ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

Pure 2 Strategy Booklet

Complete strategy booklet collected from the individual topic notes, with hard-variant coverage preserved.

Pure 2 Strategy Booklet · WMA12 · Complete

eliteigcse.com · WhatsApp +20 112 000 9622

Dr Eslam Ahmed · Cairo University Faculty of Engineering

Booklet Map

This complete booklet collects all Pure 2 individual strategy notes into one printable file. The individual topic PDFs remain available separately.

COURSE EVIDENCE Topics: 12. Topic-note pages: 132. The combined PDF also includes this cover and map.

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

Proof

Proof questions are not tricks: choose the proof shape, then make every case visible.

Proof · P2 · 01

eliteigcse.com · WhatsApp +20 112 000 9622

Dr Eslam Ahmed · Cairo University Faculty of Engineering

The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA12 website answer bank. The topic reduces to these exam moves, with hard variants kept visible on each page.

01 CASE: Proof By Cases

TRIGGER *integer proof, even/odd, positive cases*

02 COUNTER: Counterexample

TRIGGER *disprove, not always true*

03 FACTOR: Algebraic Proof

TRIGGER *show divisible, prove expression is always even*

04 INEQ: Proof By Square

TRIGGER *prove an inequality*

BANK EVIDENCE Local pages: 17 problem, 40 answer, 9 notes. Website primary entries: 15.

01 CASE: Proof By Cases

TRIGGER *integer proof, even/odd, positive cases*

BECOMES Every allowed value is placed into a complete list of cases.

FIRST LINE TO WRITE

$$n = 2k \quad \text{or} \quad n = 2k+1$$

SIMPLEST STRATEGY

1. **C**hoose the possible cases.
2. **A**lgebra each case.
3. **S**how the required factor or bound.
4. **E**nd by saying all cases are covered.

WORKED MODEL

If $n = 2k+1$, then $n^2 - 1 = (2k+1)^2 - 1 = 4k(k+1)$.

HARD VARIANTS

1. For modulo cases, use $3k, 3k+1, 3k+2$ or $4k, 4k+1, 4k+2, 4k+3$.
2. State why the cases are exhaustive before concluding.
3. If the domain is positive integers, do not include zero or negative cases.

BOTTOM LINE

A proof by cases only works when the cases cover everything.

Practice 01

SAME IDEA Use CASE: Proof By Cases.

QUESTION

> Prove that $n^2 - n$ is even for every integer n.

YOUR SOLUTION

02 COUNTER: Counterexample

TRIGGER *disprove, not always true*

BECOMES One valid example breaks the statement.

FIRST LINE TO WRITE

> choose one allowed value

SIMPLEST STRATEGY

1. **C**hoose a legal example.
2. **O**btain both sides.
3. **U**nderline the failure.
4. **N**ame it as a counterexample.
5. **T**ell the conclusion.
6. **R**eject the statement.

WORKED MODEL

> For the claim $2p + 1$ is always prime, $p = 7$ gives 15, not prime.

HARD VARIANTS

1. The example must satisfy every condition in the question before it can disprove it.
2. If the statement says positive, prime, integer, or irrational, choose within that set.
3. Write the failed conclusion explicitly, not just the calculation.

BOTTOM LINE

One counterexample is enough to disprove a universal statement.

Practice 02

SAME IDEA Use COUNTER: Counterexample.

QUESTION

Disprove: if x is irrational, then x^2 is irrational.

YOUR SOLUTION

03 FACTOR: Algebraic Proof

TRIGGER *show divisible, prove expression is always even*

BECOMES Factor the expression into the required multiple.

FIRST LINE TO WRITE

$$\text{expression} = m(\text{integer expression})$$

SIMPLEST STRATEGY

1. **F**orm the expression.
2. **A**lgebra expand.
3. **C**ollect terms.
4. **T**ake out the required factor.
5. **O**bserve the bracket is an integer.
6. **R**eport divisibility.

WORKED MODEL

$$5n^2 + n + 12 = 2(10k^2 + k + 6) \quad \text{when } n = 2k.$$

HARD VARIANTS

1. When proving not divisible, show the expression is $mq + r$ with non-zero remainder.
2. For odd/even claims, represent the variable first, then expand.
3. Do not divide by a variable unless its sign or non-zero status is guaranteed.

BOTTOM LINE

The bracket must be an integer for a divisibility proof.

Practice 03

SAME IDEA Use FACTOR: Algebraic Proof.

QUESTION

Show that $n^3 + 3n + 2$ is even when n is odd.

YOUR SOLUTION

04 INEQ: Proof By Square

TRIGGER *prove an inequality*

BECOMES Start from a square that is always non-negative.

FIRST LINE TO WRITE

$$(A-B)^2 \geq 0$$

SIMPLEST STRATEGY

1. **I**dentify the target inequality.
2. **N**ame the square to start from.
3. **E**xpand the square.
4. **Q**uietly rearrange to the target.

WORKED MODEL

$$(\sqrt{x}-\sqrt{y})^2 \geq 0 \Rightarrow x+y \geq 2\sqrt{xy}.$$

HARD VARIANTS

1. Check positivity conditions before using roots or division.
2. For all-real inequalities, complete the square is often the hidden square.
3. If asked why conditions matter, give a counterexample outside the conditions.

BOTTOM LINE

A non-negative square is a clean inequality engine.

Practice 04

SAME IDEA Use INEQ: Proof By Square.

QUESTION

Prove $x^2 + 6x + 10 > 0$ for all real x.

YOUR SOLUTION

Quick Reference

TRIGGER → FIRST ACTION

TRIGGER	FIRST ACTION
integer proof, even/odd, positive cases	$n = 2k$ or $n = 2k+1$
disprove, not always true	choose one allowed value
show divisible, prove expression is always even	expression $= m$(integer expression)
prove an inequality	$(A-B)^2 \geq 0$

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

Polynomials

Polynomials become manageable when roots, factors, and remainders are translated correctly.

Polynomials · P2 · 02

eliteigcse.com · WhatsApp +20 112 000 9622

Dr Eslam Ahmed · Cairo University Faculty of Engineering

The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA12 website answer bank. The topic reduces to these exam moves, with hard variants kept visible on each page.

01 REMA: Remainder Theorem

TRIGGER *remainder when divided by $ax - b$*

02 FACT: Factor Theorem

TRIGGER *show $ax - b$ is a factor*

03 DIV: Divide And Reduce

TRIGGER *factorise cubic, quotient, repeated roots*

04 ROOT: Build From Roots

TRIGGER *given roots, form polynomial*

BANK EVIDENCE Local pages: 18 problem, 34 answer, 15 notes. Website primary entries: 15.

01 REMA: Remainder Theorem

TRIGGER *remainder when divided by $ax - b$*

BECOMES Substitute the value that makes the divisor zero.

FIRST LINE TO WRITE

$$\text{remainder} = f\left(\frac{b}{a}\right)$$

SIMPLEST STRATEGY

1. **R**ead the divisor.
2. **E**quate it to zero.
3. **M**ake the substitution.
4. **A**nswer the remainder.

WORKED MODEL

For $2x - 3$, use $x = \frac{3}{2}$, so the remainder is $f(\frac{3}{2})$.

HARD VARIANTS

1. If the remainder is given, set $f(b/a)$ equal to that value and solve for the parameter.
2. Use exact fractions, not decimals, for the substitution.
3. If the divisor is quadratic, use division or factor it first.

BOTTOM LINE

For $ax - b$, the input is b/a.

Practice 01

SAME IDEA Use REMA: Remainder Theorem.

QUESTION

Find the remainder when $f(x) = 2x^3 - x + 4$ is divided by $2x - 1$.

YOUR SOLUTION

02 FACT: Factor Theorem

TRIGGER *show $ax - b$ is a factor*

BECOMES A zero remainder proves the factor.

FIRST LINE TO WRITE

$$f\left(\frac{b}{a}\right) = 0$$

SIMPLEST STRATEGY

1. **F**ind the zero of the divisor.
2. **A**pply the substitution.
3. **C**heck it gives zero.
4. **T**ell the factor conclusion.

WORKED MODEL

If $f(2) = 0$, then $x - 2$ is a factor.

HARD VARIANTS

1. For parameter questions, use the zero condition before factorising fully.
2. After one factor is known, divide to reduce degree.
3. Do not confuse root $x = a$ with factor $x + a$.

BOTTOM LINE

Zero remainder means exact factor.

Practice 02

SAME IDEA Use FACT: Factor Theorem.

QUESTION

Show that $x+1$ is a factor of x^3-2x^2-x+2.

YOUR SOLUTION

03 DIV: Divide And Reduce

TRIGGER *factorise cubic, quotient, repeated roots*

BECOMES Write polynomial as factor times quotient plus remainder.

FIRST LINE TO WRITE

$$f(x) = (x-a)q(x) + r$$

SIMPLEST STRATEGY

1. **D**ecide the divisor.
2. **I**nsert quotient terms.
3. **V**alue-match coefficients or divide.
4. **I**dentify the quotient.
5. **D**rop the remainder if it is zero.
6. **E**xtract remaining roots.

WORKED MODEL

$$x^3 - 6x^2 + 11x - 6 = (x-1)(x^2 - 5x + 6).$$

HARD VARIANTS

1. Comparison of coefficients is safer when the divisor has unknowns.
2. If the leading coefficient is not 1, match it before finding roots.
3. Repeated roots appear as repeated factors after division.

BOTTOM LINE

Cubic questions usually become quadratic after one factor is removed.

Practice 03

SAME IDEA Use DIV: Divide And Reduce.

QUESTION

Divide $x^3 - 4x^2 + x + 6$ by $x - 2$.

YOUR SOLUTION

04 ROOT: Build From Roots

TRIGGER *given roots, form polynomial*

BECOMES Each root gives one factor.

FIRST LINE TO WRITE

$$x = a \Rightarrow (x - a)$$

SIMPLEST STRATEGY

1. **R**ecord the roots.
2. **O**btain factors.
3. **O**rganise product.
4. **T**une the leading coefficient.

WORKED MODEL

Roots $2, -3$ give $k(x-2)(x+3)$.

HARD VARIANTS

1. If a point on the curve is given, use it to find the multiplier.
2. Complex or surd roots may arrive in conjugate pairs.
3. Check multiplicity when the graph touches the axis.

BOTTOM LINE

Roots and factors are the same information in different clothes.

Practice 04

SAME IDEA Use ROOT: Build From Roots.

QUESTION

Form a cubic with roots $1, 2, -4$ and leading coefficient 3.

YOUR SOLUTION

Quick Reference

TRIGGER → FIRST ACTION

TRIGGER	FIRST ACTION
remainder when divided by $ax - b$	remainder $= f\left(\frac{b}{a}\right)$
show $ax - b$ is a factor	$f\left(\frac{b}{a}\right) = 0$
factorise cubic, quotient, repeated roots	$f(x) = (x - a)q(x) + r$
given roots, form polynomial	$x = a \Rightarrow (x - a)$

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

Circles

Circle questions are centre-radius questions first, then tangent or intersection questions.

Circles · P2 · 03

eliteigcse.com · WhatsApp +20 112 000 9622

Dr Eslam Ahmed · Cairo University Faculty of Engineering

The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA12 website answer bank. The topic reduces to these exam moves, with hard variants kept visible on each page.

01 CENTRE: Complete The Circle

TRIGGER *equation of a circle*

02 RADIUS: Distance To Centre

TRIGGER *point on circle, radius, diameter*

03 TANG: Tangent To Circle

TRIGGER *tangent at a point*

04 MEET: Line Meets Circle

TRIGGER *intersection of line and circle*

BANK EVIDENCE Local pages: 21 problem, 38 answer, 21 notes. Website primary entries: 19.

01 CENTRE: Complete The Circle

TRIGGER *equation of a circle*

BECOMES Complete the square in x and y.

FIRST LINE TO WRITE

$$(x-a)^2+(y-b)^2=r^2$$

SIMPLEST STRATEGY

1 **C**ollect x and y terms.

2 **E**xtract half coefficients.

3 **N**ew squares created.

4 **T**idy constants.

5 **R**ead centre.

6 **E**valuate radius.

WORKED MODEL

$$x^2+y^2-6x+4y=12 \Rightarrow (x-3)^2+(y+2)^2=25.$$

HARD VARIANTS

1 If there is no x^2+y^2 coefficient 1, divide first.

2 Radius squared must be positive; check impossible circle cases.

3 Use the completed form before drawing or finding tangents.

BOTTOM LINE

The centre is read after completing both squares.

Practice 01

SAME IDEA Use CENTRE: Complete The Circle.

QUESTION

Find the centre and radius of $x^2 + y^2 + 8x - 2y = 8$.

YOUR SOLUTION

02 RADIUS: Distance To Centre

TRIGGER *point on circle, radius, diameter*

BECOMES Use distance from the centre.

FIRST LINE TO WRITE

$$r^2 = (x-a)^2 + (y-b)^2$$

SIMPLEST STRATEGY

1. **R**ead the centre.
2. **A**pply distance formula.
3. **D**ecide radius or diameter.
4. **I**nsert point.
5. **U**se squared form if cleaner.
6. **S**tate exact radius.

WORKED MODEL

Centre $(3,-2)$, point $(6,2)$: $r^2 = 3^2 + 4^2 = 25$.

HARD VARIANTS

1. For diameter endpoints, the centre is the midpoint first.
2. Use r^2 in the circle equation, not r.
3. If a point is inside/outside, compare distance squared with radius squared.

BOTTOM LINE

A point on a circle is a radius equation.

Practice 02

SAME IDEA Use RADIUS: Distance To Centre.

QUESTION

Find the circle with centre $(1, -4)$ through $(5, -1)$.

YOUR SOLUTION

03 TANG: Tangent To Circle

TRIGGER *tangent at a point*

BECOMES Radius to tangent is perpendicular.

FIRST LINE TO WRITE

$$m_t = -\frac{1}{m_r}$$

SIMPLEST STRATEGY

1. **T**ake centre and tangent point.
2. **A**ssess radius gradient.
3. **N**egative reciprocal.
4. **G**enerate line through the point.

WORKED MODEL

If radius gradient is 2, tangent gradient is $-\frac{1}{2}$.

HARD VARIANTS

1. If the radius is vertical, the tangent is horizontal.
2. If the radius is horizontal, the tangent is vertical.
3. Check the tangent point lies on the circle if it is not stated.

BOTTOM LINE

A circle tangent is a perpendicular line question.

Practice 03

SAME IDEA Use TANG: Tangent To Circle.

QUESTION

Find the tangent at $(4,5)$ to a circle with centre $(1,1)$.

YOUR SOLUTION

04 MEET: Line Meets Circle

TRIGGER *intersection of line and circle*

BECOMES Substitute the line into the circle.

FIRST LINE TO WRITE

$$(x-a)^2+(mx+c-b)^2=r^2$$

SIMPLEST STRATEGY

1. **M**ake y from the line.
2. **E**nter it into circle.
3. **E**valuate the quadratic.
4. **T**urn roots into coordinates.

WORKED MODEL

Substitute $y=x+1$ into $(x-2)^2+y^2=10$.

HARD VARIANTS

1. Tangency means the quadratic has one repeated root.
2. No intersection means discriminant is negative.
3. Keep both roots unless the question restricts the point.

BOTTOM LINE

Intersections need coordinates, not just x-values.

Practice 04

SAME IDEA Use MEET: Line Meets Circle.

QUESTION

Find where $y = x$ meets $x^2 + y^2 = 8$.

YOUR SOLUTION

Quick Reference

TRIGGER → FIRST ACTION

TRIGGER	FIRST ACTION
equation of a circle	$(x-a)^2+(y-b)^2=r^2$
point on circle, radius, diameter	$r^2=(x-a)^2+(y-b)^2$
tangent at a point	$m_t=-\dfrac{1}{m_r}$
intersection of line and circle	$(x-a)^2+(mx+c-b)^2=r^2$

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

Binomial Expansion

Binomial questions are coefficient questions: choose the term, then control powers.

Binomial Expansion · P2 · 04

eliteigcse.com · WhatsApp +20 112 000 9622

Dr Eslam Ahmed · Cairo University Faculty of Engineering

The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA12 website answer bank. The topic reduces to these exam moves, with hard variants kept visible on each page.

01 TERM: General Term

TRIGGER *expand* $(a+bx)^n$

02 COEFF: Coefficient Match

TRIGGER *coefficient of* x^k

03 RANGE: Validity Range

TRIGGER *binomial approximation*

04 APPROX: Approximate A Number

TRIGGER *use binomial to estimate*

BANK EVIDENCE Local pages: 16 problem, 36 answer, 6 notes. Website primary entries: 19.

01 TERM: General Term

TRIGGER *expand* $(a+bx)^n$

BECOMES Use the binomial term formula.

FIRST LINE TO WRITE

$$T_{r+1} = \binom{n}{r} a^{n-r}(bx)^r$$

SIMPLEST STRATEGY

1. **T**ake r from the term needed.
2. **E**nter binomial coefficient.
3. **R**aise both parts to powers.
4. **M**ultiply and simplify.

WORKED MODEL

In $(1+2x)^5$, $T_4 = \binom{5}{3}(1)^2(2x)^3 = 80x^3$.

HARD VARIANTS

1. For descending powers, solve the power of x first.
2. If the bracket contains $-x$, carry the sign with the term.
3. Use exact binomial coefficients, not calculator decimals.

BOTTOM LINE

The term formula prevents missing coefficients.

Practice 01

SAME IDEA Use TERM: General Term.

QUESTION

Find the term in x^3 of $(2+x)^6$.

YOUR SOLUTION

02 COEFF: Coefficient Match

TRIGGER *coefficient of x^k*

BECOMES Collect all contributions to the same power.

FIRST LINE TO WRITE

> coefficient of x^k

SIMPLEST STRATEGY

1. **C**hoose the power needed.
2. **O**btain the term(s).
3. **E**valuate constants.
4. **F**old signs in.
5. **F**inal coefficient only.

WORKED MODEL

> $(1+x)^4(1+2x)$: coefficient of x^2 is $6+8=14$.

HARD VARIANTS

1. For products, pair powers whose exponents add to k.
2. For unknown constants, form an equation from the coefficient.
3. Do not include the x^k in the coefficient answer.

BOTTOM LINE

Products of expansions can contribute from more than one term.

Practice 02

SAME IDEA Use COEFF: Coefficient Match.

QUESTION

Find the coefficient of x^2 in $(1+x)^5(1-2x)$.

YOUR SOLUTION

03 RANGE: Validity Range

TRIGGER *binomial approximation*

BECOMES Require the small part to have magnitude less than 1.

FIRST LINE TO WRITE

$$|u| < 1$$

SIMPLEST STRATEGY

1 **R**ewrite as $(1+u)^n$.

2 **A**sk what u is.

3 **N**ote $|u| < 1$.

4 **G**ive the interval.

5 **E**ndpoints excluded.

WORKED MODEL

$(1-3x)^{-2}$ is valid when $|-3x| < 1$, so $|x| < \frac{1}{3}$.

HARD VARIANTS

1 If the bracket is $a+bx$, factor out a first.

2 Strict inequality means endpoints are not included.

3 Use the range before trusting an approximation.

BOTTOM LINE

The range belongs to the transformed small term.

Practice 03

SAME IDEA Use RANGE: Validity Range.

QUESTION

State the range of validity for $(1+4x)^{-1}$.

YOUR SOLUTION

04 APPROX: Approximate A Number

TRIGGER *use binomial to estimate*

BECOMES Choose x so the bracket matches the number.

FIRST LINE TO WRITE

$$(1+u)^n \approx \text{first terms}$$

SIMPLEST STRATEGY

1. **A**lign the number with the bracket.
2. **P**ick a small u.
3. **P**lace u into expansion.
4. **R**ound at the end.
5. **O**bserve validity.
6. **X**exclude over-precision.

WORKED MODEL

$\sqrt{1.04} \approx 1 + \frac{1}{2}(0.04) - \frac{1}{8}(0.04)^2$.

HARD VARIANTS

1. Factor constants so the remaining bracket is close to 1.
2. Use enough terms for the requested accuracy.
3. Check that the chosen u lies inside the validity range.

BOTTOM LINE

Approximation begins by matching the bracket.

Practice 04

SAME IDEA Use APPROX: Approximate A Number.

QUESTION

Use binomial expansion to approximate $(1.02)^5$.

YOUR SOLUTION

Quick Reference

TRIGGER → FIRST ACTION

TRIGGER	FIRST ACTION
expand $(a+bx)^n$	$T_{r+1} = \binom{n}{r} a^{n-r}(bx)^r$
coefficient of x^k	coefficient of x^k
binomial approximation	$\lvert u \rvert < 1$
use binomial to estimate	$(1+u)^n \approx$ first terms

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

Arithmetic Sequences

Arithmetic sequences move by a constant difference.

Arithmetic Sequences · P2 · 05

eliteigcse.com · WhatsApp +20 112 000 9622

Dr Eslam Ahmed · Cairo University Faculty of Engineering

The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA12 website answer bank. The topic reduces to these exam moves, with hard variants kept visible on each page.

01 DIFF: Common Difference

TRIGGER *arithmetic sequence*

02 TERM: Nth Term

TRIGGER *find* u_n

03 SUM: Arithmetic Sum

TRIGGER *sum of arithmetic sequence*

04 SOLVE: Find Unknowns

TRIGGER *given two terms or a sum*

BANK EVIDENCE Local pages: 17 problem, 30 answer, 6 notes. Website primary entries: 6.

01 DIFF: Common Difference

TRIGGER *arithmetic sequence*

BECOMES Subtract consecutive terms.

FIRST LINE TO WRITE

$$d = u_{n+1} - u_n$$

SIMPLEST STRATEGY

1. **D**etect consecutive terms.
2. **I**dentify direction.
3. **F**ind the difference.
4. **F**orm the next terms.

WORKED MODEL

Sequence $7, 11, 15$ has $d = 4$.

HARD VARIANTS

1. If terms are not consecutive, divide the total change by the number of steps.
2. A decreasing sequence has negative d.
3. Check whether the sequence is arithmetic before using formulas.

BOTTOM LINE

Arithmetic means add the same amount each time.

Practice 01

SAME IDEA Use DIFF: Common Difference.

QUESTION

Find d for $12, 7, 2, \ldots$.

YOUR SOLUTION

02 TERM: Nth Term

TRIGGER *find* u_n

BECOMES Use first term plus $n-1$ differences.

FIRST LINE TO WRITE

$$u_n = a + (n-1)d$$

SIMPLEST STRATEGY

1. **T**ake first term a.
2. **E**valuate difference d.
3. **R**eplace n.
4. **M**ake the requested term or equation.

WORKED MODEL

If $a = 5, d = 3$, then $u_n = 5 + 3(n-1)$.

HARD VARIANTS

1. For missing term number, set u_n equal to the given value and solve for n.
2. Keep n as a positive integer.
3. If terms are described in context, define a and d clearly.

BOTTOM LINE

The first term is not d; it is a.

Practice 02

SAME IDEA Use TERM: Nth Term.

QUESTION

Find u_{20} when $a = 8, d = -2$.

YOUR SOLUTION

03 SUM: Arithmetic Sum

TRIGGER *sum of arithmetic sequence*

BECOMES Use average of first and last times number of terms.

FIRST LINE TO WRITE

$$S_n = \frac{n}{2}(a + l)$$

SIMPLEST STRATEGY

1 **S**ecure n.

2 **U**se first and last.

3 **M**ultiply average by count.

WORKED MODEL

$$S_{10} = \frac{10}{2}(3 + 30) = 165.$$

HARD VARIANTS

1 If last term is unknown, use $l = a + (n-1)d$ first.

2 If the sum is given, form a quadratic in n.

3 Reject non-integer or negative term counts.

BOTTOM LINE

Arithmetic sum is average term times number of terms.

Practice 03

SAME IDEA Use SUM: Arithmetic Sum.

QUESTION

Find the sum of the first 25 terms when $a = 4, d = 3$.

YOUR SOLUTION

04 SOLVE: Find Unknowns

TRIGGER *given two terms or a sum*

BECOMES Turn the sequence facts into equations.

FIRST LINE TO WRITE

$$u_p = a + (p-1)d$$

SIMPLEST STRATEGY

1. **S**et up term equations.
2. **O**rganise unknowns.
3. **L**inear solve for a, d.
4. **V**erify with given term.
5. **E**nd with requested value.

WORKED MODEL

$u_3 = 10, u_8 = 25 \Rightarrow a + 2d = 10,\ a + 7d = 25.$

HARD VARIANTS

1. With sums, equations may be quadratic in n.
2. Use simultaneous equations when two term positions are given.
3. Check the final sequence matches every given fact.

BOTTOM LINE

Two sequence facts usually find a and d.

Practice 04

SAME IDEA Use SOLVE: Find Unknowns.

QUESTION

Given $u_4 = 17, u_{10} = 41$, find a, d.

YOUR SOLUTION

Quick Reference

TRIGGER → FIRST ACTION

TRIGGER	FIRST ACTION
arithmetic sequence	$d = u_{n+1} - u_n$
find u_n	$u_n = a + (n-1)d$
sum of arithmetic sequence	$S_n = \frac{n}{2}(a+l)$
given two terms or a sum	$u_p = a + (p-1)d$

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

Geometric Sequences

Geometric sequences multiply by one constant ratio.

Geometric Sequences · P2 · 06

eliteigcse.com · WhatsApp +20 112 000 9622

Dr Eslam Ahmed · Cairo University Faculty of Engineering

The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA12 website answer bank. The topic reduces to these exam moves, with hard variants kept visible on each page.

01 RATIO: Common Ratio

TRIGGER *geometric sequence*

02 TERM: Nth Geometric Term

TRIGGER *find* u_n

03 SUM: Finite Geometric Sum

TRIGGER *sum of first n geometric terms*

04 INFTY: Sum To Infinity

TRIGGER *infinite geometric series*

BANK EVIDENCE Local pages: 25 problem, 46 answer, 7 notes. Website primary entries: 7.

01 RATIO: Common Ratio

TRIGGER *geometric sequence*

BECOMES Divide consecutive terms.

FIRST LINE TO WRITE

$$r = \frac{u_{n+1}}{u_n}$$

SIMPLEST STRATEGY

1. **R**ead consecutive terms.
2. **A**rrange next over previous.
3. **T**ake the ratio.
4. **I**nsert signs/fractions.
5. **O**btain next terms.

WORKED MODEL

Sequence $3, 6, 12$ has $r = 2$.

HARD VARIANTS

1. If terms are not consecutive, use a power of r.
2. Negative ratios alternate signs.
3. Fractional ratios create decay.

BOTTOM LINE

Geometric means multiply by the same amount.

Practice 01

SAME IDEA Use RATIO: Common Ratio.

QUESTION

Find r for $81, 27, 9, \ldots$.

YOUR SOLUTION

02 TERM: Nth Geometric Term

TRIGGER *find* u_n

BECOMES Use first term times ratio to the $n-1$.

FIRST LINE TO WRITE

$$u_n = ar^{n-1}$$

SIMPLEST STRATEGY

1. **T**ake a.
2. **E**valuate r.
3. **R**aise to $n-1$.
4. **M**ultiply and simplify.

WORKED MODEL

If $a = 5, r = 2$, $u_n = 5 \cdot 2^{n-1}$.

HARD VARIANTS

1. For missing n, use logs if needed after forming $ar^{n-1} =$ value.
2. Keep exact powers where possible.
3. Check whether n must be an integer.

BOTTOM LINE

The exponent counts jumps from the first term.

Practice 02

SAME IDEA Use TERM: Nth Geometric Term.

QUESTION

Find u_8 for $a = 3, r = \frac{1}{2}$.

YOUR SOLUTION

03 SUM: Finite Geometric Sum

TRIGGER *sum of first n geometric terms*

BECOMES Use the finite sum formula.

FIRST LINE TO WRITE

$$S_n = \frac{a(1-r^n)}{1-r}$$

SIMPLEST STRATEGY

1 **S**ecure a, r, n.

2 **U**se correct sign version.

3 **M**ake the substitution.

WORKED MODEL

$$S_5 = \frac{3(1-2^5)}{1-2} = 93.$$

HARD VARIANTS

1 If $r > 1$, the formula still works; signs cancel.

2 If $r = 1$, use $S_n = an$, not the formula.

3 If a sum is given, solve the resulting exponential equation carefully.

BOTTOM LINE

Finite geometric sums depend on r^n.

Practice 03

SAME IDEA Use SUM: Finite Geometric Sum.

QUESTION

Find S_6 for $a = 4, r = 3$.

YOUR SOLUTION

04 INFTY: Sum To Infinity

TRIGGER *infinite geometric series*

BECOMES Use the formula only when $|r| < 1$.

FIRST LINE TO WRITE

$$S_\infty = \frac{a}{1-r}$$

SIMPLEST STRATEGY

1. **I**nspect $|r|$.
2. **N**ote convergence.
3. **F**orm $a/(1-r)$.
4. **T**idy exact value.
5. **Y**ield answer.

WORKED MODEL

If $a = 10, r = \frac{1}{2}$, $S_\infty = 20$.

HARD VARIANTS

1. State the convergence condition before using the formula.
2. For word problems, interpret the limiting total.
3. Negative ratios can converge if their magnitude is less than 1.

BOTTOM LINE

No convergence, no sum to infinity.

Practice 04

SAME IDEA Use INFTY: Sum To Infinity.

QUESTION

Find S_∞ for $a = 6, r = -\frac{1}{3}$.

YOUR SOLUTION

Quick Reference

TRIGGER → FIRST ACTION

TRIGGER	FIRST ACTION
geometric sequence	$r = \frac{u_{n+1}}{u_n}$
find u_n	$u_n = ar^{n-1}$
sum of first n geometric terms	$S_n = \frac{a(1-r^n)}{1-r}$
infinite geometric series	$S_\infty = \frac{a}{1-r}$

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

Sequences & Series

Mixed series questions are about recognising the structure before choosing the formula.

Sequences & Series · P2 · 07

eliteigcse.com · WhatsApp +20 112 000 9622

Dr Eslam Ahmed · Cairo University Faculty of Engineering

The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA12 website answer bank. The topic reduces to these exam moves, with hard variants kept visible on each page.

01 SPOT: Spot The Type

TRIGGER *sequence or series unknown type*

02 SIGMA: Sigma Notation

TRIGGER $\sum$ *notation*

03 LINK: Link Term And Sum

TRIGGER *given* S_n, *find* u_n

04 MIX: Mixed Series Equation

TRIGGER *series condition, unknown* n

BANK EVIDENCE Local pages: 17 problem, 35 answer, 12 notes. Website primary entries: 10.

01 SPOT: Spot The Type

TRIGGER *sequence or series unknown type*

BECOMES Check difference and ratio.

FIRST LINE TO WRITE

$$d = u_{n+1} - u_n, \qquad r = \frac{u_{n+1}}{u_n}$$

SIMPLEST STRATEGY

1. **S**can consecutive terms.
2. **P**robe difference.
3. **O**bserve ratio.
4. **T**ake the matching formula.

WORKED MODEL

2, 6, 18 has ratio 3, so it is geometric.

HARD VARIANTS

1. Some hard questions hide terms as algebraic expressions; simplify first.
2. If neither constant difference nor ratio appears, do not force a formula.
3. State the type before applying a sum formula.

BOTTOM LINE

The type decides the formula.

Practice 01

SAME IDEA Use SPOT: Spot The Type.

QUESTION

Decide whether $5, 9, 13, \ldots$ is arithmetic or geometric.

YOUR SOLUTION

02 SIGMA: Sigma Notation

TRIGGER $\sum$ *notation*

BECOMES Read lower limit, upper limit, and term formula.

FIRST LINE TO WRITE

$$\sum_{r=1}^{n} u_r$$

SIMPLEST STRATEGY

1. **S**tart at the lower limit.
2. **I**nsert the term expression.
3. **G**enerate the first few terms if needed.
4. **M**atch to known sum.
5. **A**dd using formula.

WORKED MODEL

$$\sum_{r=1}^{n}(2r+1) = 2\sum r + \sum 1.$$

HARD VARIANTS

1. Split sigma sums term by term.
2. Use standard $\sum r$ and $\sum r^2$ only when available in the course.
3. If the lower limit is not 1, subtract the earlier terms.

BOTTOM LINE

Sigma is compressed summation, not a new topic.

Practice 02

SAME IDEA Use SIGMA: Sigma Notation.

QUESTION

Write out the first four terms of $\sum_{r=1}^{n}(3r-1)$.

YOUR SOLUTION

03 LINK: Link Term And Sum

TRIGGER *given S_n, find u_n*

BECOMES A term is the difference of two consecutive sums.

FIRST LINE TO WRITE

$$u_n = S_n - S_{n-1}$$

SIMPLEST STRATEGY

1. **L**ocate the sum formula.
2. **I**nsert n and $n-1$.
3. **N**et the difference.
4. **K**eep the final term expression.

WORKED MODEL

If $S_n = n^2$, then $u_n = n^2 - (n-1)^2 = 2n - 1$.

HARD VARIANTS

1. Check $u_1 = S_1$ after deriving the formula.
2. If S_n is quadratic, u_n is usually linear.
3. Use the term formula to test arithmetic or geometric behaviour.

BOTTOM LINE

Terms come from changes in sums.

Practice 03

SAME IDEA Use LINK: Link Term And Sum.

QUESTION

If $S_n = 3n^2 + n$, find u_n.

YOUR SOLUTION

04 MIX: Mixed Series Equation

TRIGGER *series condition, unknown n*

BECOMES Build the equation from the correct sum formula.

FIRST LINE TO WRITE

$$S_n = \text{given value}$$

SIMPLEST STRATEGY

1. **M**atch series type.
2. **I**nsert formula.
3. **X**solve the equation and check n.

WORKED MODEL

$$\frac{n}{2}(2a + (n-1)d) = 100.$$

HARD VARIANTS

1. Sum equations often become quadratics or exponentials.
2. Reject negative or non-integer values of n.
3. In context, check whether the answer should be rounded up or exact.

BOTTOM LINE

Unknown n must be a positive integer.

Practice 04

SAME IDEA Use MIX: Mixed Series Equation.

QUESTION

Find n when an arithmetic sum is 120.

YOUR SOLUTION

Quick Reference

TRIGGER → FIRST ACTION

TRIGGER	FIRST ACTION
sequence or series unknown type	$d = u_{n+1} - u_n$, $r = \dfrac{u_{n+1}}{u_n}$
$\sum$ notation	$\displaystyle\sum_{r=1}^{n} u_r$
given S_n, find u_n	$u_n = S_n - S_{n-1}$
series condition, unknown n	$S_n =$ given value

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

Modelling with Sequences & Series

Modelling questions turn real words into sequence variables and then interpret the result.

Modelling with Sequences & Series · P2 · 08

eliteigcse.com · WhatsApp +20 112 000 9622

Dr Eslam Ahmed · Cairo University Faculty of Engineering

The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA12 website answer bank. The topic reduces to these exam moves, with hard variants kept visible on each page.

01 MODEL: Define The Model

TRIGGER *context with repeated change*

02 FIT: Fit Unknowns

TRIGGER *two pieces of data*

03 LIMIT: Long-Term Limit

TRIGGER *maximum total, limiting value*

04 INTERP: Interpret The Answer

TRIGGER *least n, first time, exceeds*

BANK EVIDENCE Local pages: 29 problem, 63 answer, 4 notes. Website primary entries: 10.

01 MODEL: Define The Model

TRIGGER *context with repeated change*

BECOMES Name the first value and the repeated change.

FIRST LINE TO WRITE

$$u_n = a + (n-1)d \quad \text{or} \quad u_n = ar^{n-1}$$

SIMPLEST STRATEGY

1. **M**ark the first term.
2. **O**bserve add or multiply change.
3. **D**efine n.
4. **E**nter the right formula.
5. **L**abel units.

WORKED MODEL

> A salary increasing by 200 each year is arithmetic.

HARD VARIANTS

1. Check whether the first payment is at the start or end of the first period.
2. Use arithmetic for fixed increase and geometric for percentage change.
3. Keep units with money, population, height, or area.

BOTTOM LINE

A model is only useful if n and units are defined.

Practice 01

SAME IDEA Use MODEL: Define The Model.

QUESTION

Decide whether a saving plan adding 50 each month is arithmetic or geometric.

YOUR SOLUTION

02 FIT: Fit Unknowns

TRIGGER *two pieces of data*

BECOMES Use the data to find model constants.

FIRST LINE TO WRITE

data point $\Rightarrow$ equation

SIMPLEST STRATEGY

1 **F**ind the two facts.

2 **I**nsert each into the model.

3 **T**urn them into constants.

WORKED MODEL

$u_1 = 500, u_4 = 650 \Rightarrow d = 50.$

HARD VARIANTS

1 If data years are not consecutive, count the number of intervals.

2 Percentage decrease means $r < 1$.

3 Check the model reproduces both given values.

BOTTOM LINE

Data points determine the model.

Practice 02

SAME IDEA Use FIT: Fit Unknowns.

QUESTION

A geometric model has $u_1 = 80, u_4 = 10$. Find r.

YOUR SOLUTION

03 LIMIT: Long-Term Limit

TRIGGER *maximum total, limiting value*

BECOMES Use sum to infinity or long-term behaviour.

FIRST LINE TO WRITE

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

SIMPLEST STRATEGY

1. **L**ook for repeated percentage.
2. **I**nspect $|r| < 1$.
3. **M**ake the limiting formula.
4. **I**nterpret the value.
5. **T**ell units.

WORKED MODEL

A bounce retaining $\frac{3}{4}$ of height has total limiting distance from a geometric series.

HARD VARIANTS

1. For decay, decide whether the first term is included in the total.
2. If a threshold is asked, solve an inequality, not just a limit.
3. Round according to the real context.

BOTTOM LINE

Limits need interpretation, not just calculation.

Practice 03

SAME IDEA Use LIMIT: Long-Term Limit.

QUESTION

Find the limiting total for first term 100 and ratio 0.8.

YOUR SOLUTION

04 INTERP: Interpret The Answer

TRIGGER *least n, first time, exceeds*

BECOMES Turn algebraic solution into context language.

FIRST LINE TO WRITE

$$u_n > \text{target} \quad \text{or} \quad S_n > \text{target}$$

SIMPLEST STRATEGY

1. **I**dentify what must exceed or fall below.
2. **N**avigate the inequality.
3. **T**ake integer n.
4. **E**xplain in context.
5. **R**ound in the correct direction.
6. **P**resent units.

WORKED MODEL

If $n > 6.2$, the first whole period is $n = 7$.

HARD VARIANTS

1. For 'at least' or 'exceeds', test the boundary integer.
2. If the model starts at $n = 0$, translate n into calendar time.
3. Give a sentence, not only a number.

BOTTOM LINE

Context answers often need rounding up.

Practice 04

SAME IDEA Use INTERP: Interpret The Answer.

QUESTION

Find the first month a balance exceeds 1000.

YOUR SOLUTION

Quick Reference

TRIGGER → FIRST ACTION

TRIGGER	FIRST ACTION
context with repeated change	$u_n = a + (n-1)d$ or $u_n = ar^{n-1}$
two pieces of data	data point ⇒ equation
maximum total, limiting value	$S_\infty = \dfrac{a}{1-r}$ $(\lvert r \rvert < 1)$
least n, first time, exceeds	$u_n >$ target or $S_n >$ target

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

Laws of Logarithms

Logarithm questions reward one clean compression step before solving.

Laws of Logarithms · P2 · 09

eliteigcse.com · WhatsApp +20 112 000 9622

Dr Eslam Ahmed · Cairo University Faculty of Engineering

The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA12 website answer bank. The topic reduces to these exam moves, with hard variants kept visible on each page.

01 LAWS: Compress Logs

TRIGGER *sum or difference of logs*

02 SOLVE: Solve Log Equation

TRIGGER *log equation*

03 BASE: Change Base

TRIGGER *different bases*

04 EXP: Log To Exponential

TRIGGER *definition of log*

BANK EVIDENCE Local pages: 17 problem, 40 answer, 17 notes. Website primary entries: 20.

01 LAWS: Compress Logs

TRIGGER *sum or difference of logs*

BECOMES Use log laws to combine into one log.

FIRST LINE TO WRITE

$$\log a + \log b = \log(ab)$$

SIMPLEST STRATEGY

1. **L**ook for plus, minus, or multiplier.
2. **A**pply product, quotient, or power law.
3. **W**rite one log.
4. **S**olve or simplify.

WORKED MODEL

$$\log x + \log(x-3) = \log(x(x-3)).$$

HARD VARIANTS

1. All logs in a law must have the same base.
2. Check log arguments are positive after solving.
3. Move coefficients into powers before combining.

BOTTOM LINE

One log is easier to solve than many.

Practice 01

SAME IDEA Use LAWS: Compress Logs.

QUESTION

Combine $\log_2 x + \log_2(x-1)$.

YOUR SOLUTION

02 SOLVE: Solve Log Equation

TRIGGER *log equation*

BECOMES Compress, then remove the log.

FIRST LINE TO WRITE

$$\log_a F(x) = \log_a G(x) \Rightarrow F(x) = G(x)$$

SIMPLEST STRATEGY

1. **S**ame base confirmed.
2. **O**ne log each side.
3. **L**ift the log.
4. **V**erify domain.
5. **E**nd with valid roots.

WORKED MODEL

$$\log x + \log(x-3) = 1 \Rightarrow x(x-3) = 10.$$

HARD VARIANTS

1. If one side is a number, rewrite it as a log of a power.
2. Quadratics from logs can create invalid negative arguments.
3. Never take log of zero or negative values.

BOTTOM LINE

Log equations always finish with a domain check.

Practice 02

SAME IDEA Use SOLVE: Solve Log Equation.

QUESTION

Solve $\log_2(x+1)+\log_2 x=3$.

YOUR SOLUTION

03 BASE: Change Base

TRIGGER *different bases*

BECOMES Rewrite using a common base or change-base formula.

FIRST LINE TO WRITE

$$\log_a b = \frac{\log b}{\log a}$$

SIMPLEST STRATEGY

1. **B**ring bases into view.
2. **A**lign with powers if possible.
3. **S**witch base if needed.
4. **E**valuate or solve.

WORKED MODEL

$$\log_4 x = \frac{1}{2}\log_2 x.$$

HARD VARIANTS

1. If bases are powers of the same number, rewrite directly.
2. In equations, convert every log to the same base.
3. Keep exact logarithmic form unless decimals are requested.

BOTTOM LINE

Changing base turns unfamiliar logs into familiar ones.

Practice 03

SAME IDEA Use BASE: Change Base.

QUESTION

Write $\log_8 x$ in terms of $\log_2 x$.

YOUR SOLUTION

04 EXP: Log To Exponential

TRIGGER *definition of log*

BECOMES Translate between log and power form.

FIRST LINE TO WRITE

$$\log_a x = b \Rightarrow x = a^b$$

SIMPLEST STRATEGY

1. **E**xtract base.
2. **X**change log form for exponent form.
3. **P**lace the argument as the result.

WORKED MODEL

$$\log_3 x = 4 \Rightarrow x = 81.$$

HARD VARIANTS

1. Use exponential form after compressing logs.
2. Check the base is positive and not 1.
3. Check the argument is positive in the original equation.

BOTTOM LINE

A logarithm is an exponent.

Practice 04

SAME IDEA Use EXP: Log To Exponential.

QUESTION

Solve $\log_5(2x-1)=3$.

YOUR SOLUTION

Quick Reference

TRIGGER → FIRST ACTION

TRIGGER	FIRST ACTION
sum or difference of logs	$\log a + \log b = \log(ab)$
log equation	$\log_a F(x) = \log_a G(x) \Rightarrow F(x) = G(x)$
different bases	$\log_a b = \dfrac{\log b}{\log a}$
definition of log	$\log_a x = b \Rightarrow x = a^b$

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

Trigonometric Equations

Trig equations are interval questions: transform first, then find every valid angle.

Trigonometric Equations · P2 · 10

eliteigcse.com · WhatsApp +20 112 000 9622

Dr Eslam Ahmed · Cairo University Faculty of Engineering

The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA12 website answer bank. The topic reduces to these exam moves, with hard variants kept visible on each page.

01 CAST: Quadrant Solutions

TRIGGER *solve trig equation*

02 IDENT: Identity First

TRIGGER *quadratic trig equation or mixed functions*

03 PERIOD: Use Period

TRIGGER *solutions beyond one cycle*

04 TRANS: Transformed Trig

TRIGGER $a\sin(bx+c)+d=0$

BANK EVIDENCE Local pages: 19 problem, 38 answer, 21 notes. Website primary entries: 20.

01 CAST: Quadrant Solutions

TRIGGER *solve trig equation*

BECOMES Find reference angle and all quadrants.

FIRST LINE TO WRITE

$$\sin x = a, \quad \cos x = a, \quad \tan x = a$$

SIMPLEST STRATEGY

1. **C**alculate reference angle.
2. **A**sk sign.
3. **S**elect quadrants.
4. **T**turn into interval solutions.

WORKED MODEL

$$\cos x = -\frac{1}{2} \Rightarrow x = 120^\circ, 240^\circ.$$

HARD VARIANTS

1. For kx, solve over the expanded interval for kx, then divide.
2. Use radians if the interval is in radians.
3. Remove solutions outside the requested interval.

BOTTOM LINE

One calculator angle is rarely the whole answer.

Practice 01

SAME IDEA Use CAST: Quadrant Solutions.

QUESTION

Solve $\sin x = -\frac{1}{2}, 0 \leq x < 360°$.

YOUR SOLUTION

02 IDENT: Identity First

TRIGGER *quadratic trig equation or mixed functions*

BECOMES Use identities to make one trig function.

FIRST LINE TO WRITE

$$\sin^2 x + \cos^2 x = 1$$

SIMPLEST STRATEGY

1. **I**dentify mixed functions.
2. **D**ecide identity.
3. **E**xchange to one function.
4. **N**ormal factorise or solve.
5. **T**test interval.

WORKED MODEL

$$2\sin^2 x + \sin x - 1 = 0.$$

HARD VARIANTS

1. If factorising, set each factor equal to zero.
2. Do not divide by $\sin x$ or $\cos x$ unless you handle zero separately.
3. Check every root against the original interval.

BOTTOM LINE

A trig quadratic is still a quadratic after substitution.

Practice 02

SAME IDEA Use IDENT: Identity First.

QUESTION

Solve $2\cos^2 x - 3\cos x + 1 = 0$.

YOUR SOLUTION

03 PERIOD: Use Period

TRIGGER *solutions beyond one cycle*

BECOMES Generate repeated solutions by adding periods.

FIRST LINE TO WRITE

$$x = \alpha + 360^\circ k$$

SIMPLEST STRATEGY

1. **P**ick base solutions.
2. **E**stablish period.
3. **R**epeat using k.
4. **I**nterval filter.
5. **O**rder answers.
6. **D**egree/radian units.

WORKED MODEL

For $\tan x = 1$, $x = 45^\circ + 180^\circ k$.

HARD VARIANTS

1. Tangent period is 180° or π.
2. Transformed functions have period divided by the inside multiplier.
3. List solutions in increasing order.

BOTTOM LINE

Period gives the missing angles.

Practice 03

SAME IDEA Use PERIOD: Use Period.

QUESTION

Solve $\tan x = \sqrt{3}, 0 \leq x \leq 720°$.

YOUR SOLUTION

04 TRANS: Transformed Trig

TRIGGER $a\sin(bx+c)+d=0$

BECOMES Isolate the trig part, then solve the inside angle.

FIRST LINE TO WRITE

$$bx+c=\theta$$

SIMPLEST STRATEGY

1. **T**idy to one trig expression.
2. **R**ead the inside angle.
3. **A**llow expanded interval.
4. **N**ormal CAST solve.
5. **S**cale back to x.

WORKED MODEL

$$\sin(2x)=\frac{1}{2}\Rightarrow 2x=30^\circ,150^\circ,\ldots$$

HARD VARIANTS

1. Expand the interval for the inside expression before finding angles.
2. After dividing back, check endpoints carefully.
3. Vertical shifts must be removed before CAST.

BOTTOM LINE

Solve the inside angle first.

Practice 04

SAME IDEA Use TRANS: Transformed Trig.

QUESTION

Solve $\sin(2x - 30°) = 0.5$ for $0 \leq x \leq 180°$.

YOUR SOLUTION

Quick Reference

TRIGGER → FIRST ACTION

TRIGGER	FIRST ACTION
solve trig equation	$\sin x = a, \quad \cos x = a, \quad \tan x = a$
quadratic trig equation or mixed functions	$\sin^2 x + \cos^2 x = 1$
solutions beyond one cycle	$x = \alpha + 360^\circ k$
$a\sin(bx + c) + d = 0$	$bx + c = \theta$

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

Applications of Differentiation

Differentiation applications turn calculus into gradients, turning points, and optimisation.

Applications of Differentiation · P2 · 11

eliteigcse.com · WhatsApp +20 112 000 9622

Dr Eslam Ahmed · Cairo University Faculty of Engineering

The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA12 website answer bank. The topic reduces to these exam moves, with hard variants kept visible on each page.

01 GRAD: Gradient Function

TRIGGER *rate of change, gradient*

02 STAT: Stationary Point

TRIGGER *turning point, maximum, minimum*

03 OPTIM: Optimisation

TRIGGER *maximum area, minimum cost*

04 SKETCH: Curve Behaviour

TRIGGER *increasing, decreasing, sketch*

BANK EVIDENCE Local pages: 20 problem, 33 answer, 23 notes. Website primary entries: 20.

01 GRAD: Gradient Function

TRIGGER *rate of change, gradient*

BECOMES Differentiate and substitute.

FIRST LINE TO WRITE

$$\text{gradient} = f'(x)$$

SIMPLEST STRATEGY

1 **G**et the derivative.

2 **R**ead the x-value.

3 **A**pply substitution.

4 **D**deliver gradient or rate.

WORKED MODEL

If $y = x^3$, then $dy/dx = 3x^2$.

HARD VARIANTS

1 If a point is given, use its x-coordinate in the derivative.

2 For rates, keep units such as m s^{-1}.

3 If the gradient is given, set $f'(x)$ equal to that value.

BOTTOM LINE

The derivative is the gradient function.

Practice 01

SAME IDEA Use GRAD: Gradient Function.

QUESTION

Find the gradient of $y = x^3 - 4x$ at $x = 2$.

YOUR SOLUTION

02 STAT: Stationary Point

TRIGGER *turning point, maximum, minimum*

BECOMES Set derivative to zero.

FIRST LINE TO WRITE

$$f'(x) = 0$$

SIMPLEST STRATEGY

1. **S**et derivative zero.
2. **T**ake roots.
3. **A**dd y-coordinates.
4. **T**est type.

WORKED MODEL

$$f'(x) = 3x^2 - 12 = 0 \Rightarrow x = \pm 2.$$

HARD VARIANTS

1. Use second derivative or sign change to classify.
2. Reject roots outside a physical domain.
3. For curves with roots or fractions, simplify before differentiating.

BOTTOM LINE

Stationary points have zero gradient.

Practice 02

SAME IDEA Use STAT: Stationary Point.

QUESTION

Find the stationary point of $y = x^2 - 6x + 8$.

YOUR SOLUTION

03 OPTIM: Optimisation

TRIGGER *maximum area, minimum cost*

BECOMES Write one variable, differentiate, then test.

FIRST LINE TO WRITE

$$\frac{d}{dx}(\text{quantity}) = 0$$

SIMPLEST STRATEGY

1. **O**bject quantity named.
2. **P**ut it in one variable.
3. **T**ake derivative.
4. **I**dentify stationary value.
5. **M**ake max/min conclusion.

WORKED MODEL

Area $A = x(20 - 2x)$, so $A' = 20 - 4x = 0$.

HARD VARIANTS

1. Use constraints to remove extra variables first.
2. Check endpoints if the domain is closed.
3. State the maximum/minimum value in context, not just x.

BOTTOM LINE

Optimisation starts before calculus: define the quantity.

Practice 03

SAME IDEA Use OPTIM: Optimisation.

QUESTION

Maximise $A = x(12 - 2x)$.

YOUR SOLUTION

04 SKETCH: Curve Behaviour

TRIGGER *increasing, decreasing, sketch*

BECOMES Use derivative signs and stationary points.

FIRST LINE TO WRITE

$$f'(x) > 0 \Rightarrow \text{increasing}$$

SIMPLEST STRATEGY

1. **S**olve critical values.
2. **K**eep a sign chart.
3. **E**valuate derivative sign.
4. **T**race increasing/decreasing.
5. **C**lassify points.
6. **H**ighlight intercepts.

WORKED MODEL

If $f'(x) = (x-1)(x-4)$, the sign is positive outside and negative between.

HARD VARIANTS

1. Repeated critical roots may not change sign.
2. Combine derivative information with intercepts for sketches.
3. Use domain restrictions before making a sign chart.

BOTTOM LINE

The derivative tells the movement of the graph.

Practice 04

SAME IDEA Use SKETCH: Curve Behaviour.

QUESTION

State where $y = x^3 - 3x^2$ is increasing.

YOUR SOLUTION

Quick Reference

TRIGGER → FIRST ACTION

TRIGGER	FIRST ACTION
rate of change, gradient	gradient $= f'(x)$
turning point, maximum, minimum	$f'(x) = 0$
maximum area, minimum cost	$\frac{d}{dx}(\text{quantity}) = 0$
increasing, decreasing, sketch	$f'(x) > 0 \Rightarrow$ increasing

ELITE IGCSE MATHEMATICS
EXPERIENCE NOTES

Integration

Pure 2 integration mixes exact area, transformed estimates, and curve geometry.

Integration · P2 · 12

eliteigcse.com · WhatsApp +20 112 000 9622

Dr Eslam Ahmed · Cairo University Faculty of Engineering

The Map

This strategy booklet was mined from the local topic problem/answer PDFs, Dr Eslam's source notes, and the WMA12 website answer bank. The topic reduces to these exam moves, with hard variants kept visible on each page.

01 POWER: Integrate Powers

TRIGGER *integrate algebraic powers*

02 AREA: Area Between Curves

TRIGGER *area bounded by curves*

03 TRAP: Trapezium Estimate

TRIGGER *table of values, approximate area*

04 SHIFT: Transform Integral

TRIGGER *estimate related integral*

BANK EVIDENCE Local pages: 43 problem, 58 answer, 39 notes. Website primary entries: 29.

01 POWER: Integrate Powers

TRIGGER *integrate algebraic powers*

BECOMES Rewrite then apply the power rule.

FIRST LINE TO WRITE

$$\int ax^n\,dx = \frac{a}{n+1}x^{n+1} + c$$

SIMPLEST STRATEGY

1. **P**ut roots/fractions as powers.
2. **O**ne added to power.
3. **W**rite coefficient over new power.
4. **E**valuate limits or add c.
5. **R**ewrite exactly.

WORKED MODEL

$$\int 6x^2dx = 2x^3 + c.$$

HARD VARIANTS

1. Do not use the simple power rule for x^{-1}.
2. Simplify products or fractions before integrating.
3. Keep exact fractional powers until the final answer.

BOTTOM LINE

Most Pure 2 integrals begin by rewriting powers.

Practice 01

SAME IDEA Use POWER: Integrate Powers.

QUESTION

Integrate $3x^2 - 4x^{-2}$.

YOUR SOLUTION

02 AREA: Area Between Curves

TRIGGER *area bounded by curves*

BECOMES Find intersections, then integrate top minus bottom.

FIRST LINE TO WRITE

$$\text{Area} = \int_a^b (\text{top} - \text{bottom})\,dx$$

SIMPLEST STRATEGY

1. **A**ssign intersections.
2. **R**ead top curve.
3. **E**valuate the integral.
4. **A**nswer positive area.

WORKED MODEL

$$\int_1^3 [(2x+1) - x^2]dx.$$

HARD VARIANTS

1. If curves cross more than once, split the area.
2. Use the graph or test point to decide top curve.
3. Check restrictions such as $x > 0$ before choosing limits.

BOTTOM LINE

Area is positive geometry, not signed displacement.

Practice 02

SAME IDEA Use AREA: Area Between Curves.

QUESTION

Set up the area between $y = 2x$ and $y = x^2$.

YOUR SOLUTION

03 TRAP: Trapezium Estimate

TRIGGER *table of values, approximate area*

BECOMES Use width times end-plus-double-middle structure.

FIRST LINE TO WRITE

$$\frac{h}{2}\{y_0 + y_n + 2(y_1 + \cdots + y_{n-1})\}$$

SIMPLEST STRATEGY

1. **T**ake the strip width.
2. **R**ecord end ordinates once.
3. **A**dd middle ordinates twice.
4. **P**present rounded estimate.

WORKED MODEL

$$\frac{2}{2}\{1 + 5 + 2(2 + 3)\} = 16.$$

HARD VARIANTS

1. If a table value is unknown, set the trapezium formula equal to the given estimate.
2. Use the same estimate to transform related integrals.
3. State overestimate or underestimate from curve shape if asked.

BOTTOM LINE

The middle ordinates are doubled.

Practice 03

SAME IDEA Use TRAP: Trapezium Estimate.

QUESTION

Use a table with width 0.5 to form a trapezium estimate.

YOUR SOLUTION

04 SHIFT: Transform Integral

TRIGGER *estimate related integral*

BECOMES Rewrite the new integrand in terms of the old one.

FIRST LINE TO WRITE

$$\int (af(x)+g(x))dx = a\int f(x)dx + \int g(x)dx$$

SIMPLEST STRATEGY

1. **S**pot the old integrand.
2. **H**andle multiplier or shift.
3. **I**ntegrate extra simple part.
4. **F**old in known estimate.
5. **T**tidy units/rounding.

WORKED MODEL

$$\int (2f(x)-3)dx = 2\int f(x)dx - 3(b-a).$$

HARD VARIANTS

1. For $f(x+c)$ or $f(-x)$, check the limits by substitution.
2. Do not round the known estimate too early.
3. If the integrand includes x, integrate that exact extra part.

BOTTOM LINE

Related estimates come from algebra before integration.

Practice 04

SAME IDEA Use SHIFT: Transform Integral.

QUESTION

> If $\int_0^4 f(x)dx = 10$, estimate $\int_0^4 (3f(x)+2)dx$.

YOUR SOLUTION

Quick Reference

TRIGGER → FIRST ACTION

TRIGGER	FIRST ACTION
integrate algebraic powers	$\int ax^n\,dx = \dfrac{a}{n+1}x^{n+1} + c$
area bounded by curves	$\text{Area} = \int_a^b (\text{top} - \text{bottom})\,dx$
table of values, approximate area	$\dfrac{h}{2}\{y_0 + y_n + 2(y_1 + \cdots + y_{n-1})\}$
estimate related integral	$\int (af(x) + g(x))dx = a\int f(x)dx + \int g(x)dx$