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Elite Mathematics
Edexcel International A Level

IAL PURE 2 · WMA12

WMA12/01 October 2025 Worked Solutions

October 2025 paper rebuilt with the worked solution after each question.

9	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA12/01 October 2025

October 2025

9	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

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Question 1

4 marks | Arithmetic Sequences

1. An arithmetic series starts

$$78 + 75 + 72 + ...$$

Find

(a) the 18^{th} term in the series,

(2)

(b) the sum of the first 100 terms of the series.

(2)

WORKED SOLUTION

Question 1 · WMA12/01 October 2025

1 Identify the sequence

The first term is 78 and the common difference is $75 - 78 = -3$.

2 Find the eighteenth term

$u_{18} = 78 + 17(-3) = 27$.

3 Use the sum formula

$S_{100} = \frac{100}{2}\{2(78) + 99(-3)\}$.

4 Evaluate the sum

$S_{100} = 50(156 - 297) = 50(-141) = -7050$.

FINAL ANSWER

(a) 27.

(b) -7050.

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Question 2

6 marks | Binomial Expansion

2. The first 4 terms, in ascending powers of x, in the binomial expansion of

$$(1 + px)^{10}$$

are

$$1 + 15x + qx^2 + rx^3$$

where p, q and r are constants.

Find the value of p, the value of q and the value of r.

(6)

WORKED SOLUTION

Question 2 · WMA12/01 October 2025

1 Write the first four terms

$(1+px)^{10} = 1 + 10px + \binom{10}{2}p^2x^2 + \binom{10}{3}p^3x^3 + \cdots$.

2 Compare the x coefficient

Since the coefficient of x is 15, $10p = 15$, so $p = \dfrac{3}{2}$.

3 Find q

$q = \binom{10}{2}p^2 = 45\left(\dfrac{3}{2}\right)^2 = \dfrac{405}{4}$.

4 Find r

$r = \binom{10}{3}p^3 = 120\left(\dfrac{3}{2}\right)^3 = 405$.

FINAL ANSWER

$p = \dfrac{3}{2}$, $q = \dfrac{405}{4}$, $r = 405$.

Question 3

7 marks | Integration

3.

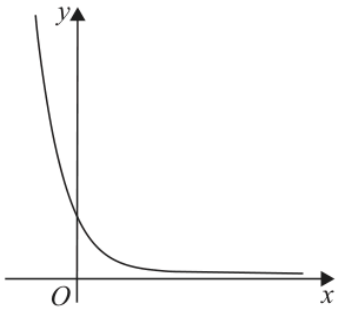


Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve with equation $y = 3 \times 2^{-x}$

The point $P(k, 300\,000)$ lies on the curve.

(a) Use logarithms to find the value of k to 2 decimal places.

(2)

x	–0.5	1	2.5	4.0	5.5	7
y	4.243	1.5	0.530	0.188	0.066	0.023

The table shows corresponding values of x and y for $y = 3 \times 2^{-x}$

The values of y are given to 3 decimal places where appropriate.

(b) (i) Use the trapezium rule, with all the values of y from the table, to find an approximate value, to 2 decimal places, for

$$\int_{-0.5}^{7} 3 \times 2^{-x}\, dx$$

(3)

(ii) Use your answer to part (b)(i) to estimate

$$\int_{-0.5}^{7} 2^{-x}\, dx + \int_{-7}^{0.5} 2^{x}\, dx$$

(2)

1 Use the point on the curve

Since $P(k, 300000)$ lies on $y = 3 \times 2^{-x}$, $300000 = 3 \times 2^{-k}$.

2 Solve with logarithms

$100000 = 2^{-k}$, so $-k = \log_2(100000)$ and $k = -\log_2(100000) = -16.609\ldots$.

3 Round k

To 2 decimal places, $k = -16.61$.

4 Find the strip width

The x values increase by 1.5, so $h = 1.5$.

5 Apply the trapezium rule

$$\int_{-0.5}^{7} 3 \times 2^{-x} dx \approx \frac{1.5}{2}\{4.243 + 0.023 + 2(1.5 + 0.530 + 0.188 + 0.066)\}.$$

6 Evaluate the estimate

This gives 6.6255, which is 6.63 to 2 decimal places.

7 Relate the first new integral

$$\int_{-0.5}^{7} 2^{-x} dx = \frac{1}{3}\int_{-0.5}^{7} 3 \times 2^{-x} dx.$$

8 Relate the second new integral

Using the substitution $u = -x$, $\int_{-7}^{0.5} 2^{x} dx = \int_{-0.5}^{7} 2^{-u} du$.

9 Estimate the total

The required estimate is therefore $\frac{2}{3}(6.6255) = 4.417$, which is 4.42 to 2 decimal places.

FINAL ANSWER

(a) $k = -16.61$.

$(b)(i)$ 6.63.

$(b)(ii)$ 4.42.

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Question 4

12 marks | Polynomials

4. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

$$f(x) = 4x^3 + 13x^2 - 10x + 8$$

(a) When $f(x)$ is divided by $(x - 2)$ the remainder is R and the quotient is $Q(x)$.

(i) Find $Q(x)$.

(ii) Find R.

(4)

(b) (i) Use the factor theorem to show that $(x + 4)$ is a factor of $f(x)$.

(ii) Hence prove, using algebra, that the equation $f(x) = 0$ has only one real solution.

(5)

(c) Find the range of values of x for which $f(x)$ is decreasing.

(3)

WORKED SOLUTION

Question 4 · WMA12/01 October 2025

1 **Divide by x minus two**

$4x^3 + 13x^2 - 10x + 8 = (x - 2)(4x^2 + 21x + 32) + 72$.

2 **State Q and R**

Hence $Q(x) = 4x^2 + 21x + 32$ and $R = 72$.

3 **Use the factor theorem**

$f(-4) = 4(-4)^3 + 13(-4)^2 - 10(-4) + 8 = -256 + 208 + 40 + 8 = 0$.

4 **Show the factor**

Since $f(-4) = 0$, $(x + 4)$ is a factor of $f(x)$.

5 **Factor the cubic**

Dividing by $(x + 4)$ gives $f(x) = (x + 4)(4x^2 - 3x + 2)$.

6 **Check the quadratic**

The discriminant of $4x^2 - 3x + 2$ is $(-3)^2 - 4(4)(2) = 9 - 32 = -23 < 0$, so this quadratic has no real roots.

7 **Conclude the real solution**

Therefore $f(x) = 0$ has only one real solution, $x = -4$.

8 **Differentiate**

$f'(x) = 12x^2 + 26x - 10 = 2(3x - 1)(2x + 5)$.

9 **Find where f is decreasing**

$f(x)$ is decreasing when $f'(x) < 0$, which is between the roots. Hence $-\frac{5}{2} < x < \frac{1}{3}$.

✓ **FINAL ANSWER**

$(a)(i)\ Q(x) = 4x^2 + 21x + 32$.

$(a)(ii)\ R = 72$.

(b) Proven; only real solution is $x = -4$.

$(c)\ -\frac{5}{2} < x < \frac{1}{3}$.

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Question 5

9 marks | Trigonometric Equations

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta \leqslant 360°$, the equation

$$4 \tan \theta + 5 \sin \theta = 0$$

giving any non-exact answers to one decimal place.

(5)

(ii) Solve, for $0 < x < \pi$, the equation

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{5}{\cos x}$$

giving the answers, in radians, to 3 significant figures.

(4)

WORKED SOLUTION

Question 5 · WMA12/01 October 2025

1 **Rewrite the first equation**

$4\tan\theta + 5\sin\theta = 0$ gives $4\dfrac{\sin\theta}{\cos\theta} + 5\sin\theta = 0$.

2 **Factor sin theta**

$\sin\theta\left(\dfrac{4}{\cos\theta} + 5\right) = 0$.

3 **Use sin theta equals zero**

In $0 < \theta \leq 360°$, $\sin\theta = 0$ gives $\theta = 180°$ or $360°$.

4 **Use the other factor**

$\dfrac{4}{\cos\theta} + 5 = 0$ gives $\cos\theta = -\dfrac{4}{5}$.

5 **Find the remaining angles**

For $0 < \theta \leq 360°$, $\cos\theta = -\dfrac{4}{5}$ gives $\theta = 143.130\ldots°$ or $216.869\ldots°$.

6 **Round part i**

The non-exact angles are $143.1°$ and $216.9°$ to one decimal place.

7 **Clear the fractions in part ii**

$\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} = \dfrac{5}{\cos x}$. Multiplying by $\sin x \cos x$ gives $\sin^2 x + \cos^2 x = 5 \sin x$.

8 **Use the identity**

$\sin^2 x + \cos^2 x = 1$, so $1 = 5 \sin x$ and $\sin x = \dfrac{1}{5}$.

9 **Solve in the interval**

For $0 < x < \pi$, $x = \sin^{-1}\left(\dfrac{1}{5}\right)$ or $x = \pi - \sin^{-1}\left(\dfrac{1}{5}\right)$.

10 **Round part ii**

Thus $x = 0.201357\ldots$ or $2.94023\ldots$, so $x = 0.201$ or 2.94 to 3 significant figures.

✓ **FINAL ANSWER**

(i) $\theta = 143.1°, 180°, 216.9°, 360°$.

(ii) $x = 0.201, 2.94$.

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Question 6

9 marks | Circles

6.

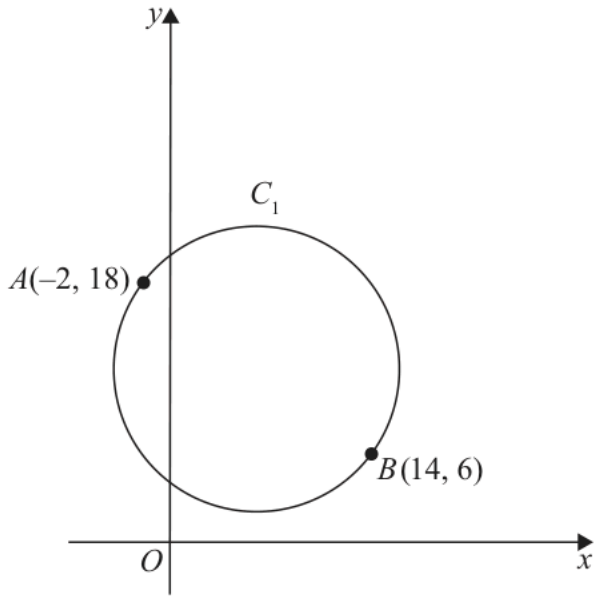


Figure 2

The points $A(-2, 18)$ and $B(14, 6)$ lie on a circle C_1 as shown in Figure 2.

Given that AB is a diameter of the circle C_1

(a) find an equation for C_1 making your method clear.

(5)

A circle C_2 has its centre at the origin.

Given that circles C_1 and C_2 touch,

(b) find possible equations for C_2

(4)

WORKED SOLUTION

Question 6 · WMA12/01 October 2025

1 Find the centre of C one

Since AB is a diameter, the centre is the midpoint of $A(-2, 18)$ and $B(14, 6)$. This is $(6, 12)$.

2 Find the radius squared

Using $A(-2, 18)$, $r^2 = (-2-6)^2 + (18-12)^2 = 64 + 36 = 100$.

3 **Write C one**

Therefore C_1 has equation $(x-6)^2+(y-12)^2=100$.

4 **Find the centre distance**

The distance from the origin to $(6,12)$ is $\sqrt{6^2+12^2}=6\sqrt{5}$.

5 **Find possible radii for C two**

Since C_1 has radius 10, a circle centred at the origin can touch it with radius $6\sqrt{5}-10$ or $6\sqrt{5}+10$.

6 **Write the equations**

Thus C_2 can be $x^2+y^2=(6\sqrt{5}-10)^2=280-120\sqrt{5}$ or $x^2+y^2=(6\sqrt{5}+10)^2=280+120\sqrt{5}$.

✓ **FINAL ANSWER**

(a) $(x-6)^2+(y-12)^2=100$.

(b) $x^2+y^2=280-120\sqrt{5}$ or $x^2+y^2=280+120\sqrt{5}$.

Question 7

10 marks | Integration

7.

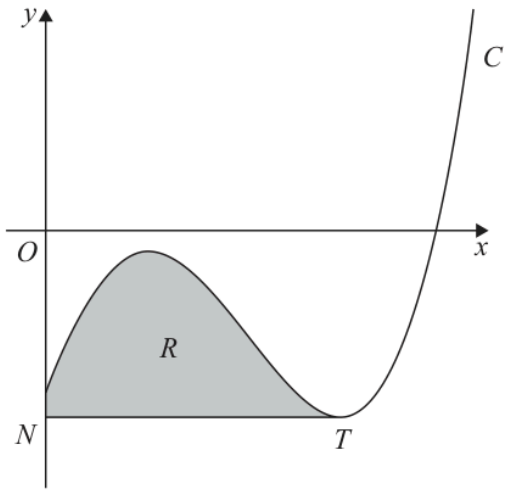


Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve C with equation

$$y = x^3 - 4x^{\frac{5}{2}} - kx^{\frac{1}{2}} + 28x - 44 \qquad x \geqslant 0$$

where k is a positive constant.

(a) Find $\dfrac{dy}{dx}$ in simplest form.

(2)

The point T, shown in Figure 3, is a minimum stationary point on C.

Given that the x coordinate of T is 9

(b) show that $k = 6$

(2)

The line through T parallel to the x-axis meets the y-axis at the point N.

The finite region R, shown shaded in Figure 3, is bounded by C, the y-axis and the line segment NT.

(c) Use algebraic integration to find the area of R, giving the answer to 3 significant figures.

(6)

1 **Differentiate**

For $y = x^3 - 4x^{5/2} - kx^{1/2} + 28x - 44$, $\dfrac{dy}{dx} = 3x^2 - 10x^{3/2} - \dfrac{k}{2}x^{-1/2} + 28$.

2 **Write in simplest form**

$\dfrac{dy}{dx} = 3x^2 - 10x^{3/2} - \dfrac{k}{2\sqrt{x}} + 28$.

3 **Use the stationary point**

At T, $x = 9$ and $\dfrac{dy}{dx} = 0$.

4 **Substitute x equals nine**

$0 = 3(9)^2 - 10(9)^{3/2} - \dfrac{k}{2\sqrt{9}} + 28 = 243 - 270 - \dfrac{k}{6} + 28$.

5 **Find k**

This simplifies to $0 = 1 - \dfrac{k}{6}$, so $k = 6$.

6 **Find the horizontal line**

With $k = 6$, the y coordinate of T is $9^3 - 4(9)^{5/2} - 6\sqrt{9} + 28(9) - 44 = -53$.

7 **Set up the area**

The line NT is $y = -53$. The region is between the curve and this line from $x = 0$ to $x = 9$.

8 **Write the integral**

$R = \displaystyle\int_0^9 \left(x^3 - 4x^{5/2} - 6x^{1/2} + 28x - 44 + 53\right)dx$.

9 **Integrate**

$R = \left[\dfrac{x^4}{4} - \dfrac{8}{7}x^{7/2} - 4x^{3/2} + 14x^2 + 9x\right]_0^9$.

10 **Evaluate**

$R = \dfrac{6939}{28} = 247.821\ldots$, so the area is 248 to 3 significant figures.

FINAL ANSWER

(a) $3x^2 - 10x^{3/2} - \dfrac{k}{2\sqrt{x}} + 28$.

(b) Proven.

(c) 248.

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Question 8

8 marks | Geometric Sequences

8. (i) A geometric series has first term a and common ratio r.

Prove that the sum of the first n terms of this series S_n is given by

$$S_n = \frac{a(1-r^n)}{1-r}$$

(3)

(ii) A liquid is to be stored in a barrel.

Due to evaporation, the volume of the liquid in the barrel at the end of each year is 8% less than the volume of the liquid in the barrel at the start of the year.

At the start of the first year, the barrel is filled with 150 litres of the liquid.

(a) Show that the amount of the liquid in the barrel at the end of 6 years is approximately 91 litres.

(2)

At the start of each year a new barrel is filled with 150 litres of the liquid so that, at the end of 40 years, there are 40 barrels containing the liquid.

(b) Calculate the total amount of the liquid, to the nearest litre, in the 40 barrels at the end of 40 years.

(3)

WORKED SOLUTION

Question 8 · WMA12/01 October 2025

1 Write S n

$S_n = a + ar + ar^2 + \cdots + ar^{n-1}$.

2 Multiply by r

$rS_n = ar + ar^2 + ar^3 + \cdots + ar^n$.

3 Subtract

$S_n - rS_n = a - ar^n = a(1 - r^n)$.

4 Rearrange

$(1 - r)S_n = a(1 - r^n)$, so $S_n = \dfrac{a(1 - r^n)}{1 - r}$.

5 **Use the yearly multiplier**

An 8% decrease each year means the multiplier is 0.92.

6 **Show the amount after six years**

After 6 years, the amount is $150(0.92)^6 = 90.953\ldots$, which is approximately 91 litres.

7 **Set up the forty barrel total**

At the end of 40 years, the amounts are $150(0.92), 150(0.92)^2, \ldots, 150(0.92)^{40}$.

8 **Sum the geometric sequence**

The total is $150(0.92)\dfrac{1-0.92^{40}}{1-0.92} = 1663.581\ldots$.

9 **Round the total**

To the nearest litre, the total amount is 1664 litres.

✓ **FINAL ANSWER**

(i) Proven.

$(ii)(a)$ Approximately 91 litres.

$(ii)(b)$ 1664 litres.

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Question 9

10 marks | Laws of Logarithms

9. **In this question you must show detailed reasoning.**
Solutions relying on calculator technology are not acceptable.

(i) Solve

$$2\log_3(4x+5) - \log_3(x+3) = 2$$

(5)

(ii) Given that $a > 0$, $b > 0$ and

$$\log_{10} a + \log_{10} b = \log_{10}(a+b)$$

(a) prove that $a = \dfrac{b}{b-1}$

(3)

(b) Hence write down the full restriction on the value of b, giving a reason for your answer.

(2)

WORKED SOLUTION

Question 9 · WMA12/01 October 2025

1 **State the domain**

The logarithms require $4x+5>0$ and $x+3>0$, so $x > -\dfrac{5}{4}$.

2 **Combine the logarithms**

$2\log_3(4x+5) - \log_3(x+3) = \log_3\left(\dfrac{(4x+5)^2}{x+3}\right)$.

3 **Use two as a logarithm**

$2 = \log_3 9$, so $\dfrac{(4x+5)^2}{x+3} = 9$.

4 **Form the quadratic**

$(4x+5)^2 = 9(x+3)$, so $16x^2+40x+25 = 9x+27$. Hence $16x^2+31x-2=0$.

5 **Solve the quadratic**

$16x^2+31x-2 = (16x-1)(x+2) = 0$, so $x = \dfrac{1}{16}$ or $x=-2$.

6 **Check the domain**

$x = -2$ is outside the domain, so $x = \frac{1}{16}$.

7 **Use the logarithm equation**

$\log_{10} a + \log_{10} b = \log_{10}(a + b)$ gives $\log_{10}(ab) = \log_{10}(a + b)$.

8 **Equate the arguments**

Since the logarithm function is one-to-one, $ab = a + b$.

9 **Rearrange for a**

$ab - a = b$, so $a(b - 1) = b$ and therefore $a = \frac{b}{b - 1}$.

10 **Restrict b**

Given $a > 0$ and $b > 0$, the fraction $\frac{b}{b - 1}$ must be positive. Since the numerator is positive, the denominator must also be positive, so $b > 1$.

✓ **FINAL ANSWER**

$(i)\ x = \frac{1}{16}$.

$(ii)(a)$ Proven.

$(ii)(b)\ b > 1$.