Elite Mathematics
Edexcel International A Level

IAL PURE 2 · WMA12

WMA12/01 January 2025 Worked Solutions

January 2025 paper rebuilt with the worked solution after each question.

10	2019–2026	Single-paper solutions
paper questions	19 exam sessions	checked by Dr Eslam



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PAST PAPER

WMA12/01 January 2025

January 2025

10	75	1 h 30
questions	marks	exam time

Each question is followed by its worked solution and final answer.

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Question 1

4 marks | Arithmetic Sequences

1. The arithmetic series S is given by

$$S = 2 + 5 + 8 + 11 + ... + 254$$

Find

(a) the number of terms in the series,

(2)

(b) the sum of the series.

(2)

WORKED SOLUTION

Question 1 · WMA12/01 January 2025

1 Identify the arithmetic series

The first term is 2, the common difference is 3, and the last term is 254.

2 Find the number of terms

Use $254 = 2 + (n-1)3$. Then $252 = 3(n-1)$, so $n - 1 = 84$ and $n = 85$.

3 Use the sum formula

$S_n = \frac{n}{2}(a + l)$, where a is the first term and l is the last term.

4 Find the sum

$S = \frac{85}{2}(2 + 254) = 85 \times 128 = 10880$.

FINAL ANSWER

(a) 85.

(b) 10880.

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Question 2

5 marks | Binomial Expansion

2. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 5x)^8$$

giving each term in simplest form.

(4)

This expansion is to be used to find an approximation for 2.05^8

(b) State the value of x that should be used.

(*There is no need to carry out this calculation.*)

(1)

WORKED SOLUTION

Question 2 · WMA12/01 January 2025

1 Write the first four terms

$(2-5x)^8 = 2^8 + 8(2^7)(-5x) + \binom{8}{2}(2^6)(-5x)^2 + \binom{8}{3}(2^5)(-5x)^3 + \cdots$.

2 Simplify each term

The first four terms are $256 - 5120x + 44800x^2 - 224000x^3$.

3 Connect the expansion to 2.05

To use $(2-5x)^8$ for 2.05^8, set $2 - 5x = 2.05$.

4 Find x

$-5x = 0.05$, so $x = -0.01$.

✓ **FINAL ANSWER**

$(a)\ 256 - 5120x + 44800x^2 - 224000x^3$.

$(b)\ x = -0.01$.

Question 3

10 marks | Applications of Differentiation

3.

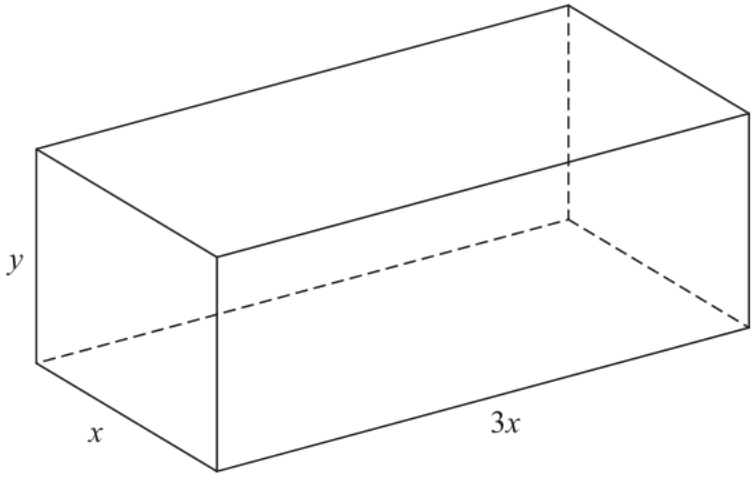


Figure 1

Figure 1 shows an open-topped container used for holding water.

The container is in the shape of a cuboid and is made of sheet metal.

The base of the container is a rectangle $3x$ metres by x metres.

The height of the container is y metres as shown in Figure 1.

Given that the capacity of the container is $120\,\text{m}^3$

(a) show that the area $A\,\text{m}^2$ of the sheet metal used to make the container is given by

$$A = Px^2 + \frac{Q}{x}$$

where P and Q are positive constants to be found.

(4)

(b) Use calculus to find the value of x for which A has a stationary value, giving your answer to 3 significant figures.

(4)

(c) Find $\frac{\mathrm{d}^2A}{\mathrm{d}x^2}$ and hence show that the value of x found in part (b) gives the minimum value of A.

(2)

1 **Use the volume**

The volume is $(3x)(x)(y) = 3x^2y$. Since the capacity is 120, $3x^2y = 120$, so $y = \dfrac{40}{x^2}$.

2 **Write the metal area**

The open top means the metal area is the base plus four sides: $A = 3x^2 + 2(3xy) + 2(xy)$.

3 **Substitute for y**

$A = 3x^2 + 8xy = 3x^2 + 8x\left(\dfrac{40}{x^2}\right) = 3x^2 + \dfrac{320}{x}$. Hence $P = 3$ and $Q = 320$.

4 **Differentiate**

$\dfrac{dA}{dx} = 6x - 320x^{-2} = 6x - \dfrac{320}{x^2}$.

5 **Use the stationary condition**

Set $\dfrac{dA}{dx} = 0$: $6x = \dfrac{320}{x^2}$, so $6x^3 = 320$.

6 **Find x**

$x^3 = \dfrac{160}{3}$, so $x = \sqrt[3]{\dfrac{160}{3}} = 3.764\ldots$. To 3 significant figures, $x = 3.76$.

7 **Differentiate again**

$\dfrac{d^2A}{dx^2} = 6 + 640x^{-3} = 6 + \dfrac{640}{x^3}$.

8 **Show the minimum**

For $x > 0$, $6 + \dfrac{640}{x^3} > 0$, so the stationary value found in part (b) is a minimum.

✓ **FINAL ANSWER**

(a) $A = 3x^2 + \dfrac{320}{x}$, so $P = 3$ and $Q = 320$.

(b) $x = 3.76$.

(c) $\dfrac{d^2A}{dx^2} = 6 + \dfrac{640}{x^3}$, minimum.

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Question 4

6 marks | Geometric Sequences

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Given that, in a particular geometric series,

- the sum of the first three terms is 70.2
- the sum to infinity is 75

find, for this series,

(a) the common ratio,

(4)

(b) the first term.

(2)

WORKED SOLUTION

Question 4 · WMA12/01 January 2025

1 **Write the two sums**

For first term a and common ratio r, $S_\infty = \dfrac{a}{1-r} = 75$ and $S_3 = \dfrac{a(1-r^3)}{1-r} = 70.2$.

2 **Divide the equations**

$\dfrac{S_3}{S_\infty} = 1 - r^3$, so $\dfrac{70.2}{75} = 1 - r^3$.

3 **Find the ratio**

$0.936 = 1 - r^3$, so $r^3 = 0.064$ and $r = 0.4$.

4 **Find the first term**

Use $\dfrac{a}{1-0.4} = 75$. Hence $a = 75(0.6) = 45$.

FINAL ANSWER

$(a)\ r = 0.4$.

$(b)\ 45$.

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Question 5

9 marks | Polynomials

5. $f(x) = 3x^3 + ax^2 - 10x + b$

where a and b are constants.

Given that $(3x - 4)$ is a factor of $f(x)$,

(a) show that $16a + 9b = 56$

(2)

Given further that when $f(x)$ is divided by $(x - 2)$ the remainder is b,

(b) find the value of a and the value of b.

(4)

(c) Hence, using algebra, fully factorise $f(x)$.

(3)

WORKED SOLUTION

Question 5 · WMA12/01 January 2025

1 Use the factor theorem

Since $3x - 4$ is a factor, $x = \frac{4}{3}$ is a root and $f\left(\frac{4}{3}\right) = 0$.

2 Show the given equation

$3\left(\frac{4}{3}\right)^3 + a\left(\frac{4}{3}\right)^2 - 10\left(\frac{4}{3}\right) + b = 0$, so $\frac{64}{9} + \frac{16a}{9} - \frac{120}{9} + b = 0$. Multiplying by 9 gives $16a + 9b = 56$.

3 Use the remainder condition

When dividing by $x - 2$, the remainder is $f(2)$. Given that the remainder is b, $f(2) = b$.

4 Find a

$f(2) = 24 + 4a - 20 + b = 4 + 4a + b$. Thus $4 + 4a + b = b$, so $a = -1$.

5 Find b

Substitute $a = -1$ into $16a + 9b = 56$: $-16 + 9b = 56$, so $b = 8$.

6 Write f of x

$f(x) = 3x^3 - x^2 - 10x + 8$.

7 **Find another factor**

$f(1) = 3 - 1 - 10 + 8 = 0$, so $(x - 1)$ is a factor.

8 **Fully factorise**

Dividing by $(x - 1)$ gives $3x^2 + 2x - 8 = (3x - 4)(x + 2)$. Hence $f(x) = (x - 1)(x + 2)(3x - 4)$.

✓ **FINAL ANSWER**

(a) Proven.

(b) $a = -1$, $b = 8$.

(c) $f(x) = (x - 1)(x + 2)(3x - 4)$.

Question 6

8 marks | Circles

6.

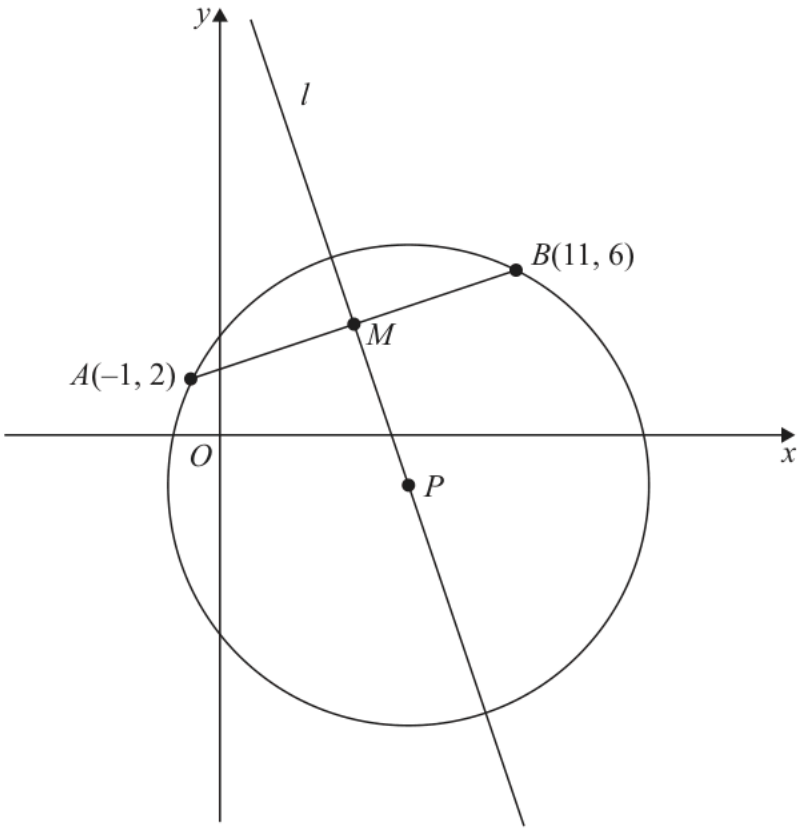


Figure 2

The point A $(-1, 2)$ and the point B $(11, 6)$ both lie on a circle with centre P.

The point M is the midpoint of AB.

Given that the line l passes through M and P, as shown in Figure 2,

(a) find an equation for l, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

Given that P has coordinates $(7, k)$, where k is a constant,

(b) find the value of k,

(1)

(c) find an equation for the circle.

(3)

1 Find the midpoint

For $A(-1,2)$ and $B(11,6)$, the midpoint is $M\left(\frac{-1+11}{2},\frac{2+6}{2}\right)=(5,4)$.

2 Find the gradient of AB

The gradient of AB is $\frac{6-2}{11-(-1)}=\frac{4}{12}=\frac{1}{3}$.

3 Use the perpendicular bisector

The centre lies on the perpendicular bisector of the chord AB, so line l has gradient -3 and passes through $M(5,4)$.

4 Find line l

$y-4=-3(x-5)$, so $y=-3x+19$.

5 Use the centre coordinates

Since $P=(7,k)$ lies on l, $k=-3(7)+19=-2$.

6 Find the radius squared

The centre is $(7,-2)$. Using point A, $r^2=(7-(-1))^2+(-2-2)^2=8^2+(-4)^2=80$.

7 Write the circle equation

Therefore the circle is $(x-7)^2+(y+2)^2=80$.

FINAL ANSWER

(a) $y=-3x+19$.

(b) $k=-2$.

(c) $(x-7)^2+(y+2)^2=80$.

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Question 7

8 marks | Laws of Logarithms

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) The table below shows values of x and y, where $y = \log_{10}(x + 5)$, for x values between -1 and 4

x	-1	0	1	2	3	4
$y = \log_{10}(x + 5)$	$\log_{10} 4$	$\log_{10} 5$	$\log_{10} 6$	$\log_{10} 7$	$\log_{10} 8$	$\log_{10} 9$

Using the trapezium rule with all the y values in the given table, show that

$$\int_{-1}^{4} \log_{10}(x + 5)\,\mathrm{d}x \approx \log_{10} k$$

where k is an integer to be found.

(3)

(ii) Find the value of a such that

$$2\log_5(5 - a) - \log_5(a + 25) = 1$$

(5)

WORKED SOLUTION

Question 7 · WMA12/01 January 2025

1 **Use the trapezium rule**

The width is 1, so

$$\int_{-1}^{4} \log_{10}(x + 5)\,dx \approx \frac{1}{2}\{\log_{10} 4 + \log_{10} 9 + 2(\log_{10} 5 + \log_{10} 6 + \log_{10} 7 + \log_{10} 8)\}.$$

2 **Combine the logarithms**

This is $\log_{10}\left(\sqrt{4 \times 9} \times 5 \times 6 \times 7 \times 8\right)$.

3 **Find k**

$\sqrt{4 \times 9} \times 5 \times 6 \times 7 \times 8 = 6 \times 1680 = 10080$, so $k = 10080$.

4 **State the domain**

The logarithms require $5 - a > 0$ and $a + 25 > 0$, so $-25 < a < 5$.

5 **Combine the logarithms**

$2\log_5(5-a) - \log_5(a+25) = \log_5\left(\dfrac{(5-a)^2}{a+25}\right)$.

6 **Use one as a logarithm**

$1 = \log_5 5$, so $\dfrac{(5-a)^2}{a+25} = 5$.

7 **Form the quadratic**

$(5-a)^2 = 5(a+25)$, so $a^2 - 10a + 25 = 5a + 125$. Hence $a^2 - 15a - 100 = 0$.

8 **Solve and check**

$(a-20)(a+5) = 0$, so $a = 20$ or $a = -5$. Only $a = -5$ is in the domain, so $a = -5$.

✓ **FINAL ANSWER**

(i) $k = 10080$.

(ii) $a = -5$.

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Question 8

5 marks | Proof

8. (i) A student states

"If x and y are irrational numbers, $x \neq y$, then xy is also irrational."

Show, by counter example, that this statement is not always true.

(1)

(ii) Prove, using algebra, that for all odd integers n, the value of the expression

$$n^3 + 3n + 2$$

is always even but never a multiple of 4

(4)

WORKED SOLUTION

Question 8 · WMA12/01 January 2025

1 **Give a counterexample**

Let $x = \sqrt{2}$ and $y = \dfrac{1}{\sqrt{2}}$. Both are irrational and $x \neq y$, but $xy = 1$, which is rational.

2 **Represent an odd integer**

Let $n = 2q + 1$, where q is an integer.

3 **Substitute into the expression**

$n^3 + 3n + 2 = (2q + 1)^3 + 3(2q + 1) + 2$.

4 **Expand and factor**

$n^3 + 3n + 2 = 8q^3 + 12q^2 + 12q + 6 = 2(4q^3 + 6q^2 + 6q + 3)$.

5 **Show it is even**

The expression has a factor of 2, so it is even.

6 **Show it is not a multiple of four**

The bracket $4q^3 + 6q^2 + 6q + 3$ is odd, because the first three terms are even and 3 is odd.

Therefore the expression is 2 times an odd integer, so it is not divisible by 4.

FINAL ANSWER

(i) For example, $x = \sqrt{2}$ and $y = \dfrac{1}{\sqrt{2}}$ give $xy = 1$.

(ii) Proven.

Question 9

12 marks | Integration

9.

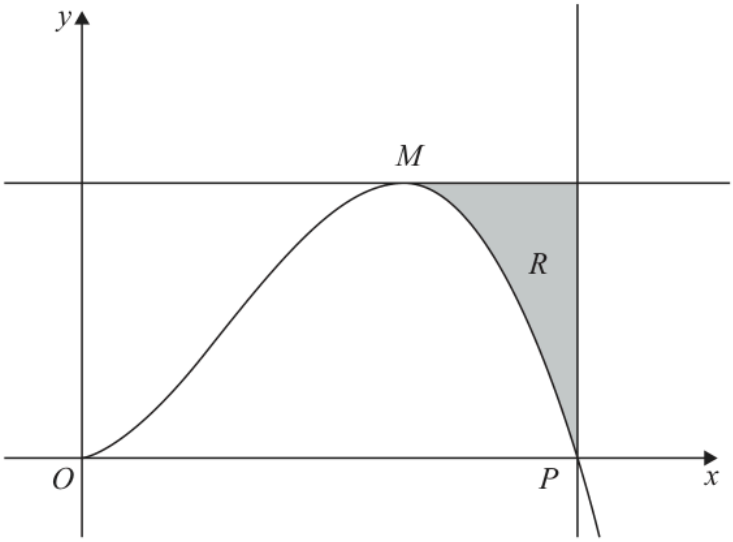


Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \frac{9x^2\left(5 - \sqrt{x}\right)}{5} \qquad x \geqslant 0$$

The curve has a turning point at the point M, as shown in Figure 3.

(a) Using calculus, find the coordinates of M.

(5)

The curve crosses the x-axis at the point P, as shown in Figure 3.

(b) Use algebra to find the x coordinate of P.

(2)

The finite region R, shown shaded in Figure 3, is bounded by the curve, the line through M parallel to the x-axis and the line through P parallel to the y-axis.

(c) Use algebraic integration to find the area of R, giving your answer to one decimal place.

(5)

1 Rewrite the curve

$$y = \frac{9x^2(5-\sqrt{x})}{5} = 9x^2 - \frac{9}{5}x^{5/2}.$$

2 Differentiate

$$\frac{dy}{dx} = 18x - \frac{9}{2}x^{3/2} = \frac{9}{2}x(4-\sqrt{x}).$$

3 Find the turning point x coordinate

For the turning point shown away from the origin, set $4-\sqrt{x}=0$. Hence $x=16$.

4 Find the y coordinate

At $x=16$, $y = \frac{9(16)^2(5-4)}{5} = \frac{2304}{5}$.

5 State M

Therefore $M\left(16, \frac{2304}{5}\right)$.

6 Find the positive x intercept

For the positive intercept, $\frac{9x^2(5-\sqrt{x})}{5} = 0$ gives $5-\sqrt{x}=0$, so $x=25$.

7 Set up the area

The top boundary is $y = \frac{2304}{5}$ and the curve is below it from $x=16$ to $x=25$. So

$$R = \int_{16}^{25}\left(\frac{2304}{5} - 9x^2 + \frac{9}{5}x^{5/2}\right)dx.$$

8 Integrate

$$R = \left[\frac{2304}{5}x - 3x^3 + \frac{18}{35}x^{7/2}\right]_{16}^{25}.$$

9 Evaluate

$R = \frac{9189}{7} = 1312.714\ldots$, so the area is 1312.7 to one decimal place.

✓ **FINAL ANSWER**

(a) $M\left(16, \frac{2304}{5}\right)$.

(b) $x_P = 25$.

(c) 1312.7.

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Question 10

8 marks | Trigonometric Equations

10. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos\theta\left(3\tan\theta+\frac{2}{\tan\theta}\right)\equiv\sin\theta+\frac{2}{\sin\theta}\qquad\theta\neq\frac{n\pi}{2}$$

(4)

(b) Hence solve, for $0<x<2\pi$, the equation

$$\cos x\left(3\tan x+\frac{2}{\tan x}\right)=4\sin x-5$$

giving your answers to 3 significant figures.

(4)

WORKED SOLUTION

Question 10 · WMA12/01 January 2025

1 **Expand the left side**

$\cos\theta\left(3\tan\theta+\dfrac{2}{\tan\theta}\right)=3\sin\theta+\dfrac{2\cos\theta}{\tan\theta}$.

2 **Rewrite the second term**

Since $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$, $\dfrac{2\cos\theta}{\tan\theta}=\dfrac{2\cos^2\theta}{\sin\theta}$.

3 **Use the identity**

$3\sin\theta+\dfrac{2\cos^2\theta}{\sin\theta}=3\sin\theta+\dfrac{2(1-\sin^2\theta)}{\sin\theta}$.

4 **Simplify**

$3\sin\theta+\dfrac{2}{\sin\theta}-2\sin\theta=\sin\theta+\dfrac{2}{\sin\theta}$, as required.

5 **Use part a**

The equation becomes $\sin x+\dfrac{2}{\sin x}=4\sin x-5$.

6 **Form a quadratic in sin x**

Multiplying by $\sin x$ gives $\sin^2 x + 2 = 4\sin^2 x - 5\sin x$, so $3\sin^2 x - 5\sin x - 2 = 0$.

7 **Solve for sin x**

$(3\sin x + 1)(\sin x - 2) = 0$. Since $\sin x = 2$ is impossible, $\sin x = -\frac{1}{3}$.

8 **Find x in the interval**

For $0 < x < 2\pi$, $\sin x = -\frac{1}{3}$ gives $x = \pi + \sin^{-1}\left(\frac{1}{3}\right)$ or $x = 2\pi - \sin^{-1}\left(\frac{1}{3}\right)$.

9 **Round the answers**

Thus $x = 3.481\ldots$ or $x = 5.943\ldots$, so to 3 significant figures $x = 3.48$ or $x = 5.94$.

✓ **FINAL ANSWER**

(a) Proven.

(b) $x = 3.48, 5.94$.