EA ELITE IAL MATHEMATICS

EDEXCEL INTERNATIONAL A LEVEL

WMA12 Pure 2 Classified Questions

190 unique questions grouped by primary topic, with cross-topic placements where a question also belongs in another chapter.

285
topic placements

WMA12
Pure 2

Question bank
standalone IAL route

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P2

TOPIC

Proof

WMA12/01 JANUARY 2021

4 marks

Question 5

Proof

5. (i) Use algebra to prove that for all $x \geqslant 0$

$$3x + 1 \geqslant 2\sqrt{3x}$$

(3)

(ii) Show that the following statement is not true.

"The sum of three consecutive prime numbers is always a multiple of 5"

(1)

WMA12/01 MAY/JUNE 2021

5 marks

Question 3

Proof

3. (i) Prove that for all single digit prime numbers, p,

$$p^3 + p \text{ is a multiple of } 10$$

(2)

(ii) Show, using algebra, that for $n \in \mathbb{N}$

$$(n + 1)^3 - n^3 \text{ is not a multiple of } 3$$

(3)

Question 9

4 marks

Proof

9. (a) Prove that for all positive values of x and y,

$$\frac{x+y}{2} \geqslant \sqrt{xy}$$

(3)

(b) Prove by counter-example that this inequality does not hold when x and y are both negative.

(1)

WMA12/01 JANUARY 2022

Question 10

5 marks

Proof

10. (i) Prove by counter example that the statement

"if p is a prime number then $2p + 1$ is also a prime number"

is not true.

(1)

(ii) Use proof by exhaustion to prove that if n is an integer then

$$5n^2 + n + 12$$

is always even.

(4)

(Total 5 marks)

Question 3

7 marks

Proof

3. (i) Show that the following statement is **false**:

$$\text{“}(n+1)^3 - n^3 \text{ is prime for all } n \in \mathbb{N}\text{”}$$

(2)

(ii) Given that the points $A(1, 0)$, $B(3, -10)$ and $C(7, -6)$ lie on a circle, prove that AB is a diameter of this circle.

(5)

WMA12/01 OCTOBER 2022

Question 1

3 marks

Proof

1. Given that a, b and c are integers greater than 0 such that

- $c = b + 2$
- $a + b + c = 10$

Prove, by exhaustion, that the product of a, b and c is always even.

You may use the table below to illustrate your answer.

(3)

You may not need to use all rows of this table.

a	b	c
	1	
	2	

Question 10

4 marks

Proof

10. A student was asked to prove by exhaustion that

if n is an integer then $2n^2 + n + 1$ is **not** divisible by 3

The start of the student's proof is shown in the box below.

Consider the case when $n = 3k$ $2n^2 + n + 1 = 18k^2 + 3k + 1 = 3(6k^2 + k) + 1$ which is not divisible by 3

Complete this proof.

(4)

Question 8

5 marks

Proof

8. (i) A student writes the following statement:

"When a and b are consecutive **prime** numbers, $a^2 + b^2$ is never a multiple of 10"

Prove by counter example that this statement is **not** true.

(2)

(ii) Given that x and y are even integers greater than 0 and less than 6, prove by exhaustion, that

$$1 < x^2 - \frac{xy}{4} < 15$$

(3)

WMA12/01 OCTOBER 2023

3 marks

Question 1

Proof

1. Given that a, b and c are integers greater than 0 such that

- $c = 3a + 1$
- $a + b + c = 15$

prove, by exhaustion, that the product abc is always a multiple of 4

You may use the table below to illustrate your answer.

(3)

You may not need to use all rows of this table.

a	b	c	abc

Question 8

6 marks

Proof

8. (i) Use a counter example to show that the following statement is **false**

"$n^2 + 3n + 1$ is prime for all $n \in \mathbb{N}$"

(2)

(ii) Use algebra to prove by exhaustion that for all $n \in \mathbb{N}$

"$n^2 - 2$ is **not** a multiple of 4"

(4)

Question 5

6 marks

Proof

5. **In this question you must show detailed reasoning.**

(a) Given that x and y are positive numbers such that

$$(x-y)^3 > x^3 - y^3$$

prove that

$$y > x$$

(4)

(b) Using a counter example, show that the result in part (a) is not true for all real numbers.

(2)

Question 11

6 marks

Proof

11. (i) Prove by counter example that the statement

"If n is a prime number then $3^n + 2$ is also a prime number."

is false.

(2)

(ii) Use proof by exhaustion to prove that if m is an integer that is **not** divisible by 3, then

$$m^2 - 1$$

is divisible by 3

(4)

Question 8

5 marks

Proof

8. (i) A student states

"If x and y are irrational numbers, $x \neq y$, then xy is also irrational."

Show, by counter example, that this statement is not always true.

(1)

(ii) Prove, using algebra, that for all odd integers n, the value of the expression

$$n^3 + 3n + 2$$

is always even but never a multiple of 4

(4)

Question 10

4 marks

Proof

10: **In this question you must show detailed reasoning.**

Use algebra to prove by exhaustion that,

for all positive integers m that are **not** multiples of 3, the value of

$$m^2 + 3m + 2$$

is always a multiple of 3

(4)

WMA12/01 JANUARY 2026

Question 6

8 marks

Proof

6. (i) Given that p and q are consecutive odd numbers, where $p > q > 0$, prove that

$$p^2 - q^2$$

is a multiple of 8

(4)

(ii) The curve C has equation

$$y = x^3 + 12x^2 + 49x + 2$$

Prove that C has no stationary points.

(4)

WMA12/01 MAY/JUNE 2023

Question 7

8 marks

Proof

Also in Proof

Primary: Applications of Differentiation

7. The height of a river above a fixed point on the riverbed was monitored over a 7-day period.

The height of the river, H metres, t days after monitoring began, was given by

$$H = \frac{\sqrt{t}}{20}(20 + 6t - t^2) + 17 \qquad 0 \leqslant t \leqslant 7$$

Given that H has a stationary value at $t = \alpha$

(a) use calculus to show that α satisfies the equation

$$5\alpha^2 - 18\alpha - 20 = 0$$

(5)

(b) Hence find the value of α, giving your answer to 3 decimal places.

(1)

(c) Use further calculus to prove that H is a maximum at this value of α.

(2)

Question 7

6 marks

Proof

Also in Proof

Primary: Circles

7. The circle C_1 has equation

$$x^2 + y^2 + 8x - 10y = 29$$

(a) (i) Find the coordinates of the centre of C_1

(ii) Find the exact value of the radius of C_1

(3)

In part (b) you must show detailed reasoning.

The circle C_2 has equation

$$(x-5)^2 + (y+8)^2 = 52$$

(b) Prove that the circles C_1 and C_2 neither touch nor intersect.

(3)

Question 9

10 marks

Proof

Also in Proof

Primary: Laws of Logarithms

9. **In this question you must show detailed reasoning.**
Solutions relying on calculator technology are not acceptable.

(i) Solve

$$2\log_3(4x + 5) - \log_3(x + 3) = 2$$

(5)

(ii) Given that $a > 0$, $b > 0$ and

$$\log_{10} a + \log_{10} b = \log_{10}(a + b)$$

(a) prove that $a = \dfrac{b}{b-1}$

(3)

(b) Hence write down the full restriction on the value of b, giving a reason for your answer.

(2)

TOPIC

Polynomials

WMA12/01 OCTOBER 2019

Question 4

8 marks

Polynomials

4. $\mathrm{f}(x) = (x - 3)(3x^2 + x + a) - 35$ where a is a constant

(a) State the remainder when $\mathrm{f}(x)$ is divided by $(x - 3)$.

(1)

Given $(3x - 2)$ is a factor of $\mathrm{f}(x)$,

(b) show that $a = -17$

(2)

(c) Using algebra and showing each step of your working, fully factorise $\mathrm{f}(x)$.

(5)

WMA12/01 JANUARY 2020

Question 3

8 marks

Polynomials

3. $$f(x) = 6x^3 + 17x^2 + 4x - 12$$

(a) Use the factor theorem to show that $(2x + 3)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, write $f(x)$ as a product of three linear factors.

(4)

(c) Solve, for $\frac{\pi}{2} < \theta < \pi$, the equation

$$6\tan^3\theta + 17\tan^2\theta + 4\tan\theta - 12 = 0$$

giving your answers to 3 significant figures.

(2)

WMA12/01 JANUARY 2021

6 marks

Question 1

Polynomials

1. $$f(x) = x^4 + ax^3 - 3x^2 + bx + 5$$

where a and b are constants.

When $f(x)$ is divided by $(x + 1)$, the remainder is 4

(a) Show that $a + b = -1$

(2)

When $f(x)$ is divided by $(x - 2)$, the remainder is -23

(b) Find the value of a and the value of b.

(4)

Question 4

8 marks

Polynomials

4.

$$f(x) = (x^2 - 2)(2x - 3) - 21$$

(a) State the value of the remainder when $f(x)$ is divided by $(2x - 3)$

(1)

(b) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$

(2)

(c) Hence,

(i) factorise $f(x)$

(ii) show that the equation $f(x) = 0$ has only one real root.

(5)

WMA12/01 JANUARY 2022

Question 5

8 marks

Polynomials

5. $$f(x) = 3x^3 + Ax^2 + Bx - 10$$

where A and B are integers.

Given that

- when $f(x)$ is divided by $(x - 1)$ the remainder is k
- when $f(x)$ is divided by $(x + 1)$ the remainder is $-10k$
- k is a constant

(a) show that

$$11A + 9B = 83$$

(3)

Given also that $(3x - 2)$ is a factor of $f(x)$,

(b) find the value of A and the value of B.

(3)

(c) Hence find the quadratic expression $g(x)$ such that

$$f(x) = (3x - 2)\,g(x)$$

(2)

WMA12/01 JANUARY 2023

Question 5

9 marks

Polynomials

5. $$f(x) = x^3 + (p + 3)x^2 - x + q$$

where p and q are constants and $p > 0$

Given that $(x - 3)$ is a factor of $f(x)$

(a) show that

$$9p + q = -51$$

(2)

Given also that when $f(x)$ is divided by $(x + p)$ the remainder is 9

(b) show that

$$3p^2 + p + q - 9 = 0$$

(2)

(c) Hence find the value of p and the value of q.

(3)

(d) Hence find a quadratic expression $g(x)$ such that

$$f(x) = (x - 3)g(x)$$

(2)

Question 2

6 marks

Polynomials

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = 4x^3 - 8x^2 + 5x + a$$

where a is a constant.

Given that $(2x - 3)$ is a factor of $f(x)$,

(a) use the factor theorem to show that $a = -3$

(2)

(b) Hence show that the equation $f(x) = 0$ has only one real root.

(4)

Question 4

9 marks

Polynomials

4. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = 4x^3 + ax^2 - 29x + b$$

where a and b are constants.

Given that $(2x + 1)$ is a factor of $f(x)$,

(a) show that

$$a + 4b = -56$$

(2)

Given also that when $f(x)$ is divided by $(x - 2)$ the remainder is -25

(b) find a second simplified equation linking a and b.

(2)

(c) Hence, using algebra and showing your working,

(i) find the value of a and the value of b,

(ii) fully factorise $f(x)$.

(5)

WMA12/01 JANUARY 2024

Question 1

3 marks

Polynomials

1. $$f(x) = ax^3 + 3x^2 - 8x + 2 \qquad \text{where } a \text{ is a constant}$$

Given that when $f(x)$ is divided by $(x - 2)$ the remainder is 3, find the value of a.

(3)

Question 4

8 marks

Polynomials

4. $$\mathrm{f}(x) = (x - 2)(2x^2 + 5x + k) + 21$$

where k is a constant.

(a) State the remainder when $\mathrm{f}(x)$ is divided by $(x - 2)$

(1)

Given that $(2x - 1)$ is a factor of $\mathrm{f}(x)$

(b) show that $k = 11$

(2)

(c) Hence

(i) fully factorise $\mathrm{f}(x)$,

(ii) find the number of real solutions of the equation

$$\mathrm{f}(x) = 0$$

giving a reason for your answer.

(5)

Question 3

7 marks

Polynomials

3. $$f(x) = 2x^3 - x^2 + Ax + B$$

where A and B are integers.

Given that when $f(x)$ is divided by $(x + 3)$ the remainder is 55

(a) show that

$$3A - B = -118$$

(2)

Given also that $(2x - 5)$ is a factor of $f(x)$,

(b) find the value of A and the value of B.

(3)

(c) Hence find the quotient when $f(x)$ is divided by $(x - 7)$

(2)

WMA12/01 JANUARY 2025

Question 5

9 marks

Polynomials

5. $$f(x) = 3x^3 + ax^2 - 10x + b$$

where a and b are constants.

Given that $(3x - 4)$ is a factor of $f(x)$,

(a) show that $16a + 9b = 56$

(2)

Given further that when $f(x)$ is divided by $(x - 2)$ the remainder is b,

(b) find the value of a and the value of b.

(4)

(c) Hence, using algebra, fully factorise $f(x)$.

(3)

WMA12/01 MAY/JUNE 2025

Question 4

9 marks

Polynomials

4: The function $f(x)$ is defined by

$$f(x) = ax^3 + bx^2 + 5x - 3$$

where a and b are constants.

Given that $(x + 3)$ is a factor of $f(x)$,

(a) show that

$$b = 3a + 2$$

(2)

Given further that when $f(x)$ is divided by $(2x - 1)$ the remainder is $\frac{7}{4}$

(b) find the value of a and the value of b.

(4)

(c) Using algebra, find the quotient and the remainder when $f(x)$ is divided by $(x - 2)$

(3)

Question 4

12 marks

Polynomials

4. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

$$f(x) = 4x^3 + 13x^2 - 10x + 8$$

(a) When f(x) is divided by (x – 2) the remainder is R and the quotient is $Q(x)$.

(i) Find $Q(x)$.

(ii) Find R.

(4)

(b) (i) Use the factor theorem to show that (x + 4) is a factor of f(x).

(ii) Hence prove, using algebra, that the equation f(x) = 0 has only one real solution.

(5)

(c) Find the range of values of x for which f(x) is decreasing.

(3)

WMA12/01 JANUARY 2026

Question 3

7 marks

Polynomials

3. (i) Given that

- $f(x) = 4x^3 + 6x + k$, where k is a constant
- $(x + 2)$ is a factor of $f(x)$

find the remainder when $f(x)$ is divided by $(x - 5)$

(4)

(ii) Find the remainder, R, and the quotient, $Q(x)$, when

$$6x^3 - 15x^2 - 21x + 8$$

is divided by $(2x + 3)$

(3)

Question 7

10 marks

Polynomials

Also in Polynomials

Primary: Laws of Logarithms

7. (a) Given that

$$3\log_3(2x-1) = 2 + \log_3(14x-25)$$

show that

$$2x^3 - 3x^2 - 30x + 56 = 0$$

(4)

(b) Show that -4 is a root of this cubic equation.

(2)

(c) Hence, using algebra and showing each step of your working, solve

$$3\log_3(2x-1) = 2 + \log_3(14x-25)$$

(4)

WMA12/01 MAY/JUNE 2022

Question 7

7 marks

Polynomials

Also in Polynomials

Primary: Integration

7. $$f(x) = Ax^3 + 6x^2 - 4x + B$$

where A and B are constants.

Given that

- $(x + 2)$ is a factor of $f(x)$
- $\int_3^5 f(x)\,dx = 176$

find the value of A and the value of B.

(7)

WMA12/01 MAY/JUNE 2022

Question 10

12 marks

Polynomials

Also in Polynomials

Primary: Circles

10. The circle C has centre $X(3, 5)$ and radius r

The line l has equation $y = 2x + k$, where k is a constant.

(a) Show that l and C intersect when

$$5x^2 + (4k - 26)x + k^2 - 10k + 34 - r^2 = 0$$

(3)

Given that l is a tangent to C,

(b) show that $5r^2 = (k + p)^2$, where p is a constant to be found.

(3)

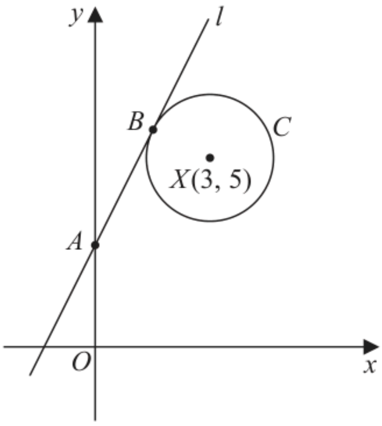

Figure 2

The line l

- cuts the y-axis at the point A
- touches the circle C at the point B

as shown in Figure 2.

Given that $AB = 2r$

(c) find the value of k

(6)

(Total 12 marks)

WMA12/01 JANUARY 2023

Question 9

8 marks

Polynomials

Also in Polynomials

Primary: Integration

9. **In this question you must show all stages of your working.**

Solutions based entirely on calculator technology are not acceptable.

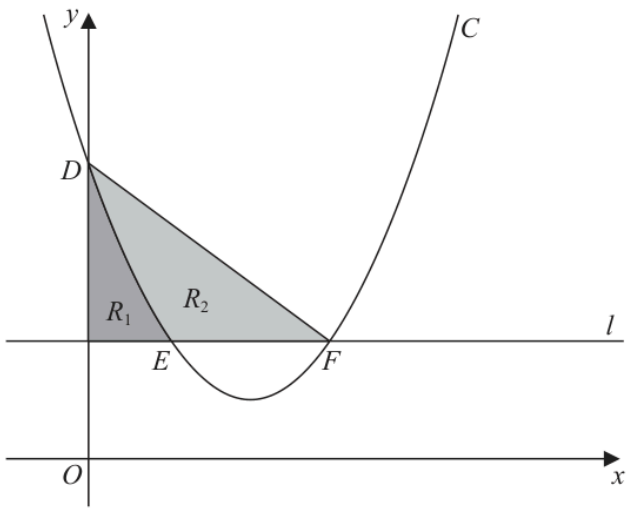

Figure 3

Figure 3 shows

- the curve C with equation $y = x^2 - 4x + 5$
- the line l with equation $y = 2$

The curve C intersects the y-axis at the point D.

(a) Write down the coordinates of D.

(1)

The curve C intersects the line l at the points E and F, as shown in Figure 3.

(b) Find the x coordinate of E and the x coordinate of F.

(2)

Shown shaded in Figure 3 is

- the region R_1 which is bounded by C, l and the y-axis
- the region R_2 which is bounded by C and the line segments EF and DF

Given that $\dfrac{\text{area of } R_1}{\text{area of } R_2} = k$, where k is a constant,

(c) use algebraic integration to find the exact value of k, giving your answer as a simplified fraction.

(5)

Question 6

8 marks

Polynomials

Also in Polynomials

Primary: Laws of Logarithms

6. (a) Given that

$$2\log_4(x+3) + \log_4 x = \log_4(4x+2) + \frac{1}{2}$$

show that

$$x^3 + 6x^2 + x - 4 = 0$$

(4)

(b) Given also that –1 is a root of the equation

$$x^3 + 6x^2 + x - 4 = 0$$

(i) use algebra to find the other two roots of the equation.

(3)

(ii) Hence solve

$$2\log_4(x+3) + \log_4 x = \log_4(4x+2) + \frac{1}{2}$$

(1)

WMA12/01 MAY/JUNE R 2024

Question 6

9 marks

Polynomials

Also in Polynomials

Primary: Applications of Differentiation

6. $$f(x) = 4x^3 + px^2 + 8x + q$$

where p and q are constants.

Given that

- $(2x + 3)$ is a factor of $f(x)$
- $f(x)$ has a remainder of -5 when divided by $(x + 2)$

(a) (i) show that $p = 10$

(ii) find the value of q.

(5)

(b) Hence find the range of values of x for which $f(x)$ is decreasing.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

WMA12/01 MAY/JUNE R 2024

Question 9

9 marks

Polynomials

Also in Polynomials

Primary: Laws of Logarithms

9. Given that

$$3\log_2(t+4) - 2\log_2(t-2) = 7$$

(a) verify that $t = 4$ is a solution of the above equation,

(2)

(b) show that

$$t^3 - 116t^2 + 560t - 448 = 0$$

(3)

(c) Hence, using algebra and showing your working, solve

$$3\log_2(t+4) - 2\log_2(t-2) = 7$$

giving each answer in simplest form.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

WMA12/01 OCTOBER 2024

Question 8

8 marks

Polynomials

Also in Polynomials

Primary: Integration

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

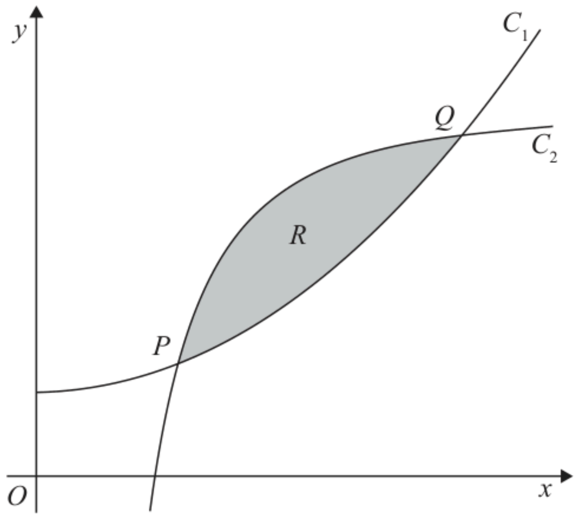

Figure 1

Figure 1 shows a sketch of part of the curve C_1 with equation

$$y = x^2 + 3 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 13 - \frac{9}{x^2} \qquad x > 0$$

The curves C_1 and C_2 intersect at the points P and Q as shown in Figure 1.

(a) Use algebra to find the x coordinate of P and the x coordinate of Q.

(4)

The finite region R, shown shaded in Figure 1, is bounded by C_1 and C_2

(b) Use algebraic integration to find the exact area of R.

(4)

TOPIC

Circles

WMA12/01 OCTOBER 2019

Question 6

7 marks

Circles

6.

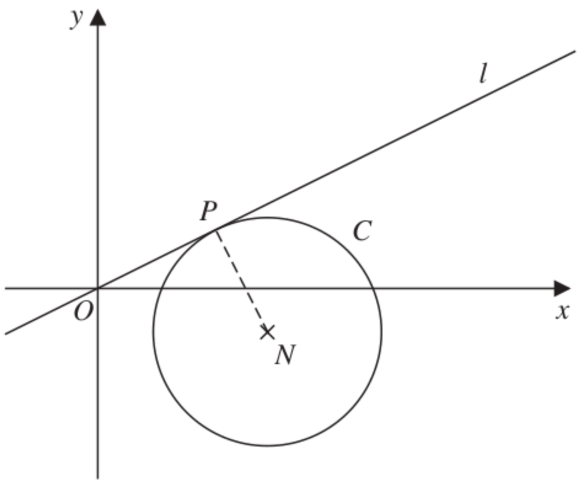

Figure 1

Figure 1 shows a sketch of a circle C with centre $N(4, -1)$.

The line l with equation $y = \frac{1}{2}x$ is a tangent to C at the point P.

Find

(a) the equation of line PN in the form $y = mx + c$, where m and c are constants,

(2)

(b) the equation of C.

(5)

Question 6

8 marks

Circles

6. The circle C has equation

$$x^2 + y^2 + 6x - 4y - 14 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C.

(3)

The line with equation $y = k$, where k is a constant, is a tangent to C.

(b) Find the possible values of k.

(2)

The line with equation $y = p$, where p is a negative constant, is a chord of C.

Given that the length of this chord is 4 units,

(c) find the value of p.

(3)

WMA12/01 JANUARY 2021

Question 9

10 marks

Circles

9. A circle C has equation

$$(x-k)^2+(y-2k)^2=k+7$$

where k is a positive constant.

(a) Write down, in terms of k,

(i) the coordinates of the centre of C,

(ii) the radius of C.

(2)

Given that the point $P(2,3)$ lies on C

(b) (i) show that $5k^2-17k+6=0$

(ii) hence find the possible values of k.

(3)

The tangent to the circle at P intersects the x-axis at point T.

Given that $k<2$

(c) calculate the exact area of triangle OPT.

(5)

Question 6

7 marks

Circles

6. A circle has equation

$$x^2 - 6x + y^2 + 8y + k = 0$$

where k is a positive constant.

Given that the x-axis is a tangent to this circle,

(a) find the value of k.

(3)

The circle meets the coordinate axes at the points R, S and T.

(b) Find the exact area of the triangle RST.

(4)

WMA12/01 OCTOBER 2021

Question 6

8 marks

Circles

6. (i) The circle C_1 has equation

$$x^2 + y^2 + 10x - 12y = k \qquad \text{where } k \text{ is a constant}$$

(a) Find the coordinates of the centre of C_1

(2)

(b) State the possible range in values for k.

(2)

(ii) The point $P(p, 0)$, the point $Q(-2, 10)$ and the point $R(8, -14)$ lie on a different circle, C_2

Given that

- p is a positive constant
- QR is a diameter of C_2

find the exact value of p.

(4)

Question 6

8 marks

Circles

6.

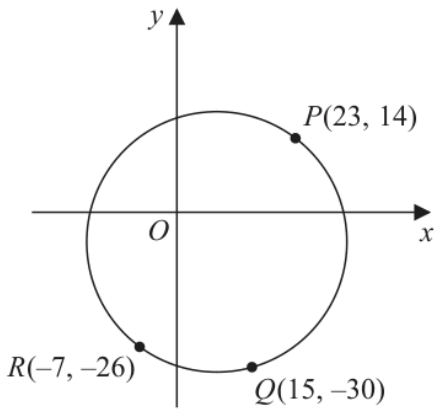

Figure 1

The points $P(23, 14)$, $Q(15, -30)$ and $R(-7, -26)$ lie on the circle C, as shown in Figure 1.

(a) Show that angle $PQR = 90°$

(2)

(b) Hence, or otherwise, find

(i) the centre of C,

(ii) the radius of C.

(3)

Given that the point S lies on C such that the distance QS is greatest,

(c) find an equation of the tangent to C at S, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

Question 10

12 marks

Circles

10. The circle C has centre $X(3, 5)$ and radius r

The line l has equation $y = 2x + k$, where k is a constant.

(a) Show that l and C intersect when

$$5x^2 + (4k - 26)x + k^2 - 10k + 34 - r^2 = 0$$

(3)

Given that l is a tangent to C,

(b) show that $5r^2 = (k + p)^2$, where p is a constant to be found.

(3)

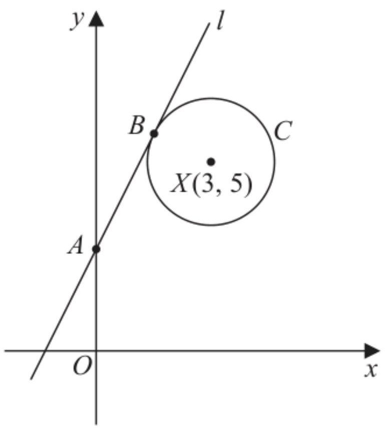

Figure 2

The line l

- cuts the y-axis at the point A
- touches the circle C at the point B

as shown in Figure 2.

Given that $AB = 2r$

(c) find the value of k

(6)

(Total 12 marks)

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Question 9

12 marks

Circles

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

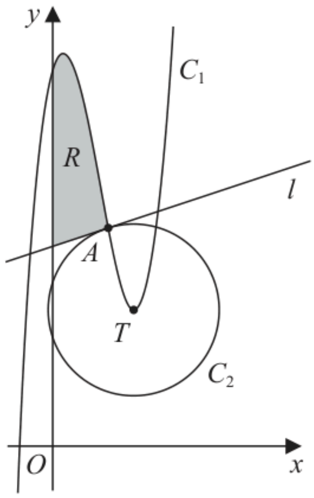

Figure 3

Figure 3 shows

- the curve C_1 with equation $y = x^3 - 5x^2 + 3x + 14$
- the circle C_2 with centre T

The point T is the minimum turning point of C_1

Using Figure 3 and calculus,

(a) find the coordinates of T

(3)

The curve C_1 intersects the circle C_2 at the point A with x coordinate 2

(b) Find an equation of the circle C_2

(3)

The line l shown in Figure 3, is the tangent to circle C_2 at A

(c) Show that an equation of l is

$$y = \frac{1}{3}x + \frac{22}{3}$$

(3)

The region R, shown shaded in Figure 3, is bounded by C_1, l and the y-axis.

(d) Find the exact area of R.

(3)

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Question 6

10 marks

Circles

6. The circle C has equation

$$x^2 + y^2 + 8x - 4y = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C.

(3)

The point P lies on C.

Given that the tangent to C at P has equation $x + 2y + 10 = 0$

(b) find the coordinates of P

(4)

(c) Find the equation of the normal to C at P, giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(3)

Question 3

7 marks

Circles

3. A circle C has centre (2, 5)

Given that the point $P(8, -3)$ lies on C

(a) (i) find the radius of C

(ii) find an equation for C

(3)

(b) Find the equation of the tangent to C at P giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(4)

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8 marks

Question 7

Circles

7.

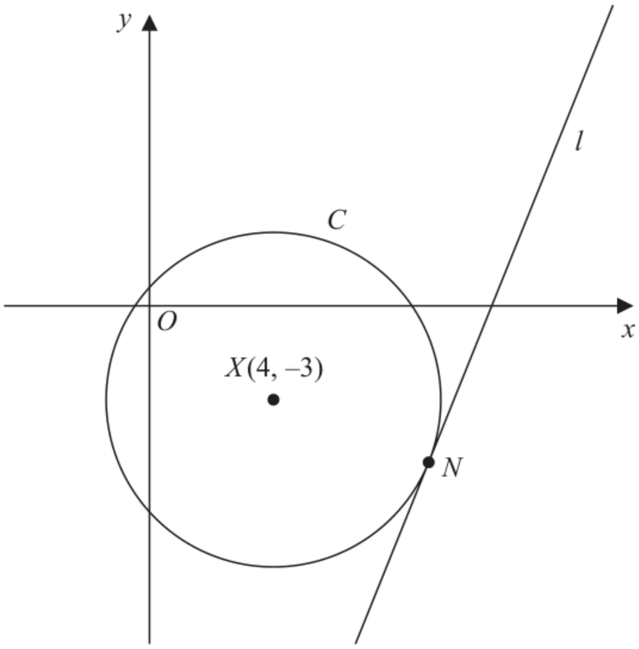

Figure 2

Figure 2 shows a sketch of

- the circle C with centre $X(4, -3)$
- the line l with equation $y = \frac{5}{2}x - \frac{55}{2}$

Given that l is the tangent to C at the point N,

(a) show that an equation for the straight line passing through X and N is

$$2x + 5y + 7 = 0$$

(3)

(b) Hence find

(i) the coordinates of N,

(ii) an equation for C.

(5)

Question 3

6 marks

Circles

3. The circle C

- has centre $A(3, 5)$
- passes through the point $B(8, -7)$

(a) Find an equation for C.

(3)

The points M and N lie on C such that MN is a chord of C.

Given that MN

- lies above the x-axis
- is parallel to the x-axis
- has length $4\sqrt{22}$

(b) find an equation for the line passing through points M and N.

(3)

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Question 7

6 marks

Circles

7. The circle C_1 has equation

$$x^2 + y^2 + 8x - 10y = 29$$

(a) (i) Find the coordinates of the centre of C_1

(ii) Find the exact value of the radius of C_1

(3)

In part (b) you must show detailed reasoning.

The circle C_2 has equation

$$(x - 5)^2 + (y + 8)^2 = 52$$

(b) Prove that the circles C_1 and C_2 neither touch nor intersect.

(3)

WMA12/01 MAY/JUNE R 2024

5 marks

Question 3

Circles

3. A circle has equation

$$x^2 + y^2 + 8x - 14y - 79 = 0$$

(a) Find

(i) the coordinates of the centre of the circle,

(ii) the radius of the circle.

(3)

Given that P is the point on the circle that is nearest the origin O,

(b) find the exact length of OP

(2)

Question 10

10 marks

Circles

10. The circle C has equation

$$x^2 + y^2 + 4x - 30y + 209 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact value of the radius of C.

(3)

The line L has equation $y = mx + 1$, where m is a constant.

Given that L is the tangent to C at the point P,

(b) show that

$$2m^2 - 7m - 22 = 0$$

(3)

(c) Hence find the possible pairs of coordinates of P.

(4)

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Question 6

8 marks

Circles

6.

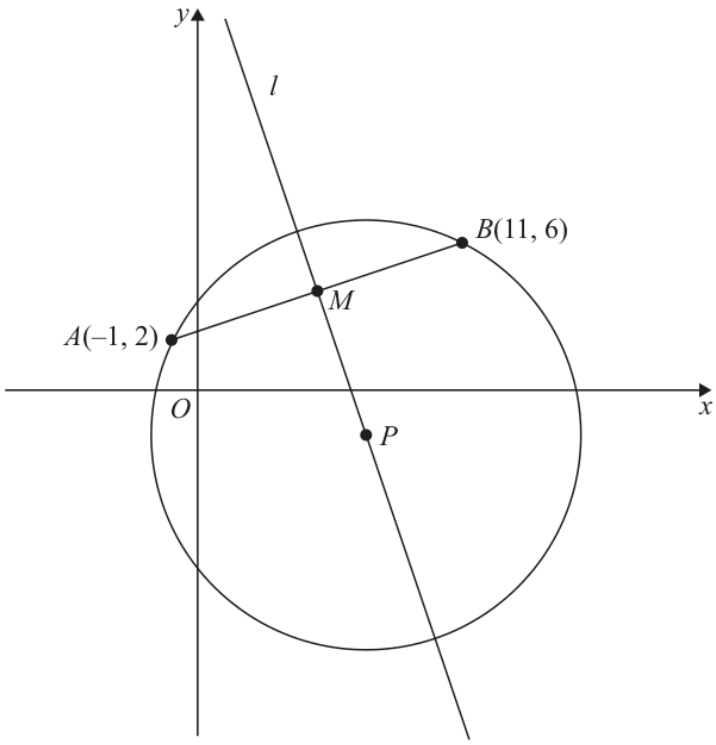

Figure 2

The point A (–1, 2) and the point B (11, 6) both lie on a circle with centre P.

The point M is the midpoint of AB.

Given that the line l passes through M and P, as shown in Figure 2,

(a) find an equation for l, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

Given that P has coordinates $(7, k)$, where k is a constant,

(b) find the value of k,

(1)

(c) find an equation for the circle.

(3)

Question 2

7 marks

Circles

2: The line joining the points $(-2, 5)$ and $(4, 15)$ is the diameter of a circle C.

(a) Find an equation for C.

(5)

(b) Hence find the exact coordinates of the point on C that is nearest the x-axis.

(2)

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Question 6

9 marks

Circles

6.

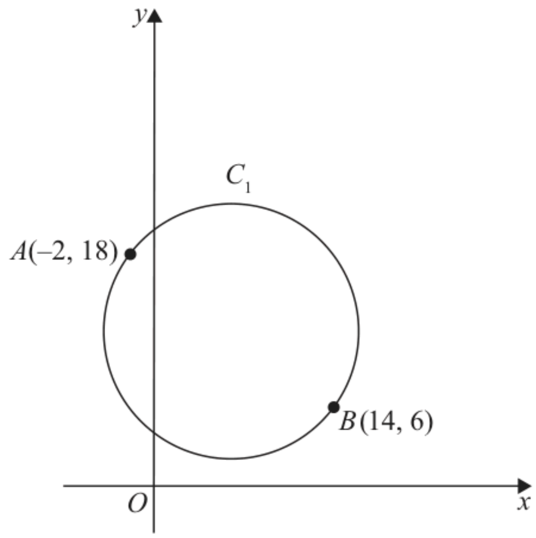

Figure 2

The points $A(-2, 18)$ and $B(14, 6)$ lie on a circle C_1 as shown in Figure 2.

Given that AB is a diameter of the circle C_1

(a) find an equation for C_1 making your method clear.

(5)

A circle C_2 has its centre at the origin.

Given that circles C_1 and C_2 touch,

(b) find possible equations for C_2

(4)

Question 5

6 marks

Circles

5. The circle C_1 has equation

$$x^2 + y^2 - 6x + 5y - 41 = 0$$

(a) Find

(i) the coordinates of the centre of C_1

(ii) the radius of C_1

(3)

The circle C_2 has

- centre $(-k, 0)$ where k is a positive constant
- radius 5

Given that circles C_1 and C_2 touch

(b) find the exact value of k.

(3)

7 marks

Circles

Also in Circles

Primary: Proof

WMA12/01 MAY/JUNE 2022

Question 3

3. (i) Show that the following statement is **false**:

$$\text{“}(n+1)^3 - n^3 \text{ is prime for all } n \in \mathbb{N}\text{”}$$

(2)

(ii) Given that the points $A(1, 0)$, $B(3, -10)$ and $C(7, -6)$ lie on a circle, prove that AB is a diameter of this circle.

(5)

TOPIC

Binomial Expansion

Question 3

6 marks

Binomial Expansion

3. (a) Find the first 4 terms, in ascending powers of x, in the binomial expansion of

$$\left(1 + \frac{x}{4}\right)^{12}$$

giving each coefficient in its simplest form.

(3)

(b) Find the term independent of x in the expansion of

$$\left(\frac{x^2 + 8}{x^5}\right)\left(1 + \frac{x}{4}\right)^{12}$$

(3)

WMA12/01 JANUARY 2020

7 marks

Question 2

Binomial Expansion

2. One of the terms in the binomial expansion of $(3 + ax)^6$, where a is a constant, is $540x^4$

(a) Find the possible values of a.

(4)

(b) Hence find the term independent of x in the expansion of

$$\left(\frac{1}{81} + \frac{1}{x^6}\right)(3 + ax)^6$$

(3)

WMA12/01 JANUARY 2021

8 marks

Question 4

Binomial Expansion

4. (a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$(2 + px)^6$$

where p is a constant. Give each term in simplest form.

(4)

Given that in the expansion of

$$\left(3 - \frac{1}{2}x\right)(2 + px)^6$$

the coefficient of x^2 is $-\frac{3}{4}$

(b) find the possible values of p.

(4)

Question 4

8 marks

Binomial Expansion

4. (a) Find, in ascending powers of x, up to and including the term in x^3, the binomial expansion of

$$\left(2 + \frac{x}{8}\right)^{13}$$

fully simplifying each coefficient.

(4)

(b) Use the answer to part (a) to find an approximation for 2.0125^{13}

Give your answer to 3 decimal places.

(3)

Without calculating 2.0125^{13}

(c) state, with a reason, whether the answer to part (b) is an overestimate or an underestimate.

(1)

Question 1

6 marks

Binomial Expansion

1. The first three terms, in ascending powers of x, of the binomial expansion of $(1 + kx)^{16}$ are

$$1, -4x \text{ and } px^2$$

where k and p are constants.

(a) Find, in simplest form,

(i) the value of k

(ii) the value of p

(3)

$$g(x) = \left(2 + \frac{16}{x}\right)(1 + kx)^{16}$$

Using the value of k found in part (a),

(b) find the term in x^2 in the expansion of $g(x)$.

(3)

WMA12/01 JANUARY 2022

7 marks

Question 3

Binomial Expansion

3. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{kx}{4}\right)^8$$

where k is a non-zero constant. Give each term in simplest form.

(4)

$$f(x) = (5 - 3x)\left(2 - \frac{kx}{4}\right)^8$$

In the expansion of $f(x)$, the constant term is 3 times the coefficient of x.

(b) Find the value of k.

(3)

WMA12/01 MAY/JUNE 2022

4 marks

Question 1

Binomial Expansion

1. Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\left(2 + \frac{3}{8}x\right)^{10}$$

Give each coefficient as an integer.

(4)

Question 2

7 marks

Binomial Expansion

2. A curve C has equation $y = \text{f}(x)$ where

$$\text{f}(x) = (2 - kx)^5$$

and k is a constant.

Given that when $\text{f}(x)$ is divided by $(4x - 5)$ the remainder is $\dfrac{243}{32}$

(a) show that $k = \dfrac{2}{5}$

(2)

(b) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{2}{5}x\right)^5$$

giving each term in simplest form.

(3)

Using the solution to part (b) and making your method clear,

(c) find the gradient of C at the point where $x = 0$

(2)

Question 3

7 marks

Binomial Expansion

3. $$f(x) = \left(2 + \frac{kx}{8}\right)^7 \quad \text{where } k \text{ is a non-zero constant}$$

(a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $f(x)$. Give each term in simplest form.

(4)

Given that, in the binomial expansion of $f(x)$, the coefficients of x, x^2 and x^3 are the first 3 terms of an arithmetic progression,

(b) find, using algebra, the possible values of k.

(Solutions relying entirely on calculator technology are not acceptable.)

(3)

WMA12/01 MAY/JUNE 2023

7 marks

Question 4

Binomial Expansion

4. The binomial expansion, in ascending powers of x, of

$$(3 + px)^5$$

where p is a constant, can be written in the form

$$A + Bx + Cx^2 + Dx^3 \ldots$$

where A, B, C and D are constants.

(a) Find the value of A

(1)

Given that

- $B = 18D$
- $p < 0$

(b) find

(i) the value of p

(ii) the value of C

(6)

Question 10

12 marks

Binomial Expansion

10. (i) (a) Find, in ascending powers of x, the 2nd, 3rd and 5th terms of the binomial expansion of

$$(3 + 2x)^6$$

(3)

For a particular value of x, these three terms form consecutive terms in a geometric series.

(b) Find this value of x.

(3)

(ii) In a **different** geometric series,

- the first term is $\sin^2\theta$
- the common ratio is $2\cos\theta$
- the sum to infinity is $\dfrac{8}{5}$

(a) Show that

$$5\cos^2\theta - 16\cos\theta + 3 = 0$$

(3)

(b) Hence find the exact value of the 2nd term in the series.

(3)

WMA12/01 JANUARY 2024

Question 2

3 marks

Binomial Expansion

2. Find the coefficient of the term in x^7 of the binomial expansion of

$$\left(\frac{3}{8} + 4x\right)^{12}$$

giving your answer in simplest form.

(3)

WMA12/01 MAY/JUNE 2024

5 marks

Question 1

Binomial Expansion

1. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\left(1-\frac{1}{6}x\right)^9$$

giving each term in simplest form.

(3)

(b) Hence find the coefficient of x^3 in the expansion of

$$(10x+3)\left(1-\frac{1}{6}x\right)^9$$

giving the answer in simplest form.

(2)

Question 4

7 marks

Binomial Expansion

4. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(3+2x)^6$$

giving each coefficient in simplest form.

(4)

(b) Hence find the coefficient of x^2 in the expansion of

$$\left(2x^2-\frac{1}{6x}\right)(3+2x)^6$$

(3)

WMA12/01 OCTOBER 2024

7 marks

Question 5

Binomial Expansion

5. (a) Find, in terms of a, the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 + ax)^6$$

where a is a non-zero constant. Give each term in simplest form.

(3)

$$\mathrm{f}(x) = \left(3 + \frac{1}{x}\right)^2 (2 + ax)^6$$

Given that the constant term in the expansion of $\mathrm{f}(x)$ is 576

(b) find the value of a.

(4)

Question 2

5 marks

Binomial Expansion

2. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 5x)^8$$

giving each term in simplest form.

(4)

This expansion is to be used to find an approximation for 2.05^8

(b) State the value of x that should be used.

(There is no need to carry out this calculation.)

(1)

Question 1

6 marks

Binomial Expansion

1: (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(1 - 4x)^7$$

giving each term in simplest form.

(4)

In the series expansion of

$$(5 + kx)(1 - 4x)^7 \qquad \text{where } k \text{ is a constant}$$

the coefficient of the term in x^2 is 1316

(b) Use the answer to part (a) to find the value of k.

(2)

Question 2

6 marks

Binomial Expansion

2. The first 4 terms, in ascending powers of x, in the binomial expansion of

$$(1 + px)^{10}$$

are

$$1 + 15x + qx^2 + rx^3$$

where p, q and r are constants.

Find the value of p, the value of q and the value of r.

(6)

Question 1

6 marks

Binomial Expansion

1. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(3 + kx)^7$$

where k is a non-zero constant. Give each term in simplest form.

(4)

Given that, in this expansion, the coefficient of x^2 is 3 times the coefficient of x^3

(b) find, in simplest form, the value of k.

(2)

TOPIC

Arithmetic Sequences

WMA12/01 OCTOBER 2021

Question 5

6 marks

Arithmetic Sequences

5. A company that owned a silver mine

- extracted 480 tonnes of silver from the mine in year 1
- extracted 465 tonnes of silver from the mine in year 2
- extracted 450 tonnes of silver from the mine in year 3

and so on, forming an arithmetic sequence.

(a) Find the mass of silver extracted in year 14

(2)

After a total of 7770 tonnes of silver was extracted, the company stopped mining.

Given that this occurred at the end of year N,

(b) show that

$$N^2 - 65N + 1036 = 0$$

(3)

(c) Hence, state the value of N.

(1)

WMA12/01 OCTOBER 2023

Question 8

7 marks

Arithmetic Sequences

8. In a large theatre there are n rows of seats, where n is a constant.

The number of seats in the first row is a, where a is a constant.

In each subsequent row there are 4 more seats than in the previous row so that

- in the 2nd row there are $(a + 4)$ seats
- in the 3rd row there are $(a + 8)$ seats
- the number of seats in each row form an **arithmetic** sequence

Given that the **total** number of seats in the first 10 rows is 360

(a) find the value of a.

(2)

Given also that the total number of seats in the n rows is 2146

(b) show that

$$n^2 + 8n - 1073 = 0$$

(2)

(c) Hence

(i) state the number of rows of seats in the theatre,

(ii) find the maximum number of seats in any one row.

(3)

Question 2

7 marks

Arithmetic Sequences

2. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

In an arithmetic series,

- the sixth term is 2
- the sum of the first ten terms is –80

For this series,

(a) find the value of the first term and the value of the common difference.

(4)

(b) Hence find the smallest value of n for which

$$S_n > 8000$$

(3)

Question 8

13 marks

Arithmetic Sequences

8. (i) (a) In an **arithmetic** series the first term is a and the common difference is d.

Show that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

(3)

(b) Hence find

$$900 + 892 + 884 + \dots + 500$$

(3)

(ii) Given that the first three terms of a **geometric** series are

$$k + 4 \qquad k - 2 \qquad 11 - k$$

where k is a constant,

(a) show that

$$2k^2 - 11k - 40 = 0$$

(3)

Given also that this series is convergent,

(b) find the value of S_∞

(4)

Question 1

4 marks

Arithmetic Sequences

1. The arithmetic series S is given by

$$S = 2 + 5 + 8 + 11 + \text{.........} + 254$$

Find

(a) the number of terms in the series,

(2)

(b) the sum of the series.

(2)

Question 1

4 marks

Arithmetic Sequences

1. An arithmetic series starts

$$78 + 75 + 72 + \ldots$$

Find

(a) the 18^{th} term in the series,

(2)

(b) the sum of the first 100 terms of the series.

(2)

12 marks

WMA12/01 OCTOBER 2019

Arithmetic Sequences

Question 9

Also in Arithmetic Sequences

Primary: Trigonometric Equations

9. **Solutions based entirely on graphical or numerical methods are not acceptable in this question.**

(i) Solve, for $0 \leqslant \theta < 180°$, the equation

$$3\sin(2\theta - 10°) = 1$$

giving your answers to one decimal place.

(4)

(ii) The first three terms of an arithmetic sequence are

$$\sin\alpha, \quad \frac{1}{\tan\alpha} \quad \text{and} \quad 2\sin\alpha$$

where α is a constant.

(a) Show that $2\cos\alpha = 3\sin^2\alpha$

(3)

Given that $\pi < \alpha < 2\pi$,

(b) find, showing all working, the value of α to 3 decimal places.

(5)

WMA12/01 JANUARY 2020

Question 8

7 marks

Arithmetic Sequences

Also in Arithmetic Sequences

Primary: Sequences & Series

8. (i) An arithmetic series has first term a and common difference d.

Prove that the sum to n terms of this series is

$$\frac{n}{2}\{2a+(n-1)d\}$$

(3)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is given by

$$u_n = 5n + 3(-1)^n$$

Find the value of

(a) u_5

(1)

(b) $\sum_{n=1}^{59} u_n$

(3)

Question 10

11 marks

Arithmetic Sequences

Also in Arithmetic Sequences

Primary: Modelling with Sequences & Series

10. In this question you must show detailed reasoning.

Owen wants to train for 12 weeks in preparation for running a marathon.

During the 12-week period he will run every Sunday and every Wednesday.

- On Sunday in week 1 he will run 15 km
- On Sunday in week 12 he will run 37 km

He considers two different 12-week training plans.

In training plan A, he will increase the distance he runs each Sunday by the same amount.

(a) Calculate the distance he will run on Sunday in week 5 under training plan A.

(3)

In training plan B, he will increase the distance he runs each Sunday by the same percentage.

(b) Calculate the distance he will run on Sunday in week 5 under training plan B.
Give your answer in km to one decimal place.

(3)

Owen will also run a fixed distance, x km, each Wednesday over the 12-week period.

Given that

- x is an integer
- the total distance that Owen will run on Sundays and Wednesdays over the 12 weeks will not exceed 360 km

(c) (i) find the maximum value of x, if he uses training plan A,

(ii) find the maximum value of x, if he uses training plan B.

(5)

(Total 11 marks)

WMA12/01 MAY/JUNE 2021

Question 1

7 marks

Arithmetic Sequences

Also in Arithmetic Sequences

Primary: Modelling with Sequences & Series

1. Adina is saving money to buy a new computer. She saves £5 in week 1, £5.25 in week 2, £5.50 in week 3 and so on until she has enough money, in total, to buy the computer.

She decides to model her savings using either an arithmetic series or a geometric series.

Using the information given,

(a) (i) state with a reason whether an arithmetic series or a geometric series should be used,

(ii) write down an expression, in terms of n, for the amount, in pounds (£), saved in week n.

(3)

Given that the computer Adina wants to buy costs £350

(b) find the number of weeks it will take for Adina to save enough money to buy the computer.

(4)

WMA12/01 JANUARY 2022

Question 8

9 marks

Arithmetic Sequences

Also in Arithmetic Sequences

Primary: Modelling with Sequences & Series

8. A metal post is repeatedly hit in order to drive it into the ground.

Given that

- on the 1st hit, the post is driven 100 mm into the ground
- on the 2nd hit, the post is driven an **additional** 98 mm into the ground
- on the 3rd hit, the post is driven an **additional** 96 mm into the ground
- the **additional** distances the post travels on each subsequent hit form an arithmetic sequence

(a) show that the post is driven an **additional** 62 mm into the ground with the 20th hit.
(1)

(b) Find the **total distance** that the post has been driven into the ground after 20 hits.
(2)

Given that for each subsequent hit after the 20th hit

- the **additional** distances the post travels form a geometric sequence with common ratio r
- on the 22nd hit, the post is driven an **additional** 60 mm into the ground

(c) find the value of r, giving your answer to 3 decimal places.
(2)

After a total of N hits, the post will have been driven more than 3 m into the ground.

(d) Find, showing all steps in your working, the smallest possible value of N.
(4)

WMA12/01 JANUARY 2023

Question 3

7 marks

Arithmetic Sequences

Also in Arithmetic Sequences

Primary: Binomial Expansion

3. $$f(x) = \left(2 + \frac{kx}{8}\right)^7 \quad \text{where } k \text{ is a non-zero constant}$$

(a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $f(x)$. Give each term in simplest form.

(4)

Given that, in the binomial expansion of $f(x)$, the coefficients of x, x^2 and x^3 are the first 3 terms of an arithmetic progression,

(b) find, using algebra, the possible values of k.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

WMA12/01 JANUARY 2024

Question 7

9 marks

Arithmetic Sequences

Also in Arithmetic Sequences

Primary: Modelling with Sequences & Series

7. Wheat is grown on a farm.

- In year 1, the farm produced 300 tonnes of wheat.
- In year 12, the farm is predicted to produce 4 000 tonnes of wheat.

Model A assumes that the amount of wheat produced on the farm will increase by the same amount each year.

(a) Using model A, find the amount of wheat produced on the farm in year 4.
Give your answer to the nearest 10 tonnes.

(3)

Model B assumes that the amount of wheat produced on the farm will increase by the same percentage each year.

(b) Using model B, find the amount of wheat produced on the farm in year 2.
Give your answer to the nearest 10 tonnes.

(3)

(c) Calculate, according to the two models, the difference between the total amounts of wheat predicted to be produced on the farm from year 1 to year 12 inclusive.
Give your answer to the nearest 10 tonnes.

(3)

WMA12/01 OCTOBER 2024

Question 7

6 marks

Arithmetic Sequences

Also in Arithmetic Sequences

Primary: Modelling with Sequences & Series

7. Jem pays money into a savings scheme, A, over a period of 300 months.

Jem pays £20 into scheme A in month 1, £20.50 in month 2, £21 in month 3 and so on, so that the amounts Jem pays each month form an arithmetic sequence.

(a) Show that Jem pays £69.50 into scheme A in month 100

(1)

(b) Find the **total** amount that Jem pays into scheme A over the period of 300 months.

(2)

Kim pays money into a different savings scheme, B, over the same period of 300 months.

In a model, the amounts Kim pays into scheme B increase by the same percentage each month, so that the amounts Kim pays each month form a geometric sequence.

Given that Kim pays

- £20 into scheme B in month 1
- £250 into scheme B in month 300

(c) use the model to calculate, to the nearest £10, the difference between the total amount paid into scheme A and the total amount paid into scheme B over the period of 300 months.

(3)

WMA12/01 MAY/JUNE 2025

Question 7

9 marks

Arithmetic Sequences

Also in Arithmetic Sequences

Primary: Sequences & Series

7: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) A **geometric** series begins

$$10 + 8 + 6.4 + \dots$$

(a) Find the sum to infinity of this series.

(2)

Given that the kth term of this series is less than 0.0005

(b) use algebra to find the smallest possible value of k.

(3)

(ii) An **arithmetic** series begins

$$850 + 843 + 836 + \dots$$

Given that the sum of the first n terms of this series is S_n
find the greatest possible value of S_n

(4)

10 marks

Arithmetic Sequences

Also in Arithmetic Sequences

Primary: Modelling with Sequences & Series

WMA12/01 JANUARY 2026

Question 9

9. A company mines lithium.

- in year 1, the company mined 12 400 tonnes of lithium
- in year 15, the company is predicted to mine 7 500 tonnes of lithium

Model A assumes that the mass of lithium the company mines will decrease by the same amount each year.

According to model A,

(a) find the mass of lithium the company will mine in year 5

(3)

(b) calculate the **total** mass of lithium the company will mine from year 1 to year 15 inclusive.

(2)

Model B assumes that the mass of lithium the company mines will decrease by the same percentage each year.

(c) Find, according to model B, the mass of lithium the company will mine in year 10. Give the answer to the nearest 10 tonnes.

(3)

According to model B, there is a limit to the **total** mass of lithium that can be mined.

(d) Calculate the value of this limit. Give the answer to the nearest 100 tonnes.

(2)

TOPIC

Geometric Sequences

Question 7

10 marks

Geometric Sequences

7. (i) A geometric sequence has first term 4 and common ratio 6
Given that the n^{th} term is greater than 10^{100}, find the minimum possible value of n.

(3)

(ii) A different geometric sequence has first term a and common ratio r.

Given that

- the second term of the sequence is −6
- the sum to infinity of the series is 25

(a) show that

$$25r^2 - 25r - 6 = 0$$

(3)

(b) Write down the solutions of

$$25r^2 - 25r - 6 = 0$$

(1)

Hence,

(c) state the value of r, giving a reason for your answer,

(1)

(d) find the sum of the first 4 terms of the series.

(2)

Question 6

8 marks

Geometric Sequences

6. In a geometric sequence $u_1, u_2, u_3, \ldots$

- the common ratio is r
- $u_2 + u_3 = 6$
- $u_4 = 8$

(a) Show that r satisfies

$$3r^2 - 4r - 4 = 0$$

(3)

Given that the geometric sequence has a sum to infinity,

(b) find u_1

(3)

(c) find S_∞

(2)

Question 8

7 marks

Geometric Sequences

8. A geometric sequence has first term a and common ratio r

Given that $S_\infty = 3a$

(a) show that $r = \frac{2}{3}$

(2)

Given also that

$$u_2 - u_4 = 16$$

where u_k is the k^{th} term of this sequence,

(b) find the value of S_{10} giving your answer to one decimal place.

(5)

Question 7

7 marks

Geometric Sequences

7. A geometric sequence has first term a and common ratio r, where $r > 0$

Given that

- the 3rd term is 20
- the 5th term is 12.8

(a) show that $r = 0.8$

(1)

(b) Hence find the value of a.

(2)

Given that the sum of the first n terms of this sequence is greater than 156

(c) find the smallest possible value of n.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

WMA12/01 MAY/JUNE 2023

Question 6

9 marks

Geometric Sequences

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A software developer released an app to download.

The numbers of downloads of the app each month, in thousands, for the first three months after the app was released were

$$2k - 15 \qquad k \qquad k + 4$$

where k is a constant.

Given that the numbers of downloads each month are modelled as a geometric series,

(a) show that $k^2 - 7k - 60 = 0$ **(2)**

(b) predict the number of downloads in the 4th month. **(4)**

The **total** number of all downloads of the app is predicted to exceed 3 million for the first time in the Nth month.

(c) Calculate the value of N according to the model. **(3)**

Question 4

6 marks

Geometric Sequences

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Given that, in a particular geometric series,

- the sum of the first three terms is 70.2
- the sum to infinity is 75

find, for this series,

(a) the common ratio,
(4)

(b) the first term.
(2)

Question 8

8 marks

Geometric Sequences

8. (i) A geometric series has first term a and common ratio r.

Prove that the sum of the first n terms of this series S_n is given by

$$S_n = \frac{a(1-r^n)}{1-r}$$

(3)

(ii) A liquid is to be stored in a barrel.

Due to evaporation, the volume of the liquid in the barrel at the end of each year is 8% less than the volume of the liquid in the barrel at the start of the year.

At the start of the first year, the barrel is filled with 150 litres of the liquid.

(a) Show that the amount of the liquid in the barrel at the end of 6 years is approximately 91 litres.

(2)

At the start of each year a new barrel is filled with 150 litres of the liquid so that, at the end of 40 years, there are 40 barrels containing the liquid.

(b) Calculate the total amount of the liquid, to the nearest litre, in the 40 barrels at the end of 40 years.

(3)

Question 2

6 marks

Geometric Sequences

Also in Geometric Sequences

Primary: Modelling with Sequences & Series

2. The adult population of a town at the start of 2019 is 25 000

A model predicts that the adult population will increase by 2% each year, so that the number of adults in the population at the start of each year following 2019 will form a geometric sequence.

(a) Find, according to the model, the adult population of the town at the start of 2032

(3)

It is also modelled that every member of the adult population gives £5 to local charity at the start of each year.

(b) Find, according to these models, the total amount of money that would be given to local charity by the adult population of the town from 2019 to 2032 inclusive. Give your answer to the nearest £1 000

(3)

WMA12/01 JANUARY 2020

Question 5

8 marks

Geometric Sequences

Also in Geometric Sequences

Primary: Modelling with Sequences & Series

5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A colony of bees is being studied.

The number of bees in the colony at the start of the study was 30 000

Three years after the start of the study, the number of bees in the colony is 34 000

A model predicts that the number of bees in the colony will increase by $p\%$ each year, so that the number of bees in the colony at the end of each year of study forms a geometric sequence.

Assuming the model,

(a) find the value of p, giving your answer to 2 decimal places.

(3)

According to the model, at the end of N years of study the number of bees in the colony exceeds 75 000

(b) Find, showing all steps in your working, the smallest integer value of N.

(5)

WMA12/01 JANUARY 2021

Question 10

11 marks

Geometric Sequences

Also in Geometric Sequences

Primary: Modelling with Sequences & Series

10. In this question you must show detailed reasoning.

Owen wants to train for 12 weeks in preparation for running a marathon.

During the 12-week period he will run every Sunday and every Wednesday.

- On Sunday in week 1 he will run 15 km
- On Sunday in week 12 he will run 37 km

He considers two different 12-week training plans.

In training plan A, he will increase the distance he runs each Sunday by the same amount.

(a) Calculate the distance he will run on Sunday in week 5 under training plan A.

(3)

In training plan B, he will increase the distance he runs each Sunday by the same percentage.

(b) Calculate the distance he will run on Sunday in week 5 under training plan B. Give your answer in km to one decimal place.

(3)

Owen will also run a fixed distance, x km, each Wednesday over the 12-week period.

Given that

- x is an integer
- the total distance that Owen will run on Sundays and Wednesdays over the 12 weeks will not exceed 360 km

(c) (i) find the maximum value of x, if he uses training plan A,

(ii) find the maximum value of x, if he uses training plan B.

(5)

(Total 11 marks)

9 marks

Geometric Sequences

Also in Geometric Sequences

Primary: Modelling with Sequences & Series

WMA12/01 JANUARY 2022

Question 8

8. A metal post is repeatedly hit in order to drive it into the ground.

Given that

- on the 1st hit, the post is driven 100 mm into the ground
- on the 2nd hit, the post is driven an **additional** 98 mm into the ground
- on the 3rd hit, the post is driven an **additional** 96 mm into the ground
- the **additional** distances the post travels on each subsequent hit form an arithmetic sequence

(a) show that the post is driven an **additional** 62 mm into the ground with the 20th hit.
(1)

(b) Find the **total distance** that the post has been driven into the ground after 20 hits.
(2)

Given that for each subsequent hit after the 20th hit

- the **additional** distances the post travels form a geometric sequence with common ratio r
- on the 22nd hit, the post is driven an **additional** 60 mm into the ground

(c) find the value of r, giving your answer to 3 decimal places.
(2)

After a total of N hits, the post will have been driven more than 3 m into the ground.

(d) Find, showing all steps in your working, the smallest possible value of N.
(4)

WMA12/01 OCTOBER 2023

Question 10

12 marks

Geometric Sequences

Also in Geometric Sequences

Primary: Binomial Expansion

10. (i) (a) Find, in ascending powers of x, the 2nd, 3rd and 5th terms of the binomial expansion of

$$(3 + 2x)^6$$

(3)

For a particular value of x, these three terms form consecutive terms in a geometric series.

(b) Find this value of x.

(3)

(ii) In a **different** geometric series,

- the first term is $\sin^2\theta$
- the common ratio is $2\cos\theta$
- the sum to infinity is $\dfrac{8}{5}$

(a) Show that

$$5\cos^2\theta - 16\cos\theta + 3 = 0$$

(3)

(b) Hence find the exact value of the 2nd term in the series.

(3)

WMA12/01 JANUARY 2024

Question 5

8 marks

Geometric Sequences

Also in Geometric Sequences

Primary: Sequences & Series

5. (i) Find the value of

$$\sum_{r=1}^{\infty} 6 \times (0.25)^r$$

(3)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 3$$

$$u_{n+1} = \frac{u_n - 3}{u_n - 2} \qquad n \in \mathbb{N}$$

(a) Show that this sequence is periodic.

(2)

(b) State the order of this sequence.

(1)

(c) Hence find

$$\sum_{n=1}^{70} u_n$$

(2)

WMA12/01 JANUARY 2024

Question 7

9 marks

Geometric Sequences

Also in Geometric Sequences

Primary: Modelling with Sequences & Series

7. Wheat is grown on a farm.

- In year 1, the farm produced 300 tonnes of wheat.
- In year 12, the farm is predicted to produce 4 000 tonnes of wheat.

Model A assumes that the amount of wheat produced on the farm will increase by the same amount each year.

(a) Using model A, find the amount of wheat produced on the farm in year 4.
Give your answer to the nearest 10 tonnes.
(3)

Model B assumes that the amount of wheat produced on the farm will increase by the same percentage each year.

(b) Using model B, find the amount of wheat produced on the farm in year 2.
Give your answer to the nearest 10 tonnes.
(3)

(c) Calculate, according to the two models, the difference between the total amounts of wheat predicted to be produced on the farm from year 1 to year 12 inclusive.
Give your answer to the nearest 10 tonnes.
(3)

WMA12/01 MAY/JUNE 2024

Question 10

8 marks

Geometric Sequences

Also in Geometric Sequences

Primary: Modelling with Sequences & Series

10. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The number of dormice and the number of voles on an island are being monitored.

Initially there are 2000 dormice on the island.

A model predicts that the number of dormice will increase by 3% each year, so that the numbers of dormice on the island at the end of each year form a geometric sequence.

(a) Find, according to the model, the number of dormice on the island 6 years after monitoring began. Give your answer to 3 significant figures.

(2)

The number of voles on the island is being monitored over the same period of time.

Given that

- 4 years after monitoring began there were 3690 voles on the island
- 7 years after monitoring began there were 3470 voles on the island
- the number of voles on the island at the end of each year is modelled as a geometric sequence

(b) find the equation of this model in the form

$$N = ab^t$$

where N is the number of voles, t years after monitoring began and a and b are constants. Give the value of a and the value of b to 2 significant figures.

(3)

When $t = T$, the number of dormice on the island is equal to the number of voles on the island.

(c) Find, according to the models, the value of T, giving your answer to one decimal place.

(3)

Question 8

8. (i) (a) In an **arithmetic** series the first term is a and the common difference is d.

Show that

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

(3)

(b) Hence find

$$900 + 892 + 884 + \ldots + 500$$

(3)

(ii) Given that the first three terms of a **geometric** series are

$$k + 4 \qquad k - 2 \qquad 11 - k$$

where k is a constant,

(a) show that

$$2k^2 - 11k - 40 = 0$$

(3)

Given also that this series is convergent,

(b) find the value of S_∞

(4)

6 marks

WMA12/01 OCTOBER 2024

Geometric Sequences

Question 7

Also in Geometric Sequences

Primary: Modelling with Sequences & Series

7. Jem pays money into a savings scheme, A, over a period of 300 months.

Jem pays £20 into scheme A in month 1, £20.50 in month 2, £21 in month 3 and so on, so that the amounts Jem pays each month form an arithmetic sequence.

(a) Show that Jem pays £69.50 into scheme A in month 100

(1)

(b) Find the **total** amount that Jem pays into scheme A over the period of 300 months.

(2)

Kim pays money into a different savings scheme, B, over the same period of 300 months.

In a model, the amounts Kim pays into scheme B increase by the same percentage each month, so that the amounts Kim pays each month form a geometric sequence.

Given that Kim pays

- £20 into scheme B in month 1
- £250 into scheme B in month 300

(c) use the model to calculate, to the nearest £10, the difference between the total amount paid into scheme A and the total amount paid into scheme B over the period of 300 months.

(3)

WMA12/01 MAY/JUNE 2025

Question 7

9 marks

Geometric Sequences

Also in Geometric Sequences

Primary: Sequences & Series

7: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) A **geometric** series begins

$$10 + 8 + 6.4 + \ldots$$

(a) Find the sum to infinity of this series.

(2)

Given that the kth term of this series is less than 0.0005

(b) use algebra to find the smallest possible value of k.

(3)

(ii) An **arithmetic** series begins

$$850 + 843 + 836 + \ldots$$

Given that the sum of the first n terms of this series is S_n
find the greatest possible value of S_n

(4)

10 marks

Geometric Sequences

Also in Geometric Sequences

Primary: Modelling with Sequences & Series

WMA12/01 JANUARY 2026

Question 9

9. A company mines lithium.

- in year 1, the company mined 12 400 tonnes of lithium
- in year 15, the company is predicted to mine 7 500 tonnes of lithium

Model *A* assumes that the mass of lithium the company mines will decrease by the same amount each year.

According to model *A*,

(a) find the mass of lithium the company will mine in year 5

(3)

(b) calculate the **total** mass of lithium the company will mine from year 1 to year 15 inclusive.

(2)

Model *B* assumes that the mass of lithium the company mines will decrease by the same percentage each year.

(c) Find, according to model *B*, the mass of lithium the company will mine in year 10. Give the answer to the nearest 10 tonnes.

(3)

According to model *B*, there is a limit to the **total** mass of lithium that can be mined.

(d) Calculate the value of this limit. Give the answer to the nearest 100 tonnes.

(2)

TOPIC

Sequences & Series

WMA12/01 JANUARY 2020

Question 8

7 marks

Sequences & Series

8. (i) An arithmetic series has first term a and common difference d.

Prove that the sum to n terms of this series is

$$\frac{n}{2}\{2a+(n-1)d\}$$

(3)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is given by

$$u_n = 5n + 3(-1)^n$$

Find the value of

(a) u_5

(1)

(b) $\sum_{n=1}^{59} u_n$

(3)

Question 8

8. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = 2(a_n + 3)^2 - 7$$
$$a_1 = p - 3$$

where p is a constant.

(a) Find an expression for a_2 in terms of p, giving your answer in simplest form.

(1)

Given that $\sum_{n=1}^{3} a_n = p + 15$

(b) find the possible values of a_2

(6)

Question 2

5 marks

Sequences & Series

2. A sequence is defined by

$$u_1 = 6$$
$$u_{n+1} = ku_n + 3$$

where k is a positive constant.

(a) Find, in terms of k, an expression for u_3

(2)

Given that $\sum_{n=1}^{3} u_n = 117$

(b) find the value of k.

(3)

Question 3

7 marks

Sequences & Series

3. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_n = \cos^2\left(\frac{n\pi}{3}\right)$$

Find the exact values of

(a) (i) a_1

(ii) a_2

(iii) a_3

(3)

(b) Hence find the exact value of

$$\sum_{n=1}^{50}\left\{n + \cos^2\left(\frac{n\pi}{3}\right)\right\}$$

You must make your method clear.

(4)

Question 11

8 marks

Sequences & Series

11. A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = b - au_n$$

$$u_1 = 3$$

where a and b are constants.

(a) Find, in terms of a and b,

(i) u_2

(ii) u_3

(2)

Given

- $\sum_{n=1}^{3} u_n = 153$
- $b = a + 9$

(b) show that

$$a^2 - 5a - 66 = 0$$

(3)

(c) Hence find the larger possible value of u_2

(3)

Question 2

5 marks

Sequences & Series

2. A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 3$$

$$u_{n+1} = 2 - \frac{4}{u_n}$$

(a) Find the value of u_2, the value of u_3 and the value of u_4

(3)

(b) Find the value of

$$\sum_{r=1}^{100} u_r$$

(2)

Question 5

8 marks

Sequences & Series

5. (i) Find the value of

$$\sum_{r=1}^{\infty} 6 \times (0.25)^r$$

(3)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 3$$
$$u_{n+1} = \frac{u_n - 3}{u_n - 2} \qquad n \in \mathbb{N}$$

(a) Show that this sequence is periodic.

(2)

(b) State the order of this sequence.

(1)

(c) Hence find

$$\sum_{n=1}^{70} u_n$$

(2)

Question 1

5 marks

Sequences & Series

1. The sequence $u_1, u_2, u_3, \ldots$ satisfies

$$u_{n+2} = 3u_{n+1} - 2u_n$$

Given that

- $u_1 = 7$
- $u_3 = 4$

(a) find the value of u_2

(2)

(b) find $\sum_{r=1}^{4} (u_r + 2r)$

(3)

Question 2

7 marks

Sequences & Series

2. A sequence of numbers $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 7$$

$$u_{n+1} = (-1)^n u_n + k$$

where k is a constant.

(a) Show that $u_5 = 7$

(3)

Given that $\sum_{r=1}^{4} u_r = 30$

(b) find the value of k.

(2)

(c) Hence find the value of $\sum_{r=1}^{150} u_r$

(2)

Question 7

9 marks

Sequences & Series

7: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) A **geometric** series begins

$$10 + 8 + 6.4 + \ldots$$

(a) Find the sum to infinity of this series.

(2)

Given that the kth term of this series is less than 0.0005

(b) use algebra to find the smallest possible value of k.

(3)

(ii) An **arithmetic** series begins

$$850 + 843 + 836 + \ldots$$

Given that the sum of the first n terms of this series is S_n
find the greatest possible value of S_n

(4)

7 marks

Sequences & Series

WMA12/01 OCTOBER 2019

Question 7

Also in Sequences & Series

Primary: Laws of Logarithms

7. Given $\log_a b = k$, find, in simplest form in terms of k,

(i) $\log_a\left(\frac{\sqrt{a}}{b}\right)$ **(2)**

(ii) $\dfrac{\log_a a^2b}{\log_a b^3}$ **(2)**

(iii) $\sum_{n=1}^{50}(k + \log_a b^n)$ **(3)**

10 marks

WMA12/01 OCTOBER 2021

Sequences & Series

Question 7

Also in Sequences & Series

Primary: Geometric Sequences

7. (i) A geometric sequence has first term 4 and common ratio 6
Given that the n^{th} term is greater than 10^{100}, find the minimum possible value of n.

(3)

(ii) A different geometric sequence has first term a and common ratio r.

Given that

- the second term of the sequence is -6
- the sum to infinity of the series is 25

(a) show that

$$25r^2 - 25r - 6 = 0$$

(3)

(b) Write down the solutions of

$$25r^2 - 25r - 6 = 0$$

(1)

Hence,

(c) state the value of r, giving a reason for your answer,

(1)

(d) find the sum of the first 4 terms of the series.

(2)

8 marks

Sequences & Series

Also in Sequences & Series

Primary: Geometric Sequences

WMA12/01 MAY/JUNE 2022

Question 6

6. In a geometric sequence $u_1, u_2, u_3, \ldots$

- the common ratio is r
- $u_2 + u_3 = 6$
- $u_4 = 8$

(a) Show that r satisfies

$$3r^2 - 4r - 4 = 0$$

(3)

Given that the geometric sequence has a sum to infinity,

(b) find u_1

(3)

(c) find S_∞

(2)

Question 8

8. A geometric sequence has first term a and common ratio r

Given that $S_\infty = 3a$

(a) show that $r = \frac{2}{3}$

(2)

Given also that

$$u_2 - u_4 = 16$$

where u_k is the k^{th} term of this sequence,

(b) find the value of S_{10} giving your answer to one decimal place.

(5)

6 marks

Sequences & Series

WMA12/01 JANUARY 2023

Question 4

Also in Sequences & Series

Primary: Laws of Logarithms

4. (i) Using the laws of logarithms, solve

$$\log_3(4x) + 2 = \log_3(5x + 7)$$

(3)

(ii) Given that

$$\sum_{r=1}^{2} \log_a\left(y^r\right) = \sum_{r=1}^{2} \left(\log_a y\right)^r \qquad y > 1,\ a > 1,\ y \neq a$$

find y in terms of a, giving your answer in simplest form.

(3)

WMA12/01 JANUARY 2023

Question 7

7 marks

Sequences & Series

Also in Sequences & Series

Primary: Geometric Sequences

7. A geometric sequence has first term a and common ratio r, where $r > 0$

Given that

- the 3rd term is 20
- the 5th term is 12.8

(a) show that $r = 0.8$

(1)

(b) Hence find the value of a.

(2)

Given that the sum of the first n terms of this sequence is greater than 156

(c) find the smallest possible value of n.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

7 marks

Sequences & Series

WMA12/01 OCTOBER 2023

Question 8

Also in Sequences & Series

Primary: Arithmetic Sequences

8. In a large theatre there are n rows of seats, where n is a constant.

The number of seats in the first row is a, where a is a constant.

In each subsequent row there are 4 more seats than in the previous row so that

- in the 2nd row there are $(a + 4)$ seats
- in the 3rd row there are $(a + 8)$ seats
- the number of seats in each row form an **arithmetic** sequence

Given that the **total** number of seats in the first 10 rows is 360

(a) find the value of a.

(2)

Given also that the total number of seats in the n rows is 2146

(b) show that

$$n^2 + 8n - 1073 = 0$$

(2)

(c) Hence

(i) state the number of rows of seats in the theatre,

(ii) find the maximum number of seats in any one row.

(3)

Question 2

2. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

In an arithmetic series,

- the sixth term is 2
- the sum of the first ten terms is –80

For this series,

(a) find the value of the first term and the value of the common difference.

(4)

(b) Hence find the smallest value of n for which

$$S_n > 8000$$

(3)

Question 1

4 marks

Sequences & Series

Also in Sequences & Series

Primary: Arithmetic Sequences

1. The arithmetic series S is given by

$$S = 2 + 5 + 8 + 11 + \dots\dots\dots + 254$$

Find

(a) the number of terms in the series,

(2)

(b) the sum of the series.

(2)

Question 4

6 marks

Sequences & Series

Also in Sequences & Series

Primary: Geometric Sequences

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Given that, in a particular geometric series,

- the sum of the first three terms is 70.2
- the sum to infinity is 75

find, for this series,

(a) the common ratio,

(4)

(b) the first term.

(2)

WMA12/01 MAY/JUNE 2025

Question 1

6 marks

Sequences & Series

Also in Sequences & Series

Primary: Binomial Expansion

1: (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(1-4x)^7$$

giving each term in simplest form.

(4)

In the series expansion of

$$(5+kx)(1-4x)^7 \qquad \text{where } k \text{ is a constant}$$

the coefficient of the term in x^2 is 1316

(b) Use the answer to part (a) to find the value of k.

(2)

WMA12/01 OCTOBER 2025

Question 1

4 marks

Sequences & Series

Also in Sequences & Series

Primary: Arithmetic Sequences

1. An arithmetic series starts

$$78 + 75 + 72 + \ldots$$

Find

(a) the 18^{th} term in the series,

(2)

(b) the sum of the first 100 terms of the series.

(2)

TOPIC

Modelling with Sequences & Series

Question 2

6 marks

Modelling with Sequences & Series

2. The adult population of a town at the start of 2019 is 25 000

A model predicts that the adult population will increase by 2% each year, so that the number of adults in the population at the start of each year following 2019 will form a geometric sequence.

(a) Find, according to the model, the adult population of the town at the start of 2032

(3)

It is also modelled that every member of the adult population gives £5 to local charity at the start of each year.

(b) Find, according to these models, the total amount of money that would be given to local charity by the adult population of the town from 2019 to 2032 inclusive. Give your answer to the nearest £1 000

(3)

Question 5

8 marks

Modelling with Sequences & Series

5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A colony of bees is being studied.

The number of bees in the colony at the start of the study was 30 000

Three years after the start of the study, the number of bees in the colony is 34 000

A model predicts that the number of bees in the colony will increase by $p\%$ each year, so that the number of bees in the colony at the end of each year of study forms a geometric sequence.

Assuming the model,

(a) find the value of p, giving your answer to 2 decimal places.

(3)

According to the model, at the end of N years of study the number of bees in the colony exceeds 75 000

(b) Find, showing all steps in your working, the smallest integer value of N.

(5)

Question 10

10. In this question you must show detailed reasoning.

Owen wants to train for 12 weeks in preparation for running a marathon.

During the 12-week period he will run every Sunday and every Wednesday.

- On Sunday in week 1 he will run 15 km
- On Sunday in week 12 he will run 37 km

He considers two different 12-week training plans.

In training plan A, he will increase the distance he runs each Sunday by the same amount.

(a) Calculate the distance he will run on Sunday in week 5 under training plan A.

(3)

In training plan B, he will increase the distance he runs each Sunday by the same percentage.

(b) Calculate the distance he will run on Sunday in week 5 under training plan B.
Give your answer in km to one decimal place.

(3)

Owen will also run a fixed distance, x km, each Wednesday over the 12-week period.

Given that

- x is an integer
- the total distance that Owen will run on Sundays and Wednesdays over the 12 weeks will not exceed 360 km

(c) (i) find the maximum value of x, if he uses training plan A,

(ii) find the maximum value of x, if he uses training plan B.

(5)

(Total 11 marks)

7 marks

Question 1

Modelling with Sequences & Series

1. Adina is saving money to buy a new computer. She saves £5 in week 1, £5.25 in week 2, £5.50 in week 3 and so on until she has enough money, in total, to buy the computer.

She decides to model her savings using either an arithmetic series or a geometric series.

Using the information given,

(a) (i) state with a reason whether an arithmetic series or a geometric series should be used,

(ii) write down an expression, in terms of n, for the amount, in pounds (£), saved in week n.

(3)

Given that the computer Adina wants to buy costs £350

(b) find the number of weeks it will take for Adina to save enough money to buy the computer.

(4)

Question 8

9 marks

Modelling with Sequences & Series

8. A metal post is repeatedly hit in order to drive it into the ground.

Given that

- on the 1st hit, the post is driven 100 mm into the ground
- on the 2nd hit, the post is driven an **additional** 98 mm into the ground
- on the 3rd hit, the post is driven an **additional** 96 mm into the ground
- the **additional** distances the post travels on each subsequent hit form an arithmetic sequence

(a) show that the post is driven an **additional** 62 mm into the ground with the 20th hit.
(1)

(b) Find the **total distance** that the post has been driven into the ground after 20 hits.
(2)

Given that for each subsequent hit after the 20th hit

- the **additional** distances the post travels form a geometric sequence with common ratio r
- on the 22nd hit, the post is driven an **additional** 60 mm into the ground

(c) find the value of r, giving your answer to 3 decimal places.
(2)

After a total of N hits, the post will have been driven more than 3 m into the ground.

(d) Find, showing all steps in your working, the smallest possible value of N.
(4)

Question 9

9. A scientist is using carbon-14 dating to determine the age of some wooden items.

The equation for carbon-14 dating an item is given by

$$N = k\lambda^t$$

where

- N grams is the amount of carbon-14 **currently** present in the item
- k grams was the **initial** amount of carbon-14 present in the item
- t is the number of years since the item was made
- λ is a constant, with $0 < \lambda < 1$

(a) Sketch the graph of N against t for $k = 1$

(2)

Given that it takes 5700 years for the amount of carbon-14 to reduce to half its initial value,

(b) show that the value of the constant λ is 0.999878 to 6 decimal places.

(2)

Given that Item A

- is known to have had 15 grams of carbon-14 present initially
- is thought to be 3250 years old

(c) calculate, to 3 significant figures, how much carbon-14 the equation predicts is currently in Item A.

(2)

Item B is known to have initially had 25 grams of carbon-14 present, but only 18 grams now remain.

(d) Use algebra to calculate the age of Item B to the nearest 100 years.

(3)

Question 7

9 marks

Modelling with Sequences & Series

7. Wheat is grown on a farm.

- In year 1, the farm produced 300 tonnes of wheat.
- In year 12, the farm is predicted to produce 4 000 tonnes of wheat.

Model A assumes that the amount of wheat produced on the farm will increase by the same amount each year.

(a) Using model A, find the amount of wheat produced on the farm in year 4. Give your answer to the nearest 10 tonnes.

(3)

Model B assumes that the amount of wheat produced on the farm will increase by the same percentage each year.

(b) Using model B, find the amount of wheat produced on the farm in year 2. Give your answer to the nearest 10 tonnes.

(3)

(c) Calculate, according to the two models, the difference between the total amounts of wheat predicted to be produced on the farm from year 1 to year 12 inclusive. Give your answer to the nearest 10 tonnes.

(3)

Question 10

8 marks

Modelling with Sequences & Series

10. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The number of dormice and the number of voles on an island are being monitored.

Initially there are 2000 dormice on the island.

A model predicts that the number of dormice will increase by 3% each year, so that the numbers of dormice on the island at the end of each year form a geometric sequence.

(a) Find, according to the model, the number of dormice on the island 6 years after monitoring began. Give your answer to 3 significant figures.

(2)

The number of voles on the island is being monitored over the same period of time.

Given that

- 4 years after monitoring began there were 3690 voles on the island
- 7 years after monitoring began there were 3470 voles on the island
- the number of voles on the island at the end of each year is modelled as a geometric sequence

(b) find the equation of this model in the form

$$N = ab^t$$

where N is the number of voles, t years after monitoring began and a and b are constants. Give the value of a and the value of b to 2 significant figures.

(3)

When $t = T$, the number of dormice on the island is equal to the number of voles on the island.

(c) Find, according to the models, the value of T, giving your answer to one decimal place.

(3)

Question 7

6 marks

Modelling with Sequences & Series

7. Jem pays money into a savings scheme, A, over a period of 300 months.

Jem pays £20 into scheme A in month 1, £20.50 in month 2, £21 in month 3 and so on, so that the amounts Jem pays each month form an arithmetic sequence.

(a) Show that Jem pays £69.50 into scheme A in month 100

(1)

(b) Find the **total** amount that Jem pays into scheme A over the period of 300 months.

(2)

Kim pays money into a different savings scheme, B, over the same period of 300 months.

In a model, the amounts Kim pays into scheme B increase by the same percentage each month, so that the amounts Kim pays each month form a geometric sequence.

Given that Kim pays

- £20 into scheme B in month 1
- £250 into scheme B in month 300

(c) use the model to calculate, to the nearest £10, the difference between the total amount paid into scheme A and the total amount paid into scheme B over the period of 300 months.

(3)

Question 9

9. A company mines lithium.

- in year 1, the company mined 12 400 tonnes of lithium
- in year 15, the company is predicted to mine 7 500 tonnes of lithium

Model A assumes that the mass of lithium the company mines will decrease by the same amount each year.

According to model A,

(a) find the mass of lithium the company will mine in year 5

(3)

(b) calculate the **total** mass of lithium the company will mine from year 1 to year 15 inclusive.

(2)

Model B assumes that the mass of lithium the company mines will decrease by the same percentage each year.

(c) Find, according to model B, the mass of lithium the company will mine in year 10. Give the answer to the nearest 10 tonnes.

(3)

According to model B, there is a limit to the **total** mass of lithium that can be mined.

(d) Calculate the value of this limit. Give the answer to the nearest 100 tonnes.

(2)

WMA12/01 JANUARY 2021

Question 7

6 marks

Modelling with Sequences & Series

Also in Modelling with Sequences & Series

Primary: Integration

7.

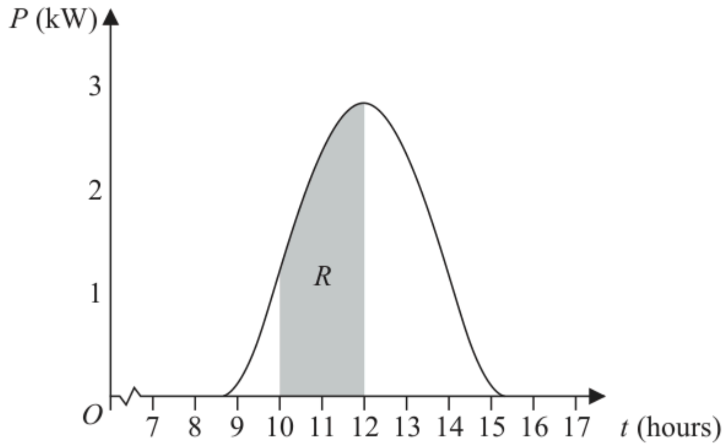

Figure 1

Solar panels are installed on the roof of a building.

The power, P, produced on a particular day, in kW, can be modelled by the equation

$$P = 0.95 + 2^{t-12} + 2^{12-t} - (t-12)^2 \qquad 8.5 \leqslant t \leqslant 15.2$$

where t is the time in hours after midnight. The graph of P against t is shown in Figure 1.

A table of values of t and P is shown below, with the values of P given to 4 significant figures where appropriate.

Time, t (hours)	10	10.5	11	11.5	12
Power, P (kW)		1.882	2.45		2.95

(a) Use the given equation to complete the table, giving the values of P to 4 significant figures where appropriate.

(2)

The amount of energy, in kWh, produced between 10:00 and 12:00 can be found by calculating the area of region R, shown shaded in Figure 1.

(b) Use the trapezium rule, with all the values of P in the completed table, to find an estimate for the amount of energy produced between 10:00 and 12:00. Give your answer to 2 decimal places.

(4)

WMA12/01 OCTOBER 2021

Question 5

6 marks

Modelling with Sequences & Series

Also in Modelling with Sequences & Series

Primary: Arithmetic Sequences

5. A company that owned a silver mine

- extracted 480 tonnes of silver from the mine in year 1
- extracted 465 tonnes of silver from the mine in year 2
- extracted 450 tonnes of silver from the mine in year 3

and so on, forming an arithmetic sequence.

(a) Find the mass of silver extracted in year 14

(2)

After a total of 7770 tonnes of silver was extracted, the company stopped mining.

Given that this occurred at the end of year N,

(b) show that

$$N^2 - 65N + 1036 = 0$$

(3)

(c) Hence, state the value of N.

(1)

WMA12/01 OCTOBER 2022

Question 4

8 marks

Modelling with Sequences & Series

Also in Modelling with Sequences & Series

Primary: Laws of Logarithms

4. The weight of a baby mammal is monitored over a 16-month period.

The weight of the mammal, w kg, is given by

$$w = \log_a(t + 5) - \log_a 4 \qquad 2 \leqslant t \leqslant 18$$

where t is the age of the mammal in months and a is a constant.

Given that the weight of the mammal was 10 kg when $t = 3$

(a) show that $a = 1.072$ correct to 3 decimal places.

(3)

Using $a = 1.072$

(b) find an equation for t in terms of w

(3)

(c) find the value of t when $w = 15$, giving your answer to 3 significant figures.

(2)

WMA12/01 MAY/JUNE 2023

Question 6

9 marks

Modelling with Sequences & Series

Also in Modelling with Sequences & Series

Primary: Geometric Sequences

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A software developer released an app to download.

The numbers of downloads of the app each month, in thousands, for the first three months after the app was released were

$$2k - 15 \qquad k \qquad k + 4$$

where k is a constant.

Given that the numbers of downloads each month are modelled as a geometric series,

(a) show that $k^2 - 7k - 60 = 0$

(2)

(b) predict the number of downloads in the 4th month.

(4)

The **total** number of all downloads of the app is predicted to exceed 3 million for the first time in the Nth month.

(c) Calculate the value of N according to the model.

(3)

WMA12/01 OCTOBER 2023

Question 6

6 marks

Modelling with Sequences & Series

Also in Modelling with Sequences & Series

Primary: Integration

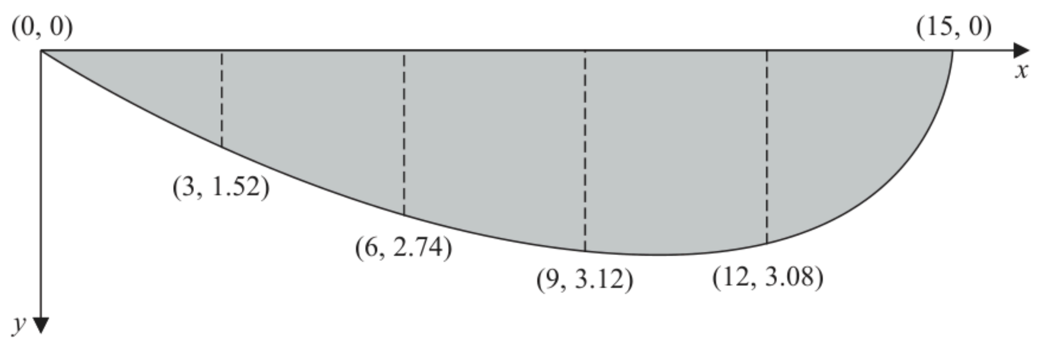

Figure 1

A river is being studied.

At one particular place, the river is 15 m wide.

The depth, y metres, of the river is measured at a point x metres from one side of the river.

Figure 1 shows a plot of the cross-section of the river and the coordinate values (x, y)

(a) Use the trapezium rule with all the y values given in Figure 1 to estimate the cross-sectional area of the river.

(3)

The water in the river is modelled as flowing at a constant speed of $1.5\,\text{m}\,\text{s}^{-1}$ across the whole of the cross-section.

(b) Use the model and the answer to part (a) to estimate the volume of water flowing through this section of the river each minute, giving your answer in m^3 to 2 significant figures.

(2)

Assuming the model,

(c) state, giving a reason for your answer, whether your answer for part (b) is an overestimate or an underestimate of the true volume of water flowing through this section of the river each minute.

(1)

WMA12/01 MAY/JUNE 2024

Question 8

12 marks

Modelling with Sequences & Series

Also in Modelling with Sequences & Series

Primary: Trigonometric Equations

8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x \leqslant \pi$, the equation

$$5\sin x \tan x + 13 = \cos x$$

giving your answer in radians to 3 significant figures.

(5)

(ii) The temperature inside a greenhouse is monitored on one particular day.

The temperature, H°C, inside the greenhouse, t hours after midnight, is modelled by the equation

$$H = 10 + 12\sin(kt + 18)^\circ \qquad 0 \leqslant t < 24$$

where k is a constant.

Use the equation of the model to answer parts (a) to (c).

Given that

- the temperature inside the greenhouse was 20 °C at 6 am
- $0 < k < 20$

(a) find all possible values for k, giving each answer to 2 decimal places.

(4)

Given further that $0 < k < 10$

(b) find the maximum temperature inside the greenhouse,

(1)

(c) find the time of day at which this maximum temperature occurs.

Give your answer to the nearest minute.

(2)

Question 5

6 marks

Modelling with Sequences & Series

Also in Modelling with Sequences & Series

Primary: Trigonometric Equations

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The depth of water, D metres, in a harbour on a particular day is given by the equation

$$D = 8 + 5\sin\left(\frac{\pi t}{6} + 3\right) \qquad 0 \leqslant t < 24$$

where t is the number of hours after **midnight**.

(a) Show that the depth of water in the harbour at 2 am is just over 4 metres.

(1)

(b) Find, to the nearest minute, the first time after **midday** when the depth of water in the harbour is exactly 6 metres.

(5)

10 marks

WMA12/01 JANUARY 2025

Modelling with Sequences & Series

Question 3

Also in Modelling with Sequences & Series

Primary: Applications of Differentiation

3.

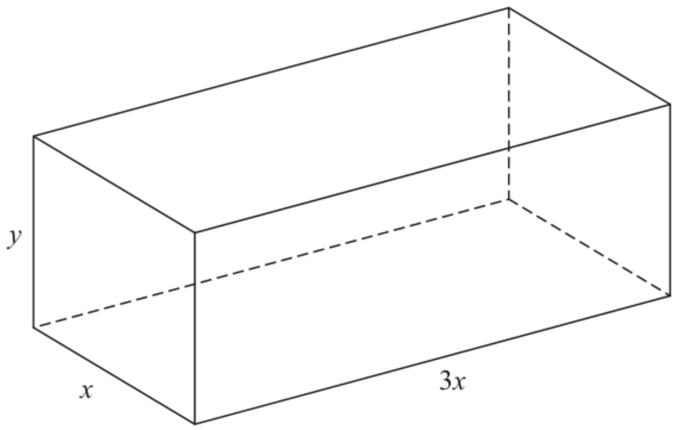

Figure 1

Figure 1 shows an open-topped container used for holding water.

The container is in the shape of a cuboid and is made of sheet metal.

The base of the container is a rectangle $3x$ metres by x metres.

The height of the container is y metres as shown in Figure 1.

Given that the capacity of the container is $120\,\text{m}^3$

(a) show that the area $A\,\text{m}^2$ of the sheet metal used to make the container is given by

$$A = Px^2 + \frac{Q}{x}$$

where P and Q are positive constants to be found.

(4)

(b) Use calculus to find the value of x for which A has a stationary value, giving your answer to 3 significant figures.

(4)

(c) Find $\frac{\mathrm{d}^2A}{\mathrm{d}x^2}$ and hence show that the value of x found in part (b) gives the minimum value of A.

(2)

WMA12/01 MAY/JUNE 2025

Question 9

11 marks

Modelling with Sequences & Series

Also in Modelling with Sequences & Series

Primary: Applications of Differentiation

9:

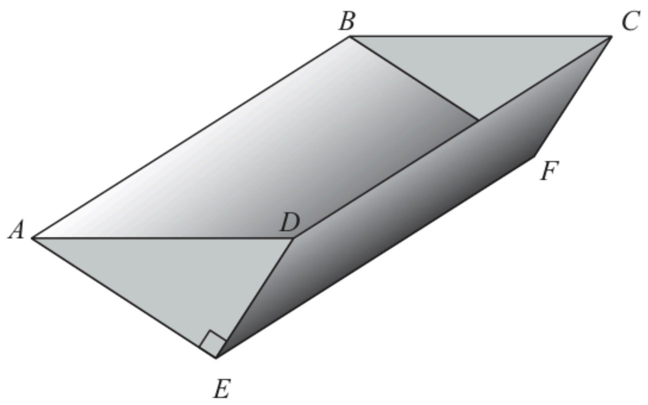

Figure 2

Figure 2 shows a sketch of an open container.

The sides *CDEF* and *ABFE* are rectangles.

The ends *ADE* and *BCF* are congruent (identical) right-angled triangles.

The container is made from metal of negligible thickness.

Given that

- $AE = BF = 3x$ metres
- $DE = CF = 2x$ metres
- $AB = DC = EF = L$ metres

and the capacity of the container is $12\,\text{m}^3$

(a) show that the area of metal used to make the container, $S\,\text{m}^2$, is given by

$$S = Px^2 + \frac{Q}{x}$$

where P and Q are positive integers to be found.

(4)

Given that x can vary,

(b) use algebraic calculus to find the minimum value of S, giving your answer to one decimal place.

(5)

(c) Justify that the value of S found in part (b) is a minimum.

(2)

8 marks

Modelling with Sequences & Series

WMA12/01 OCTOBER 2025

Question 8

Also in Modelling with Sequences & Series

Primary: Geometric Sequences

8. (i) A geometric series has first term a and common ratio r.

Prove that the sum of the first n terms of this series S_n is given by

$$S_n = \frac{a(1-r^n)}{1-r}$$

(3)

(ii) A liquid is to be stored in a barrel.

Due to evaporation, the volume of the liquid in the barrel at the end of each year is 8% less than the volume of the liquid in the barrel at the start of the year.

At the start of the first year, the barrel is filled with 150 litres of the liquid.

(a) Show that the amount of the liquid in the barrel at the end of 6 years is approximately 91 litres.

(2)

At the start of each year a new barrel is filled with 150 litres of the liquid so that, at the end of 40 years, there are 40 barrels containing the liquid.

(b) Calculate the total amount of the liquid, to the nearest litre, in the 40 barrels at the end of 40 years.

(3)

TOPIC

Laws of Logarithms

Question 7

7 marks

Laws of Logarithms

7. Given $\log_a b = k$, find, in simplest form in terms of k,

(i) $\log_a\left(\frac{\sqrt{a}}{b}\right)$ **(2)**

(ii) $\frac{\log_a a^2 b}{\log_a b^3}$ **(2)**

(iii) $\sum_{n=1}^{50}(k + \log_a b^n)$ **(3)**

Question 9

7 marks

Laws of Logarithms

9. (a) Sketch the curve with equation

$$y = 3 \times 4^x$$

showing the coordinates of any points of intersection with the coordinate axes.

(2)

The curve with equation $y = 6^{1-x}$ meets the curve with equation $y = 3 \times 4^x$ at the point P.

(b) Show that the x coordinate of P is $\dfrac{\log_{10} 2}{\log_{10} 24}$

(5)

Question 3

8 marks

Laws of Logarithms

3. (i) Solve

$$7^{x+2} = 3$$

giving your answer in the form $x = \log_7 a$ where a is a rational number in its simplest form.

(3)

(ii) Using the laws of logarithms, solve

$$1 + \log_2 y + \log_2 (y + 4) = \log_2 (5 - y)$$

(5)

WMA12/01 MAY/JUNE 2021

8 marks

Question 2

Laws of Logarithms

2.

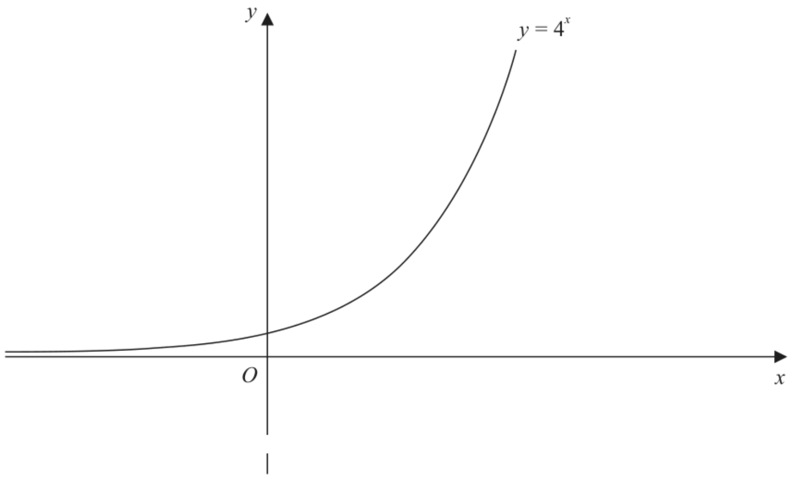

Figure 1

Figure 1 shows a sketch of the curve with equation $y = 4^x$

A copy of Figure 1, labelled Diagram 1, is shown on the next page.

(a) On Diagram 1, sketch the curve with equation

(i) $y = 2^x$

(ii) $y = 4^x - 6$

Label clearly the coordinates of any points of intersection with the coordinate axes.

(4)

The curve with equation $y = 2^x$ meets the curve with equation $y = 4^x - 6$ at the point P.

(b) Using algebra, find the exact coordinates of P.

(4)

Question 2 continued

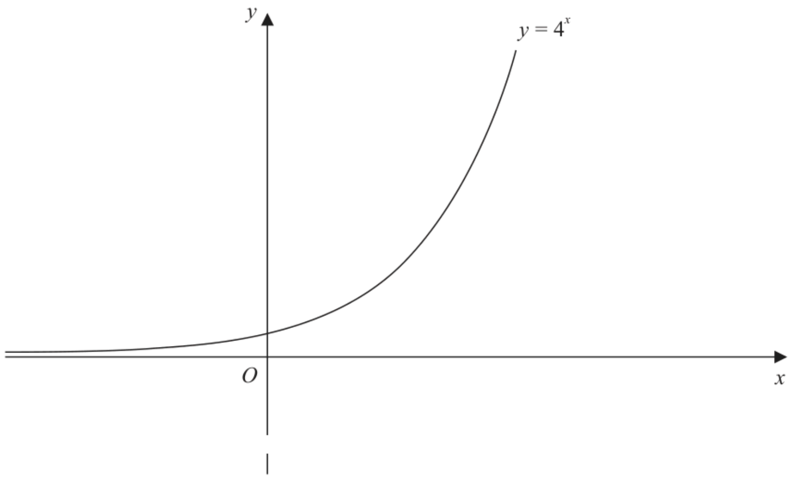

Diagram 1

Question 7

10 marks

Laws of Logarithms

7. (a) Given that

$$3\log_3(2x - 1) = 2 + \log_3(14x - 25)$$

show that

$$2x^3 - 3x^2 - 30x + 56 = 0$$

(4)

(b) Show that -4 is a root of this cubic equation.

(2)

(c) Hence, using algebra and showing each step of your working, solve

$$3\log_3(2x - 1) = 2 + \log_3(14x - 25)$$

(4)

Question 4

5 marks

Laws of Logarithms

4. Using the laws of logarithms, solve

$$\log_3(32 - 12x) = 2\log_3(1 - x) + 3$$

(5)

Question 4

6 marks

Laws of Logarithms

4. **In this question you must show all stages of your working.**

Give your answers in fully simplified surd form.

Given that a and b are positive constants, solve the simultaneous equations

$$a - b = 8$$
$$\log_4 a + \log_4 b = 3$$

(6)

WMA12/01 OCTOBER 2022

8 marks

Question 4

Laws of Logarithms

4. The weight of a baby mammal is monitored over a 16-month period.

The weight of the mammal, w kg, is given by

$$w = \log_a(t+5) - \log_a 4 \qquad 2 \leqslant t \leqslant 18$$

where t is the age of the mammal in months and a is a constant.

Given that the weight of the mammal was 10 kg when $t = 3$

(a) show that $a = 1.072$ correct to 3 decimal places.

(3)

Using $a = 1.072$

(b) find an equation for t in terms of w

(3)

(c) find the value of t when $w = 15$, giving your answer to 3 significant figures.

(2)

Question 10

7 marks

Laws of Logarithms

10. Given $a = \log_2 3$

(i) write, in simplest form, in terms of a,

(a) $\log_2 9$

(b) $\log_2 \left(\dfrac{\sqrt{3}}{16}\right)$

(3)

(ii) Solve

$$3^x \times 2^{x+4} = 6$$

giving your answer, in simplest form, in terms of a.

(4)

Question 4

6 marks

Laws of Logarithms

4. (i) Using the laws of logarithms, solve

$$\log_3(4x) + 2 = \log_3(5x + 7)$$

(3)

(ii) Given that

$$\sum_{r=1}^{2} \log_a\left(y^r\right) = \sum_{r=1}^{2} \left(\log_a y\right)^r \qquad y > 1,\ a > 1,\ y \neq a$$

find y in terms of a, giving your answer in simplest form.

(3)

WMA12/01 MAY/JUNE 2023

Question 5

5 marks

Laws of Logarithms

5. Use the laws of logarithms to solve

$$\log_2(16x) + \log_2(x + 1) = 3 + \log_2(x + 6)$$

(5)

WMA12/01 OCTOBER 2023

Question 5

6 marks

Laws of Logarithms

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve

$$3^a = 70$$

giving the answer to 3 decimal places.

(2)

(ii) Find the exact value of b such that

$$4 + 3\log_3 b = \log_3 5b$$

(4)

WMA12/01 JANUARY 2024

8 marks

Question 6

Laws of Logarithms

6. (a) Given that

$$2\log_4(x+3) + \log_4 x = \log_4(4x+2) + \frac{1}{2}$$

show that

$$x^3 + 6x^2 + x - 4 = 0$$

(4)

(b) Given also that –1 is a root of the equation

$$x^3 + 6x^2 + x - 4 = 0$$

(i) use algebra to find the other two roots of the equation.

(3)

(ii) Hence solve

$$2\log_4(x+3) + \log_4 x = \log_4(4x+2) + \frac{1}{2}$$

(1)

WMA12/01 MAY/JUNE 2024

6 marks

Question 3

Laws of Logarithms

3. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Using the laws of logarithms, solve

$$2\log_2(2-x) = 4 + \log_2(x+10)$$

(5)

(ii) Find the value of

$$\log_{\sqrt{a}} a^6$$

where a is a positive constant greater than 1

(1)

Question 9

9 marks

Laws of Logarithms

9. Given that

$$3\log_2(t+4) - 2\log_2(t-2) = 7$$

(a) verify that $t = 4$ is a solution of the above equation,

(2)

(b) show that

$$t^3 - 116t^2 + 560t - 448 = 0$$

(3)

(c) Hence, using algebra and showing your working, solve

$$3\log_2(t+4) - 2\log_2(t-2) = 7$$

giving each answer in simplest form.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

Question 6

5 marks

Laws of Logarithms

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Using the laws of logarithms, solve

$$\log_4(12 - 2x) = 2 + 2\log_4(x + 1)$$

(5)

WMA12/01 JANUARY 2025

Question 7

8 marks

Laws of Logarithms

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) The table below shows values of x and y, where $y = \log_{10}(x+5)$, for x values between -1 and 4

x	-1	0	1	2	3	4
$y = \log_{10}(x+5)$	$\log_{10}4$	$\log_{10}5$	$\log_{10}6$	$\log_{10}7$	$\log_{10}8$	$\log_{10}9$

Using the trapezium rule with all the y values in the given table, show that

$$\int_{-1}^{4} \log_{10}(x+5)\,dx \approx \log_{10} k$$

where k is an integer to be found.

(3)

(ii) Find the value of a such that

$$2\log_5(5-a) - \log_5(a+25) = 1$$

(5)

WMA12/01 MAY/JUNE 2025

10 marks

Question 6

Laws of Logarithms

6: **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(i) Solve

$$2\log_2(4-x) = 3 + \log_2\left(\frac{x+11}{2}\right)$$

(5)

(ii) The curves C_1 and C_2 with equations

$$y = 3^{2x+1} \qquad \text{and} \qquad y = 6 \times 3^x$$

meet at the point P.

Find the exact coordinates of P, writing your answer in the form $(\log_3 a, b)$ where a and b are integers.

(5)

Question 9

10 marks

Laws of Logarithms

9. **In this question you must show detailed reasoning.**
Solutions relying on calculator technology are not acceptable.

(i) Solve

$$2\log_3(4x + 5) - \log_3(x + 3) = 2$$

(5)

(ii) Given that $a > 0$, $b > 0$ and

$$\log_{10} a + \log_{10} b = \log_{10}(a + b)$$

(a) prove that $a = \dfrac{b}{b-1}$

(3)

(b) Hence write down the full restriction on the value of b, giving a reason for your answer.

(2)

WMA12/01 JANUARY 2026

Question 7

9 marks

Laws of Logarithms

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Find the exact solution of the equation

$$8^{2x-5} = 20$$

giving the answer in the form $a + b\log_2 5$ where a and b are rational constants.

(4)

(ii) Using the laws of logarithms, solve the equation

$$\log_3(13 + 2y) + 3 = 2\log_3(4 - y)$$

(5)

WMA12/01 JANUARY 2020

Question 1

7 marks

Laws of Logarithms

Also in Laws of Logarithms

Primary: Integration

1. The table below shows corresponding values of x and y for $y = \log_2(2x)$

The values of y are given to 2 decimal places as appropriate.

x	2	5	8	11	14
y	2	3.32	4	4.46	4.81

Using the trapezium rule with all the values of y in the given table,

(a) obtain an estimate for $\int_2^{14} \log_2(2x)\,\mathrm{d}x$, giving your answer to one decimal place.

(3)

Using your answer to part (a) and making your method clear, estimate

(b) (i) $\int_2^{14} \frac{\log_2(4x^2)}{5}\,\mathrm{d}x$

(ii) $\int_2^{14} \log_2\left(\frac{2}{x}\right)\mathrm{d}x$

(4)

WMA12/01 JANUARY 2020

Question 5

8 marks

Laws of Logarithms

Also in Laws of Logarithms

Primary: Modelling with Sequences & Series

5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A colony of bees is being studied.

The number of bees in the colony at the start of the study was 30 000

Three years after the start of the study, the number of bees in the colony is 34 000

A model predicts that the number of bees in the colony will increase by $p\%$ each year, so that the number of bees in the colony at the end of each year of study forms a geometric sequence.

Assuming the model,

(a) find the value of p, giving your answer to 2 decimal places.

(3)

According to the model, at the end of N years of study the number of bees in the colony exceeds 75 000

(b) Find, showing all steps in your working, the smallest integer value of N.

(5)

WMA12/01 OCTOBER 2021

Question 3

8 marks

Laws of Logarithms

Also in Laws of Logarithms

Primary: Integration

3.

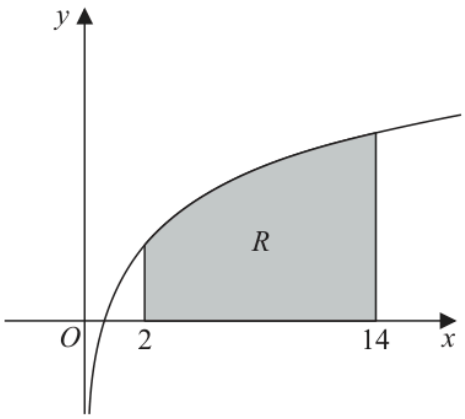

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \log_{10} x$

The region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 14$

Using the trapezium rule with four strips of equal width,

(a) show that the area of R is approximately 10.10

(3)

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of R.

(1)

(c) Using the answer to part (a) and making your method clear, estimate the value of

(i) $\displaystyle\int_2^{14} \log_{10} \sqrt{x}\, dx$

(ii) $\displaystyle\int_2^{14} \log_{10} 100x^3\, dx$

(4)

WMA12/01 MAY/JUNE 2022

Question 2

8 marks

Laws of Logarithms

Also in Laws of Logarithms

Primary: Integration

2.

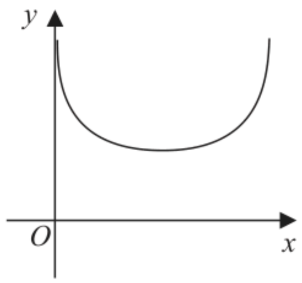

Figure 1

Figure 1 shows the graph of

$$y = 1 - \log_{10}(\sin x) \qquad 0 < x < \pi$$

where x is in radians.

The table below shows some values of x and y for this graph, with values of y given to 3 decimal places.

x	0.5	1	1.5	2	2.5	3
y	1.319		1.001		1.223	1.850

(a) Complete the table above, giving values of y to 3 decimal places.

(2)

(b) Use the trapezium rule with all the y values in the completed table to find, to 2 decimal places, an estimate for

$$\int_{0.5}^{3} \left(1 - \log_{10}(\sin x)\right) dx$$

(3)

(c) Use your answer to part (b) to find an estimate for

$$\int_{0.5}^{3} \left(3 + \log_{10}(\sin x)\right) dx$$

(3)

WMA12/01 MAY/JUNE 2022

Question 9

9 marks

Laws of Logarithms

Also in Laws of Logarithms

Primary: Modelling with Sequences & Series

9. A scientist is using carbon-14 dating to determine the age of some wooden items.

The equation for carbon-14 dating an item is given by

$$N = k\lambda^t$$

where

- N grams is the amount of carbon-14 **currently** present in the item
- k grams was the **initial** amount of carbon-14 present in the item
- t is the number of years since the item was made
- λ is a constant, with $0 < \lambda < 1$

(a) Sketch the graph of N against t for $k = 1$

(2)

Given that it takes 5700 years for the amount of carbon-14 to reduce to half its initial value,

(b) show that the value of the constant λ is 0.999878 to 6 decimal places.

(2)

Given that Item A

- is known to have had 15 grams of carbon-14 present initially
- is thought to be 3250 years old

(c) calculate, to 3 significant figures, how much carbon-14 the equation predicts is currently in Item A.

(2)

Item B is known to have initially had 25 grams of carbon-14 present, but only 18 grams now remain.

(d) Use algebra to calculate the age of Item B to the nearest 100 years.

(3)

8 marks

Laws of Logarithms

Also in Laws of Logarithms

Primary: Modelling with Sequences & Series

WMA12/01 MAY/JUNE 2024

Question 10

10. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The number of dormice and the number of voles on an island are being monitored.

Initially there are 2000 dormice on the island.

A model predicts that the number of dormice will increase by 3% each year, so that the numbers of dormice on the island at the end of each year form a geometric sequence.

(a) Find, according to the model, the number of dormice on the island 6 years after monitoring began. Give your answer to 3 significant figures.

(2)

The number of voles on the island is being monitored over the same period of time.

Given that

- 4 years after monitoring began there were 3690 voles on the island
- 7 years after monitoring began there were 3470 voles on the island
- the number of voles on the island at the end of each year is modelled as a geometric sequence

(b) find the equation of this model in the form

$$N = ab^t$$

where N is the number of voles, t years after monitoring began and a and b are constants. Give the value of a and the value of b to 2 significant figures.

(3)

When $t = T$, the number of dormice on the island is equal to the number of voles on the island.

(c) Find, according to the models, the value of T, giving your answer to one decimal place.

(3)

WMA12/01 OCTOBER 2025

Question 3

7 marks

Laws of Logarithms

Also in Laws of Logarithms

Primary: Integration

3.

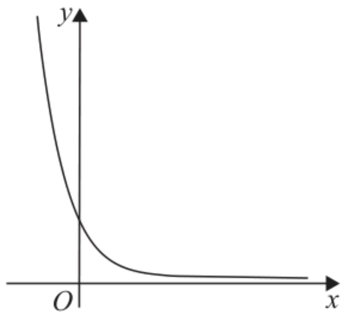

Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve with equation $y = 3 \times 2^{-x}$

The point $P(k, 300\,000)$ lies on the curve.

(a) Use logarithms to find the value of k to 2 decimal places.

(2)

x	–0.5	1	2.5	4.0	5.5	7
y	4.243	1.5	0.530	0.188	0.066	0.023

The table shows corresponding values of x and y for $y = 3 \times 2^{-x}$

The values of y are given to 3 decimal places where appropriate.

(b) (i) Use the trapezium rule, with all the values of y from the table, to find an approximate value, to 2 decimal places, for

$$\int_{-0.5}^{7} 3 \times 2^{-x}\, dx$$

(3)

(ii) Use your answer to part (b)(i) to estimate

$$\int_{-0.5}^{7} 2^{-x}\, dx + \int_{-7}^{0.5} 2^{x}\, dx$$

(2)

TOPIC

Trigonometric Equations

Question 9

9. **Solutions based entirely on graphical or numerical methods are not acceptable in this question.**

(i) Solve, for $0 \leqslant \theta < 180°$, the equation

$$3\sin(2\theta - 10°) = 1$$

giving your answers to one decimal place.

(4)

(ii) The first three terms of an arithmetic sequence are

$$\sin\alpha, \quad \frac{1}{\tan\alpha} \quad \text{and} \quad 2\sin\alpha$$

where α is a constant.

(a) Show that $2\cos\alpha = 3\sin^2\alpha$

(3)

Given that $\pi < \alpha < 2\pi$,

(b) find, showing all working, the value of α to 3 decimal places.

(5)

Question 7

7 marks

Trigonometric Equations

7. (a) Show that the equation

$$8\tan\theta = 3\cos\theta$$

may be rewritten in the form

$$3\sin^2\theta + 8\sin\theta - 3 = 0$$

(3)

(b) Hence solve, for $0 \leqslant x \leqslant 90°$, the equation

$$8\tan 2x = 3\cos 2x$$

giving your answers to 2 decimal places.

(4)

Question 6

8 marks

Trigonometric Equations

6. (a) Show that the equation

$$\frac{3\sin\theta\cos\theta}{2\sin\theta - 1} = 5\tan\theta \qquad \sin\theta \neq \frac{1}{2}$$

can be written in the form

$$3\sin^3\theta + 10\sin^2\theta - 8\sin\theta = 0$$

(4)

(b) Hence solve, for $-\frac{\pi}{4} < x < \frac{\pi}{4}$

$$\frac{3\sin 2x\cos 2x}{2\sin 2x - 1} = 5\tan 2x$$

giving your answers to 3 decimal places where appropriate.

(4)

WMA12/01 MAY/JUNE 2021

Question 8

10 marks

Trigonometric Equations

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta < 360°$, the equation

$$3\sin(\theta + 30°) = 7\cos(\theta + 30°)$$

giving your answers to one decimal place.

(4)

(ii) (a) Show that the equation

$$3\sin^3 x = 5\sin x - 7\sin x\cos x$$

can be written in the form

$$\sin x(a\cos^2 x + b\cos x + c) = 0$$

where a, b and c are constants to be found.

(b) Hence solve for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$ the equation

$$3\sin^3 x = 5\sin x - 7\sin x\cos x$$

(6)

Question 10

10 marks

Trigonometric Equations

10. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$$\tan^2\left(2x + \frac{\pi}{4}\right) = 3$$

(5)

(ii) Solve, for $0 < \theta < 360°$

$$(2\sin\theta - \cos\theta)^2 = 1$$

giving your answers, as appropriate, to one decimal place.

(5)

(Total 10 marks)

Question 7

8 marks

Trigonometric Equations

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $-90° < x < 90°$, the equation

$$3\sin(2x - 15°) = \cos(2x - 15°)$$

giving your answers to one decimal place.

(4)

(ii) Solve, for $0 < \theta < 2\pi$, the equation

$$4\sin^2\theta + 8\cos\theta = 3$$

giving your answers to 3 significant figures.

(4)

Question 5

6 marks

Trigonometric Equations

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Solve, for $-180° < \theta \leqslant 180°$, the equation

$$3\tan(\theta + 43°) = 2\cos(\theta + 43°)$$

(6)

Question 5

8 marks

Trigonometric Equations

5. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$(3\cos\theta - \tan\theta)\cos\theta = 2$$

can be written as

$$3\sin^2\theta + \sin\theta - 1 = 0$$

(3)

(b) Hence solve for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$

$$(3\cos 2x - \tan 2x)\cos 2x = 2$$

(5)

Question 8

9 marks

Trigonometric Equations

8. **In this question you must show all stages of your working.**

Solutions based entirely on calculator technology are not acceptable.

(i) Solve, for $-\frac{\pi}{2} < x < \pi$, the equation

$$5\sin(3x + 0.1) + 2 = 0$$

giving your answers, **in radians**, to 2 decimal places.

(4)

(ii) Solve, for $0 < \theta < 360°$, the equation

$$2\tan\theta\sin\theta = 5 + \cos\theta$$

giving your answers, **in degrees**, to one decimal place.

(5)

Question 9

7 marks

Trigonometric Equations

9. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$3\cos\theta(\tan\theta\sin\theta + 3) = 11 - 5\cos\theta$$

may be written as

$$3\cos^2\theta - 14\cos\theta + 8 = 0$$

(3)

(b) Hence solve, for $0 < x < 360°$

$$3\cos 2x(\tan 2x\sin 2x + 3) = 11 - 5\cos 2x$$

giving your answers to one decimal place.

(4)

Question 3

7 marks

Trigonometric Equations

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Solve, for $0 < \theta \leqslant 360°$ the equation

$$2\tan\theta + 3\sin\theta = 0$$

giving your answers, as appropriate, to one decimal place.

(5)

(b) Hence, or otherwise, find the smallest positive solution of

$$2\tan(2x + 40°) + 3\sin(2x + 40°) = 0$$

giving your answer to one decimal place.

(2)

WMA12/01 JANUARY 2024

Question 9

14 marks

Trigonometric Equations

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 \leqslant x < 360°$, the equation

$$\sin x \tan x = 5$$

giving your answers to one decimal place.

(6)

(ii)

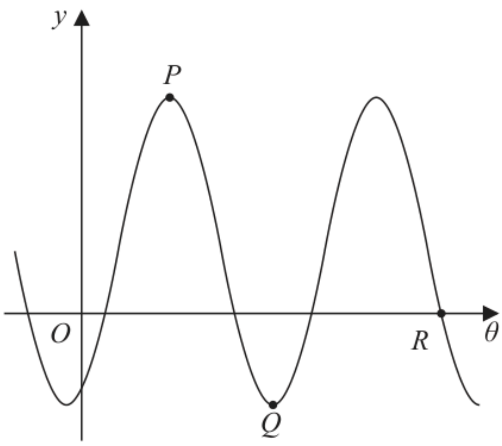

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = A\sin\left(2\theta - \frac{3\pi}{8}\right) + 2$$

where A is a constant and θ is measured in radians.

The points P, Q and R lie on the curve and are shown in Figure 1.

Given that the y coordinate of P is 7

(a) state the value of A,

(1)

(b) find the exact coordinates of Q,

(3)

(c) find the value of θ at R, giving your answer to 3 significant figures.

(4)

Question 8

12 marks

Trigonometric Equations

8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < x \leqslant \pi$, the equation

$$5\sin x \tan x + 13 = \cos x$$

giving your answer in radians to 3 significant figures.

(5)

(ii) The temperature inside a greenhouse is monitored on one particular day.

The temperature, H°C, inside the greenhouse, t hours after midnight, is modelled by the equation

$$H = 10 + 12\sin(kt + 18)° \qquad 0 \leqslant t < 24$$

where k is a constant.

Use the equation of the model to answer parts (a) to (c).

Given that

- the temperature inside the greenhouse was 20 °C at 6 am
- $0 < k < 20$

(a) find all possible values for k, giving each answer to 2 decimal places.

(4)

Given further that $0 < k < 10$

(b) find the maximum temperature inside the greenhouse,

(1)

(c) find the time of day at which this maximum temperature occurs.

Give your answer to the nearest minute.

(2)

Question 5

6 marks

Trigonometric Equations

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The depth of water, D metres, in a harbour on a particular day is given by the equation

$$D = 8 + 5\sin\left(\frac{\pi t}{6} + 3\right) \qquad 0 \leqslant t < 24$$

where t is the number of hours after **midnight**.

(a) Show that the depth of water in the harbour at 2 am is just over 4 metres.

(1)

(b) Find, to the nearest minute, the first time after **midday** when the depth of water in the harbour is exactly 6 metres.

(5)

Question 7

7 marks

Trigonometric Equations

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 \leqslant x < 2\pi$, the equation

$$3\sin x \tan x = 11 + \cos x$$

giving the answers in radians to 3 decimal places.

(5)

(ii) Given that

- $0 < \theta < 90°$
- $\cos\theta = \frac{1}{3}$

find, in simplest form, the exact value of $\tan\theta$

(2)

Question 9

7 marks

Trigonometric Equations

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\tan\theta = 3\cos\theta$$

can be written as

$$3\sin^2\theta + 2\sin\theta - 3 = 0$$

(3)

(b) Hence solve, for $-\pi < x < \pi$, the equation

$$2\tan\left(2x + \frac{\pi}{3}\right) = 3\cos\left(2x + \frac{\pi}{3}\right)$$

giving your answers to 3 significant figures.

(4)

Question 10

8 marks

Trigonometric Equations

10. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos\theta\left(3\tan\theta + \frac{2}{\tan\theta}\right) \equiv \sin\theta + \frac{2}{\sin\theta} \qquad \theta \neq \frac{n\pi}{2}$$

(4)

(b) Hence solve, for $0 < x < 2\pi$, the equation

$$\cos x\left(3\tan x + \frac{2}{\tan x}\right) = 4\sin x - 5$$

giving your answers to 3 significant figures.

(4)

Question 8

8 marks

Trigonometric Equations

8: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 \leqslant x < \pi$, the equation

$$3\tan\left(2x + \frac{\pi}{5}\right) = \sqrt{3}$$

giving the answers in radians in the form $k\pi$, where k is a rational constant to be found.

(3)

(ii) Solve, for $0 \leqslant \theta < 360°$, the equation

$$5\sin\theta\tan\theta = \cos\theta + 4$$

giving your answers, in degrees, to one decimal place.

(5)

Question 5

9 marks

Trigonometric Equations

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta \leqslant 360°$, the equation

$$4 \tan \theta + 5 \sin \theta = 0$$

giving any non-exact answers to one decimal place.

(5)

(ii) Solve, for $0 < x < \pi$, the equation

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{5}{\cos x}$$

giving the answers, in radians, to 3 significant figures.

(4)

Question 8

10 marks

Trigonometric Equations

8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 \leqslant \theta < 180$

$$4\tan^2(2\theta - 30)° + 1 = 49$$

giving the answers to one decimal place.

(5)

(ii) Solve, for $0 \leqslant x < 2\pi$

$$2\tan x \sin x + 3 = 0$$

giving the answers, in radians, in the form $k\pi$ where k is a rational constant.

(5)

WMA12/01 JANUARY 2020

Question 3

8 marks

Trigonometric Equations

Also in Trigonometric Equations

Primary: Polynomials

3. $$f(x) = 6x^3 + 17x^2 + 4x - 12$$

(a) Use the factor theorem to show that $(2x + 3)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, write $f(x)$ as a product of three linear factors.

(4)

(c) Solve, for $\frac{\pi}{2} < \theta < \pi$, the equation

$$6\tan^3\theta + 17\tan^2\theta + 4\tan\theta - 12 = 0$$

giving your answers to 3 significant figures.

(2)

WMA12/01 OCTOBER 2022

Question 3

7 marks

Trigonometric Equations

Also in Trigonometric Equations

Primary: Sequences & Series

3. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_n = \cos^2\left(\frac{n\pi}{3}\right)$$

Find the exact values of

(a) (i) a_1

(ii) a_2

(iii) a_3

(3)

(b) Hence find the exact value of

$$\sum_{n=1}^{50}\left\{n + \cos^2\left(\frac{n\pi}{3}\right)\right\}$$

You must make your method clear.

(4)

Question 10

12 marks

Trigonometric Equations

Also in Trigonometric Equations

Primary: Binomial Expansion

10. (i) (a) Find, in ascending powers of x, the 2nd, 3rd and 5th terms of the binomial expansion of

$$(3 + 2x)^6$$

(3)

For a particular value of x, these three terms form consecutive terms in a geometric series.

(b) Find this value of x.

(3)

(ii) In a **different** geometric series,

- the first term is $\sin^2\theta$
- the common ratio is $2\cos\theta$
- the sum to infinity is $\frac{8}{5}$

(a) Show that

$$5\cos^2\theta - 16\cos\theta + 3 = 0$$

(3)

(b) Hence find the exact value of the 2nd term in the series.

(3)

TOPIC

Applications of Differentiation

Question 1

1. A curve C has equation $y = 2x^2(x - 5)$

(a) Find, using calculus, the x coordinates of the stationary points of C.

(4)

(b) Hence find the values of x for which y is increasing.

(2)

Question 10

7 marks

Applications of Differentiation

10. The curve C has equation

$$y = ax^3 - 3x^2 + 3x + b$$

where a and b are constants.

Given that

- the point $(2, 5)$ lies on C
- the gradient of the curve at $(2, 5)$ is 7

(a) find the value of a and the value of b.

(4)

(b) Prove that C has no turning points.

(3)

Question 10

10 marks

Applications of Differentiation

10. A curve C has equation

$$y = 4x^3 - 9x + \frac{k}{x} \qquad x > 0$$

where k is a constant.

The point P with x coordinate $\frac{1}{2}$ lies on C.

Given that P is a stationary point of C,

(a) show that $k = -\frac{3}{2}$

(4)

(b) Determine the nature of the stationary point at P, justifying your answer.

(2)

The curve C has a second stationary point.

(c) Using algebra, find the x coordinate of this second stationary point.

(4)

(Total 10 marks)

Question 2

7 marks

Applications of Differentiation

2. A curve has equation

$$y = x^3 - x^2 - 16x + 2$$

(a) Using calculus, find the x coordinates of the stationary points of the curve.

(4)

(b) Justify, by further calculus, the nature of all of the stationary points of the curve.

(3)

WMA12/01 MAY/JUNE 2021

Question 9

10 marks

Applications of Differentiation

9.

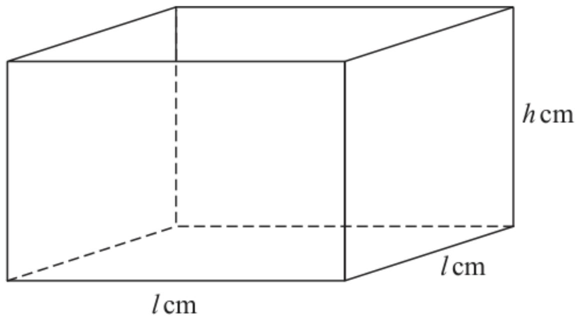

Figure 3

Figure 3 shows a sketch of a square based, open top box.

The height of the box is h cm, and the base edges each have length l cm.

Given that the volume of the box is 250 000 cm^3

(a) show that the external surface area, S cm^2, of the box is given by

$$S = \frac{250\,000}{h} + 2000\sqrt{h}$$

(3)

(b) Use algebraic differentiation to show that S has a stationary point when $h = 250^k$ where k is a rational constant to be found.

(5)

(c) Justify by further differentiation that this value of h gives the minimum external surface area of the box.

(2)

(Total 10 marks)

TOTAL FOR PAPER IS 75 MARKS

Question 8

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

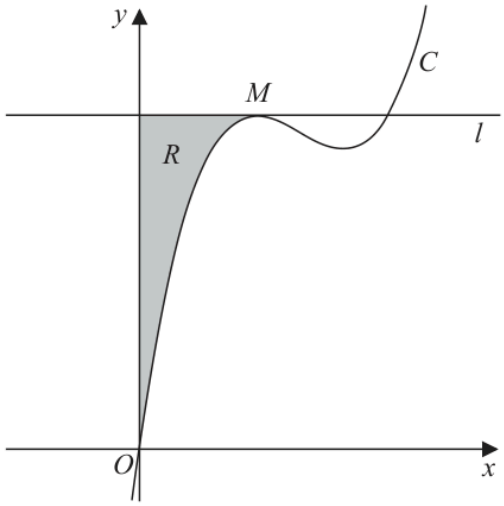

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{4}{3}x^3 - 11x^2 + kx \qquad \text{where } k \text{ is a constant}$$

The point M is the maximum turning point of C and is shown in Figure 2.

Given that the x coordinate of M is 2

(a) show that $k = 28$ **(3)**

(b) Determine the range of values of x for which y is increasing. **(2)**

The line l passes through M and is parallel to the x-axis.

The region R, shown shaded in Figure 2, is bounded by the curve C, the line l and the y-axis.

(c) Find, by algebraic integration, the exact area of R. **(5)**

8 marks

Question 2

Applications of Differentiation

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = 27x^{\frac{1}{2}} - x^{\frac{3}{2}} - 20 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving each term in simplest form.

(2)

(b) Hence find the coordinates of the stationary point of C.

(4)

(c) Find $\frac{d^2y}{dx^2}$ and hence determine the nature of the stationary point of C.

(2)

Question 8

8 marks

Applications of Differentiation

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A curve has equation

$$y = 256x^4 - 304x - 35 + \frac{27}{x^2} \qquad x \neq 0$$

(a) Find $\frac{dy}{dx}$

(3)

(b) Hence find the coordinates of the stationary points of the curve.

(5)

Question 7

7. The curve C has equation

$$y = \frac{12x^3(x-7) + 14x(13x-15)}{21\sqrt{x}} \qquad x > 0$$

(a) Write the equation of C in the form

$$y = ax^{\frac{7}{2}} + bx^{\frac{5}{2}} + cx^{\frac{3}{2}} + dx^{\frac{1}{2}}$$

where a, b, c and d are fully simplified constants.

(3)

The curve C has three turning points.

Using calculus,

(b) show that the x coordinates of the three turning points satisfy the equation

$$2x^3 - 10x^2 + 13x - 5 = 0$$

(3)

Given that the x coordinate of one of the turning points is 1

(c) find, using algebra, the exact x coordinates of the other two turning points.

(*Solutions based entirely on calculator technology are not acceptable.*)

(3)

Question 2

9 marks

Applications of Differentiation

2. **In this question you must show all stages of your working.**

Solutions based entirely on calculator technology are not acceptable.

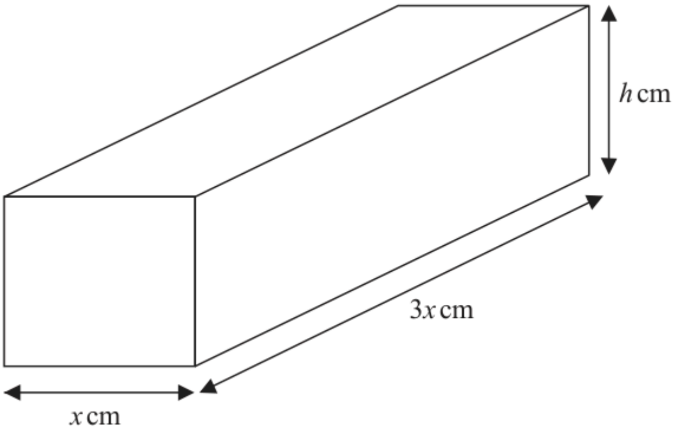

Figure 2

A brick is in the shape of a cuboid with width x cm, length $3x$ cm and height h cm, as shown in Figure 2.

The volume of the brick is $972\,\text{cm}^3$

(a) Show that the surface area of the brick, $S\,\text{cm}^2$, is given by

$$S = 6x^2 + \frac{2592}{x}$$

(3)

(b) Find $\frac{dS}{dx}$

(1)

(c) Hence find the value of x for which S is stationary.

(2)

(d) Find $\frac{d^2S}{dx^2}$ and hence show that the value of x found in part (c) gives the minimum value of S.

(2)

(e) Hence find the minimum surface area of the brick.

(1)

WMA12/01 MAY/JUNE 2023

8 marks

Question 7

Applications of Differentiation

7. The height of a river above a fixed point on the riverbed was monitored over a 7-day period.

The height of the river, H metres, t days after monitoring began, was given by

$$H = \frac{\sqrt{t}}{20}(20 + 6t - t^2) + 17 \qquad 0 \leqslant t \leqslant 7$$

Given that H has a stationary value at $t = \alpha$

(a) use calculus to show that α satisfies the equation

$$5\alpha^2 - 18\alpha - 20 = 0$$

(5)

(b) Hence find the value of α, giving your answer to 3 decimal places.

(1)

(c) Use further calculus to prove that H is a maximum at this value of α.

(2)

WMA12/01 OCTOBER 2023

12 marks

Question 9

Applications of Differentiation

9.

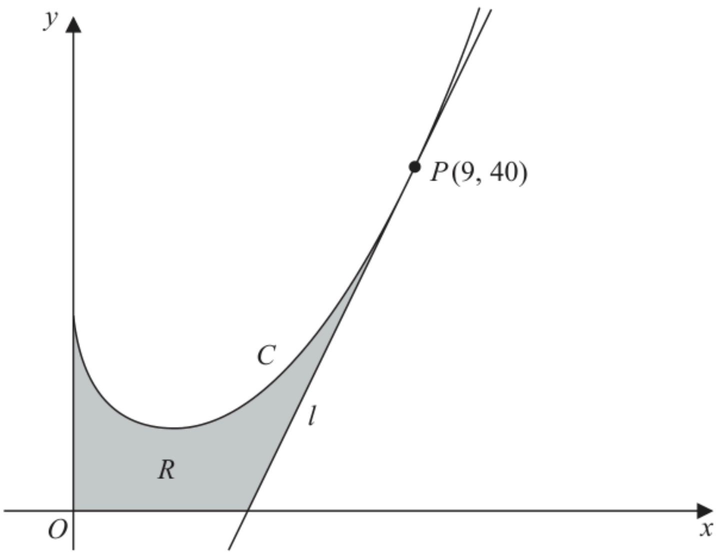

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{2}{3}x^2 - 9\sqrt{x} + 13 \qquad x \geqslant 0$$

(a) Find, using calculus, the range of values of x for which y is increasing.

(4)

The point P lies on C and has coordinates (9, 40).

The line l is the tangent to C at the point P.

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the line l, the x-axis and the y-axis.

(b) Find, using calculus, the exact area of R.

(8)

WMA12/01 JANUARY 2024

9 marks

Question 10

Applications of Differentiation

10. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

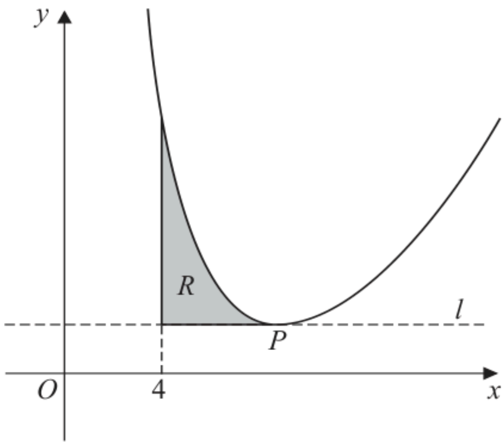

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = \frac{1}{2}x^2 + \frac{1458}{\sqrt{x^3}} - 74 \qquad x > 0$$

The point P is the only stationary point on the curve.

(a) Use calculus to show that the x coordinate of P is 9

(4)

The line l passes through the point P and is parallel to the x-axis.

The region R, shown shaded in Figure 2, is bounded by the curve, the line l and the line with equation $x = 4$

(b) Use algebraic integration to find the exact area of R.

(5)

WMA12/01 MAY/JUNE 2024

8 marks

Question 9

Applications of Differentiation

9.

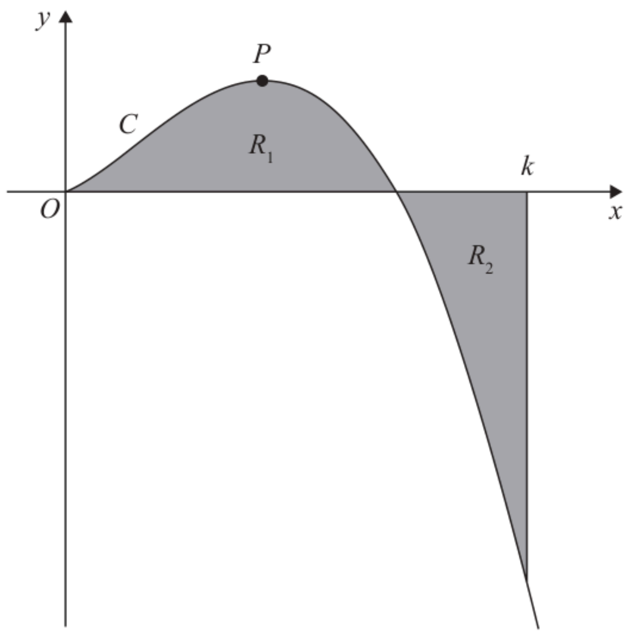

Figure 1

Figure 1 is a sketch of the curve C with equation

$$y = 2x^{\frac{3}{2}}(4 - x) \qquad x \geqslant 0$$

The point P is the stationary point of C.

(a) Find, using calculus, the x coordinate of P.

(4)

The region R_1, shown shaded in Figure 1, is bounded by C and the x-axis.

The region R_2, also shown shaded in Figure 1, is bounded by C, the x-axis and the line with equation $x = k$, where k is a constant.

Given that the area of R_1 is equal to the area of R_2

(b) find, using calculus, the exact value of k.

(4)

Question 6

9 marks

Applications of Differentiation

6. $$f(x) = 4x^3 + px^2 + 8x + q$$

where p and q are constants.

Given that

- $(2x + 3)$ is a factor of $f(x)$
- $f(x)$ has a remainder of -5 when divided by $(x + 2)$

(a) (i) show that $p = 10$

(ii) find the value of q.

(5)

(b) Hence find the range of values of x for which $f(x)$ is decreasing.

(Solutions based entirely on calculator technology are not acceptable.)

(4)

WMA12/01 MAY/JUNE R 2024

9 marks

Question 10

Applications of Differentiation

10.

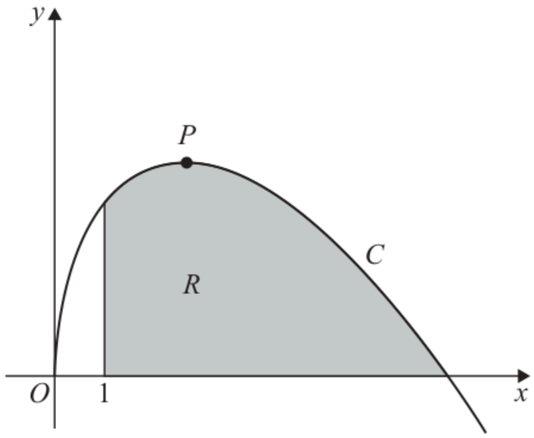

Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of part of the curve C with equation

$$y = \frac{9x - x^2}{2\sqrt{x}} \qquad x > 0$$

The point P is the stationary point on C.

(a) Find, using calculus, the x coordinate of P.

(4)

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the x-axis and the line with equation $x = 1$

(b) Using calculus, calculate the exact area of R.

(5)

Question 4

7 marks

Applications of Differentiation

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = 4x^{\frac{1}{2}} + 9x^{-\frac{1}{2}} + 3 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$ giving each term in simplest form.

(2)

(b) Hence find the x coordinate of the stationary point of C.

(2)

(c) (i) Find $\frac{d^2y}{dx^2}$ giving each term in simplest form.

(ii) Hence determine the nature of the stationary point of C, giving a reason for your answer.

(2)

(d) State the range of values of x for which y is decreasing.

(1)

Question 3

3.

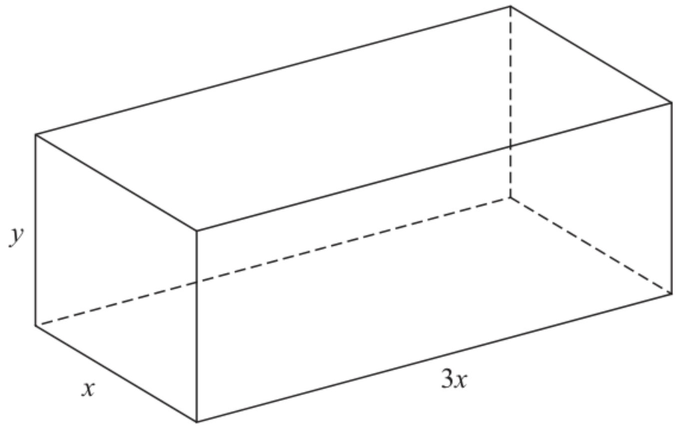

Figure 1

Figure 1 shows an open-topped container used for holding water.

The container is in the shape of a cuboid and is made of sheet metal.

The base of the container is a rectangle $3x$ metres by x metres.

The height of the container is y metres as shown in Figure 1.

Given that the capacity of the container is $120\,\text{m}^3$

(a) show that the area $A\,\text{m}^2$ of the sheet metal used to make the container is given by

$$A = Px^2 + \frac{Q}{x}$$

where P and Q are positive constants to be found.

(4)

(b) Use calculus to find the value of x for which A has a stationary value, giving your answer to 3 significant figures.

(4)

(c) Find $\frac{d^2A}{dx^2}$ and hence show that the value of x found in part (b) gives the minimum value of A.

(2)

WMA12/01 MAY/JUNE 2025

11 marks

Question 9

Applications of Differentiation

9:

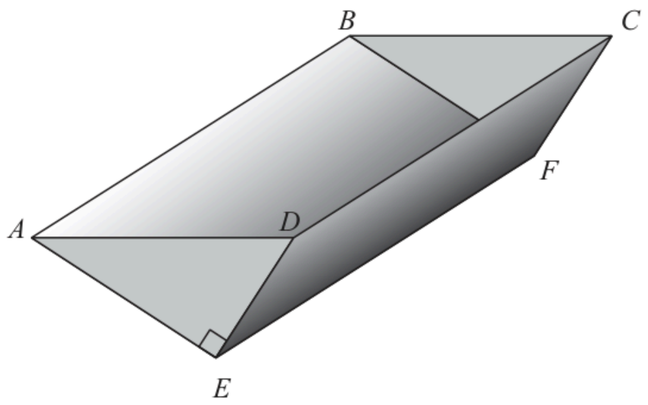

Figure 2

Figure 2 shows a sketch of an open container.

The sides *CDEF* and *ABFE* are rectangles.

The ends *ADE* and *BCF* are congruent (identical) right-angled triangles.

The container is made from metal of negligible thickness.

Given that

- $AE = BF = 3x$ metres
- $DE = CF = 2x$ metres
- $AB = DC = EF = L$ metres

and the capacity of the container is $12\,\text{m}^3$

(a) show that the area of metal used to make the container, $S\,\text{m}^2$, is given by

$$S = Px^2 + \frac{Q}{x}$$

where P and Q are positive integers to be found.

(4)

Given that x can vary,

(b) use algebraic calculus to find the minimum value of S, giving your answer to one decimal place.

(5)

(c) Justify that the value of S found in part (b) is a minimum.

(2)

Question 10

10.

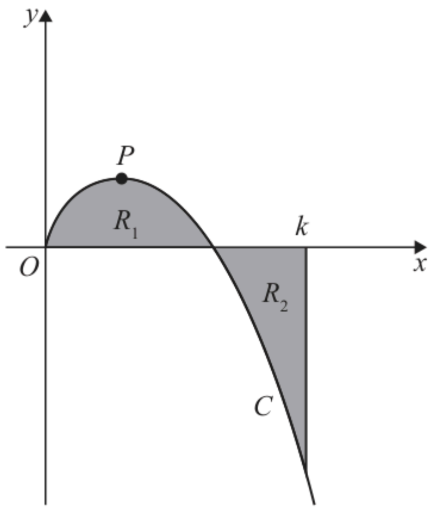

Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve C with equation

$$y = \frac{\sqrt{x}\left(100 - x^2\right)}{40} \qquad x \geqslant 0$$

The point P is a stationary point on C.

(a) Use algebraic differentiation to find the exact x coordinate of P.

(4)

The region R_1, shown shaded in Figure 2, is bounded by C and the x-axis.

The region R_2, also shown shaded in Figure 2, is bounded by C, the x-axis and the line with equation $x = k$, where k is a constant.

Given that the area of R_1 is equal to the area of R_2

(b) use algebraic integration to find the exact value of k.

(4)

Question 6

6. The circle C has equation

$$x^2 + y^2 + 6x - 4y - 14 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C.

(3)

The line with equation $y = k$, where k is a constant, is a tangent to C.

(b) Find the possible values of k.

(2)

The line with equation $y = p$, where p is a negative constant, is a chord of C.

Given that the length of this chord is 4 units,

(c) find the value of p.

(3)

WMA12/01 JANUARY 2021

Question 9

10 marks

Applications of Differentiation

Also in Applications of Differentiation

Primary: Circles

9. A circle C has equation

$$(x - k)^2 + (y - 2k)^2 = k + 7$$

where k is a positive constant.

(a) Write down, in terms of k,

(i) the coordinates of the centre of C,

(ii) the radius of C.

(2)

Given that the point $P(2, 3)$ lies on C

(b) (i) show that $5k^2 - 17k + 6 = 0$

(ii) hence find the possible values of k.

(3)

The tangent to the circle at P intersects the x-axis at point T.

Given that $k < 2$

(c) calculate the exact area of triangle OPT.

(5)

WMA12/01 MAY/JUNE 2021

Question 6

7 marks

Applications of Differentiation

Also in Applications of Differentiation

Primary: Circles

6. A circle has equation

$$x^2 - 6x + y^2 + 8y + k = 0$$

where k is a positive constant.

Given that the x-axis is a tangent to this circle,

(a) find the value of k.

(3)

The circle meets the coordinate axes at the points R, S and T.

(b) Find the exact area of the triangle RST.

(4)

WMA12/01 JANUARY 2022

Question 6

8 marks

Applications of Differentiation

Also in Applications of Differentiation

Primary: Circles

6.

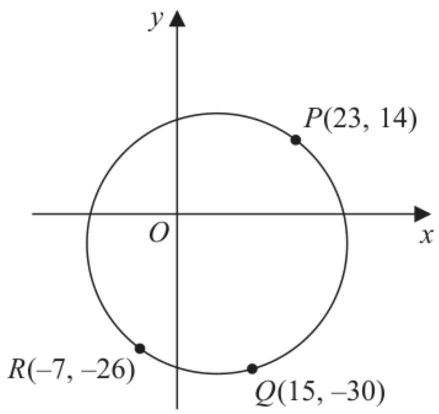

Figure 1

The points $P(23, 14)$, $Q(15, -30)$ and $R(-7, -26)$ lie on the circle C, as shown in Figure 1.

(a) Show that angle $PQR = 90°$

(2)

(b) Hence, or otherwise, find

(i) the centre of C,

(ii) the radius of C.

(3)

Given that the point S lies on C such that the distance QS is greatest,

(c) find an equation of the tangent to C at S, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

10 marks

WMA12/01 JANUARY 2022

Applications of Differentiation

Question 9

Also in Applications of Differentiation

Primary: Integration

9.

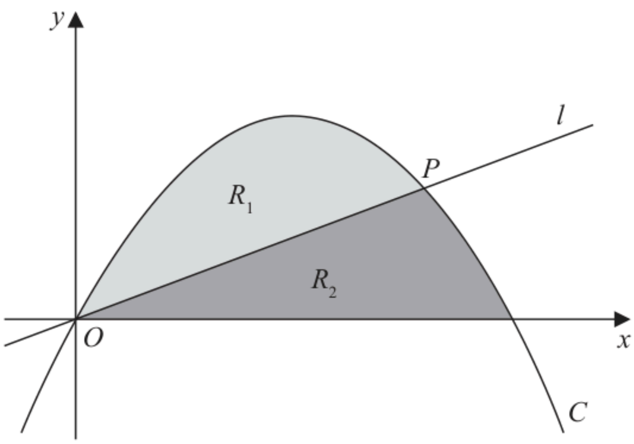

Figure 2

Figure 2 shows

- the curve C with equation $y = x - x^2$
- the line l with equation $y = mx$, where m is a constant and $0 < m < 1$

The line and the curve intersect at the origin O and at the point P.

(a) Find, in terms of m, the coordinates of P.

(2)

The region R_1, shown shaded in Figure 2, is bounded by C and l.

(b) Show that the area of R_1 is

$$\frac{(1 - m)^3}{6}$$

(5)

The region R_2, also shown shaded in Figure 2, is bounded by C, the x-axis and l.

Given that the area of R_1 is equal to the area of R_2

(c) find the exact value of m.

(3)

WMA12/01 OCTOBER 2022

Question 2

7 marks

Applications of Differentiation

Also in Applications of Differentiation

Primary: Binomial Expansion

2. A curve C has equation $y = \text{f}(x)$ where

$$\text{f}(x) = (2 - kx)^5$$

and k is a constant.

Given that when $\text{f}(x)$ is divided by $(4x - 5)$ the remainder is $\dfrac{243}{32}$

(a) show that $k = \dfrac{2}{5}$

(2)

(b) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{2}{5}x\right)^5$$

giving each term in simplest form.

(3)

Using the solution to part (b) and making your method clear,

(c) find the gradient of C at the point where $x = 0$

(2)

WMA12/01 OCTOBER 2022

Question 6

7 marks

Applications of Differentiation

Also in Applications of Differentiation

Primary: Integration

6. The curve C_1 has equation $y = f(x)$.

A table of values of x and y for $y = f(x)$ is shown below, with the y values rounded to 4 decimal places where appropriate.

x	0	0.5	1	1.5	2
y	3	2.6833	2.4	2.1466	1.92

(a) Use the trapezium rule with all the values of y in the table to find an approximation for

$$\int_0^2 f(x)\,dx$$

giving your answer to 3 decimal places.

(3)

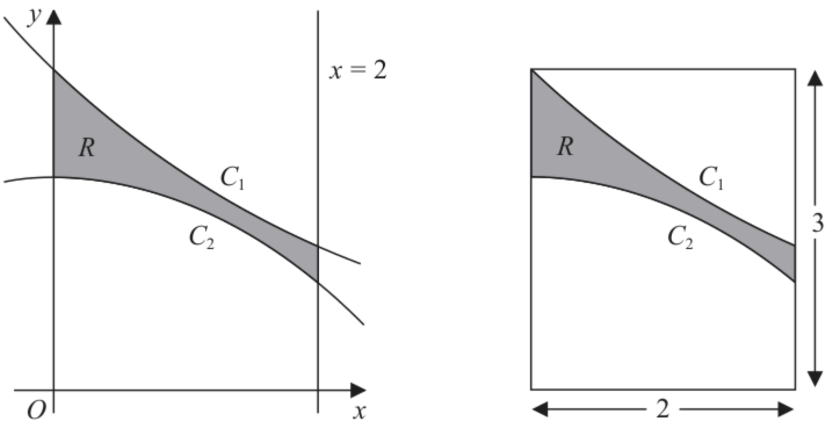

Figure 1 **Figure 2**

The region R, shown shaded in Figure 1, is bounded by

- the curve C_1
- the curve C_2 with equation $y = 2 - \frac{1}{4}x^2$
- the line with equation $x = 2$
- the y-axis

The region R forms part of the design for a logo shown in Figure 2.

The design consists of the shaded region R inside a rectangle of width 2 and height 3

Using calculus and the answer to part (a),

(b) calculate an estimate for the percentage of the logo which is shaded.

(4)

WMA12/01 OCTOBER 2022

Question 9

12 marks

Applications of Differentiation

Also in Applications of Differentiation

Primary: Circles

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

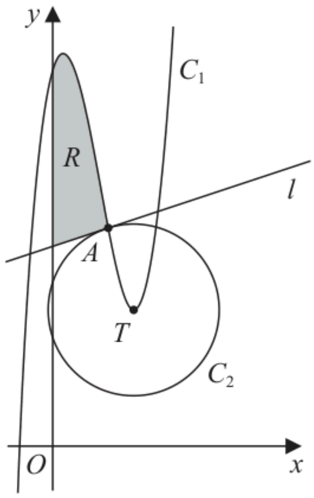

Figure 3

Figure 3 shows

- the curve C_1 with equation $y = x^3 - 5x^2 + 3x + 14$
- the circle C_2 with centre T

The point T is the minimum turning point of C_1

Using Figure 3 and calculus,

(a) find the coordinates of T

(3)

The curve C_1 intersects the circle C_2 at the point A with x coordinate 2

(b) Find an equation of the circle C_2

(3)

The line l shown in Figure 3, is the tangent to circle C_2 at A

(c) Show that an equation of l is

$$y = \frac{1}{3}x + \frac{22}{3}$$

(3)

The region R, shown shaded in Figure 3, is bounded by C_1, l and the y-axis.

(d) Find the exact area of R.

(3)

WMA12/01 JANUARY 2023

Question 6

10 marks

Applications of Differentiation

Also in Applications of Differentiation

Primary: Circles

6. The circle C has equation

$$x^2 + y^2 + 8x - 4y = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C.

(3)

The point P lies on C.

Given that the tangent to C at P has equation $x + 2y + 10 = 0$

(b) find the coordinates of P

(4)

(c) Find the equation of the normal to C at P, giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(3)

WMA12/01 MAY/JUNE 2023

Question 3

7 marks

Applications of Differentiation

Also in Applications of Differentiation

Primary: Circles

3. A circle C has centre $(2, 5)$

Given that the point $P(8, -3)$ lies on C

(a) (i) find the radius of C

(ii) find an equation for C

(3)

(b) Find the equation of the tangent to C at P giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(4)

WMA12/01 MAY/JUNE 2023

Question 10

10 marks

Applications of Differentiation

Also in Applications of Differentiation

Primary: Integration

10. The curve C has equation

$$y = \frac{(x-k)^2}{\sqrt{x}} \qquad x > 0$$

where k is a **positive** constant.

(a) Show that

$$\int_1^{16} \frac{(x-k)^2}{\sqrt{x}}\,\mathrm{d}x = ak^2 + bk + \frac{2046}{5}$$

where a and b are integers to be found.

(5)

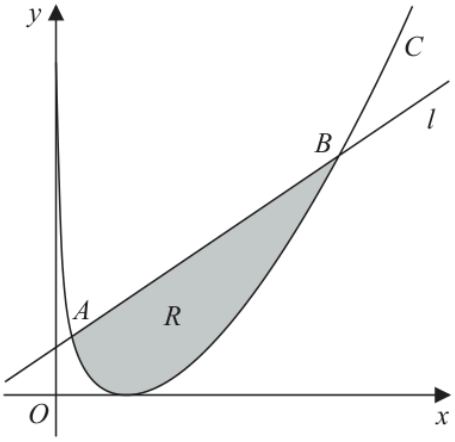

Figure 1

Figure 1 shows a sketch of the curve C and the line l.

Given that l intersects C at the point $A(1, 9)$ and at the point $B(16, q)$ where q is a constant,

(b) show that $k = 4$

(2)

The region R, shown shaded in Figure 1, is bounded by C and l

Using the answers to parts (a) and (b),

(c) find the area of region R

(3)

WMA12/01 OCTOBER 2023

Question 7

8 marks

Applications of Differentiation

Also in Applications of Differentiation

Primary: Circles

7.

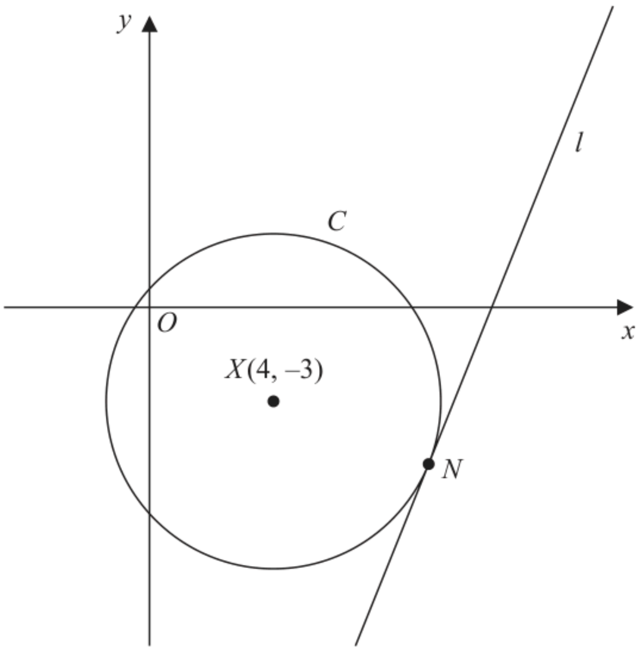

Figure 2

Figure 2 shows a sketch of

- the circle C with centre $X(4, -3)$
- the line l with equation $y = \frac{5}{2}x - \frac{55}{2}$

Given that l is the tangent to C at the point N,

(a) show that an equation for the straight line passing through X and N is

$$2x + 5y + 7 = 0$$

(3)

(b) Hence find

(i) the coordinates of N,

(ii) an equation for C.

(5)

WMA12/01 OCTOBER 2024

Question 10

10 marks

Applications of Differentiation

Also in Applications of Differentiation

Primary: Circles

10. The circle C has equation

$$x^2 + y^2 + 4x - 30y + 209 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact value of the radius of C.

(3)

The line L has equation $y = mx + 1$, where m is a constant.

Given that L is the tangent to C at the point P,

(b) show that

$$2m^2 - 7m - 22 = 0$$

(3)

(c) Hence find the possible pairs of coordinates of P.

(4)

WMA12/01 JANUARY 2025

Question 9

12 marks

Applications of Differentiation

Also in Applications of Differentiation

Primary: Integration

9.

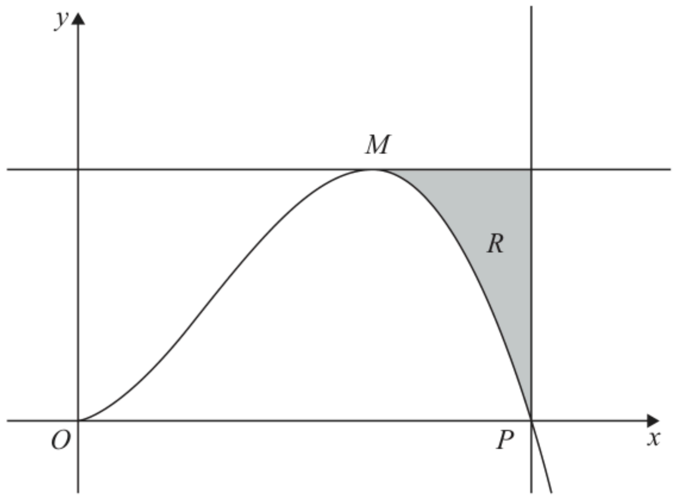

Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \frac{9x^2\left(5 - \sqrt{x}\right)}{5} \qquad x \geqslant 0$$

The curve has a turning point at the point M, as shown in Figure 3.

(a) Using calculus, find the coordinates of M.

(5)

The curve crosses the x-axis at the point P, as shown in Figure 3.

(b) Use algebra to find the x coordinate of P.

(2)

The finite region R, shown shaded in Figure 3, is bounded by the curve, the line through M parallel to the x-axis and the line through P parallel to the y-axis.

(c) Use algebraic integration to find the area of R, giving your answer to one decimal place.

(5)

Question 4

12 marks

Applications of Differentiation

Also in Applications of Differentiation

Primary: Polynomials

4. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

$$f(x) = 4x^3 + 13x^2 - 10x + 8$$

(a) When $f(x)$ is divided by $(x - 2)$ the remainder is R and the quotient is $Q(x)$.

(i) Find $Q(x)$.

(ii) Find R.

(4)

(b) (i) Use the factor theorem to show that $(x + 4)$ is a factor of $f(x)$.

(ii) Hence prove, using algebra, that the equation $f(x) = 0$ has only one real solution.

(5)

(c) Find the range of values of x for which $f(x)$ is decreasing.

(3)

WMA12/01 OCTOBER 2025

Question 7

10 marks

Applications of Differentiation

Also in Applications of Differentiation

Primary: Integration

7.

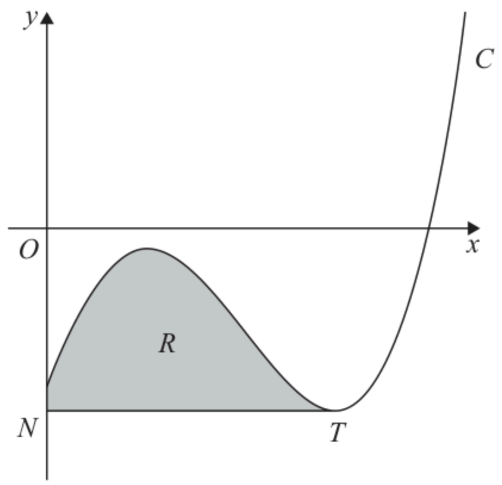

Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve C with equation

$$y = x^3 - 4x^{\frac{5}{2}} - kx^{\frac{1}{2}} + 28x - 44 \qquad x \geqslant 0$$

where k is a positive constant.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in simplest form.

(2)

The point T, shown in Figure 3, is a minimum stationary point on C.

Given that the x coordinate of T is 9

(b) show that $k = 6$

(2)

The line through T parallel to the x-axis meets the y-axis at the point N.

The finite region R, shown shaded in Figure 3, is bounded by C, the y-axis and the line segment NT.

(c) Use algebraic integration to find the area of R, giving the answer to 3 significant figures.

(6)

WMA12/01 JANUARY 2026

Question 6

8 marks

Applications of Differentiation

Also in Applications of Differentiation

Primary: Proof

6. (i) Given that p and q are consecutive odd numbers, where $p > q > 0$, prove that

$$p^2 - q^2$$

is a multiple of 8

(4)

(ii) The curve C has equation

$$y = x^3 + 12x^2 + 49x + 2$$

Prove that C has no stationary points.

(4)

TOPIC

Integration

WMA12/01 OCTOBER 2019

Question 5

7 marks

Integration

5. (a) Given $0 < a < 1$, sketch the curve with equation

$$y = a^x$$

showing the coordinates of the point at which the curve crosses the y-axis.

(2)

x	2	2.5	3	3.5	4
y	4.25	6.427	9.125	12.34	16.06

The table above shows corresponding values of x and y for $y = x^2 + \left(\frac{1}{2}\right)^x$

The values of y are given to 4 significant figures as appropriate.

Using the trapezium rule with all the values of y in the given table,

(b) obtain an estimate for $\displaystyle\int_2^4 \left(x^2 + \left(\frac{1}{2}\right)^x\right) \mathrm{d}x$

(3)

Using your answer to part (b) and making your method clear, estimate

(c) $\displaystyle\int_2^4 \left(x(x-3) + \left(\frac{1}{2}\right)^x\right) \mathrm{d}x$

(2)

WMA12/01 OCTOBER 2019

Question 8

9 marks

Integration

8. Solutions relying on calculator technology are not acceptable in this question.

(i)

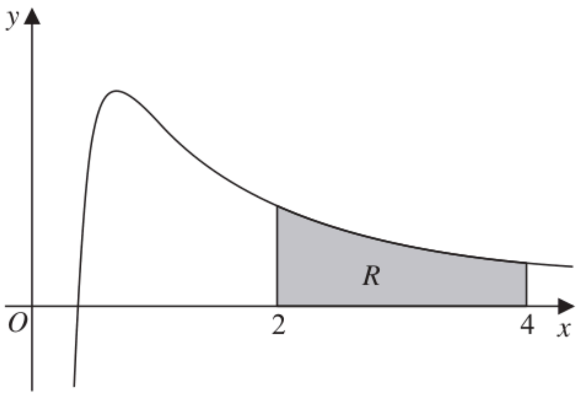

Figure 2

Figure 2 shows a sketch of part of a curve with equation

$$y = \frac{8\sqrt{x} - 5}{2x^2} \qquad x > 0$$

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$

Find the exact area of R.

(5)

(ii) Find the value of the constant k such that

$$\int_{-3}^{6} \left(\frac{1}{2}x^2 + k\right) \mathrm{d}x = 55$$

(4)

WMA12/01 JANUARY 2020

7 marks

Question 1

Integration

1. The table below shows corresponding values of x and y for $y = \log_2(2x)$

The values of y are given to 2 decimal places as appropriate.

x	2	5	8	11	14
y	2	3.32	4	4.46	4.81

Using the trapezium rule with all the values of y in the given table,

(a) obtain an estimate for $\int_2^{14} \log_2(2x)\,\mathrm{d}x$, giving your answer to one decimal place.

(3)

Using your answer to part (a) and making your method clear, estimate

(b) (i) $\int_2^{14} \frac{\log_2(4x^2)}{5}\,\mathrm{d}x$

(ii) $\int_2^{14} \log_2\left(\frac{2}{x}\right)\mathrm{d}x$

(4)

WMA12/01 JANUARY 2020

Question 4

6 marks

Integration

4.

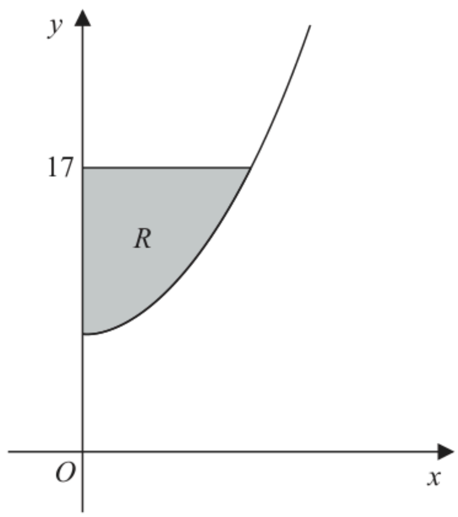

Figure 1

Figure 1 shows a sketch of the curve with equation

$$y = 2x^2 + 7 \qquad x \geqslant 0$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the y-axis and the line with equation $y = 17$

Find the exact area of R.

(6)

WMA12/01 JANUARY 2021

6 marks

Question 7

Integration

7.

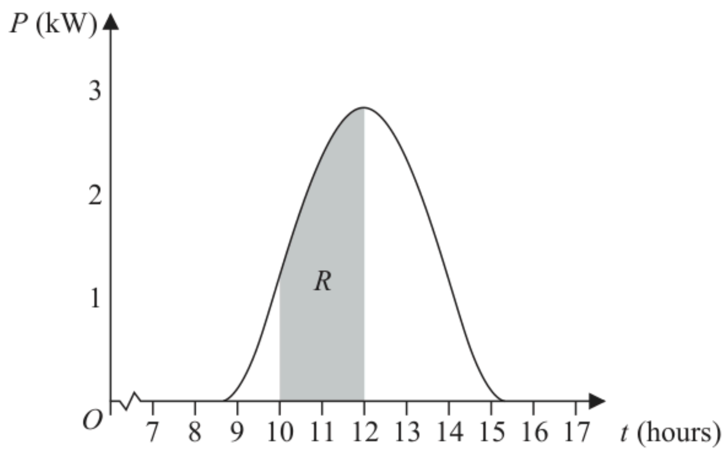

Figure 1

Solar panels are installed on the roof of a building.

The power, P, produced on a particular day, in kW, can be modelled by the equation

$$P = 0.95 + 2^{t-12} + 2^{12-t} - (t-12)^2 \qquad 8.5 \leqslant t \leqslant 15.2$$

where t is the time in hours after midnight. The graph of P against t is shown in Figure 1.

A table of values of t and P is shown below, with the values of P given to 4 significant figures where appropriate.

Time, t (hours)	10	10.5	11	11.5	12
Power, P (kW)		1.882	2.45		2.95

(a) Use the given equation to complete the table, giving the values of P to 4 significant figures where appropriate.

(2)

The amount of energy, in kWh, produced between 10:00 and 12:00 can be found by calculating the area of region R, shown shaded in Figure 1.

(b) Use the trapezium rule, with all the values of P in the completed table, to find an estimate for the amount of energy produced between 10:00 and 12:00. Give your answer to 2 decimal places.

(4)

Question 5

10 marks

Integration

5.

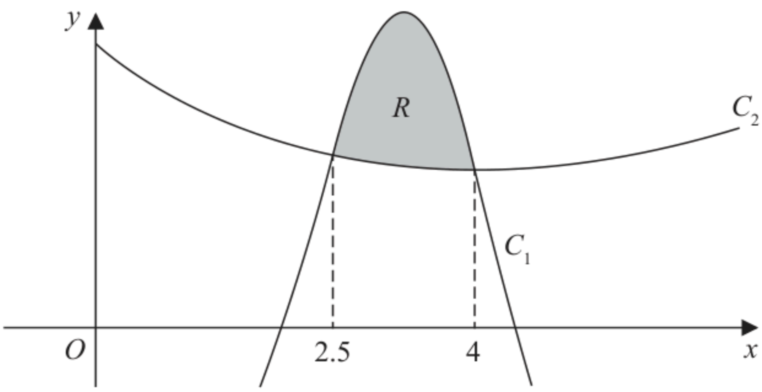

Figure 2

Figure 2 shows a sketch of part of the graph of the curves C_1 and C_2

The curves intersect when $x = 2.5$ and when $x = 4$

A table of values for some points on the curve C_1 is shown below, with y values given to 3 decimal places as appropriate.

x	2.5	2.75	3	3.25	3.5	3.75	4
y	5.453	7.764	9.375	9.964	9.367	7.626	5

Using the trapezium rule with all the values of y in the table,

(a) find, to 2 decimal places, an estimate for the area bounded by the curve C_1, the line with equation $x = 2.5$, the x-axis and the line with equation $x = 4$

(4)

The curve C_2 has equation

$$y = x^{\frac{3}{2}} - 3x + 9 \qquad x > 0$$

(b) Find $\displaystyle\int \left(x^{\frac{3}{2}} - 3x + 9 \right) \mathrm{d}x$

(3)

The region R, shown shaded in Figure 2, is bounded by the curves C_1 and C_2

(c) Use the answers to part (a) and part (b) to find, to one decimal place, an estimate for the area of the region R.

(3)

Question 3

8 marks

Integration

3.

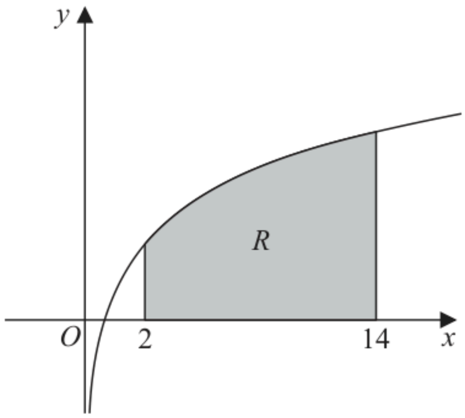

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \log_{10} x$

The region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 14$

Using the trapezium rule with four strips of equal width,

(a) show that the area of R is approximately 10.10

(3)

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of R.

(1)

(c) Using the answer to part (a) and making your method clear, estimate the value of

(i) $\int_2^{14} \log_{10} \sqrt{x}\, dx$

(ii) $\int_2^{14} \log_{10} 100x^3\, dx$

(4)

WMA12/01 JANUARY 2022

Question 1

7 marks

Integration

1. The table below shows corresponding values of x and y for

$$y = 2^{5-\sqrt{x}}$$

The values of y are given to 3 decimal places.

x	5	5.5	6	6.5	7
y	6.792	6.298	5.858	5.466	5.113

Using the trapezium rule with all the values of y in the given table,

(a) obtain an estimate for

$$\int_5^7 2^{5-\sqrt{x}}\,dx$$

giving your answer to 2 decimal places.

(3)

(b) Using your answer to part (a) and making your method clear, estimate

(i) $\int_5^7 2^{6-\sqrt{x}}\,dx$

(ii) $\int_5^7 \left(3 + 2^{5-\sqrt{x}}\right)dx$

(4)

WMA12/01 JANUARY 2022

10 marks

Question 9

Integration

9.

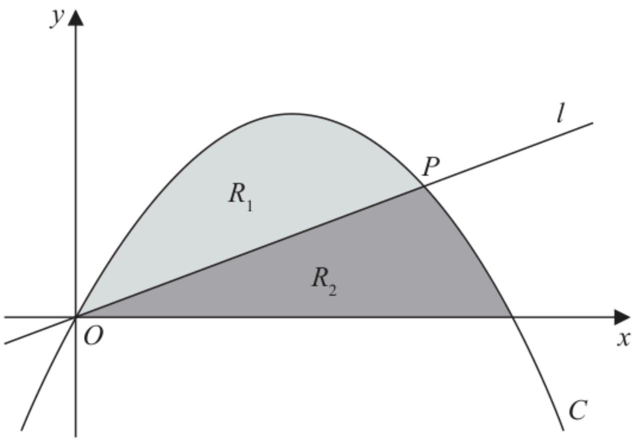

Figure 2

Figure 2 shows

- the curve C with equation $y = x - x^2$
- the line l with equation $y = mx$, where m is a constant and $0 < m < 1$

The line and the curve intersect at the origin O and at the point P.

(a) Find, in terms of m, the coordinates of P.

(2)

The region R_1, shown shaded in Figure 2, is bounded by C and l.

(b) Show that the area of R_1 is

$$\frac{(1-m)^3}{6}$$

(5)

The region R_2, also shown shaded in Figure 2, is bounded by C, the x-axis and l.

Given that the area of R_1 is equal to the area of R_2

(c) find the exact value of m.

(3)

WMA12/01 MAY/JUNE 2022

Question 2

8 marks

Integration

2.

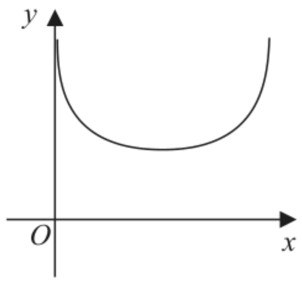

Figure 1

Figure 1 shows the graph of

$$y = 1 - \log_{10}(\sin x) \qquad 0 < x < \pi$$

where x is in radians.

The table below shows some values of x and y for this graph, with values of y given to 3 decimal places.

x	0.5	1	1.5	2	2.5	3
y	1.319		1.001		1.223	1.850

(a) Complete the table above, giving values of y to 3 decimal places.

(2)

(b) Use the trapezium rule with all the y values in the completed table to find, to 2 decimal places, an estimate for

$$\int_{0.5}^{3} \left(1 - \log_{10}(\sin x)\right) dx$$

(3)

(c) Use your answer to part (b) to find an estimate for

$$\int_{0.5}^{3} \left(3 + \log_{10}(\sin x)\right) dx$$

(3)

Question 7

7 marks

Integration

7. $$f(x) = Ax^3 + 6x^2 - 4x + B$$

where A and B are constants.

Given that

- $(x + 2)$ is a factor of $f(x)$
- $\int_3^5 f(x)\,dx = 176$

find the value of A and the value of B.

(7)

WMA12/01 OCTOBER 2022

Question 6

7 marks

Integration

6. The curve C_1 has equation $y = f(x)$.

A table of values of x and y for $y = f(x)$ is shown below, with the y values rounded to 4 decimal places where appropriate.

x	0	0.5	1	1.5	2
y	3	2.6833	2.4	2.1466	1.92

(a) Use the trapezium rule with all the values of y in the table to find an approximation for

$$\int_0^2 f(x)\,dx$$

giving your answer to 3 decimal places.

(3)

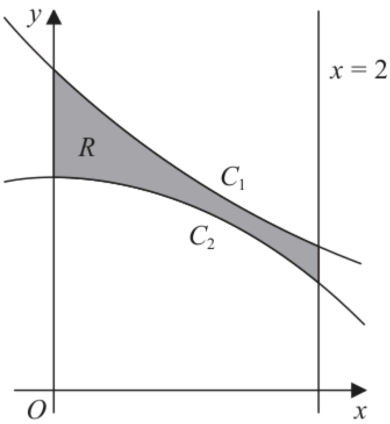

Figure 1

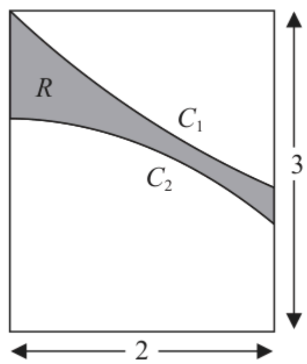

Figure 2

The region R, shown shaded in Figure 1, is bounded by

- the curve C_1
- the curve C_2 with equation $y = 2 - \frac{1}{4}x^2$
- the line with equation $x = 2$
- the y-axis

The region R forms part of the design for a logo shown in Figure 2.

The design consists of the shaded region R inside a rectangle of width 2 and height 3

Using calculus and the answer to part (a),

(b) calculate an estimate for the percentage of the logo which is shaded.

(4)

WMA12/01 JANUARY 2023

6 marks

Question 1

Integration

1.

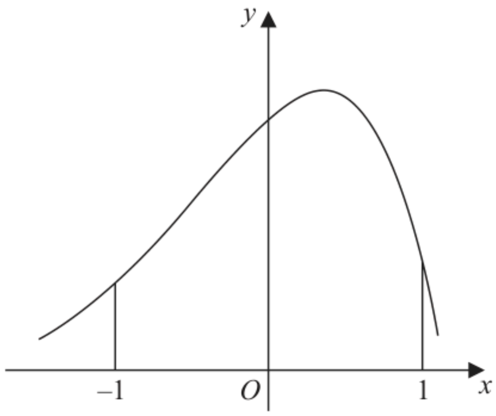

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$

The table below shows some corresponding values of x and y for this curve.

The values of y are given to 3 decimal places.

x	–1	–0.5	0	0.5	1
y	2.287	4.470	6.719	7.291	2.834

Using the trapezium rule with all the values of y in the given table,

(a) obtain an estimate for

$$\int_{-1}^{1} \mathrm{f}(x)\,\mathrm{d}x$$

giving your answer to 2 decimal places.

(3)

(b) Use your answer to part (a) to estimate

(i) $\displaystyle\int_{-1}^{1} (\mathrm{f}(x) - 2)\,\mathrm{d}x$

(ii) $\displaystyle\int_{1}^{3} \mathrm{f}(x-2)\,\mathrm{d}x$

(3)

WMA12/01 JANUARY 2023

Question 9

8 marks

Integration

9. **In this question you must show all stages of your working.**

Solutions based entirely on calculator technology are not acceptable.

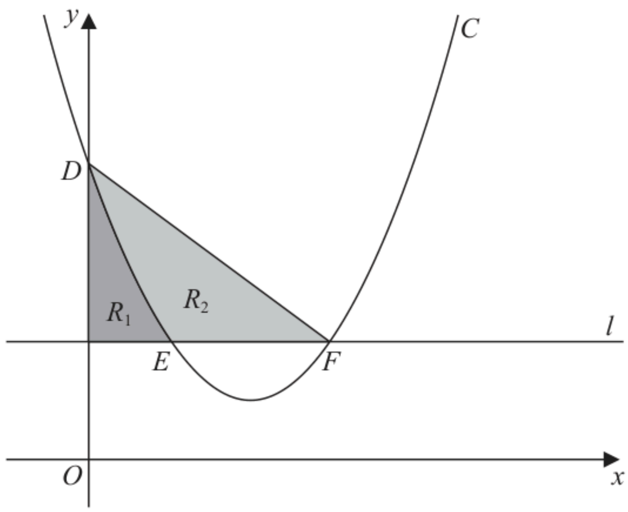

Figure 3

Figure 3 shows

- the curve C with equation $y = x^2 - 4x + 5$
- the line l with equation $y = 2$

The curve C intersects the y-axis at the point D.

(a) Write down the coordinates of D.

(1)

The curve C intersects the line l at the points E and F, as shown in Figure 3.

(b) Find the x coordinate of E and the x coordinate of F.

(2)

Shown shaded in Figure 3 is

- the region R_1 which is bounded by C, l and the y-axis
- the region R_2 which is bounded by C and the line segments EF and DF

Given that $\dfrac{\text{area of } R_1}{\text{area of } R_2} = k$, where k is a constant,

(c) use algebraic integration to find the exact value of k, giving your answer as a simplified fraction.

(5)

WMA12/01 MAY/JUNE 2023

3 marks

Question 1

Integration

1. The continuous curve C has equation $y = \mathrm{f}(x)$.

A table of values of x and y for $y = \mathrm{f}(x)$ is shown below.

x	4.0	4.2	4.4	4.6	4.8	5.0
y	9.2	8.4556	3.8512	5.0342	7.8297	8.6

Use the trapezium rule with all the values of y in the table to find an approximation for

$$\int_4^5 \mathrm{f}(x)\,\mathrm{d}x$$

giving your answer to 3 decimal places.

(3)

WMA12/01 MAY/JUNE 2023

10 marks

Question 10

Integration

10. The curve C has equation

$$y = \frac{(x-k)^2}{\sqrt{x}} \qquad x > 0$$

where k is a **positive** constant.

(a) Show that

$$\int_1^{16} \frac{(x-k)^2}{\sqrt{x}}\,dx = ak^2 + bk + \frac{2046}{5}$$

where a and b are integers to be found.

(5)

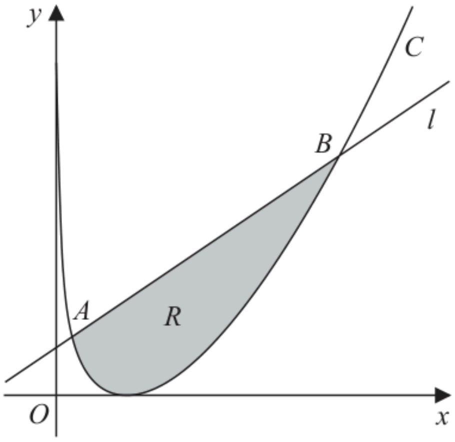

Figure 1

Figure 1 shows a sketch of the curve C and the line l.

Given that l intersects C at the point $A(1, 9)$ and at the point $B(16, q)$ where q is a constant,

(b) show that $k = 4$

(2)

The region R, shown shaded in Figure 1, is bounded by C and l

Using the answers to parts (a) and (b),

(c) find the area of region R

(3)

WMA12/01 OCTOBER 2023

Question 6

6 marks

Integration

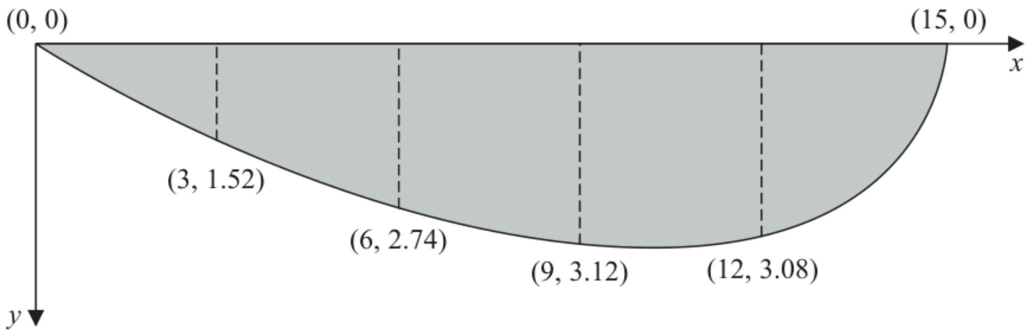

Figure 1

A river is being studied.

At one particular place, the river is 15 m wide.

The depth, y metres, of the river is measured at a point x metres from one side of the river.

Figure 1 shows a plot of the cross-section of the river and the coordinate values (x, y)

(a) Use the trapezium rule with all the y values given in Figure 1 to estimate the cross-sectional area of the river.

(3)

The water in the river is modelled as flowing at a constant speed of $1.5\,\text{m s}^{-1}$ across the whole of the cross-section.

(b) Use the model and the answer to part (a) to estimate the volume of water flowing through this section of the river each minute, giving your answer in m^3 to 2 significant figures.

(2)

Assuming the model,

(c) state, giving a reason for your answer, whether your answer for part (b) is an overestimate or an underestimate of the true volume of water flowing through this section of the river each minute.

(1)

WMA12/01 JANUARY 2024

9 marks

Question 4

Integration

4. (a) Sketch the curve with equation

$$y = a^{-x} + 4$$

where a is a constant and $a > 1$

On your sketch show

- the coordinates of the point of intersection of the curve with the y-axis
- the equation of the asymptote to the curve.

(3)

x	–4	–1.5	1	3.5	6	8.5
y	13	6.280	4.577	4.146	4.037	4.009

The table above shows corresponding values of x and y for $y = 3^{-\frac{1}{2}x} + 4$

The values of y are given to four significant figures, as appropriate.

Using the trapezium rule with all the values of y in the table,

(b) find an approximate value for

$$\int_{-4}^{8.5} \left(3^{-\frac{1}{2}x} + 4\right) dx$$

giving your answer to two significant figures.

(3)

(c) **Using the answer to part (b)**, find an approximate value for

(i) $\int_{-4}^{8.5} \left(3^{-\frac{1}{2}x}\right) dx$

(ii) $\int_{-4}^{8.5} \left(3^{-\frac{1}{2}x} + 4\right) dx + \int_{-8.5}^{4} \left(3^{\frac{1}{2}x} + 4\right) dx$

(3)

WMA12/01 MAY/JUNE 2024

9 marks

Question 6

Integration

6. (a) Sketch the curve with equation

$$y = a^x + 4$$

where a is a positive constant greater than 1

On your sketch, show

- the coordinates of the point of intersection of the curve with the y-axis
- the equation of the asymptote of the curve

(3)

x	2	2.3	2.6	2.9	3.2	3.5
y	0	0.3246	0.8629	1.6643	2.7896	4.3137

The table shows corresponding values of x and y for

$$y = 2^x - 2x$$

with the values of y given to 4 decimal places as appropriate.

Using the trapezium rule with all the values of y in the given table,

(b) obtain an estimate for $\int_2^{3.5} \left(2^x - 2x\right) \mathrm{d}x$, giving your answer to 2 decimal places.

(3)

(c) Using your answer to part (b) and making your method clear, estimate

(i) $\int_2^{3.5} \left(2^x + 2x\right) \mathrm{d}x$

(ii) $\int_2^{3.5} \left(2^{x+1} - 4x\right) \mathrm{d}x$

(3)

Question 2

5 marks

Integration

2. The table shows corresponding values of x and y for a continuous curve with equation $y = \mathrm{f}(x)$ between $x = -4$ and $x = 5$, where a is a constant.

x	-4	-2.5	-1	0.5	2	3.5	5
y	4.16	2.91	a	1.73	1.37	1.43	2.28

The trapezium rule is used with all the y values in the table to find an approximation for

$$\int_{-4}^{5} \mathrm{f}(x)\,\mathrm{d}x$$

Given that the value of this approximation is 19.3

(a) find the value of the constant a to 3 significant figures.

(3)

(b) Use the given answer of 19.3 to find an approximate value for

$$\int_{-4}^{5} \left(2\mathrm{f}(x) - 3\right)\mathrm{d}x$$

(2)

WMA12/01 OCTOBER 2024

Question 1

5 marks

Integration

1. A continuous curve has equation $y = \mathrm{f}(x)$.

A table of values of x and y for $y = \mathrm{f}(x)$ is shown below.

x	0.5	1.75	3	4.25	5.5
y	3.479	6.101	7.448	6.823	5.182

Using the trapezium rule with all the values of y in the given table,

(a) find an estimate for

$$\int_{0.5}^{5.5} \mathrm{f}(x)\,\mathrm{d}x$$

giving your answer to one decimal place.

(3)

(b) Using your answer to part (a) and making your method clear, estimate

$$\int_{0.5}^{5.5} \left(\mathrm{f}(x) + 4x\right)\mathrm{d}x$$

(2)

WMA12/01 OCTOBER 2024

Question 8

8 marks

Integration

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

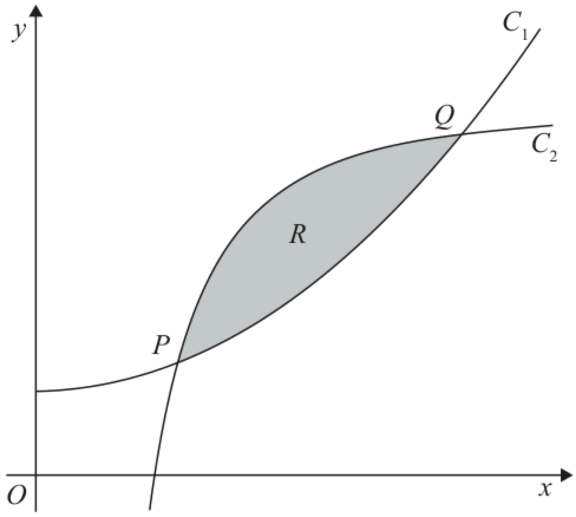

Figure 1

Figure 1 shows a sketch of part of the curve C_1 with equation

$$y = x^2 + 3 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 13 - \frac{9}{x^2} \qquad x > 0$$

The curves C_1 and C_2 intersect at the points P and Q as shown in Figure 1.

(a) Use algebra to find the x coordinate of P and the x coordinate of Q.

(4)

The finite region R, shown shaded in Figure 1, is bounded by C_1 and C_2

(b) Use algebraic integration to find the exact area of R.

(4)

WMA12/01 JANUARY 2025

Question 9

12 marks

Integration

9.

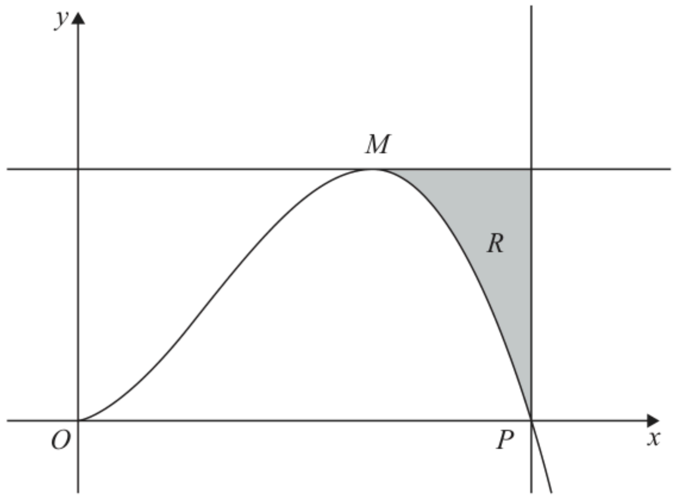

Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \frac{9x^2\left(5 - \sqrt{x}\right)}{5} \qquad x \geqslant 0$$

The curve has a turning point at the point M, as shown in Figure 3.

(a) Using calculus, find the coordinates of M.

(5)

The curve crosses the x-axis at the point P, as shown in Figure 3.

(b) Use algebra to find the x coordinate of P.

(2)

The finite region R, shown shaded in Figure 3, is bounded by the curve, the line through M parallel to the x-axis and the line through P parallel to the y-axis.

(c) Use algebraic integration to find the area of R, giving your answer to one decimal place.

(5)

WMA12/01 MAY/JUNE 2025

Question 3

5 marks

Integration

3:

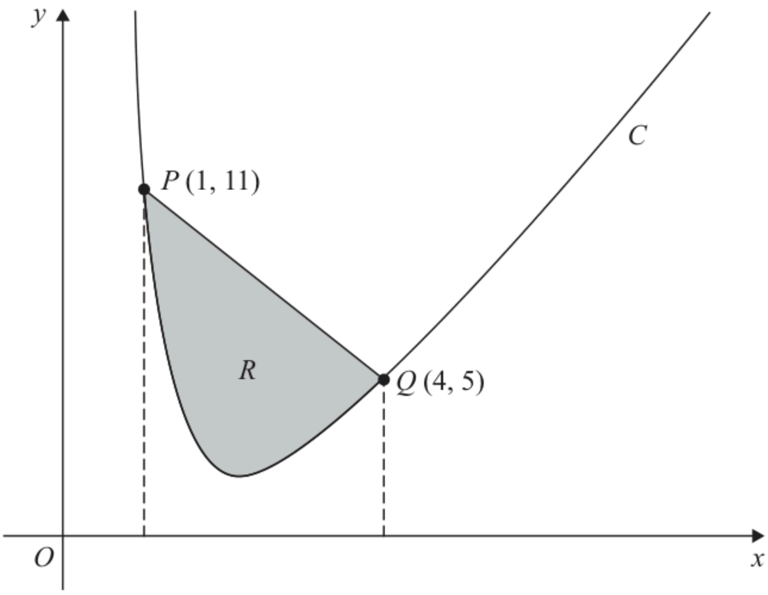

Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve C with equation

$$y = 3x + \frac{16}{x^2} - 8 \qquad x > 0$$

The points $P(1, 11)$ and $Q(4, 5)$ lie on C and are shown in Figure 1.

The region R, shown shaded in Figure 1, is bounded by C and line segment PQ.

Use algebraic integration to find the area of R.

(5)

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6 marks

Question 5

Integration

x	-2	-0.5	1	2.5	4	5.5	7
y	12	4.243	1.5	0.530	0.188	0.066	0.023

The table above shows corresponding values of x and y for

$$y = 3\left(\frac{1}{2}\right)^x$$

The values of y are given to 3 decimal places as appropriate.

(a) Using the trapezium rule with all the values of y in the given table, obtain an estimate for

$$\int_{-2}^{7} 3\left(\frac{1}{2}\right)^x \, dx$$

giving the answer to one decimal place.

(3)

Using the answer to part (a) and making your method clear, estimate

(b) (i) $\displaystyle\int_{-2}^{7} 3\left(\frac{1}{2}\right)^{x+2} \, dx$

(ii) $\displaystyle\int_{-2}^{7} \left(2^{-x} + 2x\right) dx$

(3)

WMA12/01 OCTOBER 2025

7 marks

Question 3

Integration

3.

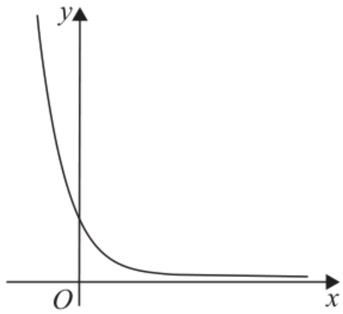

Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve with equation $y = 3 \times 2^{-x}$

The point $P(k, 300\,000)$ lies on the curve.

(a) Use logarithms to find the value of k to 2 decimal places.

(2)

x	-0.5	1	2.5	4.0	5.5	7
y	4.243	1.5	0.530	0.188	0.066	0.023

The table shows corresponding values of x and y for $y = 3 \times 2^{-x}$

The values of y are given to 3 decimal places where appropriate.

(b) (i) Use the trapezium rule, with all the values of y from the table, to find an approximate value, to 2 decimal places, for

$$\int_{-0.5}^{7} 3 \times 2^{-x}\,dx$$

(3)

(ii) Use your answer to part (b)(i) to estimate

$$\int_{-0.5}^{7} 2^{-x}\,dx + \int_{-7}^{0.5} 2^{x}\,dx$$

(2)

Question 7

10 marks

Integration

7.

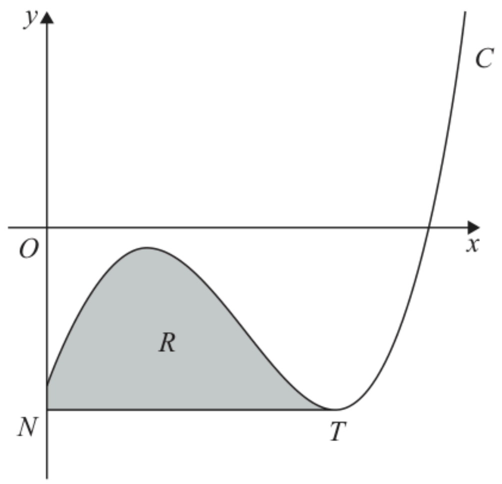

Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve C with equation

$$y = x^3 - 4x^{\frac{5}{2}} - kx^{\frac{1}{2}} + 28x - 44 \qquad x \geqslant 0$$

where k is a positive constant.

(a) Find $\frac{dy}{dx}$ in simplest form.
(2)

The point T, shown in Figure 3, is a minimum stationary point on C.

Given that the x coordinate of T is 9

(b) show that $k = 6$
(2)

The line through T parallel to the x-axis meets the y-axis at the point N.

The finite region R, shown shaded in Figure 3, is bounded by C, the y-axis and the line segment NT.

(c) Use algebraic integration to find the area of R, giving the answer to 3 significant figures.
(6)

WMA12/01 JANUARY 2026

Question 2

6 marks

Integration

2.

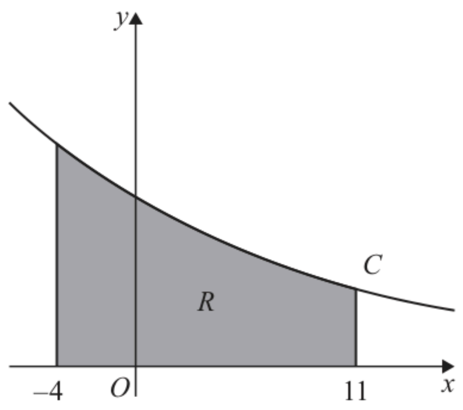

Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = 2^{-0.1x}$

The table below shows corresponding values of x and y for $y = 2^{-0.1x}$

The y values are given to 4 decimal places.

x	–4	–1	2	5	8	11
y	1.3195	1.0718	0.8706	0.7071	0.5743	0.4665

The region R, shown shaded in Figure 1, is bounded by C, the x-axis and the lines with equations $x = -4$ and $x = 11$

(a) Use the trapezium rule with all the values of y in the table to find an estimate for the area of R. Give the answer to 2 decimal places.

(3)

(b) State how you would use the trapezium rule to get a more accurate estimate for the true area of R.

(1)

Using the answer to part (a) and showing your working,

(c) estimate the value of

$$\int_{-4}^{11} 2^{3-0.1x}\, dx$$

(2)

Question 4

5 marks

Integration

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Given that k is a positive constant and

$$\int_{k}^{2k} \left(\frac{12}{x^2} + 4 \right) \mathrm{d}x = 14$$

find the possible values of k.

(5)

10 marks

WMA12/01 JANUARY 2021

Integration

Question 9

Also in Integration

Primary: Circles

9. A circle C has equation

$$(x - k)^2 + (y - 2k)^2 = k + 7$$

where k is a positive constant.

(a) Write down, in terms of k,

(i) the coordinates of the centre of C,

(ii) the radius of C.

(2)

Given that the point $P(2, 3)$ lies on C

(b) (i) show that $5k^2 - 17k + 6 = 0$

(ii) hence find the possible values of k.

(3)

The tangent to the circle at P intersects the x-axis at point T.

Given that $k < 2$

(c) calculate the exact area of triangle OPT.

(5)

WMA12/01 MAY/JUNE 2021

Question 6

7 marks

Integration

Also in Integration

Primary: Circles

6. A circle has equation

$$x^2 - 6x + y^2 + 8y + k = 0$$

where k is a positive constant.

Given that the x-axis is a tangent to this circle,

(a) find the value of k.

(3)

The circle meets the coordinate axes at the points R, S and T.

(b) Find the exact area of the triangle RST.

(4)

10 marks

Integration

Also in Integration

Primary: Applications of Differentiation

WMA12/01 MAY/JUNE 2021

Question 9

9.

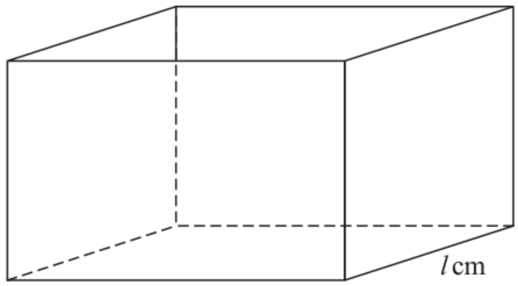

Figure 3

Figure 3 shows a sketch of a square based, open top box.

The height of the box is h cm, and the base edges each have length l cm.

Given that the volume of the box is 250 000 cm^3

(a) show that the external surface area, $S\,\text{cm}^2$, of the box is given by

$$S = \frac{250\,000}{h} + 2000\sqrt{h}$$

(3)

(b) Use algebraic differentiation to show that S has a stationary point when $h = 250^k$ where k is a rational constant to be found.

(5)

(c) Justify by further differentiation that this value of h gives the minimum external surface area of the box.

(2)

(Total 10 marks)

WMA12/01 OCTOBER 2021

Question 8

10 marks

Integration

Also in Integration

Primary: Applications of Differentiation

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

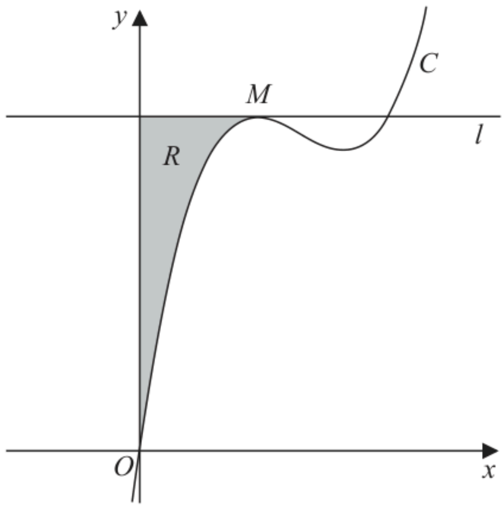

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{4}{3}x^3 - 11x^2 + kx \qquad \text{where } k \text{ is a constant}$$

The point M is the maximum turning point of C and is shown in Figure 2.

Given that the x coordinate of M is 2

(a) show that $k = 28$

(3)

(b) Determine the range of values of x for which y is increasing.

(2)

The line l passes through M and is parallel to the x-axis.

The region R, shown shaded in Figure 2, is bounded by the curve C, the line l and the y-axis.

(c) Find, by algebraic integration, the exact area of R.

(5)

WMA12/01 OCTOBER 2022

Question 9

12 marks

Integration

Also in Integration

Primary: Circles

9. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

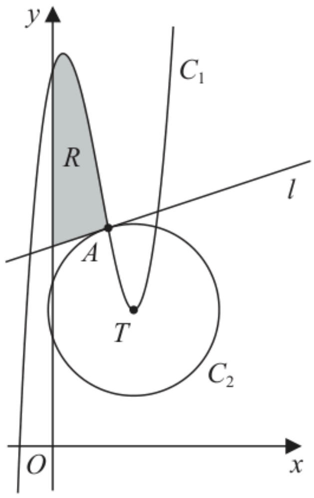

Figure 3

Figure 3 shows

- the curve C_1 with equation $y = x^3 - 5x^2 + 3x + 14$
- the circle C_2 with centre T

The point T is the minimum turning point of C_1

Using Figure 3 and calculus,

(a) find the coordinates of T

(3)

The curve C_1 intersects the circle C_2 at the point A with x coordinate 2

(b) Find an equation of the circle C_2

(3)

The line l shown in Figure 3, is the tangent to circle C_2 at A

(c) Show that an equation of l is

$$y = \frac{1}{3}x + \frac{22}{3}$$

(3)

The region R, shown shaded in Figure 3, is bounded by C_1, l and the y-axis.

(d) Find the exact area of R.

(3)

Question 4

4. The binomial expansion, in ascending powers of x, of

$$(3 + px)^5$$

where p is a constant, can be written in the form

$$A + Bx + Cx^2 + Dx^3 \ldots$$

where A, B, C and D are constants.

(a) Find the value of A

(1)

Given that

- $B = 18D$
- $p < 0$

(b) find

(i) the value of p

(ii) the value of C

(6)

WMA12/01 OCTOBER 2023

Question 9

12 marks

Integration

Also in Integration

Primary: Applications of Differentiation

9.

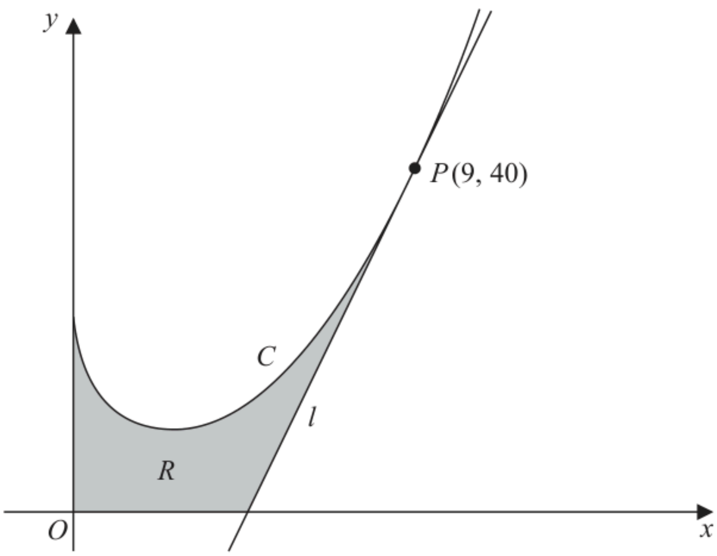

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{2}{3}x^2 - 9\sqrt{x} + 13 \qquad x \geqslant 0$$

(a) Find, using calculus, the range of values of x for which y is increasing.

(4)

The point P lies on C and has coordinates $(9, 40)$.

The line l is the tangent to C at the point P.

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the line l, the x-axis and the y-axis.

(b) Find, using calculus, the exact area of R.

(8)

WMA12/01 JANUARY 2024

Question 10

9 marks

Integration

Also in Integration

Primary: Applications of Differentiation

10. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

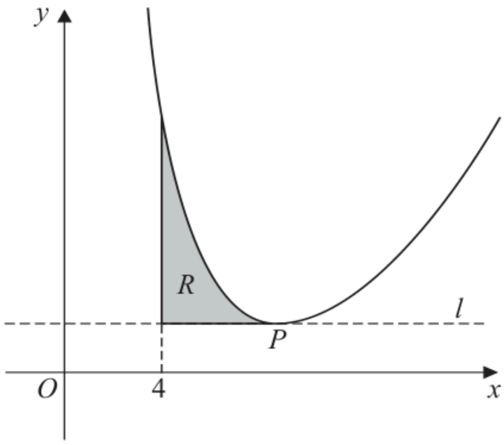

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = \frac{1}{2}x^2 + \frac{1458}{\sqrt{x^3}} - 74 \qquad x > 0$$

The point P is the only stationary point on the curve.

(a) Use calculus to show that the x coordinate of P is 9

(4)

The line l passes through the point P and is parallel to the x-axis.

The region R, shown shaded in Figure 2, is bounded by the curve, the line l and the line with equation $x = 4$

(b) Use algebraic integration to find the exact area of R.

(5)

WMA12/01 MAY/JUNE 2024

Question 9

8 marks

Integration

Also in Integration

Primary: Applications of Differentiation

9.

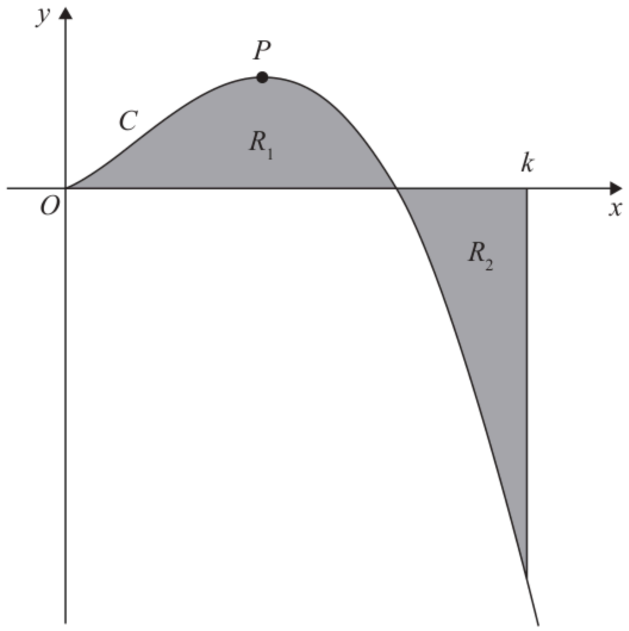

Figure 1

Figure 1 is a sketch of the curve C with equation

$$y = 2x^{\frac{3}{2}}(4 - x) \qquad x \geqslant 0$$

The point P is the stationary point of C.

(a) Find, using calculus, the x coordinate of P.

(4)

The region R_1, shown shaded in Figure 1, is bounded by C and the x-axis.

The region R_2, also shown shaded in Figure 1, is bounded by C, the x-axis and the line with equation $x = k$, where k is a constant.

Given that the area of R_1 is equal to the area of R_2

(b) find, using calculus, the exact value of k.

(4)

Question 10

9 marks

Integration

Also in Integration

Primary: Applications of Differentiation

10.

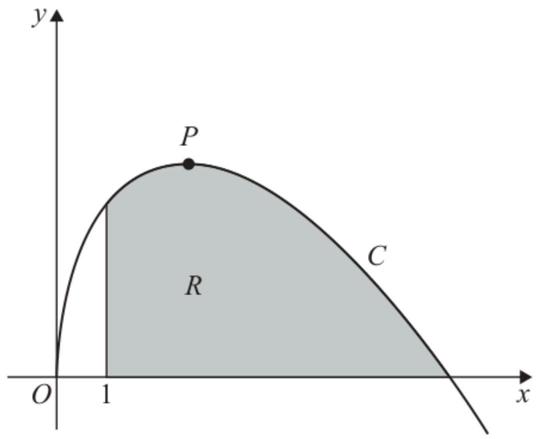

Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of part of the curve C with equation

$$y = \frac{9x - x^2}{2\sqrt{x}} \qquad x > 0$$

The point P is the stationary point on C.

(a) Find, using calculus, the x coordinate of P.

(4)

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the x-axis and the line with equation $x = 1$

(b) Using calculus, calculate the exact area of R.

(5)

WMA12/01 JANUARY 2025

Question 7

8 marks

Integration

Also in Integration

Primary: Laws of Logarithms

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) The table below shows values of x and y, where $y = \log_{10}(x+5)$, for x values between –1 and 4

x	–1	0	1	2	3	4
$y = \log_{10}(x+5)$	$\log_{10}4$	$\log_{10}5$	$\log_{10}6$	$\log_{10}7$	$\log_{10}8$	$\log_{10}9$

Using the trapezium rule with all the y values in the given table, show that

$$\int_{-1}^{4} \log_{10}(x+5)\,dx \approx \log_{10} k$$

where k is an integer to be found.

(3)

(ii) Find the value of a such that

$$2\log_5(5-a) - \log_5(a+25) = 1$$

(5)

WMA12/01 JANUARY 2026

Question 10

8 marks

Integration

Also in Integration

Primary: Applications of Differentiation

10.

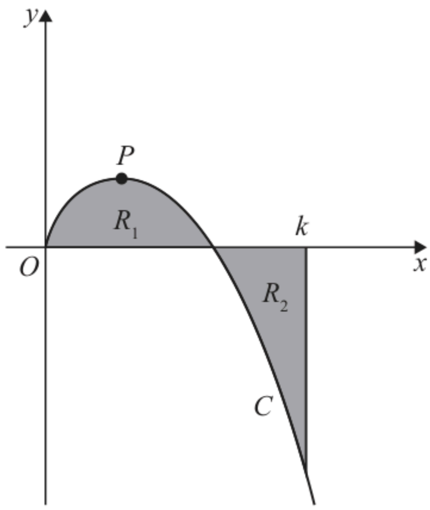

Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve C with equation

$$y = \frac{\sqrt{x}\left(100 - x^2\right)}{40} \qquad x \geqslant 0$$

The point P is a stationary point on C.

(a) Use algebraic differentiation to find the exact x coordinate of P.

(4)

The region R_1, shown shaded in Figure 2, is bounded by C and the x-axis.

The region R_2, also shown shaded in Figure 2, is bounded by C, the x-axis and the line with equation $x = k$, where k is a constant.

Given that the area of R_1 is equal to the area of R_2

(b) use algebraic integration to find the exact value of k.

(4)