Elite Mathematics
Edexcel International A Level

IAL PURE 2 · WMA12

WMA12/01 January 2026 Worked Solutions

January 2026 paper rebuilt with the worked solution after each question.

10	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA12/01 January 2026

January 2026

10	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA12/01 JANUARY 2026

Question 1

6 marks | Binomial Expansion

1. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(3 + kx)^7$$

where k is a non-zero constant. Give each term in simplest form.

(4)

Given that, in this expansion, the coefficient of x^2 is 3 times the coefficient of x^3

(b) find, in simplest form, the value of k.

(2)

WORKED SOLUTION

Question 1 · WMA12/01 January 2026

1 Use the binomial expansion

For $(3 + kx)^7$, the first four terms are $3^7 + \binom{7}{1}3^6(kx) + \binom{7}{2}3^5(kx)^2 + \binom{7}{3}3^4(kx)^3$.

2 Simplify the coefficients

This gives $2187 + 5103kx + 5103k^2x^2 + 2835k^3x^3$.

3 Compare the required coefficients

The coefficient of x^2 is $5103k^2$ and the coefficient of x^3 is $2835k^3$.

4 Solve for k

Since $5103k^2 = 3(2835k^3)$ and $k \neq 0$, divide by k^2 to get $5103 = 8505k$, hence $k = \dfrac{5103}{8505} = \dfrac{3}{5}$.

FINAL ANSWER

$(a)\ 2187 + 5103kx + 5103k^2x^2 + 2835k^3x^3.$ $\quad (b)\ k = \dfrac{3}{5}.$

Question 2

6 marks | Integration

2.

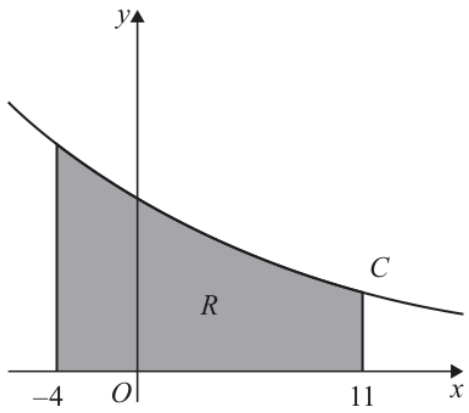


Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = 2^{-0.1x}$

The table below shows corresponding values of x and y for $y = 2^{-0.1x}$

The y values are given to 4 decimal places.

x	–4	–1	2	5	8	11
y	1.3195	1.0718	0.8706	0.7071	0.5743	0.4665

The region R, shown shaded in Figure 1, is bounded by C, the x-axis and the lines with equations $x = -4$ and $x = 11$

(a) Use the trapezium rule with all the values of y in the table to find an estimate for the area of R. Give the answer to 2 decimal places.

(3)

(b) State how you would use the trapezium rule to get a more accurate estimate for the true area of R.

(1)

Using the answer to part (a) and showing your working,

(c) estimate the value of

$$\int_{-4}^{11} 2^{3-0.1x}\,\mathrm{d}x$$

(2)

1 Find the strip width

The x values increase by 3, so $h = 3$.

2 Apply the trapezium rule

$A \approx \frac{3}{2}\{1.3195 + 0.4665 + 2(1.0718 + 0.8706 + 0.7071 + 0.5743)\}$.

3 Calculate the estimated area

$A \approx \frac{3}{2}(8.2336) = 12.3504$, so the area is 12.35 to 2 decimal places.

4 Improve the estimate

A more accurate estimate would use more trapezia, so the strip width is smaller.

5 Use the scale factor in the integral

Since $2^{3-0.1x} = 8 \cdot 2^{-0.1x}$, the required estimate is $8(12.35) = 98.8$.

FINAL ANSWER

(a) 12.35. (b) Use narrower strips, or more trapezia.

(c) 98.8.

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Question 3

7 marks | Polynomials

3. (i) Given that

- $f(x) = 4x^3 + 6x + k$, where k is a constant
- $(x + 2)$ is a factor of $f(x)$

find the remainder when $f(x)$ is divided by $(x - 5)$

(4)

(ii) Find the remainder, R, and the quotient, $Q(x)$, when

$$6x^3 - 15x^2 - 21x + 8$$

is divided by $(2x + 3)$

(3)

WORKED SOLUTION

Question 3 · WMA12/01 January 2026

1 Use the factor theorem

Since $(x + 2)$ is a factor of $f(x) = 4x^3 + 6x + k$, we have $f(-2) = 0$.

2 Find k

$4(-2)^3 + 6(-2) + k = 0$, so $-32 - 12 + k = 0$ and $k = 44$.

3 Find the required remainder

The remainder on division by $(x - 5)$ is $f(5) = 4(5)^3 + 6(5) + 44 = 574$.

4 Set up the division

Write $6x^3 - 15x^2 - 21x + 8 = (2x + 3)(Ax^2 + Bx + C) + R$.

5 Match coefficients

Comparing coefficients gives $A = 3$, $B = -12$, and $C = \frac{15}{2}$.

6 Find the remainder

Then $(2x + 3)\left(3x^2 - 12x + \frac{15}{2}\right) = 6x^3 - 15x^2 - 21x + \frac{45}{2}$, so $R = 8 - \frac{45}{2} = -\frac{29}{2}$.

FINAL ANSWER

$(i)\ 574. \quad (ii)\ Q(x) = 3x^2 - 12x + \dfrac{15}{2},\ R = -\dfrac{29}{2}.$

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Question 4

5 marks | Integration

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Given that k is a positive constant and

$$\int_k^{2k} \left(\frac{12}{x^2} + 4 \right) \mathrm{d}x = 14$$

find the possible values of k.

(5)

WORKED SOLUTION

Question 4 · WMA12/01 January 2026

1 Integrate first

$\int \left(\frac{12}{x^2} + 4 \right) dx = \int (12x^{-2} + 4)\, dx = -\frac{12}{x} + 4x.$

2 Apply the limits

$\left[-\frac{12}{x} + 4x \right]_k^{2k} = \left(-\frac{6}{k} + 8k \right) - \left(-\frac{12}{k} + 4k \right) = \frac{6}{k} + 4k.$

3 Form a quadratic

Given the integral is 14, $\frac{6}{k} + 4k = 14$. Multiplying by k gives $4k^2 - 14k + 6 = 0$, so $2k^2 - 7k + 3 = 0$.

4 Solve for k

$(2k - 1)(k - 3) = 0$, hence $k = \frac{1}{2}$ or $k = 3$. Both are positive, so both are valid.

✓ FINAL ANSWER

$k = \frac{1}{2}$ or $k = 3$.

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Question 5

6 marks | Circles

5. The circle C_1 has equation

$$x^2 + y^2 - 6x + 5y - 41 = 0$$

(a) Find

(i) the coordinates of the centre of C_1

(ii) the radius of C_1

(3)

The circle C_2 has

- centre $(-k, 0)$ where k is a positive constant
- radius 5

Given that circles C_1 and C_2 touch

(b) find the exact value of k.

(3)

WORKED SOLUTION

Question 5 · WMA12/01 January 2026

1 **Complete the square**

$x^2 - 6x = (x-3)^2 - 9$ and $y^2 + 5y = \left(y + \frac{5}{2}\right)^2 - \frac{25}{4}$.

2 **Write the circle in centre-radius form**

The equation becomes $(x-3)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{225}{4}$.

3 **Read the centre and radius**

The centre is $\left(3, -\frac{5}{2}\right)$ and the radius is $\sqrt{\frac{225}{4}} = \frac{15}{2}$.

4 **Use the touching condition**

The centre of C_2 is $(-k, 0)$ and its radius is 5. Since the circles touch externally, the distance between centres is $\frac{15}{2} + 5 = \frac{25}{2}$.

5 **Solve for k**

$(k+3)^2+\left(\frac{5}{2}\right)^2=\left(\frac{25}{2}\right)^2$, so $(k+3)^2=150$. Since $k>0$, $k+3=5\sqrt{6}$, hence $k=5\sqrt{6}-3$.

FINAL ANSWER

$(a)(i)\ (3,-\frac{5}{2}).\quad (a)(ii)\ \frac{15}{2}.\quad (b)\ k=5\sqrt{6}-3.$

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Question 6

8 marks | Proof

6. (i) Given that p and q are consecutive odd numbers, where $p > q > 0$, prove that

$$p^2 - q^2$$

is a multiple of 8

(4)

(ii) The curve C has equation

$$y = x^3 + 12x^2 + 49x + 2$$

Prove that C has no stationary points.

(4)

WORKED SOLUTION

Question 6 · WMA12/01 January 2026

1 Represent the consecutive odd numbers

Let $q = 2n - 1$ and $p = 2n + 1$, where n is a positive integer. Then $p > q > 0$ and the numbers are consecutive odd numbers.

2 Subtract the squares

$p^2 - q^2 = (2n+1)^2 - (2n-1)^2$.

3 Simplify the expression

$p^2 - q^2 = (4n^2 + 4n + 1) - (4n^2 - 4n + 1) = 8n$.

4 Conclude the proof

Since n is an integer, $8n$ is a multiple of 8, so $p^2 - q^2$ is a multiple of 8.

5 Differentiate the curve

For $y = x^3 + 12x^2 + 49x + 2$, $\dfrac{dy}{dx} = 3x^2 + 24x + 49$.

6 Show the gradient is never zero

$3x^2 + 24x + 49 = 3(x+4)^2 + 1$. This is always positive for real x.

7 State the conclusion

Since $\dfrac{dy}{dx}$ is never 0, the curve has no stationary points.

FINAL ANSWER

(i) Proven.

(ii) C has no stationary points.

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Question 7

9 marks | Laws of Logarithms

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Find the exact solution of the equation

$$8^{2x-5} = 20$$

giving the answer in the form $a + b\log_2 5$ where a and b are rational constants.

(4)

(ii) Using the laws of logarithms, solve the equation

$$\log_3(13 + 2y) + 3 = 2\log_3(4 - y)$$

(5)

WORKED SOLUTION

Question 7 · WMA12/01 January 2026

1 Take logarithms base 2

From $8^{2x-5} = 20$, take $\log_2$ of both sides to get $(2x-5)\log_2 8 = \log_2 20$.

2 Simplify the logarithms

Since $\log_2 8 = 3$ and $\log_2 20 = \log_2(4 \cdot 5) = 2 + \log_2 5$, we get $3(2x-5) = 2 + \log_2 5$.

3 Solve for x

$6x - 15 = 2 + \log_2 5$, so $6x = 17 + \log_2 5$ and $x = \frac{17}{6} + \frac{1}{6}\log_2 5$.

4 Use log laws

$3 = \log_3 27$ and $2\log_3(4-y) = \log_3(4-y)^2$.

5 Remove the logarithms

$\log_3(13+2y) + \log_3 27 = \log_3(4-y)^2$, so $27(13+2y) = (4-y)^2$.

6 Solve the quadratic

$351 + 54y = y^2 - 8y + 16$, hence $y^2 - 62y - 335 = 0 = (y-67)(y+5)$.

7 Check the logarithm domains

We need $13 + 2y > 0$ and $4 - y > 0$. Therefore $y = 67$ is not valid, and the solution is $y = -5$.

FINAL ANSWER

$(i)\ x = \dfrac{17}{6} + \dfrac{1}{6}\log_2 5. \quad (ii)\ y = -5.$

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Question 8

10 marks | Trigonometric Equations

8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 \leqslant \theta < 180$

$$4\tan^2(2\theta - 30)° + 1 = 49$$

giving the answers to one decimal place.

(5)

(ii) Solve, for $0 \leqslant x < 2\pi$

$$2\tan x \sin x + 3 = 0$$

giving the answers, in radians, in the form $k\pi$ where k is a rational constant.

(5)

WORKED SOLUTION

Question 8 · WMA12/01 January 2026

1 Make the tangent expression the subject

$4\tan^2(2\theta - 30°) + 1 = 49$ gives $\tan^2(2\theta - 30°) = 12$.

2 Find the possible angle values

Let $\alpha = 2\theta - 30°$. Since $0 \leq \theta < 180°$, $-30° \leq \alpha < 330°$. The solutions of $\tan\alpha = \pm\sqrt{12}$ in this range are approximately $73.9°, 106.1°, 253.9°, 286.1°$.

3 Return to theta

$\theta = \dfrac{\alpha + 30°}{2}$, giving $\theta = 51.9°, 68.1°, 141.9°, 158.1°$ to one decimal place.

4 Rewrite tan x

$2\tan x \sin x + 3 = 0$ becomes $2\left(\dfrac{\sin x}{\cos x}\right)\sin x + 3 = 0$.

5 Form a quadratic in cos x

Multiplying by $\cos x$ gives $2\sin^2 x + 3\cos x = 0$. Using $\sin^2 x = 1 - \cos^2 x$,
$2(1 - \cos^2 x) + 3\cos x = 0$, so $2\cos^2 x - 3\cos x - 2 = 0$.

6 **Solve for x**

$(2\cos x + 1)(\cos x - 2) = 0$. Since $\cos x = 2$ is impossible, $\cos x = -\frac{1}{2}$. In $0 \le x < 2\pi$, $x = \frac{2\pi}{3}$ or $x = \frac{4\pi}{3}$.

FINAL ANSWER

$(i)\ \theta = 51.9°,\ 68.1°,\ 141.9°,\ 158.1°.$ $(ii)\ x = \frac{2\pi}{3},\ \frac{4\pi}{3}.$

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Question 9

10 marks | Modelling with Sequences & Series

9. A company mines lithium.

- in year 1, the company mined 12 400 tonnes of lithium
- in year 15, the company is predicted to mine 7 500 tonnes of lithium

Model A assumes that the mass of lithium the company mines will decrease by the same amount each year.

According to model A,

(a) find the mass of lithium the company will mine in year 5

(3)

(b) calculate the **total** mass of lithium the company will mine from year 1 to year 15 inclusive.

(2)

Model B assumes that the mass of lithium the company mines will decrease by the same percentage each year.

(c) Find, according to model B, the mass of lithium the company will mine in year 10. Give the answer to the nearest 10 tonnes.

(3)

According to model B, there is a limit to the **total** mass of lithium that can be mined.

(d) Calculate the value of this limit. Give the answer to the nearest 100 tonnes.

(2)

WORKED SOLUTION

Question 9 · WMA12/01 January 2026

1 **Model A as an arithmetic sequence**

For Model A, let the common difference be d. Since year 15 is the 15th term, $7500 = 12400 + 14d$.

2 **Find year 5 for Model A**

$d = \dfrac{7500 - 12400}{14} = -350$. Year 5 is $12400 + 4(-350) = 11000$ tonnes.

3 **Find the total from year 1 to year 15**

$S_{15} = \dfrac{15}{2}(12400 + 7500) = 149250$ tonnes.

4 **Model B as a geometric sequence**

Let the common ratio be r. Then $7500 = 12400r^{14}$, so $r = \left(\frac{7500}{12400}\right)^{1/14} \approx 0.964723$.

5 **Find year 10 for Model B**

Year 10 is $12400r^{9} \approx 8975.26$, which is 8980 tonnes to the nearest 10 tonnes.

6 **Find the limiting total**

Since $0 < r < 1$, the sum to infinity is $S_{\infty} = \frac{12400}{1 - r} \approx 351508.1$, so the limit is 351500 tonnes to the nearest 100 tonnes.

✓ **FINAL ANSWER**

(a) 11000 tonnes.

(b) 149250 tonnes.

(c) 8980 tonnes.

(d) 351500 tonnes.

Question 10

8 marks | Applications of Differentiation

10.

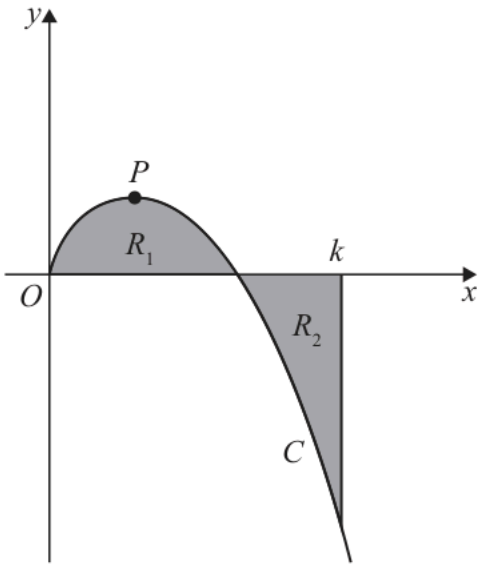


Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve C with equation

$$y = \frac{\sqrt{x}\left(100 - x^2\right)}{40} \qquad x \geqslant 0$$

The point P is a stationary point on C.

(a) Use algebraic differentiation to find the exact x coordinate of P.

(4)

The region R_1, shown shaded in Figure 2, is bounded by C and the x-axis.

The region R_2, also shown shaded in Figure 2, is bounded by C, the x-axis and the line with equation $x = k$, where k is a constant.

Given that the area of R_1 is equal to the area of R_2

(b) use algebraic integration to find the exact value of k.

(4)

1 **Rewrite the curve**

$y = \dfrac{\sqrt{x}(100 - x^2)}{40} = \dfrac{5}{2}x^{1/2} - \dfrac{1}{40}x^{5/2}$.

2 **Differentiate**

$\dfrac{dy}{dx} = \dfrac{5}{4}x^{-1/2} - \dfrac{1}{16}x^{3/2}$.

3 **Use the stationary point**

At P, $\dfrac{dy}{dx} = 0$. Thus $\dfrac{5}{4}x^{-1/2} = \dfrac{1}{16}x^{3/2}$. Multiplying by $16x^{1/2}$ gives $20 = x^2$.

4 **Find the x-coordinate**

Since $x \geq 0$, $x = \sqrt{20} = 2\sqrt{5}$.

5 **Integrate the curve**

$\displaystyle\int y\,dx = \int \left(\frac{5}{2}x^{1/2} - \frac{1}{40}x^{5/2}\right)dx = \frac{5}{3}x^{3/2} - \frac{1}{140}x^{7/2}$.

6 **Use equal areas**

The signed area from 0 to k is zero because the positive area R_1 equals the negative area R_2 in magnitude.

7 **Solve for k**

$\dfrac{5}{3}k^{3/2} - \dfrac{1}{140}k^{7/2} = 0$. Since $k > 0$, divide by $k^{3/2}$ to get $\dfrac{5}{3} - \dfrac{k^2}{140} = 0$, so $k^2 = \dfrac{700}{3}$.

8 **Write the exact value**

$k = \sqrt{\dfrac{700}{3}} = \dfrac{10\sqrt{21}}{3}$.

✓ **FINAL ANSWER**

$(a)\ x = 2\sqrt{5}$. $\quad (b)\ k = \dfrac{10\sqrt{21}}{3}$.