Elite Mathematics
Edexcel International A Level

IAL PURE 2 · WMA12

WMA12/01 May/June 2025 Worked Solutions

May/June 2025 paper rebuilt with the worked solution after each question.

10	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA12/01 May/June 2025

May/June 2025

10	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

WMA12/01 MAY/JUNE 2025

Question 1

6 marks | Binomial Expansion

1: (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(1 - 4x)^7$$

giving each term in simplest form.

(4)

In the series expansion of

$$(5 + kx)(1 - 4x)^7 \qquad \text{where } k \text{ is a constant}$$

the coefficient of the term in x^2 is 1316

(b) Use the answer to part (a) to find the value of k.

(2)

WORKED SOLUTION

Question 1 · WMA12/01 May/June 2025

1 Write the binomial terms

$(1-4x)^7 = 1 + 7(-4x) + \binom{7}{2}(-4x)^2 + \binom{7}{3}(-4x)^3 + \cdots$.

2 Simplify

The first four terms are $1 - 28x + 336x^2 - 2240x^3$.

3 Find the x squared terms

In $(5 + kx)(1 - 4x)^7$, the x^2 coefficient comes from $5(336x^2)$ and $(kx)(-28x)$.

4 Use the given coefficient

$5(336) - 28k = 1316$, so $1680 - 28k = 1316$.

5 Find k

$28k = 364$, hence $k = 13$.

FINAL ANSWER

$(a)\ 1 - 28x + 336x^2 - 2240x^3$.

$(b)\ k = 13$.

WMA12/01 MAY/JUNE 2025

Question 2

7 marks | Circles

2: The line joining the points $(-2, 5)$ and $(4, 15)$ is the diameter of a circle C.

(a) Find an equation for C. **(5)**

(b) Hence find the exact coordinates of the point on C that is nearest the x-axis. **(2)**

WORKED SOLUTION

Question 2 · WMA12/01 May/June 2025

1 Find the centre

The centre is the midpoint of $(-2, 5)$ and $(4, 15)$, so it is $\left(\frac{-2+4}{2}, \frac{5+15}{2}\right) = (1, 10)$.

2 Find the radius squared

Using $(-2, 5)$, $r^2 = (-2-1)^2 + (5-10)^2 = 9 + 25 = 34$.

3 Write the circle equation

Therefore C has equation $(x-1)^2 + (y-10)^2 = 34$.

4 Find the nearest point to the x axis

The nearest point to the x-axis is vertically below the centre by one radius.

5 Write the coordinates

The radius is $\sqrt{34}$, so the point is $\left(1, 10-\sqrt{34}\right)$.

FINAL ANSWER

(a) $(x-1)^2 + (y-10)^2 = 34$.

(b) $\left(1, 10-\sqrt{34}\right)$.

WMA12/01 MAY/JUNE 2025

Question 3

5 marks | Integration

3:

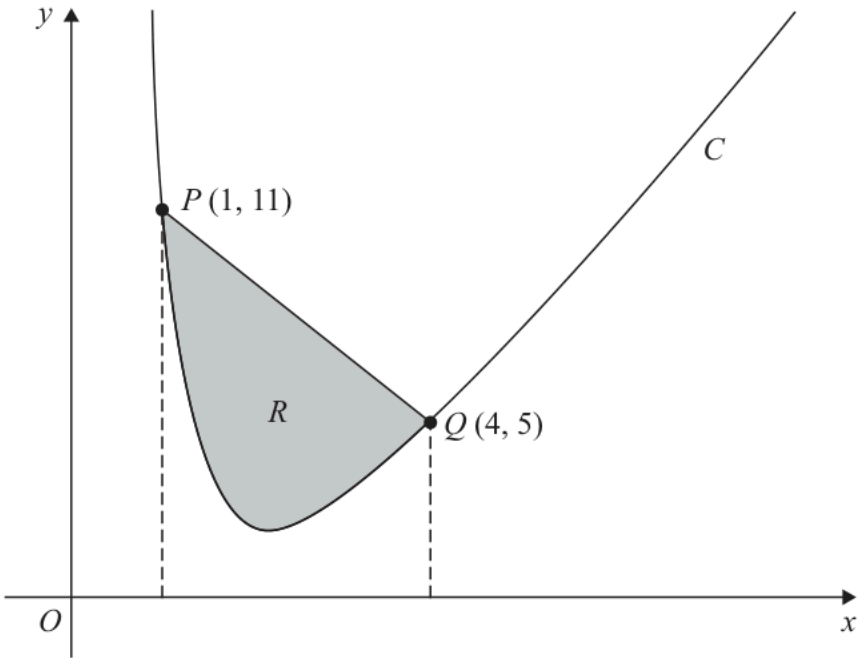


Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve C with equation

$$y = 3x + \frac{16}{x^2} - 8 \qquad x > 0$$

The points $P(1, 11)$ and $Q(4, 5)$ lie on C and are shown in Figure 1.

The region R, shown shaded in Figure 1, is bounded by C and line segment PQ.

Use algebraic integration to find the area of R.

(5)

WORKED SOLUTION

Question 3 · WMA12/01 May/June 2025

1 **Find the line PQ**

The gradient of PQ is $\frac{5-11}{4-1} = -2$, so the line through $P(1, 11)$ is $y - 11 = -2(x - 1)$.

2 **Write the line equation**

Thus PQ has equation $y = -2x + 13$.

3 **Set up the area**

From $x = 1$ to $x = 4$, the line segment is above the curve. Hence

$$R = \int_1^4 \left[(-2x + 13) - \left(3x + \frac{16}{x^2} - 8\right)\right] dx.$$

4 **Simplify the integrand**

$$R = \int_1^4 (21 - 5x - 16x^{-2})\, dx.$$

5 **Integrate**

$$R = \left[21x - \frac{5}{2}x^2 + \frac{16}{x}\right]_1^4.$$

6 **Evaluate**

$$R = (84 - 40 + 4) - \left(21 - \frac{5}{2} + 16\right) = 48 - \frac{69}{2} = \frac{27}{2}.$$

✓ **FINAL ANSWER**

$$\frac{27}{2}.$$

WMA12/01 MAY/JUNE 2025

Question 4

9 marks | Polynomials

4: The function f(x) is defined by

$$f(x) = ax^3 + bx^2 + 5x - 3$$

where a and b are constants.

Given that $(x + 3)$ is a factor of f(x),

(a) show that

$$b = 3a + 2$$

(2)

Given further that when f(x) is divided by $(2x - 1)$ the remainder is $\frac{7}{4}$

(b) find the value of a and the value of b.

(4)

(c) Using algebra, find the quotient and the remainder when f(x) is divided by $(x - 2)$

(3)

WORKED SOLUTION

Question 4 · WMA12/01 May/June 2025

1 Use the factor theorem

Since $x + 3$ is a factor, $f(-3) = 0$.

2 Show the relation

$f(-3) = -27a + 9b - 15 - 3 = 0$, so $9b = 27a + 18$ and therefore $b = 3a + 2$.

3 Use the remainder theorem

When dividing by $2x - 1$, substitute $x = \frac{1}{2}$. The remainder is $\frac{7}{4}$, so $f\left(\frac{1}{2}\right) = \frac{7}{4}$.

4 Form the second equation

$\frac{a}{8} + \frac{b}{4} + \frac{5}{2} - 3 = \frac{7}{4}$. Multiplying by 8 gives $a + 2b - 4 = 14$, so $a + 2b = 18$.

5 Solve for a and b

Substitute $b = 3a + 2$ into $a + 2b = 18$: $a + 6a + 4 = 18$, so $a = 2$ and $b = 8$.

6 **Write f of x**

$f(x) = 2x^3 + 8x^2 + 5x - 3$.

7 **Divide by x minus two**

$2x^3 + 8x^2 + 5x - 3 = (x - 2)(2x^2 + 12x + 29) + 55$.

8 **State the result**

The quotient is $2x^2 + 12x + 29$ and the remainder is 55.

✓ **FINAL ANSWER**

(a) Proven.

(b) $a = 2$, $b = 8$.

(c) Quotient $2x^2 + 12x + 29$, remainder 55.

WMA12/01 MAY/JUNE 2025

Question 5

6 marks | Integration

x	–2	–0.5	1	2.5	4	5.5	7
y	12	4.243	1.5	0.530	0.188	0.066	0.023

The table above shows corresponding values of x and y for

$$y = 3\left(\frac{1}{2}\right)^x$$

The values of y are given to 3 decimal places as appropriate.

(a) Using the trapezium rule with all the values of y in the given table, obtain an estimate for

$$\int_{-2}^{7} 3\left(\frac{1}{2}\right)^x \, dx$$

giving the answer to one decimal place.

(3)

Using the answer to part (a) and making your method clear, estimate

(b) (i) $\displaystyle\int_{-2}^{7} 3\left(\frac{1}{2}\right)^{x+2} \, dx$

(ii) $\displaystyle\int_{-2}^{7} \left(2^{-x} + 2x\right) dx$

(3)

WORKED SOLUTION

Question 5 · WMA12/01 May/June 2025

1 Find the strip width

The x values increase by 1.5, so $h = 1.5$.

2 Apply the trapezium rule

$$\int_{-2}^{7} 3\left(\frac{1}{2}\right)^x dx \approx \frac{1.5}{2}\{12 + 0.023 + 2(4.243 + 1.5 + 0.530 + 0.188 + 0.066)\}.$$

3 **Evaluate**

This gives 18.80775, which is 18.8 to one decimal place.

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4 **Use the power shift**

$3\left(\frac{1}{2}\right)^{x+2} = \frac{1}{4} \cdot 3\left(\frac{1}{2}\right)^{x}$.

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5 **Estimate part b i**

So $\int_{-2}^{7} 3\left(\frac{1}{2}\right)^{x+2} dx \approx \frac{1}{4}(18.80775) = 4.7019375$, which is 4.7.

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6 **Rewrite part b ii**

Since $3\left(\frac{1}{2}\right)^{x} = 3 \cdot 2^{-x}$, we have $2^{-x} = \frac{1}{3} \cdot 3\left(\frac{1}{2}\right)^{x}$.

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7 **Estimate part b ii**

$\int_{-2}^{7} (2^{-x} + 2x)dx \approx \frac{18.80775}{3} + [x^2]_{-2}^{7} = 6.26925 + 45 = 51.26925$, which is 51.3.

✓ **FINAL ANSWER**

(a) 18.8.

$(b)(i)$ 4.7.

$(b)(ii)$ 51.3.

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Question 6

10 marks | Laws of Logarithms

6: **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(i) Solve

$$2\log_2(4-x) = 3 + \log_2\left(\frac{x+11}{2}\right)$$

(5)

(ii) The curves C_1 and C_2 with equations

$$y = 3^{2x+1} \qquad \text{and} \qquad y = 6 \times 3^x$$

meet at the point P.

Find the exact coordinates of P, writing your answer in the form $(\log_3 a, b)$ where a and b are integers.

(5)

WORKED SOLUTION

Question 6 · WMA12/01 May/June 2025

1 **State the domain**

The logarithms require $4 - x > 0$ and $\frac{x+11}{2} > 0$, so $-11 < x < 4$.

2 **Use log laws**

$2\log_2(4-x) = \log_2((4-x)^2)$ and $3 + \log_2\left(\frac{x+11}{2}\right) = \log_2\left(8 \cdot \frac{x+11}{2}\right)$.

3 **Equate the arguments**

Thus $(4-x)^2 = 4(x+11)$.

4 **Solve the quadratic**

$x^2 - 8x + 16 = 4x + 44$, so $x^2 - 12x - 28 = 0$. Hence $(x-14)(x+2) = 0$.

5 **Check the domain**

The root 14 is outside the domain, so $x = -2$.

6 **Equate the curves**

At P, $3^{2x+1} = 6 \times 3^x$.

7 **Let u equal 3 to the x**

Let $u = 3^x$. Then $3^{2x+1} = 3u^2$, so $3u^2 = 6u$.

8 **Find u**

Since $u > 0$, divide by $3u$ to get $u = 2$. Hence $3^x = 2$ and $x = \log_3 2$.

9 **Find y**

$y = 6 \times 3^x = 6 \times 2 = 12$, so $P = (\log_3 2, 12)$.

✓ **FINAL ANSWER**

$(i)\ x = -2$.

$(ii)\ P = (\log_3 2, 12)$.

WMA12/01 MAY/JUNE 2025

Question 7

9 marks | Sequences & Series

7: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) A **geometric** series begins

$$10 + 8 + 6.4 + \ldots$$

(a) Find the sum to infinity of this series.

(2)

Given that the kth term of this series is less than 0.0005

(b) use algebra to find the smallest possible value of k.

(3)

(ii) An **arithmetic** series begins

$$850 + 843 + 836 + \ldots$$

Given that the sum of the first n terms of this series is S_n
find the greatest possible value of S_n

(4)

WORKED SOLUTION

Question 7 · WMA12/01 May/June 2025

1 **Identify the geometric sequence**

The first term is 10 and the common ratio is $\frac{8}{10} = 0.8$.

2 **Find the sum to infinity**

$S_\infty = \frac{10}{1 - 0.8} = 50$.

3 **Write the kth term**

The kth term is $10(0.8)^{k-1}$.

4 **Use the inequality**

$10(0.8)^{k-1} < 0.0005$, so $(0.8)^{k-1} < 0.00005$.

5 **Solve with logarithms**

Taking logarithms gives $(k-1)\log(0.8) < \log(0.00005)$. Since $\log(0.8) < 0$,

$k - 1 > \dfrac{\log(0.00005)}{\log(0.8)} = 44.381\ldots.$

6 **Find the smallest integer**

Thus $k > 45.381\ldots$, so the smallest possible value is $k = 46$.

7 **Write the arithmetic sum**

For $850 + 843 + 836 + \cdots$, the first term is 850 and the common difference is -7.

8 **Form S n**

$S_n = \dfrac{n}{2}\{2(850) + (n-1)(-7)\} = \dfrac{n}{2}(1707 - 7n).$

9 **Find the best integer n**

This quadratic has its maximum near $n = \dfrac{1707}{14} = 121.928\ldots$, so check $n = 122$.

10 **Find the greatest sum**

$S_{122} = \dfrac{122}{2}(1707 - 854) = 61(853) = 52033.$

✓ **FINAL ANSWER**

$(i)(a)\ 50.$

$(i)(b)\ 46.$

$(ii)\ 52033.$

WMA12/01 MAY/JUNE 2025

Question 8

8 marks | Trigonometric Equations

8: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 \leqslant x < \pi$, the equation

$$3\tan\left(2x + \frac{\pi}{5}\right) = \sqrt{3}$$

giving the answers in radians in the form $k\pi$, where k is a rational constant to be found.

(3)

(ii) Solve, for $0 \leqslant \theta < 360°$, the equation

$$5\sin\theta\tan\theta = \cos\theta + 4$$

giving your answers, in degrees, to one decimal place.

(5)

WORKED SOLUTION

Question 8 · WMA12/01 May/June 2025

1 **Solve the tangent equation**

$3\tan\left(2x + \frac{\pi}{5}\right) = \sqrt{3}$ gives $\tan\left(2x + \frac{\pi}{5}\right) = \frac{1}{\sqrt{3}}$.

2 **Write the general angle**

Therefore $2x + \frac{\pi}{5} = \frac{\pi}{6} + n\pi$.

3 **Find x**

$2x = -\frac{\pi}{30} + n\pi$, so $x = -\frac{\pi}{60} + \frac{n\pi}{2}$.

4 **Use the interval**

For $0 \leq x < \pi$, the possible values are $x = \frac{29\pi}{60}$ and $x = \frac{59\pi}{60}$.

5 **Rewrite the second equation**

Using $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $5\sin\theta\tan\theta = \cos\theta + 4$ becomes $\frac{5\sin^2\theta}{\cos\theta} = \cos\theta + 4$.

6 **Form an equation in cos theta**

Multiplying by $\cos\theta$ gives $5\sin^2\theta = \cos^2\theta + 4\cos\theta$. Using $\sin^2\theta = 1 - \cos^2\theta$ gives $6\cos^2\theta + 4\cos\theta - 5 = 0$.

7 **Solve for cos theta**

$\cos\theta = \dfrac{-2 \pm \sqrt{34}}{6}$. The negative option is less than -1, so reject it.

8 **Find theta**

$\cos\theta = \dfrac{-2 + \sqrt{34}}{6}$ gives $\theta = 50.320\ldots°$ or $309.679\ldots°$.

9 **Round the answers**

To one decimal place, $\theta = 50.3°$ or $309.7°$.

✓ **FINAL ANSWER**

$(i)\ x = \dfrac{29\pi}{60}, \dfrac{59\pi}{60}$.

$(ii)\ \theta = 50.3°, 309.7°$.

Question 9

11 marks | Applications of Differentiation

9:

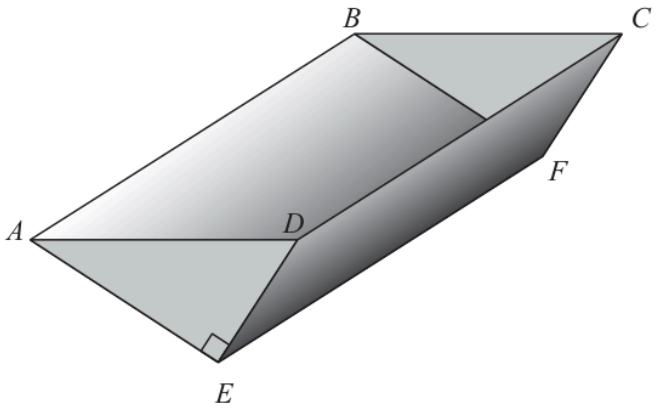


Figure 2

Figure 2 shows a sketch of an open container.

The sides $CDEF$ and $ABFE$ are rectangles.

The ends ADE and BCF are congruent (identical) right-angled triangles.

The container is made from metal of negligible thickness.

Given that

- $AE = BF = 3x$ metres
- $DE = CF = 2x$ metres
- $AB = DC = EF = L$ metres

and the capacity of the container is $12\,\text{m}^3$

(a) show that the area of metal used to make the container, $S\,\text{m}^2$, is given by

$$S = Px^2 + \frac{Q}{x}$$

where P and Q are positive integers to be found.

(4)

Given that x can vary,

(b) use algebraic calculus to find the minimum value of S, giving your answer to one decimal place.

(5)

(c) Justify that the value of S found in part (b) is a minimum.

(2)

1 **Find the triangular end area**

Each end is a right-angled triangle with perpendicular sides $3x$ and $2x$, so its area is $\frac{1}{2}(3x)(2x) = 3x^2$.

2 **Use the capacity**

The volume is cross-section area times length, so $3x^2L = 12$ and $L = \frac{4}{x^2}$.

3 **Write the metal area**

There are two triangular ends with total area $6x^2$, and two rectangular sides with total area $3xL + 2xL = 5xL$.

4 **Substitute for L**

$S = 6x^2 + 5x\left(\frac{4}{x^2}\right) = 6x^2 + \frac{20}{x}$. Hence $P = 6$ and $Q = 20$.

5 **Differentiate**

$\frac{dS}{dx} = 12x - 20x^{-2} = 12x - \frac{20}{x^2}$.

6 **Find the stationary value of x**

Set $\frac{dS}{dx} = 0$: $12x = \frac{20}{x^2}$, so $x^3 = \frac{5}{3}$.

7 **Evaluate S**

At $x = \sqrt[3]{\frac{5}{3}}$, $S = 6x^2 + \frac{20}{x} = 25.3029\ldots$, so the minimum value is 25.3 to one decimal place.

8 **Differentiate again**

$\frac{d^2S}{dx^2} = 12 + 40x^{-3} = 12 + \frac{40}{x^3}$.

9 **Justify the minimum**

Since $x > 0$, $12 + \frac{40}{x^3} > 0$, so the stationary value found is a minimum.

FINAL ANSWER

(a) $S = 6x^2 + \frac{20}{x}$, so $P = 6$ and $Q = 20$.

(b) 25.3.

(c) Minimum justified.

WMA12/01 MAY/JUNE 2025

Question 10

4 marks | Proof

10: **In this question you must show detailed reasoning.**

Use algebra to prove by exhaustion that,

for all positive integers m that are **not** multiples of 3, the value of

$$m^2 + 3m + 2$$

is always a multiple of 3

(4)

WORKED SOLUTION

Question 10 · WMA12/01 May/June 2025

1 **List the possible cases**

A positive integer that is not a multiple of 3 must have the form $m = 3q + 1$ or $m = 3q + 2$.

2 **Case one**

If $m = 3q + 1$, then

$m^2 + 3m + 2 = (3q + 1)^2 + 3(3q + 1) + 2 = 9q^2 + 15q + 6 = 3(3q^2 + 5q + 2)$.

3 **Case two**

If $m = 3q + 2$, then

$m^2 + 3m + 2 = (3q + 2)^2 + 3(3q + 2) + 2 = 9q^2 + 21q + 12 = 3(3q^2 + 7q + 4)$.

4 **Conclude by exhaustion**

Both possible cases give a factor of 3, so $m^2 + 3m + 2$ is always a multiple of 3.

✓ **FINAL ANSWER**

Proven.