0$$

(4)

(b) Given also that -1 is a root of the equation

$$x^3 + 6x^2 + x - 4 = 0$$

(i) use algebra to find the other two roots of the equation.

(3)

(ii) Hence solve

$$2\log_4(x+3) + \log_4 x = \log_4(4x+2) + \frac{1}{2}$$

(1)

Question 9

9 marks | Laws of Logarithms

9. Given that

$$3\log_2(t+4) - 2\log_2(t-2) = 7$$

(a) verify that $t = 4$ is a solution of the above equation,

(2)

(b) show that

$$t^3 - 116t^2 + 560t - 448 = 0$$

(3)

(c) Hence, using algebra and showing your working, solve

$$3\log_2(t+4) - 2\log_2(t-2) = 7$$

giving each answer in simplest form.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

Question 6

5 marks | Laws of Logarithms

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Using the laws of logarithms, solve

$$\log_4(12 - 2x) = 2 + 2\log_4(x + 1)$$

(5)

Question 7

8 marks | Laws of Logarithms

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) The table below shows values of x and y, where $y = \log_{10}(x + 5)$, for x values between -1 and 4

x	-1	0	1	2	3	4
$y = \log_{10}(x + 5)$	$\log_{10} 4$	$\log_{10} 5$	$\log_{10} 6$	$\log_{10} 7$	$\log_{10} 8$	$\log_{10} 9$

Using the trapezium rule with all the y values in the given table, show that

$$\int_{-1}^{4} \log_{10}(x + 5)\,dx \approx \log_{10} k$$

where k is an integer to be found.

(3)

(ii) Find the value of a such that

$$2\log_5(5 - a) - \log_5(a + 25) = 1$$

(5)

Question 6

10 marks | Laws of Logarithms

6: **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(i) Solve

$$2\log_2(4-x) = 3 + \log_2\left(\frac{x+11}{2}\right)$$

(5)

(ii) The curves C_1 and C_2 with equations

$$y = 3^{2x+1} \qquad \text{and} \qquad y = 6 \times 3^x$$

meet at the point P.

Find the exact coordinates of P, writing your answer in the form $(\log_3 a, b)$ where a and b are integers.

(5)

Question 9

10 marks | Laws of Logarithms

9. **In this question you must show detailed reasoning.**
Solutions relying on calculator technology are not acceptable.

(i) Solve

$$2\log_3(4x+5) - \log_3(x+3) = 2$$

(5)

(ii) Given that $a > 0$, $b > 0$ and

$$\log_{10} a + \log_{10} b = \log_{10}(a+b)$$

(a) prove that $a = \dfrac{b}{b-1}$

(3)

(b) Hence write down the full restriction on the value of b, giving a reason for your answer.

(2)

Question 7

9 marks | Laws of Logarithms

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Find the exact solution of the equation

$$8^{2x-5} = 20$$

giving the answer in the form $a + b\log_2 5$ where a and b are rational constants.

(4)

(ii) Using the laws of logarithms, solve the equation

$$\log_3(13 + 2y) + 3 = 2\log_3(4 - y)$$

(5)

Question 9

9 marks | Laws of Logarithms | Main topic: Modelling with Sequences & Series

9. A scientist is using carbon-14 dating to determine the age of some wooden items.

The equation for carbon-14 dating an item is given by

$$N = k\lambda^t$$

where

- N grams is the amount of carbon-14 **currently** present in the item
- k grams was the **initial** amount of carbon-14 present in the item
- t is the number of years since the item was made
- λ is a constant, with $0 < \lambda < 1$

(a) Sketch the graph of N against t for $k = 1$

(2)

Given that it takes 5700 years for the amount of carbon-14 to reduce to half its initial value,

(b) show that the value of the constant λ is 0.999878 to 6 decimal places.

(2)

Given that Item A

- is known to have had 15 grams of carbon-14 present initially
- is thought to be 3250 years old

(c) calculate, to 3 significant figures, how much carbon-14 the equation predicts is currently in Item A.

(2)

Item B is known to have initially had 25 grams of carbon-14 present, but only 18 grams now remain.

(d) Use algebra to calculate the age of Item B to the nearest 100 years.

(3)

Question 10

8 marks | Laws of Logarithms | Main topic: Modelling with Sequences & Series

10. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The number of dormice and the number of voles on an island are being monitored.

Initially there are 2000 dormice on the island.

A model predicts that the number of dormice will increase by 3% each year, so that the numbers of dormice on the island at the end of each year form a geometric sequence.

(a) Find, according to the model, the number of dormice on the island 6 years after monitoring began. Give your answer to 3 significant figures.

(2)

The number of voles on the island is being monitored over the same period of time.

Given that
- 4 years after monitoring began there were 3690 voles on the island
- 7 years after monitoring began there were 3470 voles on the island
- the number of voles on the island at the end of each year is modelled as a geometric sequence

(b) find the equation of this model in the form

$$N = ab^t$$

where N is the number of voles, t years after monitoring began and a and b are constants. Give the value of a and the value of b to 2 significant figures.

(3)

When $t = T$, the number of dormice on the island is equal to the number of voles on the island.

(c) Find, according to the models, the value of T, giving your answer to one decimal place.

(3)

TOPIC

Trigonometric Equations

Every past-paper question on this topic, October 2019 to January 2026.

Question 9

12 marks | Trigonometric Equations

9. Solutions based entirely on graphical or numerical methods are not acceptable in this question.

(i) Solve, for $0 \leqslant \theta < 180°$, the equation

$$3\sin(2\theta - 10°) = 1$$

giving your answers to one decimal place.

(4)

(ii) The first three terms of an arithmetic sequence are

$$\sin\alpha, \quad \frac{1}{\tan\alpha} \quad \text{and} \quad 2\sin\alpha$$

where α is a constant.

(a) Show that $2\cos\alpha = 3\sin^2\alpha$

(3)

Given that $\pi < \alpha < 2\pi$,

(b) find, showing all working, the value of α to 3 decimal places.

(5)

Question 7

7 marks | Trigonometric Equations

7. (a) Show that the equation

$$8\tan\theta = 3\cos\theta$$

may be rewritten in the form

$$3\sin^2\theta + 8\sin\theta - 3 = 0$$

(3)

(b) Hence solve, for $0 \leqslant x \leqslant 90°$, the equation

$$8\tan 2x = 3\cos 2x$$

giving your answers to 2 decimal places.

(4)

Question 6

8 marks | Trigonometric Equations

6. (a) Show that the equation

$$\frac{3\sin\theta\cos\theta}{2\sin\theta - 1} = 5\tan\theta \qquad \sin\theta \neq \frac{1}{2}$$

can be written in the form

$$3\sin^3\theta + 10\sin^2\theta - 8\sin\theta = 0$$

(4)

(b) Hence solve, for $-\frac{\pi}{4} < x < \frac{\pi}{4}$

$$\frac{3\sin 2x\cos 2x}{2\sin 2x - 1} = 5\tan 2x$$

giving your answers to 3 decimal places where appropriate.

(4)

Question 8

10 marks | Trigonometric Equations

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta < 360°$, the equation

$$3\sin(\theta + 30°) = 7\cos(\theta + 30°)$$

giving your answers to one decimal place.

(4)

(ii) (a) Show that the equation

$$3\sin^3 x = 5\sin x - 7\sin x\cos x$$

can be written in the form

$$\sin x(a\cos^2 x + b\cos x + c) = 0$$

where a, b and c are constants to be found.

(b) Hence solve for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$ the equation

$$3\sin^3 x = 5\sin x - 7\sin x\cos x$$

(6)

Question 10

10 marks | Trigonometric Equations

10. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$$\tan^2\left(2x + \frac{\pi}{4}\right) = 3$$

(5)

(ii) Solve, for $0 < \theta < 360°$

$$(2\sin\theta - \cos\theta)^2 = 1$$

giving your answers, as appropriate, to one decimal place.

(5)

(Total 10 marks)

Question 7

8 marks | Trigonometric Equations

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $-90° < x < 90°$, the equation

$$3\sin(2x - 15°) = \cos(2x - 15°)$$

giving your answers to one decimal place.

(4)

(ii) Solve, for $0 < \theta < 2\pi$, the equation

$$4\sin^2\theta + 8\cos\theta = 3$$

giving your answers to 3 significant figures.

(4)

Question 8

9 marks | Trigonometric Equations

8. **In this question you must show all stages of your working.**

Solutions based entirely on calculator technology are not acceptable.

(i) Solve, for $-\frac{\pi}{2} < x < \pi$, the equation

$$5\sin(3x + 0.1) + 2 = 0$$

giving your answers, **in radians**, to 2 decimal places.

(4)

(ii) Solve, for $0 < \theta < 360°$, the equation

$$2\tan\theta\sin\theta = 5 + \cos\theta$$

giving your answers, **in degrees**, to one decimal place.

(5)

Question 9

7 marks | Trigonometric Equations

9. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$3\cos\theta(\tan\theta\sin\theta+3)=11-5\cos\theta$$

may be written as

$$3\cos^2\theta-14\cos\theta+8=0$$

(3)

(b) Hence solve, for $0<x<360°$

$$3\cos 2x(\tan 2x\sin 2x+3)=11-5\cos 2x$$

giving your answers to one decimal place.

(4)

Question 9

14 marks | Trigonometric Equations

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 \leqslant x < 360°$, the equation

$$\sin x \tan x = 5$$

giving your answers to one decimal place.

(6)

(ii)

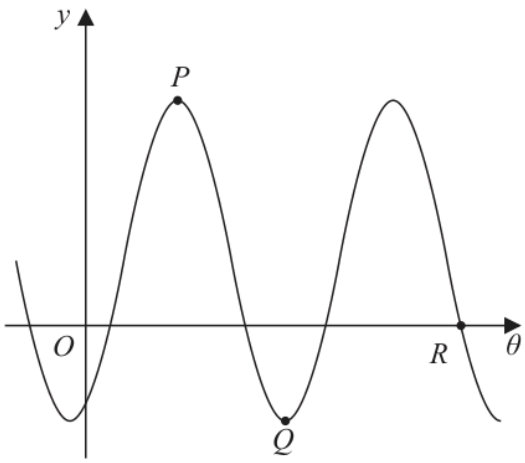


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = A\sin\left(2\theta - \frac{3\pi}{8}\right) + 2$$

where A is a constant and θ is measured in radians.

The points P, Q and R lie on the curve and are shown in Figure 1.

Given that the y coordinate of P is 7

(a) state the value of A,

(1)

(b) find the exact coordinates of Q,

(3)

(c) find the value of θ at R, giving your answer to 3 significant figures.

(4)

Question 8

12 marks | Trigonometric Equations

8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x \leqslant \pi$, the equation

$$5\sin x \tan x + 13 = \cos x$$

giving your answer in radians to 3 significant figures.

(5)

(ii) The temperature inside a greenhouse is monitored on one particular day.

The temperature, $H°C$, inside the greenhouse, t hours after midnight, is modelled by the equation

$$H = 10 + 12\sin(kt + 18)° \qquad 0 \leqslant t < 24$$

where k is a constant.

Use the equation of the model to answer parts (a) to (c).

Given that

- the temperature inside the greenhouse was 20 °C at 6 am
- $0 < k < 20$

(a) find all possible values for k, giving each answer to 2 decimal places.

(4)

Given further that $0 < k < 10$

(b) find the maximum temperature inside the greenhouse,

(1)

(c) find the time of day at which this maximum temperature occurs.

Give your answer to the nearest minute.

(2)

Question 7

7 marks | Trigonometric Equations

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 \leqslant x < 2\pi$, the equation

$$3\sin x \tan x = 11 + \cos x$$

giving the answers in radians to 3 decimal places.

(5)

(ii) Given that

- $0 < \theta < 90°$
- $\cos\theta = \frac{1}{3}$

find, in simplest form, the exact value of $\tan\theta$

(2)

Question 9

7 marks | Trigonometric Equations

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\tan\theta = 3\cos\theta$$

can be written as

$$3\sin^2\theta + 2\sin\theta - 3 = 0$$

(3)

(b) Hence solve, for $-\pi < x < \pi$, the equation

$$2\tan\left(2x + \frac{\pi}{3}\right) = 3\cos\left(2x + \frac{\pi}{3}\right)$$

giving your answers to 3 significant figures.

(4)

Question 10

8 marks | Trigonometric Equations

10. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos\theta\left(3\tan\theta+\frac{2}{\tan\theta}\right)\equiv\sin\theta+\frac{2}{\sin\theta}\qquad\theta\neq\frac{n\pi}{2}$$

(4)

(b) Hence solve, for $0<x<2\pi$, the equation

$$\cos x\left(3\tan x+\frac{2}{\tan x}\right)=4\sin x-5$$

giving your answers to 3 significant figures.

(4)

Question 8

8 marks | Trigonometric Equations

8: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 \leqslant x < \pi$, the equation

$$3\tan\left(2x + \frac{\pi}{5}\right) = \sqrt{3}$$

giving the answers in radians in the form $k\pi$, where k is a rational constant to be found.

(3)

(ii) Solve, for $0 \leqslant \theta < 360°$, the equation

$$5\sin\theta\tan\theta = \cos\theta + 4$$

giving your answers, in degrees, to one decimal place.

(5)

Question 8

10 marks | Trigonometric Equations

8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 \leqslant \theta < 180$

$$4\tan^2(2\theta - 30)° + 1 = 49$$

giving the answers to one decimal place.

(5)

(ii) Solve, for $0 \leqslant x < 2\pi$

$$2\tan x \sin x + 3 = 0$$

giving the answers, in radians, in the form $k\pi$ where k is a rational constant.

(5)

Question 10

12 marks | Trigonometric Equations | Main topic: Binomial Expansion

10. (i) (a) Find, in ascending powers of x, the 2nd, 3rd and 5th terms of the binomial expansion of

$$(3 + 2x)^6$$

(3)

For a particular value of x, these three terms form consecutive terms in a geometric series.

(b) Find this value of x.

(3)

(ii) In a **different** geometric series,

- the first term is $\sin^2\theta$
- the common ratio is $2\cos\theta$
- the sum to infinity is $\frac{8}{5}$

(a) Show that

$$5\cos^2\theta - 16\cos\theta + 3 = 0$$

(3)

(b) Hence find the exact value of the 2nd term in the series.

(3)

TOPIC

Applications of Differentiation

Every past-paper question on this topic, October 2019 to January 2026.

Question 10

7 marks | Applications of Differentiation

10. The curve C has equation

$$y = ax^3 - 3x^2 + 3x + b$$

where a and b are constants.

Given that

- the point $(2, 5)$ lies on C
- the gradient of the curve at $(2, 5)$ is 7

(a) find the value of a and the value of b.

(4)

(b) Prove that C has no turning points.

(3)

Question 10

10 marks | Applications of Differentiation

10. A curve C has equation

$$y = 4x^3 - 9x + \frac{k}{x} \qquad x > 0$$

where k is a constant.

The point P with x coordinate $\frac{1}{2}$ lies on C.

Given that P is a stationary point of C,

(a) show that $k = -\frac{3}{2}$

(4)

(b) Determine the nature of the stationary point at P, justifying your answer.

(2)

The curve C has a second stationary point.

(c) Using algebra, find the x coordinate of this second stationary point.

(4)

(Total 10 marks)

Question 9

10 marks | Applications of Differentiation

9.

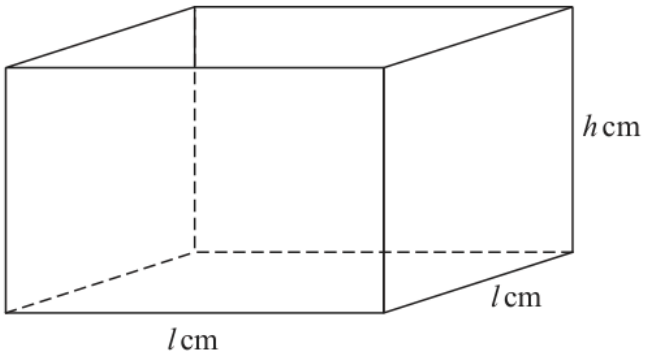


Figure 3

Figure 3 shows a sketch of a square based, open top box.

The height of the box is h cm, and the base edges each have length l cm.

Given that the volume of the box is $250\,000\,\text{cm}^3$

(a) show that the external surface area, $S\,\text{cm}^2$, of the box is given by

$$S = \frac{250\,000}{h} + 2000\sqrt{h}$$

(3)

(b) Use algebraic differentiation to show that S has a stationary point when $h = 250^k$ where k is a rational constant to be found.

(5)

(c) Justify by further differentiation that this value of h gives the minimum external surface area of the box.

(2)

(Total 10 marks)

TOTAL FOR PAPER IS 75 MARKS

Question 8

10 marks | Applications of Differentiation

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

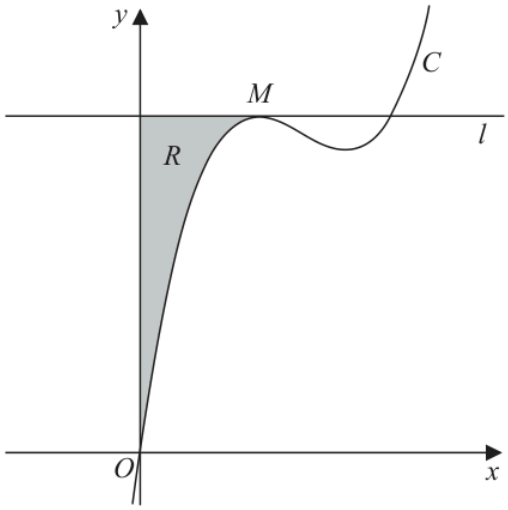


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{4}{3}x^3 - 11x^2 + kx \qquad \text{where } k \text{ is a constant}$$

The point M is the maximum turning point of C and is shown in Figure 2.

Given that the x coordinate of M is 2

(a) show that $k = 28$

(3)

(b) Determine the range of values of x for which y is increasing.

(2)

The line l passes through M and is parallel to the x-axis.

The region R, shown shaded in Figure 2, is bounded by the curve C, the line l and the y-axis.

(c) Find, by algebraic integration, the exact area of R.

(5)

Question 8

8 marks | Applications of Differentiation

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A curve has equation

$$y = 256x^4 - 304x - 35 + \frac{27}{x^2} \qquad x \neq 0$$

(a) Find $\frac{dy}{dx}$

(3)

(b) Hence find the coordinates of the stationary points of the curve.

(5)

Question 7

9 marks | Applications of Differentiation

7. The curve C has equation

$$y = \frac{12x^3(x-7) + 14x(13x-15)}{21\sqrt{x}} \qquad x > 0$$

(a) Write the equation of C in the form

$$y = ax^{\frac{7}{2}} + bx^{\frac{5}{2}} + cx^{\frac{3}{2}} + dx^{\frac{1}{2}}$$

where a, b, c and d are fully simplified constants.

(3)

The curve C has three turning points.

Using calculus,

(b) show that the x coordinates of the three turning points satisfy the equation

$$2x^3 - 10x^2 + 13x - 5 = 0$$

(3)

Given that the x coordinate of one of the turning points is 1

(c) find, using algebra, the exact x coordinates of the other two turning points.

(*Solutions based entirely on calculator technology are not acceptable.*)

(3)

Question 7

8 marks | Applications of Differentiation

7. The height of a river above a fixed point on the riverbed was monitored over a 7-day period.

The height of the river, H metres, t days after monitoring began, was given by

$$H = \frac{\sqrt{t}}{20}(20 + 6t - t^2) + 17 \qquad 0 \leqslant t \leqslant 7$$

Given that H has a stationary value at $t = \alpha$

(a) use calculus to show that α satisfies the equation

$$5\alpha^2 - 18\alpha - 20 = 0$$

(5)

(b) Hence find the value of α, giving your answer to 3 decimal places.

(1)

(c) Use further calculus to prove that H is a maximum at this value of α.

(2)

Question 9

12 marks | Applications of Differentiation

9.

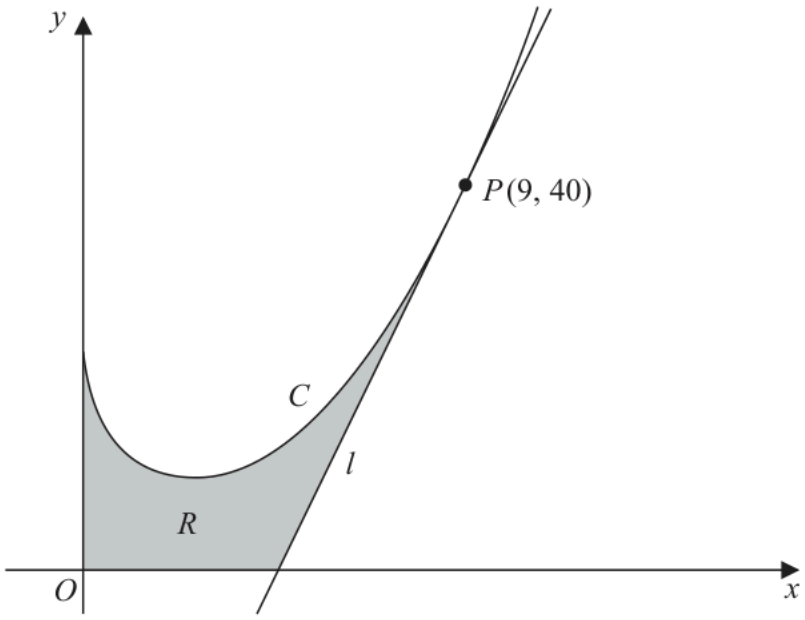


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{2}{3}x^2 - 9\sqrt{x} + 13 \qquad x \geqslant 0$$

(a) Find, using calculus, the range of values of x for which y is increasing.

(4)

The point P lies on C and has coordinates (9, 40).

The line l is the tangent to C at the point P.

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the line l, the x-axis and the y-axis.

(b) Find, using calculus, the exact area of R.

(8)

Question 10

9 marks | Applications of Differentiation

10. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

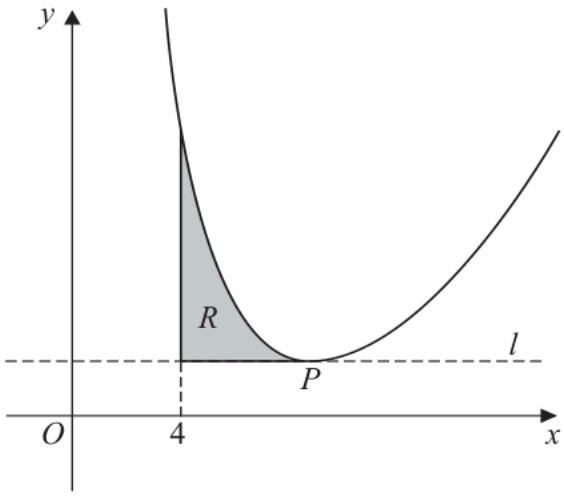


Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = \frac{1}{2}x^2 + \frac{1458}{\sqrt{x^3}} - 74 \qquad x > 0$$

The point P is the only stationary point on the curve.

(a) Use calculus to show that the x coordinate of P is 9

(4)

The line l passes through the point P and is parallel to the x-axis.

The region R, shown shaded in Figure 2, is bounded by the curve, the line l and the line with equation $x = 4$

(b) Use algebraic integration to find the exact area of R.

(5)

Question 9

8 marks | Applications of Differentiation

9.

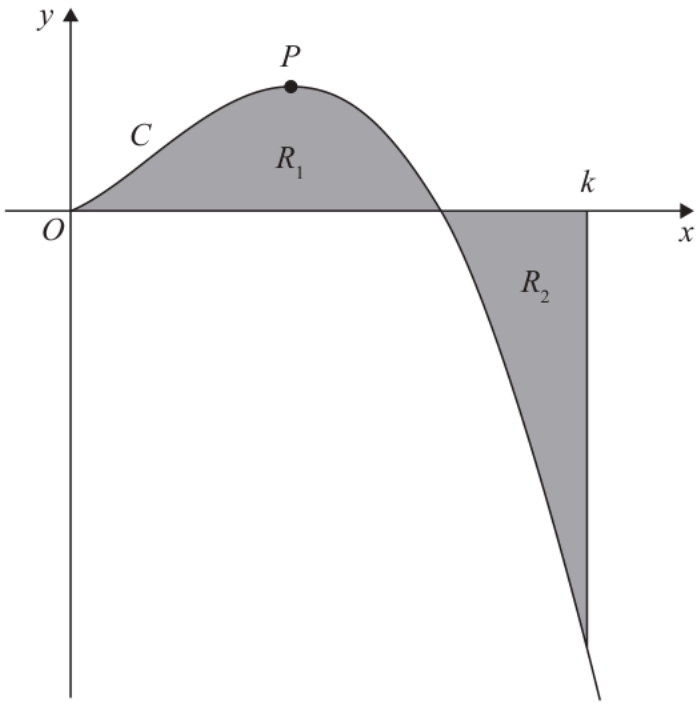


Figure 1

Figure 1 is a sketch of the curve C with equation

$$y = 2x^{\frac{3}{2}}(4 - x) \qquad x \geqslant 0$$

The point P is the stationary point of C.

(a) Find, using calculus, the x coordinate of P.

(4)

The region R_1, shown shaded in Figure 1, is bounded by C and the x-axis.

The region R_2, also shown shaded in Figure 1, is bounded by C, the x-axis and the line with equation $x = k$, where k is a constant.

Given that the area of R_1 is equal to the area of R_2

(b) find, using calculus, the exact value of k.

(4)

Question 6

9 marks | Applications of Differentiation

6. $f(x) = 4x^3 + px^2 + 8x + q$

where p and q are constants.

Given that

- $(2x + 3)$ is a factor of $f(x)$
- $f(x)$ has a remainder of -5 when divided by $(x + 2)$

(a) (i) show that $p = 10$

(ii) find the value of q.

(5)

(b) Hence find the range of values of x for which $f(x)$ is decreasing.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

Question 10

9 marks | Applications of Differentiation

10.

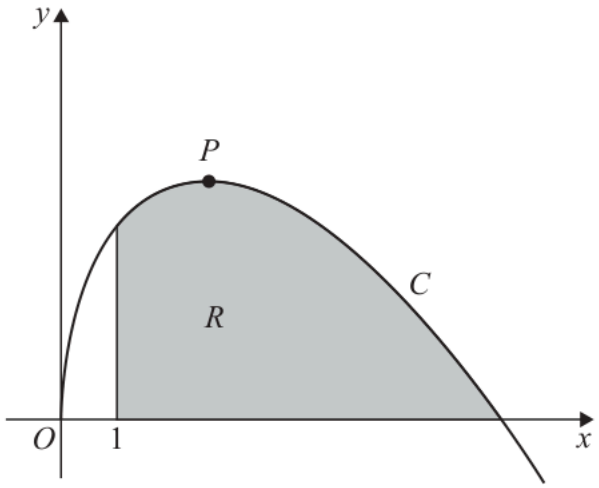


Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of part of the curve C with equation

$$y = \frac{9x - x^2}{2\sqrt{x}} \qquad x > 0$$

The point P is the stationary point on C.

(a) Find, using calculus, the x coordinate of P.

(4)

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the x-axis and the line with equation $x = 1$

(b) Using calculus, calculate the exact area of R.

(5)

Question 9

11 marks | Applications of Differentiation

9:

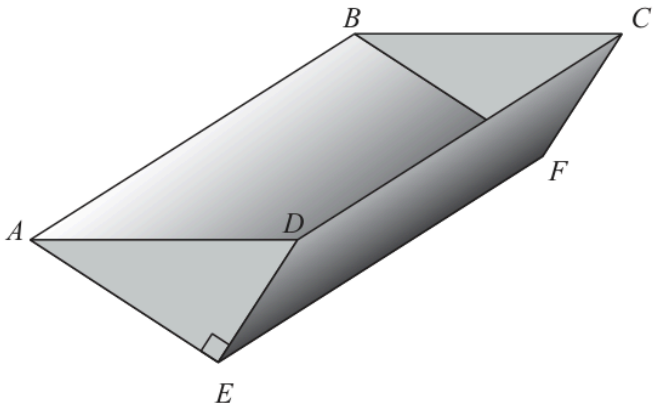


Figure 2

Figure 2 shows a sketch of an open container.

The sides $CDEF$ and $ABFE$ are rectangles.

The ends ADE and BCF are congruent (identical) right-angled triangles.

The container is made from metal of negligible thickness.

Given that

- $AE = BF = 3x$ metres
- $DE = CF = 2x$ metres
- $AB = DC = EF = L$ metres

and the capacity of the container is $12\,\text{m}^3$

(a) show that the area of metal used to make the container, $S\,\text{m}^2$, is given by

$$S = Px^2 + \frac{Q}{x}$$

where P and Q are positive integers to be found.

(4)

Given that x can vary,

(b) use algebraic calculus to find the minimum value of S, giving your answer to one decimal place.

(5)

(c) Justify that the value of S found in part (b) is a minimum.

(2)

Question 10

8 marks | Applications of Differentiation

10.

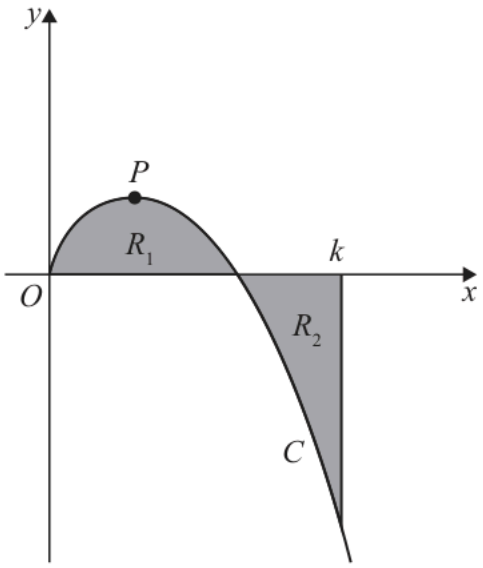


Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve C with equation

$$y = \frac{\sqrt{x}\left(100 - x^2\right)}{40} \qquad x \geqslant 0$$

The point P is a stationary point on C.

(a) Use algebraic differentiation to find the exact x coordinate of P.

(4)

The region R_1, shown shaded in Figure 2, is bounded by C and the x-axis.

The region R_2, also shown shaded in Figure 2, is bounded by C, the x-axis and the line with equation $x = k$, where k is a constant.

Given that the area of R_1 is equal to the area of R_2

(b) use algebraic integration to find the exact value of k.

(4)

Question 6

8 marks | Applications of Differentiation | Main topic: Circles

6. The circle C has equation

$$x^2 + y^2 + 6x - 4y - 14 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C.

(3)

The line with equation $y = k$, where k is a constant, is a tangent to C.

(b) Find the possible values of k.

(2)

The line with equation $y = p$, where p is a negative constant, is a chord of C.

Given that the length of this chord is 4 units,

(c) find the value of p.

(3)

Question 9

10 marks | Applications of Differentiation | Main topic: Circles

9. A circle C has equation

$$(x - k)^2 + (y - 2k)^2 = k + 7$$

where k is a positive constant.

(a) Write down, in terms of k,

(i) the coordinates of the centre of C,

(ii) the radius of C.

(2)

Given that the point $P(2, 3)$ lies on C

(b) (i) show that $5k^2 - 17k + 6 = 0$

(ii) hence find the possible values of k.

(3)

The tangent to the circle at P intersects the x-axis at point T.

Given that $k < 2$

(c) calculate the exact area of triangle OPT.

(5)

Question 6

7 marks | Applications of Differentiation | Main topic: Circles

6. A circle has equation

$$x^2 - 6x + y^2 + 8y + k = 0$$

where k is a positive constant.

Given that the x-axis is a tangent to this circle,

(a) find the value of k.

(3)

The circle meets the coordinate axes at the points R, S and T.

(b) Find the exact area of the triangle RST.

(4)

Question 6

8 marks | Applications of Differentiation | Main topic: Circles

6.

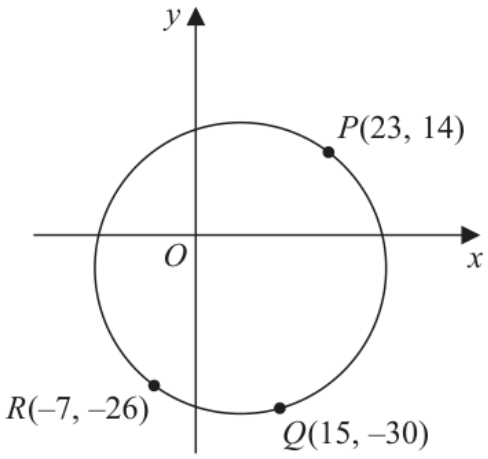


Figure 1

The points $P(23, 14)$, $Q(15, -30)$ and $R(-7, -26)$ lie on the circle C, as shown in Figure 1.

(a) Show that angle $PQR = 90°$

(2)

(b) Hence, or otherwise, find

(i) the centre of C,

(ii) the radius of C.

(3)

Given that the point S lies on C such that the distance QS is greatest,

(c) find an equation of the tangent to C at S, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

Question 9

10 marks | Applications of Differentiation | Main topic: Integration

9.

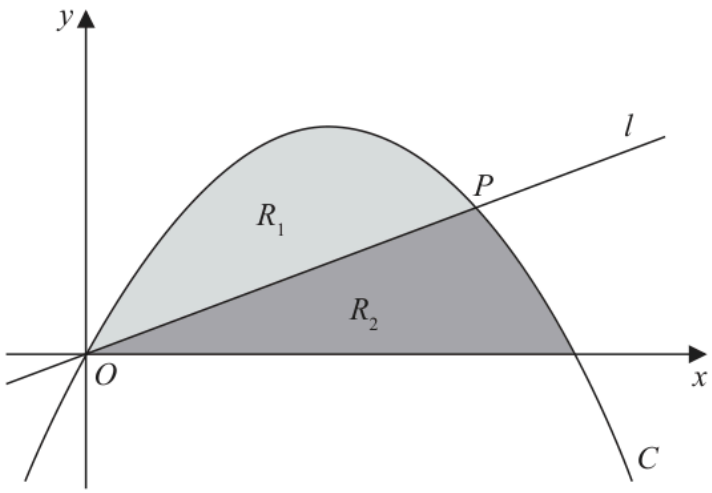


Figure 2

Figure 2 shows

- the curve C with equation $y = x - x^2$
- the line l with equation $y = mx$, where m is a constant and $0 < m < 1$

The line and the curve intersect at the origin O and at the point P.

(a) Find, in terms of m, the coordinates of P.

(2)

The region R_1, shown shaded in Figure 2, is bounded by C and l.

(b) Show that the area of R_1 is

$$\frac{(1-m)^3}{6}$$

(5)

The region R_2, also shown shaded in Figure 2, is bounded by C, the x-axis and l.

Given that the area of R_1 is equal to the area of R_2

(c) find the exact value of m.

(3)

Question 6

7 marks | Applications of Differentiation | Main topic: Integration

6. The curve C_1 has equation $y = f(x)$.

A table of values of x and y for $y = f(x)$ is shown below, with the y values rounded to 4 decimal places where appropriate.

x	0	0.5	1	1.5	2
y	3	2.6833	2.4	2.1466	1.92

(a) Use the trapezium rule with all the values of y in the table to find an approximation for

$$\int_0^2 f(x)\,dx$$

giving your answer to 3 decimal places.

(3)

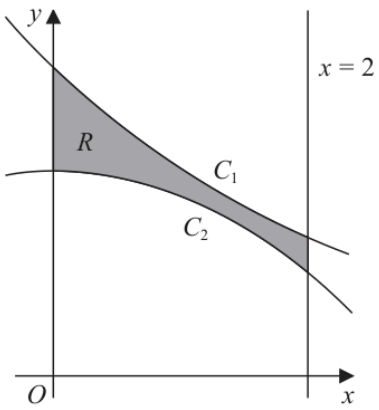


Figure 1

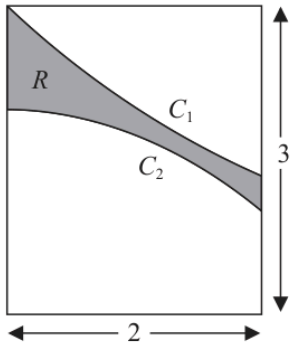


Figure 2

The region R, shown shaded in Figure 1, is bounded by

- the curve C_1
- the curve C_2 with equation $y = 2 - \frac{1}{4}x^2$
- the line with equation $x = 2$
- the y-axis

The region R forms part of the design for a logo shown in Figure 2.

The design consists of the shaded region R inside a rectangle of width 2 and height 3

Using calculus and the answer to part (a),

(b) calculate an estimate for the percentage of the logo which is shaded.

(4)

Question 9

12 marks | Applications of Differentiation | Main topic: Circles

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

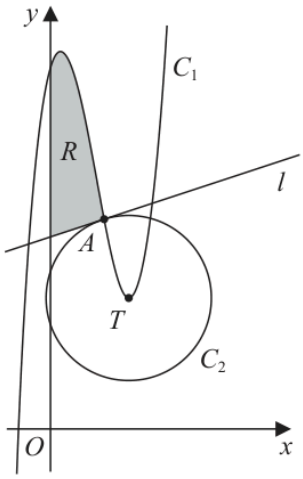


Figure 3

Figure 3 shows

- the curve C_1 with equation $y = x^3 - 5x^2 + 3x + 14$
- the circle C_2 with centre T

The point T is the minimum turning point of C_1

Using Figure 3 and calculus,

(a) find the coordinates of T

(3)

The curve C_1 intersects the circle C_2 at the point A with x coordinate 2

(b) Find an equation of the circle C_2

(3)

The line l shown in Figure 3, is the tangent to circle C_2 at A

(c) Show that an equation of l is

$$y = \frac{1}{3}x + \frac{22}{3}$$

(3)

The region R, shown shaded in Figure 3, is bounded by C_1, l and the y-axis.

(d) Find the exact area of R.

(3)

Question 6

10 marks | Applications of Differentiation | Main topic: Circles

6. The circle C has equation

$$x^2 + y^2 + 8x - 4y = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C.

(3)

The point P lies on C.

Given that the tangent to C at P has equation $x + 2y + 10 = 0$

(b) find the coordinates of P

(4)

(c) Find the equation of the normal to C at P, giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(3)

Question 10

10 marks | Applications of Differentiation | Main topic: Integration

10. The curve C has equation

$$y = \frac{(x-k)^2}{\sqrt{x}} \qquad x > 0$$

where k is a **positive** constant.

(a) Show that

$$\int_1^{16} \frac{(x-k)^2}{\sqrt{x}}\,\mathrm{d}x = ak^2 + bk + \frac{2046}{5}$$

where a and b are integers to be found.

(5)

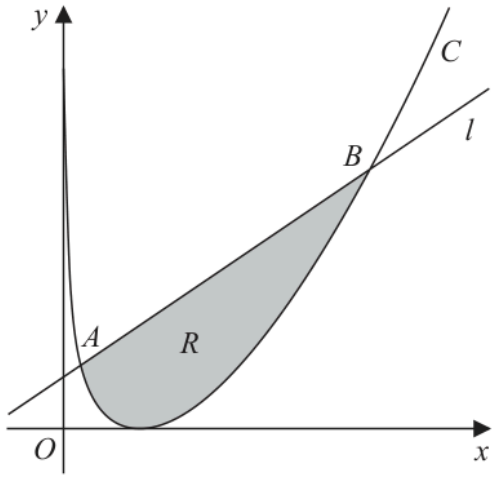


Figure 1

Figure 1 shows a sketch of the curve C and the line l.

Given that l intersects C at the point $A(1, 9)$ and at the point $B(16, q)$ where q is a constant,

(b) show that $k = 4$

(2)

The region R, shown shaded in Figure 1, is bounded by C and l

Using the answers to parts (a) and (b),

(c) find the area of region R

(3)

Question 7

8 marks | Applications of Differentiation | Main topic: Circles

7.

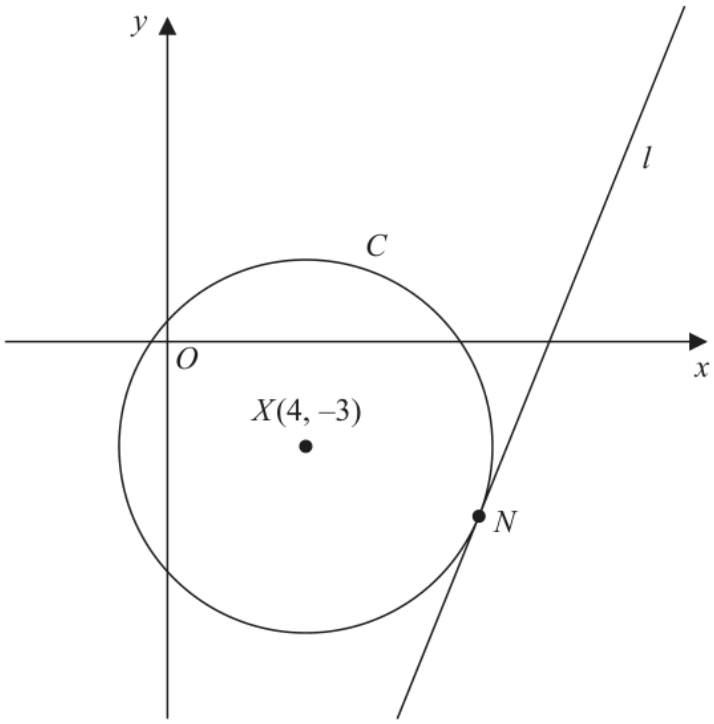


Figure 2

Figure 2 shows a sketch of

- the circle C with centre $X(4, -3)$
- the line l with equation $y = \frac{5}{2}x - \frac{55}{2}$

Given that l is the tangent to C at the point N,

(a) show that an equation for the straight line passing through X and N is

$$2x + 5y + 7 = 0$$

(3)

(b) Hence find

(i) the coordinates of N,

(ii) an equation for C.

(5)

Question 10

10 marks | Applications of Differentiation | Main topic: Circles

10. The circle C has equation

$$x^2 + y^2 + 4x - 30y + 209 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact value of the radius of C.

(3)

The line L has equation $y = mx + 1$, where m is a constant.

Given that L is the tangent to C at the point P,

(b) show that

$$2m^2 - 7m - 22 = 0$$

(3)

(c) Hence find the possible pairs of coordinates of P.

(4)

Question 9

12 marks | Applications of Differentiation | Main topic: Integration

9.

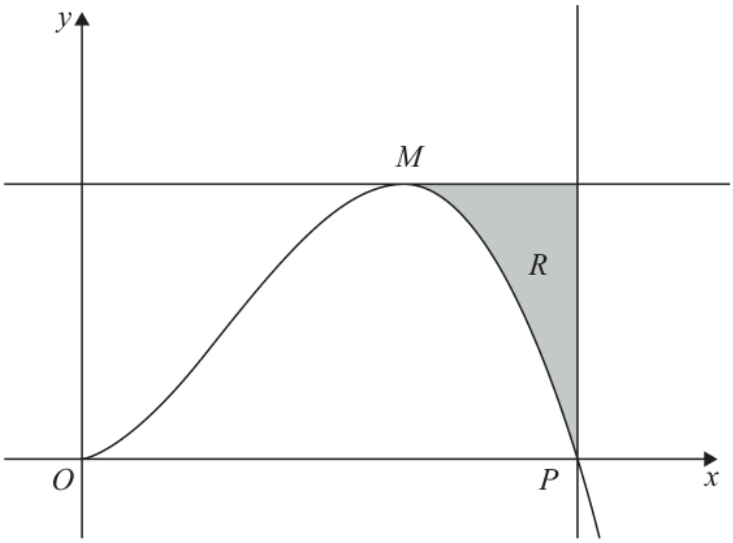


Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \frac{9x^2\left(5 - \sqrt{x}\right)}{5} \qquad x \geqslant 0$$

The curve has a turning point at the point M, as shown in Figure 3.

(a) Using calculus, find the coordinates of M.

(5)

The curve crosses the x-axis at the point P, as shown in Figure 3.

(b) Use algebra to find the x coordinate of P.

(2)

The finite region R, shown shaded in Figure 3, is bounded by the curve, the line through M parallel to the x-axis and the line through P parallel to the y-axis.

(c) Use algebraic integration to find the area of R, giving your answer to one decimal place.

(5)

Question 7

10 marks | Applications of Differentiation | Main topic: Integration

7.

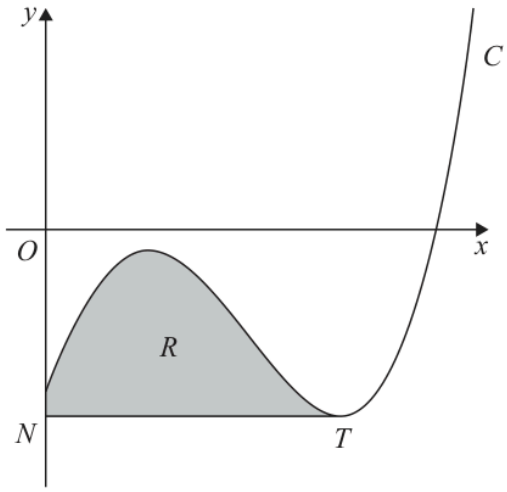


Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve C with equation

$$y = x^3 - 4x^{\frac{5}{2}} - kx^{\frac{1}{2}} + 28x - 44 \qquad x \geqslant 0$$

where k is a positive constant.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in simplest form.

(2)

The point T, shown in Figure 3, is a minimum stationary point on C.

Given that the x coordinate of T is 9

(b) show that $k = 6$

(2)

The line through T parallel to the x-axis meets the y-axis at the point N.

The finite region R, shown shaded in Figure 3, is bounded by C, the y-axis and the line segment NT.

(c) Use algebraic integration to find the area of R, giving the answer to 3 significant figures.

(6)

WMA12/01 JANUARY 2026

Question 6

8 marks | Applications of Differentiation | Main topic: Proof

6. (i) Given that p and q are consecutive odd numbers, where $p > q > 0$, prove that

$$p^2 - q^2$$

is a multiple of 8

(4)

(ii) The curve C has equation

$$y = x^3 + 12x^2 + 49x + 2$$

Prove that C has no stationary points.

(4)

Dr Eslam Ahmed · eliteigcse.com

TOPIC

Integration

Every past-paper question on this topic, October 2019 to January 2026.

Question 8

9 marks | Integration

8. Solutions relying on calculator technology are not acceptable in this question.

(i)

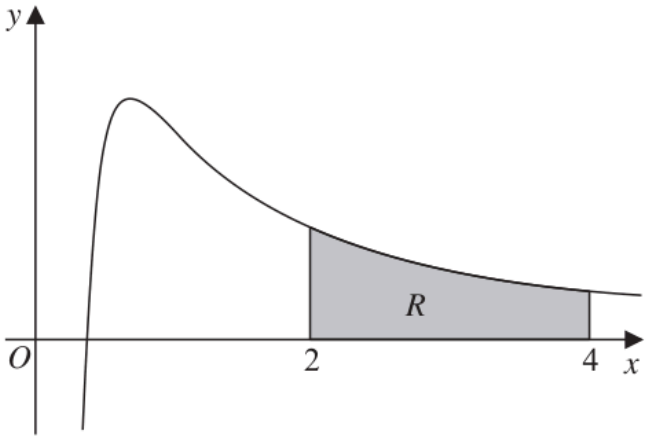


Figure 2

Figure 2 shows a sketch of part of a curve with equation

$$y = \frac{8\sqrt{x} - 5}{2x^2} \qquad x > 0$$

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$

Find the exact area of R.

(5)

(ii) Find the value of the constant k such that

$$\int_{-3}^{6} \left(\frac{1}{2}x^2 + k \right) \mathrm{d}x = 55$$

(4)

Question 7

6 marks | Integration

7.

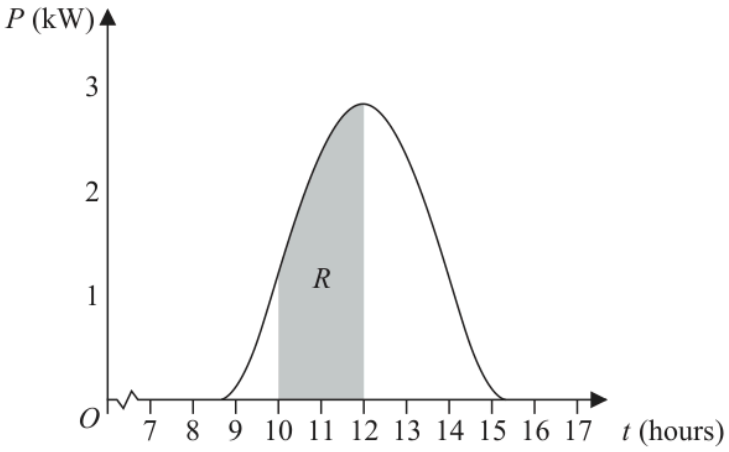


Figure 1

Solar panels are installed on the roof of a building.

The power, P, produced on a particular day, in kW, can be modelled by the equation

$$P = 0.95 + 2^{t-12} + 2^{12-t} - (t-12)^2 \qquad 8.5 \leqslant t \leqslant 15.2$$

where t is the time in hours after midnight. The graph of P against t is shown in Figure 1.

A table of values of t and P is shown below, with the values of P given to 4 significant figures where appropriate.

Time, t (hours)	10	10.5	11	11.5	12
Power, P (kW)		1.882	2.45		2.95

(a) Use the given equation to complete the table, giving the values of P to 4 significant figures where appropriate.

(2)

The amount of energy, in kWh, produced between 10:00 and 12:00 can be found by calculating the area of region R, shown shaded in Figure 1.

(b) Use the trapezium rule, with all the values of P in the completed table, to find an estimate for the amount of energy produced between 10:00 and 12:00. Give your answer to 2 decimal places.

(4)

Question 9

10 marks | Integration

9.

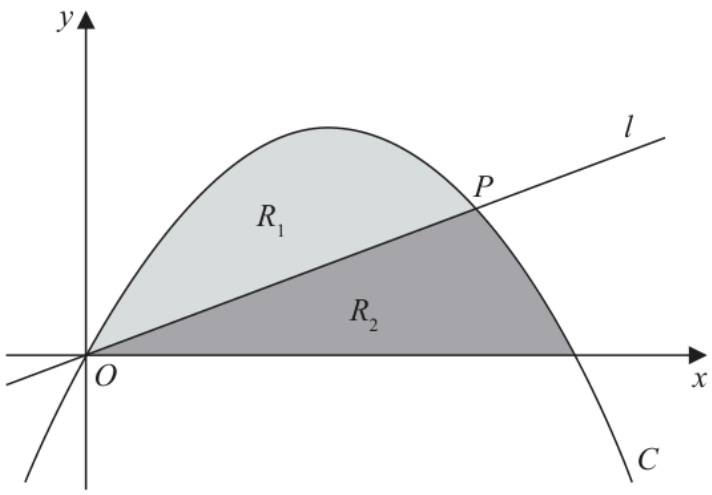


Figure 2

Figure 2 shows

- the curve C with equation $y = x - x^2$
- the line l with equation $y = mx$, where m is a constant and $0 < m < 1$

The line and the curve intersect at the origin O and at the point P.

(a) Find, in terms of m, the coordinates of P.

(2)

The region R_1, shown shaded in Figure 2, is bounded by C and l.

(b) Show that the area of R_1 is

$$\frac{(1-m)^3}{6}$$

(5)

The region R_2, also shown shaded in Figure 2, is bounded by C, the x-axis and l.

Given that the area of R_1 is equal to the area of R_2

(c) find the exact value of m.

(3)

Question 7

7 marks | Integration

7. $$f(x) = Ax^3 + 6x^2 - 4x + B$$

where A and B are constants.

Given that

- $(x + 2)$ is a factor of $f(x)$
- $\int_3^5 f(x)\,dx = 176$

find the value of A and the value of B.

(7)

Question 6

7 marks | Integration

6. The curve C_1 has equation $y = f(x)$.

A table of values of x and y for $y = f(x)$ is shown below, with the y values rounded to 4 decimal places where appropriate.

x	0	0.5	1	1.5	2
y	3	2.6833	2.4	2.1466	1.92

(a) Use the trapezium rule with all the values of y in the table to find an approximation for

$$\int_0^2 f(x)\,dx$$

giving your answer to 3 decimal places.

(3)

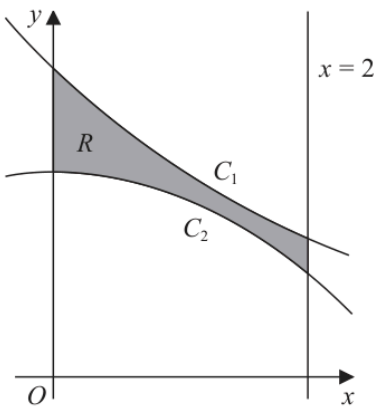


Figure 1

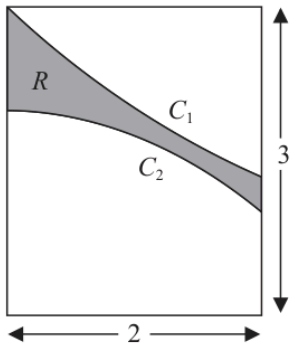


Figure 2

The region R, shown shaded in Figure 1, is bounded by

- the curve C_1
- the curve C_2 with equation $y = 2 - \frac{1}{4}x^2$
- the line with equation $x = 2$
- the y-axis

The region R forms part of the design for a logo shown in Figure 2.

The design consists of the shaded region R inside a rectangle of width 2 and height 3

Using calculus and the answer to part (a),

(b) calculate an estimate for the percentage of the logo which is shaded.

(4)

Question 9

8 marks | Integration

9. **In this question you must show all stages of your working.**

Solutions based entirely on calculator technology are not acceptable.

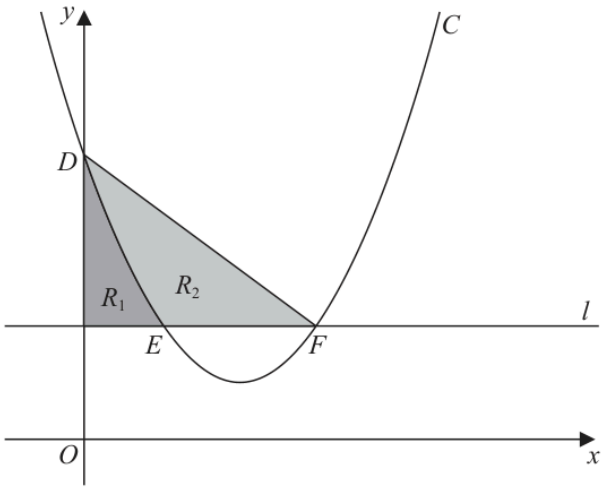


Figure 3

Figure 3 shows

- the curve C with equation $y = x^2 - 4x + 5$
- the line l with equation $y = 2$

The curve C intersects the y-axis at the point D.

(a) Write down the coordinates of D.

(1)

The curve C intersects the line l at the points E and F, as shown in Figure 3.

(b) Find the x coordinate of E and the x coordinate of F.

(2)

Shown shaded in Figure 3 is

- the region R_1 which is bounded by C, l and the y-axis
- the region R_2 which is bounded by C and the line segments EF and DF

Given that $\dfrac{\text{area of } R_1}{\text{area of } R_2} = k$, where k is a constant,

(c) use algebraic integration to find the exact value of k, giving your answer as a simplified fraction.

(5)

Question 10

10 marks | Integration

10. The curve C has equation

$$y = \frac{(x-k)^2}{\sqrt{x}} \qquad x > 0$$

where k is a **positive** constant.

(a) Show that

$$\int_1^{16} \frac{(x-k)^2}{\sqrt{x}}\,\mathrm{d}x = ak^2 + bk + \frac{2046}{5}$$

where a and b are integers to be found.

(5)

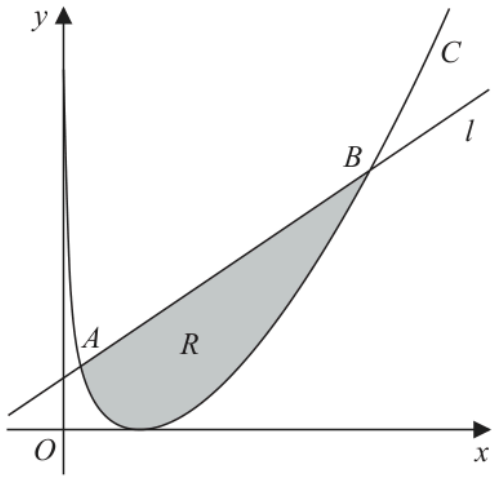


Figure 1

Figure 1 shows a sketch of the curve C and the line l.

Given that l intersects C at the point $A(1, 9)$ and at the point $B(16, q)$ where q is a constant,

(b) show that $k = 4$

(2)

The region R, shown shaded in Figure 1, is bounded by C and l

Using the answers to parts (a) and (b),

(c) find the area of region R

(3)

Question 6

6 marks | Integration

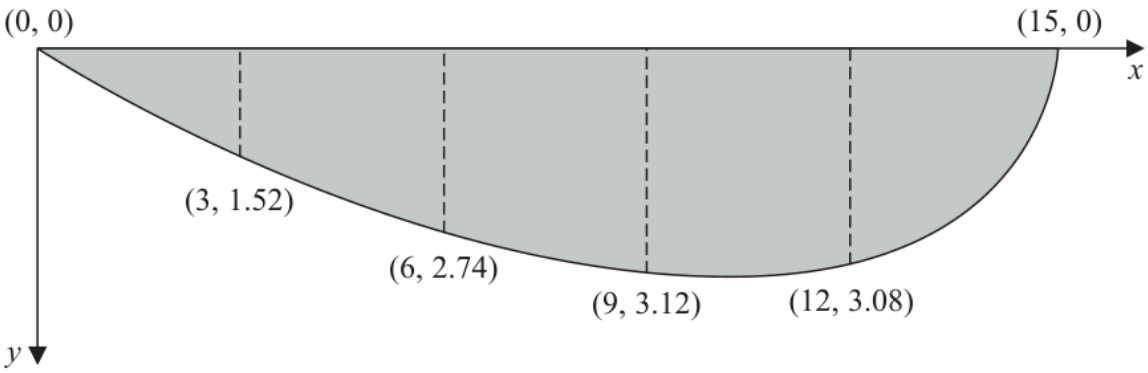


Figure 1

A river is being studied.

At one particular place, the river is 15 m wide.

The depth, y metres, of the river is measured at a point x metres from one side of the river.

Figure 1 shows a plot of the cross-section of the river and the coordinate values (x, y)

(a) Use the trapezium rule with all the y values given in Figure 1 to estimate the cross-sectional area of the river.

(3)

The water in the river is modelled as flowing at a constant speed of $1.5\,\text{m}\,\text{s}^{-1}$ across the whole of the cross-section.

(b) Use the model and the answer to part (a) to estimate the volume of water flowing through this section of the river each minute, giving your answer in m^3 to 2 significant figures.

(2)

Assuming the model,

(c) state, giving a reason for your answer, whether your answer for part (b) is an overestimate or an underestimate of the true volume of water flowing through this section of the river each minute.

(1)

Question 6

9 marks | Integration

6. (a) Sketch the curve with equation

$$y = a^x + 4$$

where a is a positive constant greater than 1

On your sketch, show

- the coordinates of the point of intersection of the curve with the y-axis
- the equation of the asymptote of the curve

(3)

x	2	2.3	2.6	2.9	3.2	3.5
y	0	0.3246	0.8629	1.6643	2.7896	4.3137

The table shows corresponding values of x and y for

$$y = 2^x - 2x$$

with the values of y given to 4 decimal places as appropriate.

Using the trapezium rule with all the values of y in the given table,

(b) obtain an estimate for $\int_2^{3.5} \left(2^x - 2x\right) \mathrm{d}x$, giving your answer to 2 decimal places.

(3)

(c) Using your answer to part (b) and making your method clear, estimate

(i) $\int_2^{3.5} \left(2^x + 2x\right) \mathrm{d}x$

(ii) $\int_2^{3.5} \left(2^{x+1} - 4x\right) \mathrm{d}x$

(3)

Question 8

8 marks | Integration

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

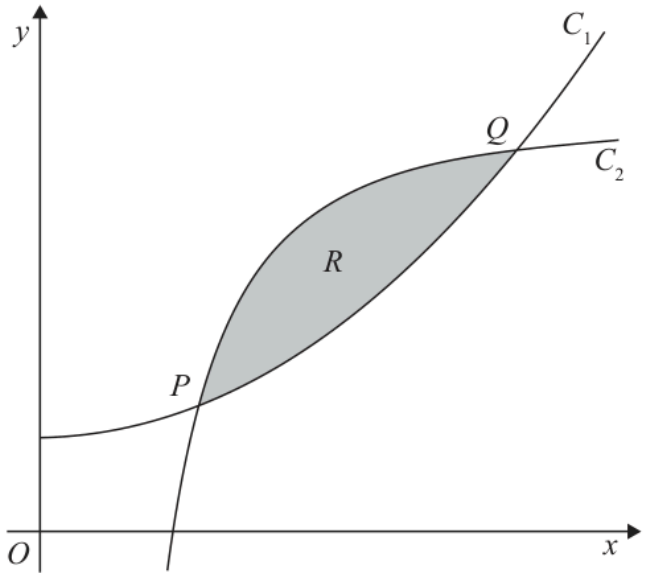


Figure 1

Figure 1 shows a sketch of part of the curve C_1 with equation

$$y = x^2 + 3 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 13 - \frac{9}{x^2} \qquad x > 0$$

The curves C_1 and C_2 intersect at the points P and Q as shown in Figure 1.

(a) Use algebra to find the x coordinate of P and the x coordinate of Q.

(4)

The finite region R, shown shaded in Figure 1, is bounded by C_1 and C_2

(b) Use algebraic integration to find the exact area of R.

(4)

Question 9

12 marks | Integration

9.

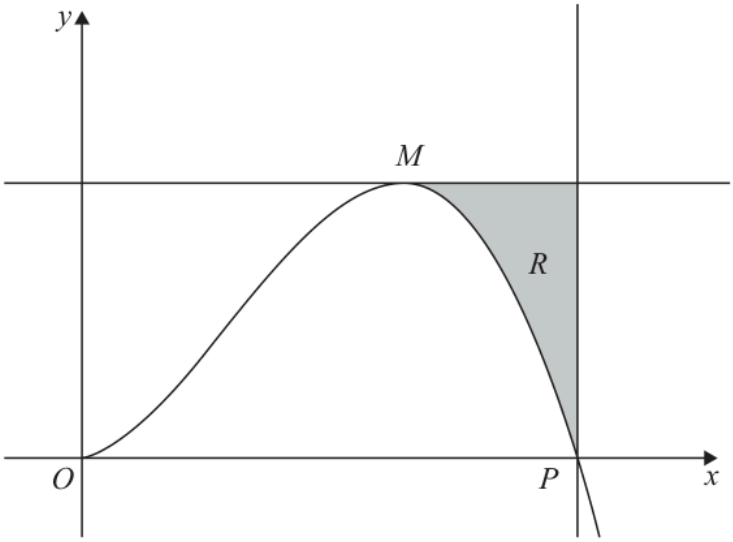


Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \frac{9x^2\left(5 - \sqrt{x}\right)}{5} \qquad x \geqslant 0$$

The curve has a turning point at the point M, as shown in Figure 3.

(a) Using calculus, find the coordinates of M.

(5)

The curve crosses the x-axis at the point P, as shown in Figure 3.

(b) Use algebra to find the x coordinate of P.

(2)

The finite region R, shown shaded in Figure 3, is bounded by the curve, the line through M parallel to the x-axis and the line through P parallel to the y-axis.

(c) Use algebraic integration to find the area of R, giving your answer to one decimal place.

(5)

Question 7

10 marks | Integration

7.

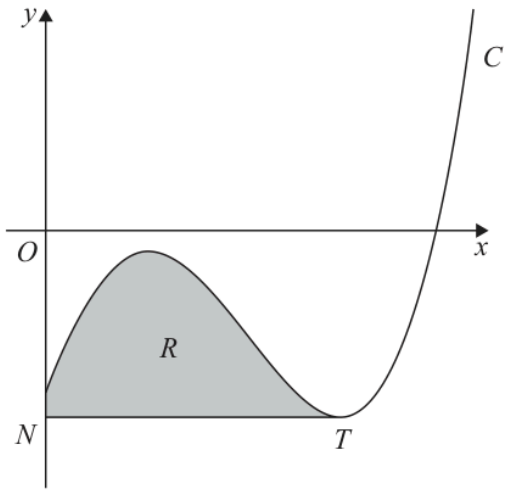


Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve C with equation

$$y = x^3 - 4x^{\frac{5}{2}} - kx^{\frac{1}{2}} + 28x - 44 \qquad x \geqslant 0$$

where k is a positive constant.

(a) Find $\dfrac{dy}{dx}$ in simplest form.

(2)

The point T, shown in Figure 3, is a minimum stationary point on C.

Given that the x coordinate of T is 9

(b) show that $k = 6$

(2)

The line through T parallel to the x-axis meets the y-axis at the point N.

The finite region R, shown shaded in Figure 3, is bounded by C, the y-axis and the line segment NT.

(c) Use algebraic integration to find the area of R, giving the answer to 3 significant figures.

(6)

Question 9

10 marks | Integration | Main topic: Circles

9. A circle C has equation

$$(x - k)^2 + (y - 2k)^2 = k + 7$$

where k is a positive constant.

(a) Write down, in terms of k,

(i) the coordinates of the centre of C,

(ii) the radius of C.

(2)

Given that the point $P(2, 3)$ lies on C

(b) (i) show that $5k^2 - 17k + 6 = 0$

(ii) hence find the possible values of k.

(3)

The tangent to the circle at P intersects the x-axis at point T.

Given that $k < 2$

(c) calculate the exact area of triangle OPT.

(5)

Question 6

7 marks | Integration | Main topic: Circles

6. A circle has equation

$$x^2 - 6x + y^2 + 8y + k = 0$$

where k is a positive constant.

Given that the x-axis is a tangent to this circle,

(a) find the value of k.

(3)

The circle meets the coordinate axes at the points R, S and T.

(b) Find the exact area of the triangle RST.

(4)

Question 9

10 marks | Integration

Main topic: Applications of Differentiation

9.

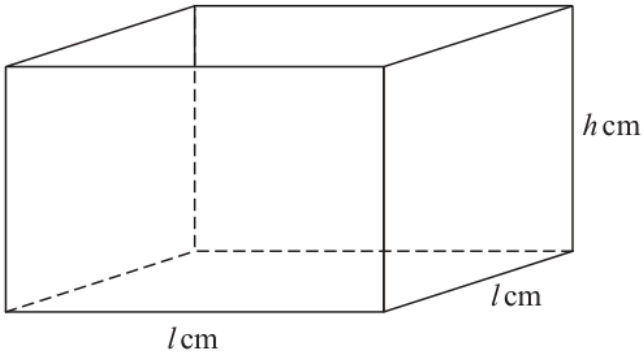


Figure 3

Figure 3 shows a sketch of a square based, open top box.

The height of the box is h cm, and the base edges each have length l cm.

Given that the volume of the box is $250\,000\,\text{cm}^3$

(a) show that the external surface area, $S\,\text{cm}^2$, of the box is given by

$$S = \frac{250\,000}{h} + 2000\sqrt{h}$$

(3)

(b) Use algebraic differentiation to show that S has a stationary point when $h = 250^k$ where k is a rational constant to be found.

(5)

(c) Justify by further differentiation that this value of h gives the minimum external surface area of the box.

(2)

(Total 10 marks)

TOTAL FOR PAPER IS 75 MARKS

Question 8

10 marks | Integration

Main topic: Applications of Differentiation

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

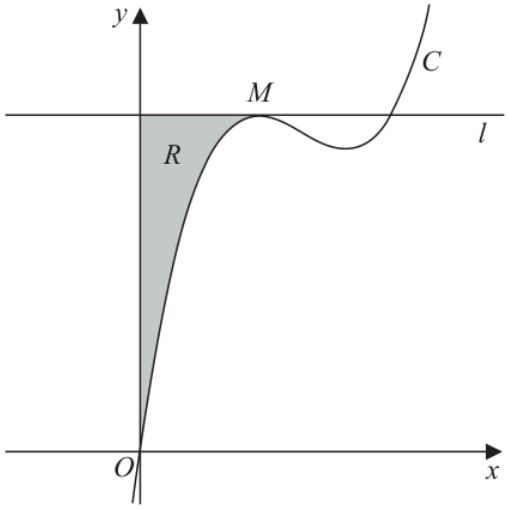


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{4}{3}x^3 - 11x^2 + kx \qquad \text{where } k \text{ is a constant}$$

The point M is the maximum turning point of C and is shown in Figure 2.

Given that the x coordinate of M is 2

(a) show that $k = 28$

(3)

(b) Determine the range of values of x for which y is increasing.

(2)

The line l passes through M and is parallel to the x-axis.

The region R, shown shaded in Figure 2, is bounded by the curve C, the line l and the y-axis.

(c) Find, by algebraic integration, the exact area of R.

(5)

Question 9

12 marks | Integration | Main topic: Circles

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

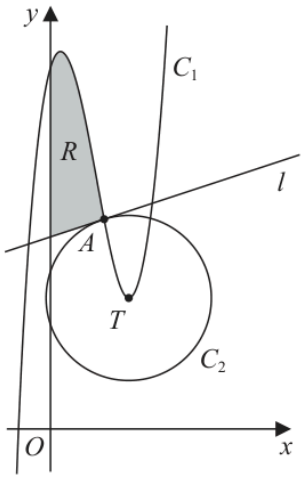


Figure 3

Figure 3 shows

- the curve C_1 with equation $y = x^3 - 5x^2 + 3x + 14$
- the circle C_2 with centre T

The point T is the minimum turning point of C_1

Using Figure 3 and calculus,

(a) find the coordinates of T

(3)

The curve C_1 intersects the circle C_2 at the point A with x coordinate 2

(b) Find an equation of the circle C_2

(3)

The line l shown in Figure 3, is the tangent to circle C_2 at A

(c) Show that an equation of l is

$$y = \frac{1}{3}x + \frac{22}{3}$$

(3)

The region R, shown shaded in Figure 3, is bounded by C_1, l and the y-axis.

(d) Find the exact area of R.

(3)

Question 9

12 marks | Integration

Main topic: Applications of Differentiation

9.

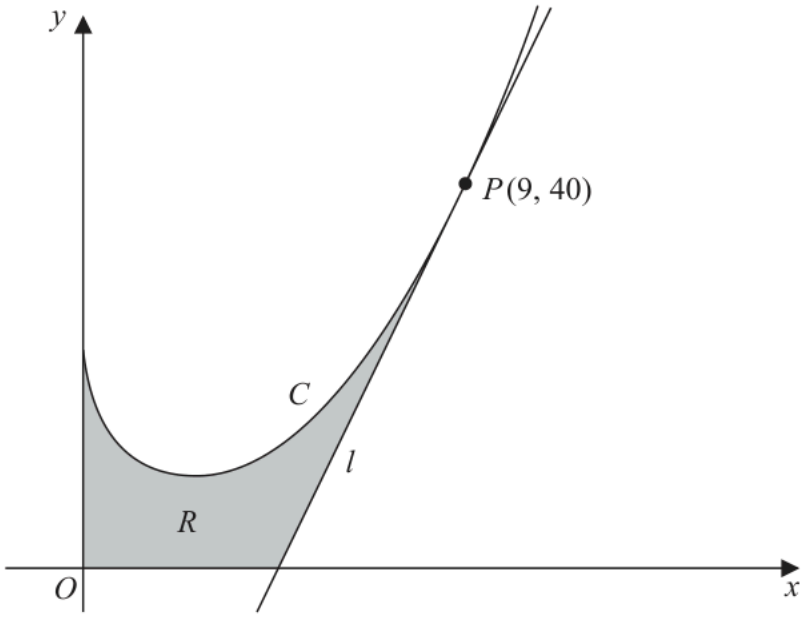


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{2}{3}x^2 - 9\sqrt{x} + 13 \qquad x \geqslant 0$$

(a) Find, using calculus, the range of values of x for which y is increasing.

(4)

The point P lies on C and has coordinates (9, 40).

The line l is the tangent to C at the point P.

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the line l, the x-axis and the y-axis.

(b) Find, using calculus, the exact area of R.

(8)

Question 10

9 marks | Integration

Main topic: Applications of Differentiation

10. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

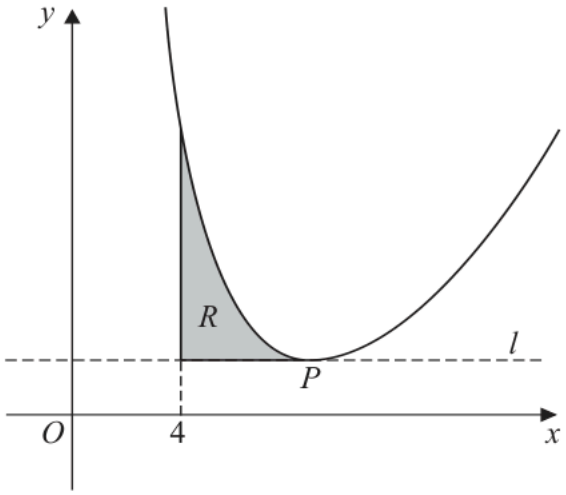


Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = \frac{1}{2}x^2 + \frac{1458}{\sqrt{x^3}} - 74 \qquad x > 0$$

The point P is the only stationary point on the curve.

(a) Use calculus to show that the x coordinate of P is 9

(4)

The line l passes through the point P and is parallel to the x-axis.

The region R, shown shaded in Figure 2, is bounded by the curve, the line l and the line with equation $x = 4$

(b) Use algebraic integration to find the exact area of R.

(5)

Question 9

8 marks | Integration

Main topic: Applications of Differentiation

9.

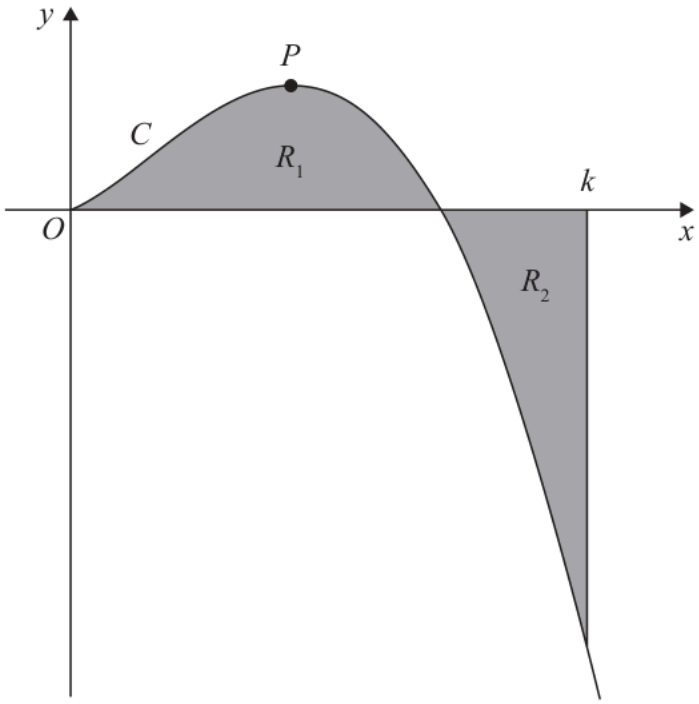


Figure 1

Figure 1 is a sketch of the curve C with equation

$$y = 2x^{\frac{3}{2}}(4 - x) \qquad x \geqslant 0$$

The point P is the stationary point of C.

(a) Find, using calculus, the x coordinate of P.

(4)

The region R_1, shown shaded in Figure 1, is bounded by C and the x-axis.

The region R_2, also shown shaded in Figure 1, is bounded by C, the x-axis and the line with equation $x = k$, where k is a constant.

Given that the area of R_1 is equal to the area of R_2

(b) find, using calculus, the exact value of k.

(4)

Question 10

9 marks | Integration

Main topic: Applications of Differentiation

10.

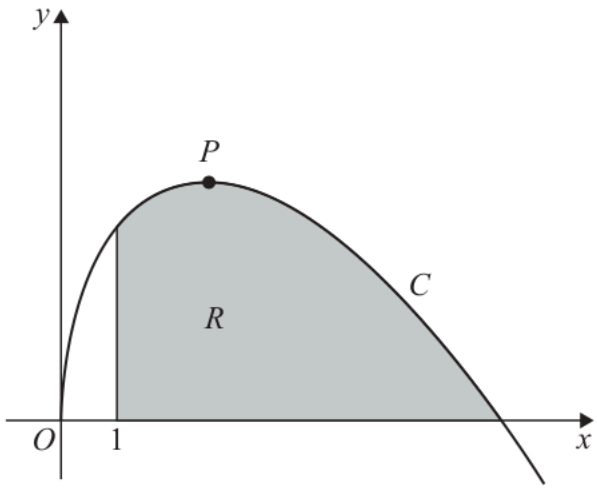


Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of part of the curve C with equation

$$y = \frac{9x - x^2}{2\sqrt{x}} \qquad x > 0$$

The point P is the stationary point on C.

(a) Find, using calculus, the x coordinate of P.

(4)

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the x-axis and the line with equation $x = 1$

(b) Using calculus, calculate the exact area of R.

(5)

Question 7

8 marks | Integration | Main topic: Laws of Logarithms

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) The table below shows values of x and y, where $y = \log_{10}(x + 5)$, for x values between -1 and 4

x	-1	0	1	2	3	4
$y = \log_{10}(x + 5)$	$\log_{10} 4$	$\log_{10} 5$	$\log_{10} 6$	$\log_{10} 7$	$\log_{10} 8$	$\log_{10} 9$

Using the trapezium rule with all the y values in the given table, show that

$$\int_{-1}^{4} \log_{10}(x + 5)\,dx \approx \log_{10} k$$

where k is an integer to be found.

(3)

(ii) Find the value of a such that

$$2\log_5(5 - a) - \log_5(a + 25) = 1$$

(5)

Question 10

8 marks | Integration

Main topic: Applications of Differentiation

10.

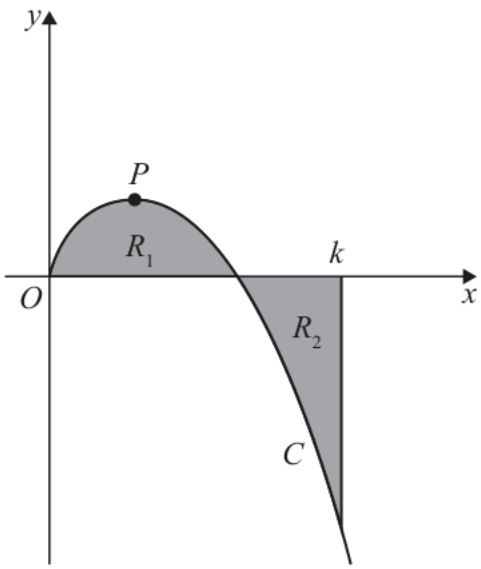


Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve C with equation

$$y = \frac{\sqrt{x}\left(100 - x^2\right)}{40} \qquad x \geqslant 0$$

The point P is a stationary point on C.

(a) Use algebraic differentiation to find the exact x coordinate of P.

(4)

The region R_1, shown shaded in Figure 2, is bounded by C and the x-axis.

The region R_2, also shown shaded in Figure 2, is bounded by C, the x-axis and the line with equation $x = k$, where k is a constant.

Given that the area of R_1 is equal to the area of R_2

(b) use algebraic integration to find the exact value of k.

(4)